Normalised Multi-jet Cross Sections Using Regularised Unfolding and Extraction of $\alpha_s(M_Z)$ in Deep-Inelastic Scattering at High $Q^2$ at HERA

Daniel Britzger
on behalf of the H1 Collaboration

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Jetproduction in DIS
Regularized Unfolding
Normalized Multijet Cross Sections
Determination of $\alpha_s$
Summary
Jet production in DIS

Jet Production in Leading Order

Analysis is performed in Breit frame: $2x_{Bj}P + k = 0$

Only hard QCD processes generate considerable $P_T$ in the Breit frame

Leading order: n-jet production $\sim \alpha_s^{n-1}$

Direct sensitivity to $\alpha_s$ and gluon density
**Multijet Measurement**

**Phase Space**
- **Neutral current phase space**
  - $150 < Q^2 < 15000 \text{ GeV}^2$
  - $0.2 < y < 0.7$
- **Jet phase space**
  - $-1.0 < \eta_{\text{Lab}} < 2.5$
- **Inclusive Jet**
  - $7.0 < P_T < 50 \text{ GeV}$
- **Dijet and Trijet**
  - $5 < P_T < 50 \text{ GeV}$
  - $7 < \langle P_T \rangle < 50 \text{ GeV}$
  - $M_{12} > 16 \text{ GeV}$

**Kinematics**
- **DIS Event = scattered electron**
  - $Q^2 = -q^2 = (k - k')^2$
  - $y = q \cdot P / k \cdot P$ (Inelasticity)
- **Jet kinematics**
  - $p_T = \text{jet transverse momentum in Breit frame}$
  - $\langle P_T \rangle = \text{Average transverse momentum of 2 (resp. 3) jets}$
  - $\eta = \text{pseudorapidity of Jet}$
  - $M_{12} = \text{invariant mass of two leading jets}$

**Normalized Jet Measurement**
- **Normalized Jets**
  - $d\sigma_{\text{Jet}} / d\sigma_{\text{DIS}}$
- **Four Ingredients**
  - (inclusive) DIS Measurement $d\sigma/dQ^2$
  - Inclusive Jet $d\sigma/dQ^2 dP_T$
  - Dijet $d\sigma/dQ^2 d\langle P_T \rangle$
  - Trijet $d\sigma/dQ^2 d\langle P_T \rangle$
Correction for Detector Effects

**Aim**

Cross section on particle level
Correction for detector effects

**Detector Response**

Steeply falling $P_T$ and $Q^2$ spectra
Boost to Breit frame
Finite resolution

**Bin-by-bin correction**

Detector correction based on bin-wise correction factors
Some error propagation from migrations

---

**Migrations are getting relevant**

-> not respected by bin-by-bin method

**Full/Correct error propagation is difficult**
Regularized Unfolding using TUnfold

Regularized Unfolding
(Migration) Matrix $A$ is describing detector response

\[ \tilde{m} = A \cdot \tilde{x} \]

$m$: measured distribution (Detector level)
$x$: 'true' distribution (Particle level)

Find particle level $x$ by analytic minimization of $\chi^2$ as function of $x$

\[ \chi^2(x) = \frac{1}{2} (m - Ax)^T V^{-1} (m - Ax) + \tau^2 \cdot L \]

TUnfold v17 (S.Schmitt)

Regularisation parameter $\tau$ suppresses large fluctuations/errors in comparison to direct matrix inversion

Unfolding of four measurements at once
NC-DIS, Inclusive Jet, Dijet und Trijet
-> same dataset, same events
-> Covariance matrix $V$ contains correlations
Schematic Definition of Migration Matrix

**Inclusive Jet**

Jet algorithm runs on
- Detector level
- Particle level
- Matching with geometrical measure

\[ r \equiv \sqrt{(\varphi_1 - \varphi_2)^2 + (\eta_1 - \eta_2)^2} < 0.9 \]

Multidimensional unfolding in kinematic variables
- \( Q^2, p_T, y \)

Different jet multiplicities
- 'only' particle level jets \( \rightarrow \) Efficiency correction
- 'only' detector level jets \( \rightarrow \)
- are estimated using NC-DIS events
Schematic Definition of Migration Matrix

Extending Migration Matrix with NC DIS Measurement

Neutral Current (NC) DIS
   Each single event
   Just electron kinematics
   2-dim Unfolding: $Q^2$, $y$

Two Measurements
   Taking Correlations into account

'label Detector level' jets
   Are estimated with electron kinematics
   Preserve normalization with $-\beta$

