Fully massive scheme for jet production in DIS

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Motivation

Two basic approaches for the heavy quarks in DIS:

- Fixed Flavour Number Scheme (FFNS)

  Given heavy quark $\mathbb{Q}$ is produced from light flavours; there is no PDF for $\mathbb{Q}$

  $\Rightarrow$ applicable when $Q^2 \sim m_{\mathbb{Q}}^2$
- Zero-Mass Scheme (ZM)

  When $Q^2 \gg m_{\mathbb{Q}}^2$ the mass can be neglected, however there are large logs of the ratio $m_{\mathbb{Q}}^2/Q^2$ in every order of α_s and resummation is needed. Effectively the PDF for the heavy quark appears.

It is highly desired to have a reliable description in the intermediate region $0 < m_{\mathbb{Q}}^2/Q^2 < 1$.

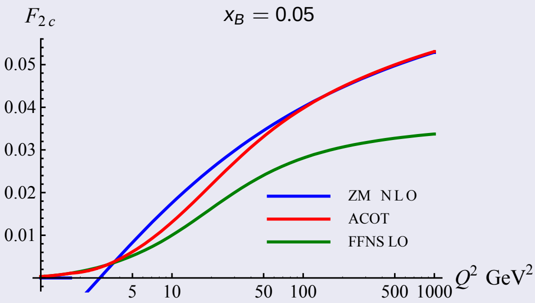
$\Rightarrow$ General Mass Schemes (GM)

there are solutions for inclusive DIS, for instance:

- ACOT scheme (Aivazis-Collins-Olness-Tung)
- Thorne-Roberts

Motivation (cont.)

Example: charm structure function



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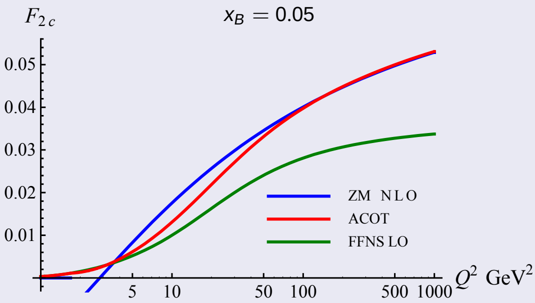


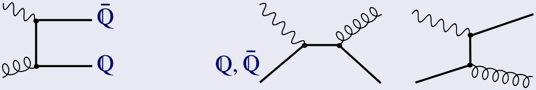
Our work

General Mass Scheme for jet production

Jets with heavy quarks (Introduction)

LO contribution to dijet production

Two types of processes: boson-gluon fusion (BGF) and quark scattering (QS)



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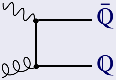
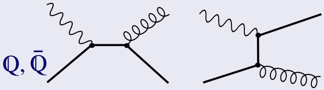
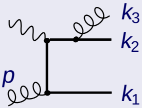
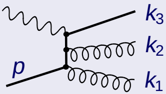




NLO real contribution

for instance:





- Integration over the region of phase space where k_1 is collinear to p leads to the $\log\left(Q^2/m_{\mathbb{Q}}^2\right)$ terms

  $\Rightarrow$ unreliable results when $Q^2 \gg m_{\mathbb{Q}}^2$ (quasi-collinear initial state singularities)

- additional problem: jet cross sections require special methods of cancelling the IR singularities between real and virtual contributions

  $\Rightarrow$ existing methods do not take into account massive initial state splitting processes

Jets with heavy quarks (Introduction cont.)

One of the methods: Dipole Subtraction Method (S. Catani, M. Seymour)

- originally for massless partons
- extended to photon radiation off heavy fermions, also in the initial state (S. Dittmaier)
  - no QCD splittings
  - IR singularities regularized by photon mass
- extended to QCD with heavy quarks in the final state (S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi)
  - no massive initial states; no initial state $g \to Q\overline{Q}$ splittings

$\Rightarrow$ not suited for constructing the GM scheme

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Essential part of our work

Extension of the Dipole Subtraction Method to massive initial state QCD splittings

Dipole Subtraction Method

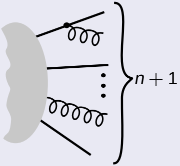
Basics

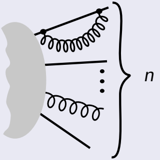
The cross section for a production of n jets

$$\sigma^{\mathrm{LO}} = \int |\mathcal{M}_n|^2 F_n \, d\Gamma_n,$$

$$\sigma^{\mathrm{NLO}} = \int d\Gamma_{n+1} \left|\mathcal{M}_{n+1}\right|^2 F_{n+1} + \int d\Gamma_n \left(\mathcal{M}_n^{\mathrm{loop}}\right)^2 F_n.$$

Add and subtract auxiliary contribution $\int d\Gamma_{n+1} \left|\mathcal{M}^{\mathrm{sub}}_{n+1}\right|^2 F_n$ such that it mimics all the singularities of real ME





$$\sigma^{\mathrm{NLO}} = \int d\Gamma_{n+1} \left\{ \left|\mathcal{M}_{n+1}\right|^2 F_{n+1} - \left|\mathcal{M}^{\mathrm{sub}}_{n+1}\right|^2 F_n \right\}$$
$$+ \int d\Gamma_n \left\{ \left(\mathcal{M}_n^{\mathrm{loop}}\right)^2 + \int d\phi \left|\mathcal{M}^{\mathrm{sub}}_{n+1}\right|^2 \right\} F_n$$

- in the singular regions $F_{n+1} = F_n$ (infrared safety)
- factorization of the phase space $d\Gamma_{n+1} = d\Gamma_n \otimes d\phi$
- dipole splitting function $\hat{V}$: $\left|\mathcal{M}^{\mathrm{sub}}_{n+1}\right|^2 = \hat{V} \otimes |\mathcal{M}_n|^2$
- analytical integration: $\int d\phi \hat{V}(\phi)$

Fully massive Dipole Subtraction Method

Main steps

- factorization of phase space with initial state (IS) massive parton

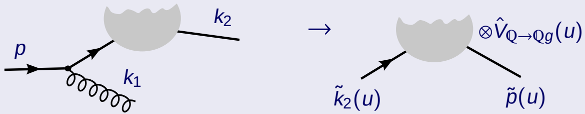
  The presence of a massive IS parton substantially complicates the procedure: the Dipole Method introduces new scaled on-shell momenta that enter reduced matrix element $\rightarrow$ the scaling variables are not the same as the Sudakov variables.

