Heavy Quark Production in the ACOT Scheme Beyond NLO

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We analyze the properties of the ACOT scheme for heavy quark production and make use of the $\overline{MS}$ massless results at NNLO and N^{3}LO for the structure functions F_2 and F_L in neutral current deep-inelastic scattering to estimate the higher order corrections. The dominant heavy quark mass effects at higher orders can be taken into account using the massless Wilson coefficients together with an appropriate slow-rescaling prescription implementing the phase space constraints. Combining the exact ACOT scheme at NLO with these expressions should provide a good approximation to the full calculation in the ACOT scheme at NNLO and N^{3}LO.

1 Introduction

With the ever-increasing precision of the experimental data, the production of heavy quarks in high energy processes has become an increasingly important subject. As theoretical calculations and parton distribution function (PDF) evolution are progressing to next-to-next-to-leading order (NNLO) of QCD, there is a clear need to formulate and also implement the heavy quark schemes at this order and beyond. The most important case is arguably the heavy quark treatment in inclusive deep-inelastic scattering (DIS) since the very precise HERA data for DIS structure functions and cross sections form the backbone of any modern global analysis of PDFs. Here, the heavy quarks contribute up to 30% or 40% to the structure functions at small momentum fractions x. Extending the heavy quark schemes to higher orders is therefore necessary for extracting precise PDFs and hence for precise predictions of observables at the LHC. Additionally, it is theoretically important to have a general pQCD framework including heavy quarks which is valid to all orders in perturbation theory over a wide range of hard energy scales. The results of this study form the basis for using the ACOT scheme in NNLO global analyses and for future comparisons with precision data for DIS structure functions.

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*Presented by F.I. Olness.

2 ACOT Scheme

The ACOT renormalization scheme [1] provides a mechanism to incorporate the heavy quark mass into the theoretical calculation of heavy quark production both kinematically and dynamically. In 1998 Collins [2] extended the factorization theorem to address the case of heavy quarks; this work provided the theoretical foundation that allows us to reliably compute heavy quark processes throughout the full kinematic realm.

The ACOT prescription is to just calculate the massive partonic cross sections and perform the factorization using the quark mass as regulator. The ACOT scheme does not need any observable–dependent extra contributions or any regulators to smooth the transition between the high and low scale regions.

In Ref. [3] we demonstrated using the NLO full ACOT scheme that the dominant mass effects are those coming from the phase space which can be taken into account via a generalized slow-rescaling $\chi(n)$-prescription.[1] Assuming that a similar relation remains true at higher orders, one can construct the following approximation to the ACOT result up to N^{3}LO ($\mathcal{O}(\alpha_S^3)$):

$$\mathrm{ACOT}[\mathcal{O}(\alpha_S^{0+1+2+3})] \quad \simeq \quad \mathrm{ACOT}[\mathcal{O}(\alpha_S^{0+1})] + \mathrm{ZMVFNS}_{\chi(\mathrm{n})}[\mathcal{O}(\alpha_{\mathrm{S}}^{2+3})] \tag{1}$$

In this equation, "ACOT" generically represents any variant of the ACOT scheme (ACOT, S-ACOT, S-ACOT$_\chi$); for the results presented in Sec. 3, we will use the fully massive ACOT scheme with all masses retained out to NLO. The ZMVFNS$_{\chi(\mathrm{n})}$ term uses the massless Wilson coefficients at $\mathcal{O}(\alpha\,\alpha_S^2)$ and $\mathcal{O}(\alpha\,\alpha_S^3)$. This approximation is necessary as not all the necessary massive Wilson coefficients at $\mathcal{O}(\alpha\,\alpha_S^2)$ and $\mathcal{O}(\alpha\,\alpha_S^3)$ have been computed.

3 Results

We now present the results of our calculation extending the ACOT scheme to NNLO and N^{3}LO. The details of the numerical calculations are presented in Ref. [3]. We have used the QCDNUM program [4] for the DGLAP evolution, and the Fortran subroutines provided by Andreas Vogt for the higher order Wilson coefficients. We choose $m_c = 1.3$ GeV, $m_b = 4.5$ GeV, $\alpha_S(M_Z) = 0.118$.

In Figures 1a and 1b we display the fractional contributions for the final-state quarks to the structure functions F_2 and F_L, respectively, for selected x values as a function of Q. We observe that for large x and low Q the heavy flavor contributions are minimal. but these can grow quickly as we move to smaller x and larger Q. For example, at $x = 10^{-5}$ and large Q we see the F_2 contributions of the u-quark and c-quark are comparable (as they both couple with a factor 4/9), and the d-quark and s-quark are comparable (as they both couple with a factor 1/9).

In Figure 2a we display the results for F_2 vs. Q computed at various orders. For large x (c.f. $x = 0.1$) we find the perturbative calculation is particularly stable; we see that the LO result is within 20% of the others at small Q, and within 5% at large Q. The NLO is within 2% at small Q, and indistinguishable from the NNLO and N^{3}LO for Q values above ~ 10 GeV. The NNLO and N^{3}LO results are essentially identical throughout the kinematic range.

In Figure 2b we display the results for F_L vs. Q computed at various orders. In contrast to F_2, we find the NLO corrections are large for F_L; this is because the LO F_L contribution

[1] Specifically, $\chi(n) = x[1 + (n\,m/Q)^2]$, where m is the quark mass.