Always $1.5 \times$ more bins on detector level than particle level
Extended phase space on detector level
Schematic Definition of Migration Matrix

Dijet and Trijet Measurement
Measurement of 'event properties' of an event 'class'

\[ d\sigma/dQ^2dp_T > \]

Migrations in 'kinematic' variable \( <p_T> \), \( Q^2 \) and \( y \)

Define 'Dijet'
\[ M_{12}, \eta_{Jet}, p_{T,Jet} \] phase space cuts
… and two jets

Respect Migrations into/out of these cuts
# Full Schematic Migration Matrix

## Migration Matrix

<table>
<thead>
<tr>
<th></th>
<th>Particle level</th>
<th>Detector level</th>
<th>DIS$^2$ Events ($Q^2, y$)</th>
<th>Incl. Jet $p_T, Q^2, y, (\eta)$</th>
<th>Dijet $Q^2, \langle p_T \rangle_2, y,$ Dijet-cuts</th>
<th>Trijet $Q^2, \langle p_T \rangle_3, y,$ Trijet-cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_1$</td>
<td></td>
<td>$E$</td>
<td>Reconstructed jets without match to generator level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_2$</td>
<td></td>
<td>$B_1$</td>
<td>Reconstructed Dijet events which are not generated as Dijet event</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_3$</td>
<td></td>
<td>$B_2$</td>
<td>Reconstructed Trijet events which are not generated as Trijet event</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon_J$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Matrix dimension: $2205 \times 671$ Entries  
Covariance Matrix: $2205 \times 2205$ Entries
Monte Carlo Test

Pull distributions
Corrected vs. true distribution

\[ P_i = \frac{x_i^{\text{Unfold}} - x_i^{\text{true}}}{\Delta x_i} \]

Two Incl. DIS Models
- Rapgap (MEPS)
- Django (CDM)

Statistically independent samples

Checking
Unfolding with same model
Unfolding with 'other' model

Compare
- Bin-by-bin
  Based on bin-wise correction factors
- Regularized unfolding
Data unfolding
Comparison to bin-by-bin method

H1 Data

$A_{\text{Unfold}} = A_{\text{Rapgap}} + A_{\text{Django}}$

Bias also in data
Uncertainties are larger

$\Rightarrow$ But knowledge of correlations

Pull between two Correction Methods

- Inclusive Jet: Mean $-0.87 \pm 0.17$
- Dijet: Mean $-0.86 \pm 0.11$
- Trijet: Mean $-0.74 \pm 0.10$
Correlation Matrix

Types of Correlations
Correlations resulting from unfolding
Intrinsic physical correlations
- Between measurements
- Within Inclusive Jet

Useful for
Normalized cross sections
$\sigma_{\text{Jet}} / \sigma_{\text{NC}}$
Combined fit to all jet data
Normalized Multijet cross sections

H1

Normalized Inclusive Jet | Normalised Dijet | Normalised Trijet

σ_{Jet}/σ_{NC} \times 10^i

\begin{align*}
\text{NLO} \otimes c_{\text{had}} \\
\text{NLOJet++ and fastNLO} \\
\text{QCDNUM} \\
\text{CT10, } \alpha_s = 0.118
\end{align*}

- 150 < Q^2 < 200 \text{ GeV}^2 \\
  (i = 10)

- 200 < Q^2 < 270 \text{ GeV}^2 \\
  (i = 8)

- 270 < Q^2 < 400 \text{ GeV}^2 \\
  (i = 6)

- 400 < Q^2 < 700 \text{ GeV}^2 \\
  (i = 4)

- 700 < Q^2 < 5000 \text{ GeV}^2 \\
  (i = 2)

- 5000 < Q^2 < 15000 \text{ GeV}^2 \\
  (i = 0)

H1-Prel-12-031

28. Februar 2012

Daniel Britzger - DIS 2012
\( \alpha_s \) Extraction: Fitting technique

NLO calculations depend on PDF and \( \alpha_s \)

- Keep PDF fixed and determine \( \alpha_s (M_Z) \)

Jet cross sections

- NLOJET++, FastNLO v2.0
- \( \mu_r^2 = (Q^2 + E_T^2)/2 \)
- \( \mu_f^2 = Q^2 \)

NC-DIS cross sections

- QCDNUM
- \( \mu_f^2 = \mu_r^2 = Q^2 \)

