Top Quark Pair Production beyond the Next-to-Leading Order

Valentin Ahrens[1], *Andrea Ferroglia*[2], *Matthias Neubert*[1], *Ben D. Pecjak*[1], *Li Lin Yang*[3]

[1]Institut für Physik (THEP), Johannes Gutenberg-Universität, D-55099 Mainz, Germany
[2]New York City College of Technology, 300 Jay Street, Brooklyn, NY 11201, USA
[3]Institute for Theoretical Physics, University of Zürich, CH-8057 Zürich, Switzerland

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We report on recent calculations of the total cross section and differential distributions of top quark pair production at hardon colliders, including the invariant mass distribution, the transverse momentum and rapidity distributions, as well as the forward-backward asymmetry. The calculations are based on soft gluon resummation at the next-to-next-to-leading logarithmic accuracy.

1 Introduction

Top quark pair production is a benchmark process at hadron colliders such as the Tevatron and the LHC. Its special role in the physics program of these experiments makes it crucial to have precise QCD predictions for the total and differential cross sections. The starting point for such predictions is the next-to-leading order (NLO) calculations of the total and differential cross sections carried out more than two decades ago [1]. Since higher-order corrections to these results as estimated through scale variations are expected to be as large as 10-15%, it would be desirable to extend the calculations beyond NLO. Here there are two paths. One is to calculate the full next-to-next-to-leading order (NNLO) cross section. This is indeed an active area of research and was discussed at this conference by Alexander Mitov, with the first numerical result for the total cross section in the $q\bar{q}$ channel available in [2]. Another is to use techniques from soft gluon resummation to calculate what are argued to be the dominant corrections at NNLO and beyond. Such resummed calculations are the subject of this talk.

2 Soft gluon resummation and approximate NNLO

Soft gluon resummation is a rich field with a long history and it is far beyond the scope of this talk to give a detailed review. Instead, we will briefly explain the main ideas and the different conventions used in the literature.

The basic idea of resummation can be conveyed through the following schematic picture. In certain kinematic regions (the so-called "threshold" regions), the differential partonic cross sections $d\hat{\sigma}$ receive logarithmically enhanced corrections in the form $\alpha_s^n L^m$ at each order in perturbation theory, where $m \leq 2n$ and L represent some logarithms[1] which become large in

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[1] usually of some conjugate variable in the Mellin or Laplace moment space

the limit of soft gluon emission. When L is so large such that $\alpha_s L \sim 1$, the perturbation series needs to be re-organized so that these large terms are resummed to all orders in α_s. This is achieved using techniques of re-factorization and renormalization-group evolution, and in the end one can show that the partonic cross section can be written in the form (with L counted as $1/\alpha_s$)

$$d\hat{\sigma} = (c_0 + \alpha_s c_1 + \cdots) \exp\left[\frac{g_0}{\alpha_s} + g_1 + \alpha_s g_2 + \cdots\right],$$

where the coefficients c_i and g_i do not contain any large logarithms. The number of terms included in the exponent and in the prefactor defines a certain "logarithmic accuracy", with the terms shown above being the next-to-next-to-leading logarithmic (NNLL) order. Alternative to resummation, one can also use the knowledge to recover the leading terms at higher orders. With the information from the exact NLO result and the NNLL resummation (as is the case for top quark pair production), one can determine the terms $\alpha_s^2 L^m$ with $m = 1, 2, 3, 4$ in the NNLO corrections. These "NLO+NNLL resummed" and "approximate NNLO" results are the starting point of our phenomenological analyses in [4], which will be presented in the next section. Finally, a method to obtain some information about the missing constant term $\alpha_s^2 L^0$ was proposed in [5], with numerical results in preparation.

Name	Observable	Threshold limit
production threshold	σ	$\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$
pair-invariant-mass (PIM)	$d\sigma/dM_{t\bar{t}}d\theta$	$(1-z) \equiv (1 - M_{t\bar{t}}^2/\hat{s}) \to 0$
single-particle-inclusive (1PI)	$d\sigma/dp_T dy$	$s_4 \equiv \hat{s} + \hat{t}_1 + \hat{u}_1 \to 0$

Table 1: The three cases in which soft gluon resummation has been applied in top quark pair production.

Before going into numerics, let us emphasize that soft gluon resummation is a very generic method which can be applied to many observables, where each observable is associated with a specific definition of "threshold". In top quark pair production at hadron colliders, all applications in the literature can be grouped into one of the three cases listed in Table 1. The production threshold is the limit where the top and anti-top quarks are nearly at rest, which can only be applied for the total inclusive cross section. Besides logarithmic enhancement from soft gluon emissions, one must take into account Coulomb gluon exchanges in this case, which result in terms of the form $\ln \beta^m/\beta^n$. A simultaneous resummation of both type of contributions at NNLL accuracy has been performed in [3]. The PIM and 1PI threshold, on the other hand, can be applied to certain differential distributions as indicated in Table 1. Of course, starting from these two distributions, one may also perform a partial integration or full integration to obtain observables such as the forward-backward asymmetry and the total cross section. In the following, we will employ PIM and 1PI kinematics, whichever is appropriate for the specific observables.