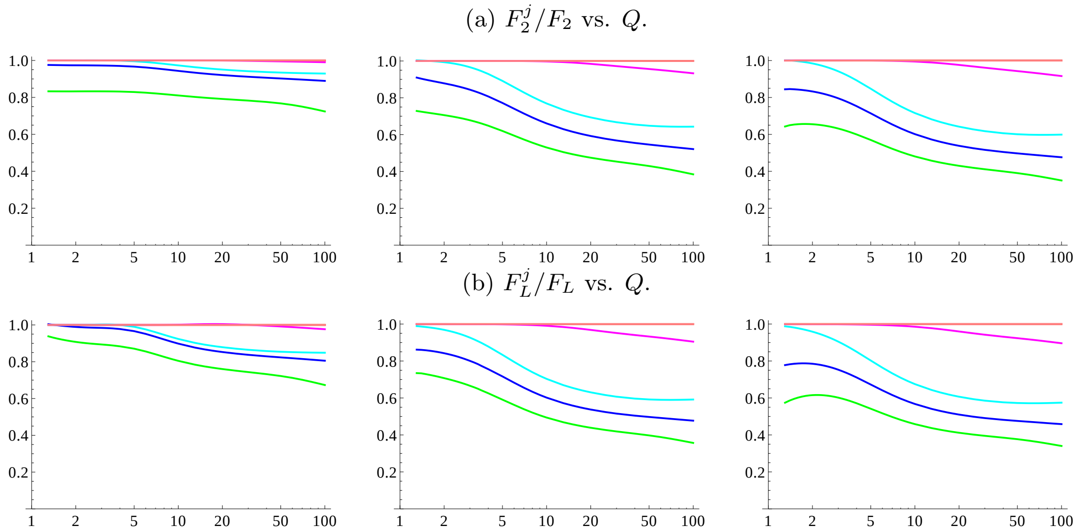


Figure 1: Fractional contribution for each quark flavor to $F^j_{2,L}/F_{2,L}$ vs. Q at N^{3}LO for fixed $x = \{10^{-1}, 10^{-3}, 10^{-5}\}$ (left to right). Results are displayed for $n = 2$ scaling. Reading from the bottom, we have the cumulative contributions from the $j = \{u, d, s, c, b\}$ (green, blue, cyan, magenta, pink).

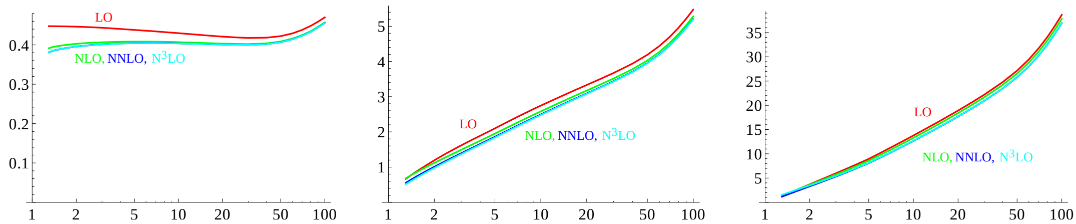
(which violates the Callan-Gross relation) is suppressed by (m^2/Q^2) compared to the dominant gluon contributions which enter at NLO. Consequently, we observe (as expected) that the LO result for F_L receives large contributions from the higher order terms. Essentially, the NLO is the first non-trivial order for F_L, and the subsequent contributions then converge. While the calculation of F_L is certainly more challenging, the relevant kinematic range probed by HERA the theoretical calculation is generally stable.

4 Conclusions

We extended the ACOT calculation for DIS structure functions to N^{3}LO by combining the exact ACOT result at NLO with a $\chi(n)$-rescaling for the higher order terms; this allows us to include the leading mass dependence at NNLO and N^{3}LO.

We studied the F_2 and F_L structure functions as a function of x and Q. We examined the flavor decomposition of these structure functions, and verified that the heavy quarks were appropriately suppressed in the low Q region. We found the results for F_2 were very stable across the full kinematic range for $\{x, Q\}$, and the contributions from the NNLO and N^{3}LO terms were small. For F_L, the higher order terms gave a proportionally larger contribution (due to the suppression of the LO term from the Callan-Gross relation); nevertheless, the contributions from the NNLO and N^{3}LO terms were generally small in the region probed by HERA. Using the results of this calculation we can obtain precise predictions for the inclusive F_2 and F_L structure functions which can be used to analyze the HERA data.

(a) F_2 vs. Q.



(b) F_L vs. Q.

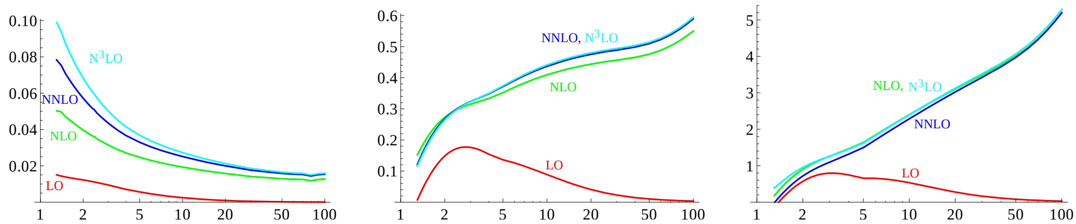


Figure 2: $F_{2,L}$ vs. Q at {LO, NLO, NNLO, N^{3}LO} (red, green, blue, cyan) for fixed $x = \{10^{-1}, 10^{-3}, 10^{-5}\}$ (left to right) for $n = 2$ scaling.

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