NLO × Had. corrections

PDF: CT10

Hessian Method

- Minimise \( \chi^2 \)
- TMinuit

\[
\chi^2 (\alpha_s, \varepsilon_k) = \tilde{\sigma}^T \cdot V^{-1} \cdot \tilde{\sigma} + \sum_k \varepsilon_k^2
\]

\[
\tilde{\sigma}_i = \sigma_i^{\text{Data}} - \sigma_i^{\text{Theo}} (\alpha_s, f(\alpha_s, \text{fix})) \cdot (1 - \sum_k \Delta_{i,k}(\varepsilon_k))
\]

Usage of full covariance matrix \( V \) from unfolding

- All correlations are respected in fit

Systematic errors as penalty terms

28. Februar 2012

Daniel Britzger - DIS 2012
\( \alpha_s \) Fits to Individual Measurements

**Normalized Inclusive Jet**

\[
\alpha_s = 0.1197 \pm 0.0008 \text{ (exp)} \pm 0.0014 \text{ (PDF)} \pm 0.0011 \text{ (had)} \pm 0.0053 \text{ (theo)}
\]

\[
\chi^2 / \text{ndf} = 28.7 / 23 = 1.24
\]

**Fit Quality**
- Reasonable \( \chi^2 / \text{ndf} \)
- Penalty parameters -1<\( \varepsilon <1 \)

**Experimental Error**
- Cancellations of some systematic errors
- All normalization uncertainties cancel

**Theoretical/systematic errors**
Repeated fit with offset method

**PDF Error**
- Error on cross section from PDF Eigenvectors
- Neglecting correlations from EV

**Hadronisation Error**
- Error on hadronisation correction
- Sherpa
  - Lund Fragmentation
  - Cluster Fragmentation

**Theory Error**
- Scan \( \mu_r \) and \( \mu_f \) independently
  - \( 0.5 < c < 2.0 \)
  - Largest and lowest cross section
- Add error on cross section from \( \mu_r \) and \( \mu_f \) variation in quadrature
\( \alpha_s \) Fits to Individual Measurements

**Normalized Inclusive Jet**
\[
\alpha_s = 0.1197 \pm 0.0008 \text{ (exp)} \pm 0.0014 \text{ (PDF)} \pm 0.0011 \text{ (had)} \pm 0.0053 \text{ (theo)} \\
\chi^2 / \text{ndf} = 28.7 / 23 = 1.24
\]

**Normalized Dijet**
\[
\alpha_s = 0.1142 \pm 0.0010 \text{ (exp)} \pm 0.0016 \text{ (PDF)} \pm 0.0009 \text{ (had)} \pm 0.0048 \text{ (theo)} \\
\chi^2 / \text{ndf} = 27.0 / 23 = 1.17
\]

**Normalized Trijet**
\[
\alpha_s = 0.1185 \pm 0.0018 \text{ (exp)} \pm 0.0013 \text{ (PDF)} \pm 0.0016 \text{ (had)} \pm 0.0042 \text{ (theo)} \\
\chi^2 / \text{ndf} = 12.0 / 16 = 0.75
\]

Reasonable \( \chi^2 / \text{ndf} \) for each fit

Large tension between Incl. Jet and Dijet
Similar in previous H1 and ZEUS analyses

**Combined Fit**
\[
\alpha_s = 0.1177 +/- 0.0008 \text{ (exp)} \\
\chi^2 / \text{ndf} = 104.608 / 64 = 1.634
\]

**Fit Quality Multijets**
Very bad \( \chi^2 / \text{ndf} \) for combined fit
Because of tension between Incl. Jet and Dijet
$\alpha_s$ Fits to Multijet Measurements

**Combined fit to**
Normalized Inclusive Jet, Normalized Dijet, Normalized Trijet

**Higher orders**
k-factor can be interpreted as indicator for higher orders

$$k = \frac{\sigma_{NLO}}{\sigma_{LO}}$$

Inclusive Jets are more sensitive in pQCD than Dijets
Multijets: $1.05 < k < 1.45$

**Normalized Multijet (k-factor < 1.3)**

$$\alpha_s = 0.1163 \pm 0.0011 \text{ (exp)} \pm 0.0014 \text{ (PDF)} \pm 0.0008 \text{ (had)} \pm 0.0039 \text{ (theo)}$$

$$\chi^2 / ndf = 53.2 / 41 = 1.30$$

**Cut on k-factor**
Demanding NLO corrections $< 30\%$
Trade-off between #bins and fast convergence of perturbative series
Keeping 42 out of 65 bins
Comparision of $\alpha_s$ values

