Spin structure of the "forward" charge-exchange reaction $n + p \to p + n$ and the deuteron charge-exchange breakup $d + p \to (pp) + n$

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Isotopic structure of *NN*-scattering

- Taking into account the isotopic invariance, the nucleon-nucleon scattering is described by the following operator:

$$\hat{f}(\mathbf{p},\mathbf{p}') = \hat{a}(\mathbf{p},\mathbf{p}') + \hat{b}(\mathbf{p},\mathbf{p}')\,\hat{\boldsymbol{\tau}}^{(1)}\hat{\boldsymbol{\tau}}^{(2)} \qquad (1)\ .$$

Here $\hat{\tau}^{(1)}$ and $\hat{\tau}^{(2)}$ are vector Pauli operators in the isotopic space, $\hat{a}(\mathbf{p},\mathbf{p}')$ and $\hat{b}(\mathbf{p},\mathbf{p}')$ are 4-row matrices in the spin space of two nucleons; $\mathbf{p}$ and $\mathbf{p}'$ are the initial and final momenta in the c.m. frame, the directions of $\mathbf{p}'$ are defined within the solid angle in the c.m. frame, corresponding to the front hemisphere.

- One should note that the process of elastic neutron-proton scattering into the back hemisphere is interpreted as the charge-exchange process $n + p \rightarrow p + n$.

- According to (1), the matrices of amplitudes of proton-proton, neutron-neutron and neutron-proton scattering take the form:

$$\hat{f}_{pp\to pp}(\mathbf{p},\mathbf{p}') = \hat{f}_{nn\to nn}(\mathbf{p},\mathbf{p}') = \hat{a}(\mathbf{p},\mathbf{p}') + \hat{b}(\mathbf{p},\mathbf{p}') \quad ;$$

$$\hat{f}_{np\to np}(\mathbf{p},\mathbf{p}') = \hat{a}(\mathbf{p},\mathbf{p}') - \hat{b}(\mathbf{p},\mathbf{p}') \qquad (2)\ ;$$

meantime, the matrix of amplitudes of the charge transfer process is as follows:

$$\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}') = 2\hat{b}(\mathbf{p},\mathbf{p}') = \hat{f}_{pp\to pp}(\mathbf{p},\mathbf{p}') - \hat{f}_{np\to np}(\mathbf{p},\mathbf{p}') \qquad (3).$$

- It should be stressed that the differential cross-section of the charge-exchange reaction, defined in the front hemisphere $0 \le \theta \le \frac{\pi}{2},\ 0 \le \varphi \le 2\pi$ (here θ is the angle between the momenta of initial neutron and final proton, φ is the azimuthal angle), should coincide with the differential cross-section of the elastic neutron-proton scattering into the back hemisphere by the angle $\tilde{\theta} = \pi - \theta$ at the azimuthal angle $\tilde{\varphi} = \pi + \varphi$ in the c.m. frame.

- Due to the antisymmetry of the state of two fermions with respect to the total permutation, including the permutation of momenta ($\mathbf{p}' \to -\mathbf{p}'$), permutation of spin projections and permutation of isotopic projections ($p \leftrightarrow n$) , the following relation between the amplitudes $\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}')$ and $\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')$ holds [1] :

$$\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}') = -\hat{P}^{(1,2)}\,\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}') \qquad (4)\,,$$

where $\hat{P}^{(1,2)}$ is the operator of permutation of spin projections of two particles with equal spins; the matrix elements of this operator are [2]:

$$\langle\, m_1' m_2' \,|\, \hat{P}^{(1,2)} \,|\, m_1 m_2 \,\rangle = \delta_{m_1' m_2}\;\delta_{m_2' m_1} \,.$$

- For particles with spin ½ [1,2]:

$$\hat{P}^{(1,2)} = \frac{1}{2}\left(\hat{I}^{(1,2)} + \hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)} \right) \qquad (5),$$

where $\hat{I}^{(1,2)}$ is the four-row unit matrix, $\hat{\boldsymbol{\sigma}}^{(1)}, \hat{\boldsymbol{\sigma}}^{(2)}$ - vector Pauli operators. It is evident that $\hat{P}^{(1,2)}$ is the unitary and Hermitian operator:

$$\hat{P}^{(1,2)} = \hat{P}^{(1,2)+}, \quad \hat{P}^{(1,2)}\hat{P}^{(1,2)+} = \hat{I}^{(1,2)} \qquad (6)\,.$$

- Taking into account the relations (5) and (6), the following matrix equality holds:

$$\hat{f}^{+}_{np\to pn}(\mathbf{p},\mathbf{p}')\,\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}') = \hat{f}^{+}_{np\to np}(\mathbf{p},-\mathbf{p}')\,\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}') \qquad (7).$$

As a result, the differential cross-sections of the charge-exchange process $n + p \to p + n$ and the elastic np -scattering in the corresponding back hemisphere coincide at any polarizations of initial nucleons:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(\mathbf{p},\mathbf{p}') = \frac{d\sigma_{np\to pn}}{d\Omega}(\mathbf{p},\mathbf{p}') \qquad (8).$$

However, the separation into the spin-dependent and spin-independent parts is different for the amplitudes

$\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}')$ and $\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')$!