3 Total and differential cross sections

In this section we present our predictions for the total and differential cross sections, which are based on the series of works in [4]. In all numerical results we adopt $m_t = 173.1$ GeV, and use

	Tevatron	LHC7	LHC8	LHC14
NLO	$6.72^{+0.41+0.47}_{-0.76-0.45}$	159^{+20+14}_{-21-13}	228^{+28+19}_{-30-17}	$889^{+107+66}_{-106-58}$
NNLO approx.	$6.63^{+0.07+0.63}_{-0.41-0.48}$	155^{+8+14}_{-9-14}	221^{+12+19}_{-12-19}	855^{+52+60}_{-38-59}

Table 2: The total cross sections (in pb) at the Tevatron and the LHC for different collider energies. The first errors are perturbative uncertainties, and the second errors are PDF+α_s uncertainties.

MSTW2008 PDF sets.

We first show our results for the total cross sections[2] in Table 2. The approximate NNLO results are computed by combining the approximate NNLO formula from PIM and 1PI kinematics. The first errors are perturbative uncertainties, which for the NLO results are estimated by varying μ_f and μ_r up and down by a factor of 2, with the default being $\mu_f = \mu_r = m_t$. For the approximate NNLO results, besides scale variation, we also use the difference between PIM and 1PI kinematics as an additional source of perturbative uncertainties. We also show the uncertainties associated with the experimental determination of the PDFs and the strong coupling constant, which are estimated following the prescription in [6] at 90% CL. Compared to the NLO results, the approximate NNLO corrections do not change the central values very much, while the perturbative uncertainties are reduced a lot.

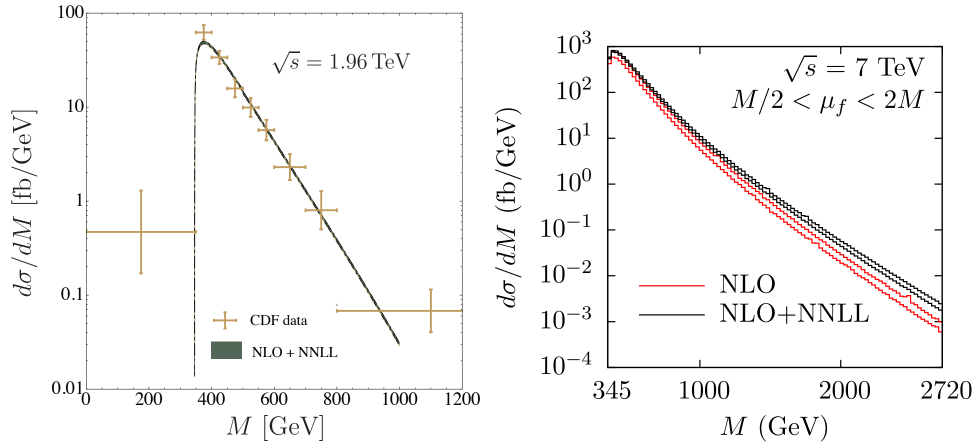

Figure 1: The invariant mass distributions at the Tevatron compared with CDF data (left) and at the LHC with $\sqrt{s} = 7$ TeV (right).

We now turn to differential distributions. A particularly interesting observable is the invariant mass distribution of the $t\bar{t}$ pair, which is very sensitive to contributions from new heavy resonances. We show in Figure 1 our NLO+NNLL predictions at the Tevatron (left) and the LHC with $\sqrt{s} = 7$ TeV (right). Our predictions at the Tevatron agree quite well with the measurements from the CDF collaboration [7]. For the LHC, we observe large corrections over

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[2] These are obtained using the numerical program TopNNLO, which can be downloaded at `http://www.physik.uzh.ch/~llyang/TopNNLO.tar.gz`

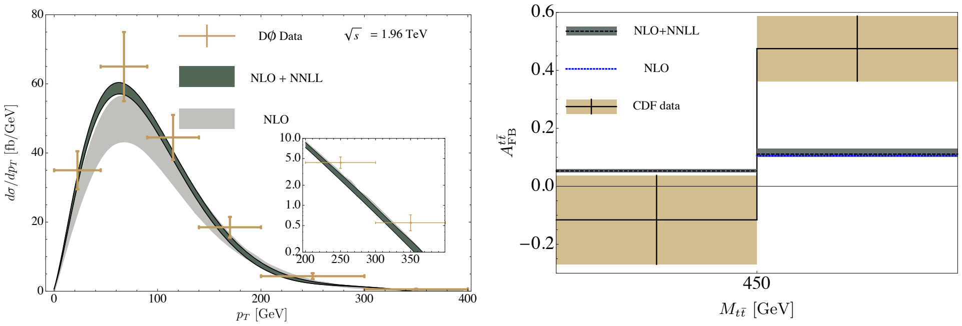

Figure 2: Left: the transverse momentum distribution of the top quark at the Tevatron compared with D0 data. Right: The invariant mass dependent forward-backward asymmetry at the Tevatron.

the NLO predictions in the high invariant mass region, with the shape being slightly distorted, which is important for new physics searches. In Figure 2 we show another two distributions at the Tevatron: the transverse momentum (p_T) distribution of the top quark, and the invariant-mass-dependent forward-backward asymmetry. Our result for the p_T distribution is shown together with the NLO result and the D0 data [8]. It is apparent that the NNLL resummation improves the agreement between the theoretical prediction and the experimental measurement. The forward-backward asymmetry, on the other hand, was found by the CDF and D0 collaborations [9] to be in tension with theoretical predictions, especially in the high invariant mass region. Here resummation only mildly increases the asymmetry, so that the discrepancy between theory and experiment calls for other explanations.

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