Uncertainties: exp. ———— theo. …………

- **H1 norm. multijets at high $Q^2$** (this talk)
  - H1-prelim-12-031

- **H1 norm. inclusive jet at high $Q^2$** (this talk)
  - H1-prelim-12-031

- **H1 norm. dijet at high $Q^2$** (this talk)
  - H1-prelim-12-031

- **H1 norm. trijet at high $Q^2$** (this talk)
  - H1-prelim-12-031

- **H1 norm. multijets at high $Q^2$**

- **H1 multijets at low $Q^2$**

- **ZEUS inclusive jet at high $Q^2$**
  - ZEUS-prel-10-002

- **ZEUS dijet at high $Q^2$**
  - Phys. Lett B 507, 70 (2001)

- **Aleph 3-jet rate, NNLO**

- **World average**
Summary

Regularized unfolding
Simultaneous unfolding of multiple measurements
Unfolding of (Jet-)Multiplicities
Observables of event classes
Normalized cross sections

Normalized Multijet Measurement
Normalized jet cross sections at High $Q^2$
Normalized Inclusive Jet
Normalized Dijet
Normalized Trijet

$\alpha_s$ Determination
$\alpha_s$ Fit with unfolded data
Full covariance matrix
Competitive experimental error
Theoretical errors dominate

$0.1163 \pm 0.0011$
Backup
### Schematic Definition of Migration Matrix

<table>
<thead>
<tr>
<th>Measurement</th>
<th>NC DIS</th>
<th>Inclusive Jet</th>
<th>Dijet</th>
<th>Trijet</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td>Measurement of scattered electron kinematics (Event properties)</td>
<td>Measurement of jet multiplicities and jet kinematics</td>
<td>Measurement of event properties of an event class</td>
<td>Measurement of event properties of an event class</td>
</tr>
<tr>
<td><strong>Cross section</strong></td>
<td>$0.2 &lt; y &lt; 0.7$ \ $-1.0 &lt; \eta &lt; 2.5$</td>
<td>$d\sigma/dQ^2$</td>
<td>$d\sigma/dQ^2dp_T$</td>
<td>$d\sigma/dQ^2dp_{T_2}$</td>
</tr>
<tr>
<td><strong>Special characteristic</strong></td>
<td>Each event</td>
<td>(geometric) matching of 'jets' on on particle and detector level</td>
<td>Cuts for 'Dijet' definition: $M_{12}$, $\eta_{Jet}$, $p_{T,Jet2}$, $p_{T,Jet1}$</td>
<td>Cuts for 'Trijet' definition: $M_{12}$, $\eta_{Jet}$, $p_{T,Jet3}$, $p_{T,Jet1}$</td>
</tr>
<tr>
<td><strong>Considered migrations</strong></td>
<td>$Q^2$, $y$</td>
<td>$Q^2$, $y$, $p_T$</td>
<td>$Q^2$, $y$, $&lt;p_T&gt;_2$, into/out of Dijet-class</td>
<td>$Q^2$, $y$, $&lt;p_T&gt;_3$, into/out of Trijet-class</td>
</tr>
<tr>
<td><strong>Through NC DIS estimated migrations</strong></td>
<td>Measured jets without matching jet on particle level</td>
<td>Measured Dijet events which do not fulfill any Dijet class definition</td>
<td>Measured Trijet events which do not fulfill any Trijet class definition</td>
<td></td>
</tr>
</tbody>
</table>
Regularized Unfolding using TUnfold

**Aim**
- Cross section on particle level
- Correction for detector effects

**Detector Response**
- Steeply falling $P_T$ spectra
- Boost to Breit frame
- Finite resolution
- $\rightarrow$ Migrations are getting relevant

**Bin-by-bin correction**
- Detector correction based on bin-wise correction factors
- Some error propagation from migrations

**Regularized Unfolding**
(Migration) Matrix $A$ describes detector response

$$\tilde{m} = A \cdot \tilde{x}$$

$m$: measured distribution
$x$: 'true' distribution

Determine particle level $x$ by analytic minimization of $\chi^2$ as function of $x$

$$\chi^2(x) = \frac{1}{2} (m - Ax)^T V^{-1} (m - Ax) + \tau^2 \cdot \chi_L^2(x)$$

Regularisation parameter $\tau$ suppresses large fluctuations/errors in comparison to direct matrix inversion

**Unfolding of four measurements at once**
- NC-DIS, Inclusive Jet, Dijet und Trijet
- $\rightarrow$ same dataset, same events
- $\rightarrow$ Covariance matrix $V$ contains correlations