Nucleon charge-exchange process at zero angle

- Now let us investigate in detail the nucleon charge transfer reaction $n + p \to p + n$ at zero angle. In the c.m. frame of the (np) system, the amplitude of the nucleon charge transfer in the "forward" direction $\hat{f}_{np \to pn}(0)$ has the following spin structure:

$$\hat{f}_{np\to pn}(0) = c_1 \hat{I}^{(1,2)} + c_2 [\hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)} - (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l})] + c_3 (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l}) \quad (9),$$

  where $\mathbf{l}$ is the unit vector directed along the incident neutron momentum. In so doing, the second term in Eq. (9) describes the spin-flip effect, and the third term characterizes the difference between the amplitudes with the parallel and antiparallel orientations of the neutron and proton spins.
- The spin structure of the amplitude of the elastic neutron-proton scattering in the "backward" direction is analogous:

$$\hat{f}_{np\to np}(\pi) = \tilde{c}_1 \hat{I}^{(1,2)} + \tilde{c}_2 [\hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)} - (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l})] + \tilde{c}_3 (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l}) \quad (10)$$

- However, the coefficients $\tilde{c}$ in Eq.(10) do not coincide with the coefficients c in Eq.(9). According to Eq.(4), the connection between the amplitudes $\hat{f}_{np\to pn}(0)$ and $\hat{f}_{np\to np}(\pi)$ is the following:

$$\hat{f}_{np\to pn}(0) = -\hat{P}^{(1,2)}\,\hat{f}_{np\to np}(\pi) \qquad (11),$$

where the unitary operator $\hat{P}^{(1,2)}$ is determined by Eq. (5) .

As a result of calculations with Pauli matrices, we obtain :

$$c_1 = -\frac{1}{2}(\tilde{c}_1 + 2\tilde{c}_2 + \tilde{c}_3); \;\; c_2 = -\frac{1}{2}(\tilde{c}_1 - \tilde{c}_3); \;\; c_3 = -\frac{1}{2}(\tilde{c}_1 - 2\tilde{c}_2 + \tilde{c}_3). \qquad (12)$$

Hence, it follows from here that the "forward" differential cross-section of the nucleon charge-exchange reaction $n + p \to p + n$ for unpolarized initial nucleons is described by the expression:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = |c_1|^2 + 2|c_2|^2 + |c_3|^2 = \frac{1}{4}|\tilde{c}_1 + 2\tilde{c}_2 + \tilde{c}_3|^2 + \frac{1}{2}|\tilde{c}_1 - \tilde{c}_3|^2 + \frac{1}{4}|\tilde{c}_1 - 2\tilde{c}_2 + \tilde{c}_3|^2 = |\tilde{c}_1|^2 + 2|\tilde{c}_2|^2 + |\tilde{c}_3|^2 . \qquad (13)$$

Thus,

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = \frac{d\sigma_{np\to np}}{d\Omega}(\pi) ,$$

just as it must be in accordance with the relation (8).

Spin-independent and spin-dependent parts of the cross-section of the reaction $n + p \to p + n$ at zero angle

- It is clear that the amplitudes of the proton-proton and neutron-proton elastic scattering at zero angle have the structure (9) with the replacements $c_1, c_2, c_3 \to c_1^{(pp)}, c_2^{(pp)}, c_3^{(pp)}$ and $c_1, c_2, c_3 \to c_1^{(np)}, c_2^{(np)}, c_3^{(np)}$, respectively.

It follows from the isotopic invariance (see Eq. (3)) that

$$c_1 = c_1^{(pp)} - c_1^{(np)}, \quad c_2 = c_2^{(pp)} - c_2^{(np)}, \quad c_3 = c_3^{(pp)} - c_3^{(np)} \qquad (14).$$

In accordance with the optical theorem, the following relation holds, taking into account Eq. (14):

$$\frac{4\pi}{k}\,\mathrm{Im}\,c_1 = \frac{4\pi}{k}\left(\mathrm{Im}\,c_1^{(pp)} - \mathrm{Im}\,c_1^{(np)}\right) = \sigma_{pp} - \sigma_{np} \qquad (15)$$

where σ_{pp} and σ_{np} are the total cross-sections of interaction of two unpolarized protons and of an unpolarized neutron with unpolarized proton, respectively (due to the isotopic invariance, $\sigma_{pp} = \sigma_{nn}$), $k = |\mathbf{p}| = |\mathbf{p}'|$ is the modulus of neutron momentum in the c.m. frame of the colliding nucleons (we use the unit system with $\hbar = c = 1$).