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\( \alpha_s \) Extraction: Fitting technique

NLO calculations depend on PDF and \( \alpha_s \)
- Keep PDF fixed and determine \( \alpha_s \) (M\(_Z\))
- Neglect correlations between gluon and \( \alpha_s \)

Theory Cross Sections
- Jet cross sections
  - NLOJET++
  - FastNLO v2.0
  - \( \mu_r^2 = (Q^2 + E_T^2)/2 \)
  - \( \mu_f^2 = Q^2 \)
- NC-DIS cross sections
  - QCDNUM
  - \( \mu_f^2 = \mu_r^2 = Q^2 \)
- NLO × Had. corrections
- LHAPDF
  - PDF: CT10
  - \( \alpha_s \) evolution code (M\(_Z\) etc…)

Hessian Method
- Minimise \( \chi^2 \)
- TMinuit

\[
\chi^2(\alpha_s, \epsilon_k) = \tilde{\sigma}^T \cdot V^{-1} \cdot \tilde{\sigma} + \sum_{k}^{SysErr} \epsilon_k^2
\]

\[
\tilde{\sigma}_i = \sigma_i^{Data} - \sigma_i^{Theo}(\alpha_s, f(\alpha_s, fix))(1 - \sum_{k}^{SysErr} \Delta_{i,k}(\epsilon_k))
\]

\[
\Delta_{i,k}(\epsilon_k) = \frac{|\Delta_{i,k}^+| + |\Delta_{i,k}^-|}{2} \cdot \epsilon_k + \frac{|\Delta_{i,k}^+| - |\Delta_{i,k}^-|}{2} \cdot \epsilon_k^2
\]

Usage of full covariance matrix V from unfolding
- All correlations are respected in fit

Systematic errors as penalty terms
Multijet Measurement

**Kinematics**

- **DIS Event = scattered electron**
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**Phase Space**

- **Neutral current phase space**
  - $150 < Q^2 < 15000 \text{ GeV}^2$
  - $0.2 < y < 0.7$

- **Jet phase space**
  - $-1.0 < \eta_{\text{Lab}} < 2.5$

- **Inclusive Jet**
  - $7.0 < P_T < 50 \text{ GeV}$

- **Dijet and Trijet**
  - $5 < P_T < 50 \text{ GeV}$
  - $7 < <P_T> 50 \text{ GeV}$
  - $M_{12} > 16 \text{ GeV}$

**Four Measurements**

- **(inclusive) DIS Measurement**
  - $d\sigma/dQ^2$
- **Inclusive Jets**
  - $d\sigma/dQ^2 dP_T$
- **Dijet**
  - $d\sigma/dQ^2 d<P_T>$
- **Trijet**
  - $d\sigma/dQ^2 d<P_T>$

**Detector Correction**

- **Aim**
  - Cross section on particle level
  - Correction of detector effects

- **Bin-by-bin correction**
  - Detector correction based on bin-wise correction factors
  - Error propagation for 'gain' events

- **Regularized unfolding**
  - This talk, this measurement
# Bin-by-bin versus Unfolding

Inclusive jet cross sections, four $P_T$ bins in the first $Q^2$ bin

<table>
<thead>
<tr>
<th>bin label</th>
<th>$\sigma_{\text{jet}}$ (pb)</th>
<th>$\delta_{\text{stat}}$ (%)</th>
<th>$\delta_{\text{unc}}$ (%)</th>
<th>$\delta_{\text{tot}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1\alpha$</td>
<td>7.25 $10^{1}$</td>
<td>0.9</td>
<td>1.5</td>
<td>3.9</td>
</tr>
<tr>
<td>$1\beta$</td>
<td>3.22 $10^{1}$</td>
<td>1.3</td>
<td>2.0</td>
<td>4.4</td>
</tr>
<tr>
<td>$1\gamma$</td>
<td>7.48 $10^{0}$</td>
<td>2.6</td>
<td>2.9</td>
<td>5.8</td>
</tr>
<tr>
<td>$1\delta$</td>
<td>8.95 $10^{-1}$</td>
<td>7.6</td>
<td>3.7</td>
<td>9.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>bin label</th>
<th>$\sigma_{\text{jet}}$ (pb)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$1\alpha$</td>
<td>6.98 $10^{1}$</td>
<td>2.4</td>
<td>1.8</td>
<td>4.8</td>
</tr>
<tr>
<td>$1\beta$</td>
<td>3.11 $10^{1}$</td>
<td>3.1</td>
<td>2.7</td>
<td>6.0</td>
</tr>
<tr>
<td>$1\gamma$</td>
<td>7.64 $10^{0}$</td>
<td>5.5</td>
<td>3.5</td>
<td>8.1</td>
</tr>
<tr>
<td>$1\delta$</td>
<td>8.58 $10^{-1}$</td>
<td>16.8</td>
<td>5.5</td>
<td>18.8</td>
</tr>
</tbody>
</table>

In case of unfolding the statistical uncertainty increased by more than 100%.