- Taking into account Eqs. (9), (13) and (15), the differential cross-section of the process $n + p \rightarrow p + n$ in the "forward" direction for unpolarized nucleons can be presented in the following form, distinguishing the spin-independent and spin-dependent parts :

$$\frac{d\sigma_{np\rightarrow pn}}{d\Omega}(0) = |c_1|^2 + 2|c_2|^2 + |c_3|^2 = \frac{d\sigma^{(si)}_{np\rightarrow pn}}{d\Omega}(0) + \frac{d\sigma^{(sd)}_{np\rightarrow pn}}{d\Omega}(0) \; . \qquad (16)$$

In doing so, the spin-independent part $\frac{d\sigma^{(si)}_{np\rightarrow pn}}{d\Omega}(0)$ in Eq.(16) is determined by the difference of total cross-sections of the unpolarized proton-proton and neutron-proton interaction:

$$\frac{d\sigma^{(si)}_{np\rightarrow pn}}{d\Omega}(0) = |c_1|^2 = \frac{k^2}{16\pi^2}(\sigma_{pp} - \sigma_{np})^2(1 + \alpha^2) \qquad (17),$$

where $\alpha = \mathrm{Re}\, c_1 / \mathrm{Im}\, c_1$. The spin-dependent part of the cross-section of the "forward" charge-exchange process is

$$\frac{d\sigma^{(sd)}_{np\rightarrow pn}}{d\Omega}(0) = 2|c_2|^2 + |c_3|^2 \qquad (18).$$

- Meantime, according to Eqs. (10), (12) and (13), the spin-dependent part of the cross-section of the "backward" elastic *np* -scattering is

$$\frac{d\sigma^{(sd)}_{np\to np}}{d\Omega}(\pi) = 2|\tilde{c}_2|^2 + |\tilde{c}_3|^2 \qquad (19).$$

We see that

$$\frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0) \neq \frac{d\sigma^{(sd)}_{np\to np}}{d\Omega}(\pi).$$

Further it is advisable to deal with the differential cross-section $\left.\frac{d\sigma}{dt}\right|_{t=0}$, being a relativistic invariant ($t = -(p_1 - p_2)^2 = (\mathbf{p} - \mathbf{p}')^2 - (E - E')^2$ is the square of the 4-dimensional transferred momentum).

In the c.m. frame we have: $t = 2k^2(1-\cos\theta)$ and $\frac{d\sigma}{dt} = \frac{\pi}{k^2}\frac{d\sigma}{d\Omega}$.

So, in this representation, the spin-independent and spin-dependent parts of the differential cross-section of the "forward" charge transfer process $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ are as follows:

$$\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} = \frac{\pi}{k^2}|c_1|^2 \quad , \quad \left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0} = \frac{\pi}{k^2}(2|c_2|^2 + |c_3|^2) \quad ,$$

and we may write, instead of Eq. (16)

$$\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = \left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0} + \frac{1}{16\pi}(\sigma_{pp} - \sigma_{np})^2(1+\alpha^2) \qquad (20).$$

- Now it should be noted that, in the framework of the impulse approach, there exists a simple connection between the spin-dependent part of the differential cross-section of the charge-exchange reaction $n + p \to p + n$ at zero angle $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$ (not the "backward" elastic neutron-proton scattering, see Section 2) and the differential cross-section of the deuteron charge-exchange breakup $d + p \to (pp) + n$ in the "forward" direction $\left.\frac{d\sigma_{dp\to(pp)n}}{dt}\right|_{t=0}$ at the deuteron momentum $\mathbf{k}_d = 2\mathbf{k}_n$ ($\mathbf{k}_n$ is the initial neutron momentum).

In the case of unpolarized particles we have [3-5]:

$$\left.\frac{d\sigma_{dp\to(pp)n}}{dt}\right|_{t=0} = \frac{2}{3}\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0} \qquad (21)$$

- It is easy to understand also that, due to the isotopic invariance, the same relation (like Eq. (21)) takes place for the process $p + d \to n + (pp)$ at the proton laboratory momentum $\mathbf{k}_p = \mathbf{k}_n$ and for the process $n + d \to p + (nn)$ at the neutron laboratory momentum $\mathbf{k}_n$.