But correlated and anti-correlated bins, so comparison is not totally meaningful.

The systematic uncertainties are very similar between the unfolded and bin-by-bin corrected results.
k- factors

inclusive jets

Dijets

Trijets
k-factors

inclusive jets

Dijets

Trijets
**α_s Fits to Individual Measurements**

**Experimental Error**
- Cancellations of some systematic errors
- All normalization uncertainties cancel

**Fit Quality**
- Reasonable $\chi^2 / \text{ndf}$
- Penalty parameters $-1 < \varepsilon < 1$

### Normalized Inclusive Jets

$$\alpha_s = 0.1197 \pm 0.0008 \text{ (exp)} \pm 0.0014 \text{ (PDF)} \pm 0.0011 \text{ (had)} \pm 0.0053 \text{ (theo)}$$

$$\chi^2 / \text{ndf} = 28.7/23 = 1.24$$

### Normalized Dijets

$$\alpha_s = 0.1142 \pm 0.0010 \text{ (exp)} \pm 0.0016 \text{ (PDF)} \pm 0.0009 \text{ (had)} \pm 0.0048 \text{ (theo)}$$

$$\chi^2 / \text{ndf} = 27.0/23 = 1.17$$

### Normalized Trijets

$$\alpha_s = 0.1185 \pm 0.0018 \text{ (exp)} \pm 0.0013 \text{ (PDF)} \pm 0.0016 \text{ (had)} \pm 0.0042 \text{ (theo)}$$

$$\chi^2 / \text{ndf} = 12.0/16 = 0.75$$

**PDF Error**
- Error on cross section from PDF Eigenvectors
- Neglecting correlations from EV

**Hadronisation Error**
- Error on hadronisation correction
  - Sherpa
    - Lund Fragmentation
    - Cluster Fragmentation

**Theory Error**
- Scan $m_{\tau}$ and $m_{\bar{f}}$ independently
  - $0.5 < c < 2.0$
  - Largest and lowest cross section
- Add error on cross section from $m_{\tau}$ and $m_{\bar{f}}$ variation in quadrature
$\alpha_s$ Fits to Individual Measurements

Normalized Inclusive Jets

$\alpha_s = 0.1197 \pm 0.0008 \text{ (exp)} \pm 0.0014 \text{ (PDF)} \pm 0.0011 \text{ (had)} \pm 0.0053 \text{ (theo)}$

$\chi^2 / \text{ndf} = 28.7 / 23 = 1.24$

Fit Quality
- Reasonable $\chi^2 / \text{ndf}$
- Penalty parameters $-1 < \varepsilon < 1$

Experimental Error
- Cancellations of some systematic errors
- All normalization uncertainties cancel

Theoretical/systematic errors
Repeated fit with offset method

PDF Error
- Error on cross section from PDF Eigenvectors
- Neglecting correlations from EV

Hadronisation Error
- Error on hadronisation correction
- Sherpa
  - Lund Fragmentation
  - Cluster Fragmentation

Theory Error
- Scan $\mu_r$ and $\mu_f$ independently
  - $0.5 < c < 2.0$
- Largest and lowest cross section
- Add error on cross section from $\mu_r$ and $\mu_f$ variation in quadrature
Errors on normalized cross sections

Some systematic errors cancel for normalized measurement

Here

Dijet $dQ^2 dp_T$ normalized with NCDIS
The y-distribution (inelasticity)

Binning on detector level
0.08, 0.15, 0.45, 0.70, 0.95
very little statistics above 0.7
Electron energy cut on 11.0 GeV
causes "cutoff" in y ~0.7
change would imply a restudy of the trigger efficiency
lowering of el. energy cut increases radiative corrections

In unfolding we basically would have to fill
1.5 to 2 yps bin
15 Q2 bins
10, 9, 7 bins for pT
from this little statistics ???

Problem:
almost every event that is generated with y>0.7 is not reconstructed
We would have to unfold quite some information out of almost no data

Solution:
and we don't "generate jets" above 0.7
we don't measure jets (pt) for high yps side bin