- Thus, in principle, taking into account Eqs. (20) and (21), the modulus of the ratio of the real and imaginary parts of the spin-independent charge transfer amplitude at zero angle ($|\alpha|$) may be determined using the experimental data on the total cross-sections of interaction of unpolarized nucleons and on the differential cross-sections of the "forward" nucleon charge transfer process and the charge-exchange breakup of an unpolarized deuteron $d + p \to (pp) + n$ in the "forward" direction.

- At present there are not yet final *reliable* experimental data on the differential cross-section of the deuteron charge-exchange breakup on a proton. However, the analysis shows: if we suppose that the real part of the spin-independent amplitude of charge transfer $n + p \to p + n$ at zero angle is smaller or of the same order as compared with the imaginary part $(\alpha^2 \le 1)$, then it follows from the available experimental data on the differential cross-section of charge transfer $\left.\frac{d\sigma_{np \to pn}}{dt}\right|_{t=0}$

  and the data on the total cross-sections σ_{pp} and σ_{np} that the main contribution into the cross-section $\left.\frac{d\sigma_{np \to pn}}{dt}\right|_{t=0}$

  is provided namely by the spin-dependent part $\left.\frac{d\sigma^{(sd)}_{np \to pn}}{dt}\right|_{t=0}$.

- If the differential cross-section $\frac{d\sigma}{dt}$ is given in the units of $mbn \Big/ \left(\frac{GeV}{c}\right)^2$ and the total cross-sections are given in *mbn* , then the spin-independent part of the "forward" charge transfer cross-section may be expressed in the form :

$$\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \approx 0.0512 \ (\sigma_{pp} - \sigma_{np})^2 (1 + \alpha^2) \ . \qquad (22)$$

Using (22) and the data from the works [6-8], we obtain the estimates of the ratio $\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$

at different values of the neutron laboratory momentum k_n :

1) $k_n = 0.7 \ \frac{GeV}{c}$; $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = 268 \ mbn \Big/ \left(\frac{GeV}{c}\right)^2$;

$\sigma_{pp} - \sigma_{np} = -22.6 \ mbn$; $\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} \approx 0.1 \ (1+\alpha^2)$.

2) $k_n = 1.7 \dfrac{GeV}{c}$; $\left.\dfrac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = 37.6 \ mbn \Big/ \left(\dfrac{GeV}{c}\right)^2$;

$\sigma_{pp} - \sigma_{np} = 10 \ mbn$; $\left.\dfrac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\dfrac{d\sigma_{np\to pn}}{dt}\right|_{t=0} \approx 0.136 \ (1+\alpha^2)$.

3) $k_n = 2.5 \dfrac{GeV}{c}$; $\left.\dfrac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = 17.85 \ mbn \Big/ \left(\dfrac{GeV}{c}\right)^2$;

$\sigma_{pp} - \sigma_{np} = 5.5 \ mbn$; $\left.\dfrac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \Big/ \left.\dfrac{d\sigma_{np\to pn}}{dt}\right|_{t=0} \approx 0.085 \ (1+\alpha^2)$.

So, it is well seen that, assuming $\alpha^2 \le 1$, the spin-dependent part $\left.\dfrac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$ provides at least (70 ÷ 90) % of the total magnitude of the "forward" charge transfer cross-section.

- The preliminary experimental data on the differential cross-section of "forward" deuteron charge-exchange breakup $d + p \to (pp) + n$, obtained recently in Dubna (JINR, Laboratory of High Energy Physics), also confirm the conclusion about the predominant role of the spin-dependent part of the differential cross-section of the nucleon charge-exchange reaction $n + p \to p + n$ in the "forward" direction.

Summary

1. Theoretical investigation of the structure of the nucleon charge transfer process $n + p \to p + n$ is performed on the basis of the isotopic invariance of the nucleon-nucleon scattering amplitude.
2. The nucleon charge-exchange reaction at zero angle is analyzed. Due to the optical theorem, the spin-independent part of the differential cross-section of the "forward" nucleon charge-exchange process $n + p \to p + n$ for unpolarized particles is connected with the difference of total cross-sections of unpolarized proton-proton and neutron-proton scattering.
3. The spin-dependent part of the differential cross-section of neutron-proton charge-exchange reaction at zero angle is proportional to the differential cross-section of "forward" deuteron charge-exchange breakup. Analysis of the existing data shows that the main contribution into the differential cross-section of the nucleon charge transfer reaction at zero angle is provided namely by the spin-dependent part.

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Thank you !