- construction of the dipole splitting functions $\hat{V}$ that mimics the IS and final state quasi-collinear singularities of ME

- integration of the massive dipole splitting functions
  - "plus" distributions with ordinary support are not sufficient
  - IS massive parton affects the integrals of the massless final state dipole splitting functions (eg. $g \rightarrow gg$)

Fully massive Dipole Subtraction Method (cont.)

Example: initial state $\mathbb{Q} \to \mathbb{Q}g$ splitting



- "dipole" momenta:

$$\tilde{k}_2^\mu(u) = w\,(k_1 + k_2)^\mu - u p^\mu, \qquad \tilde{p}^\mu(u) = (w-1)\,(k_1 + k_2)^\mu - (u-1)\,p^\mu$$

- dipole splitting function:

$$\hat{V}_{\mathbb{Q}\to\mathbb{Q}g}(u) = 8\pi\mu_r^{2\varepsilon}\alpha_s C_F \left[\frac{2}{uv^2 + z} + (1-\varepsilon)\,u - 2 - \frac{(1-u)\,m_{\mathbb{Q}}^2}{k_1 \cdot p} \right]$$

- the integral:

$$\int d\phi(u)\,\hat{V}_{\mathbb{Q}\to\mathbb{Q}g}(u) = \frac{1}{\varepsilon}\delta(u)\,\mathcal{I}_{\text{pole}} + \mathcal{I}_{\text{coll}}(u) + \mathcal{I}_{\text{remn}}(u)$$

It is not infra-red safe:

$$\mathcal{I}_{\text{coll}}(u) \underset{m_{\mathbb{Q}}\to 0}{\longrightarrow} \left(\frac{1 + (1-u)^2}{u} \right)_+ \log \frac{m_{\mathbb{Q}}^2}{Q^2} + \text{finite terms}$$

Factorization of the quasi-singularities

The IR sensitive terms have to be factorized out by means of the (quasi-)collinear subtraction terms constructed using the densities of partons inside a parton $f_{ab}(z)$

- eg. for massless partons in $\overline{\text{MS}}$ scheme

$$f_{ab}^{\overline{\text{MS}}}(z) = \frac{\alpha_s}{2\pi}\left(-\frac{1}{\varepsilon}\right)P_{ab}(z)$$

  where $P_{ab}(z)$ are ordinary massless splitting functions

- for massive partons in $\overline{\text{MS}}$ we have for instance

$$f_{g\mathbb{Q}}^{\overline{\text{MS}}}(z) = \frac{\alpha_s}{2\pi} P_{g\mathbb{Q}}(z) \log\frac{\mu^2}{m_{\mathbb{Q}}^2}$$

$$f_{\mathbb{Q}\mathbb{Q}}^{\overline{\text{MS}}}(z) = \frac{\alpha_s}{2\pi}\left\{P_{\mathbb{Q}\mathbb{Q}}(z)\left[\log\frac{\mu^2}{m_{\mathbb{Q}}^2} - 2\log(1-z) - 1\right]\right\}_+$$

  They are obtained using the Feynman rules for PDFs (Collins, Soper) and $\overline{\text{MS}}$ renormalization scheme (globally one uses CWZ scheme (Collins-Wilczek-Zee))

  ⇒ PDFs undergo the standard massless DGLAP equations

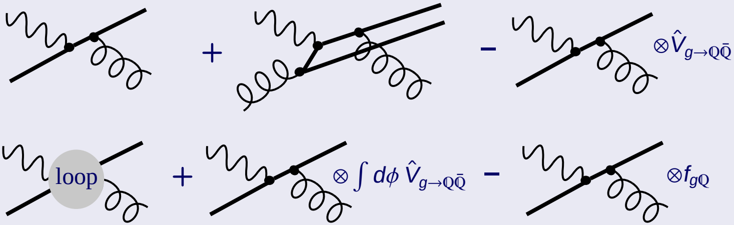
⇒ This is essentially the ACOT scheme applied to jets.

General Mass Scheme for Jets

The structure of GM scheme

- Dipole Subtraction Method supplied with all QCD massive initial state splittings
- set of (quasi-)collinear subtraction terms constructed according to massive factorization scheme (ACOT)

Schematic example



We have checked that in the massless limit our dipole formulae are infra-red safe and reduce to the known massless results (Catani et al.).

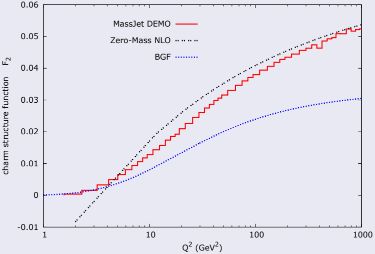
Summary

General Mass Scheme - consistent method of treating heavy quarks; applies when the factorization scale is comparable to the mass as well as it is much larger

- already in use for inclusive DIS
- our contribution: jet production

- We have partially implemented our method in a dedicated C++ MC program for numerical integration of matrix elements (using FOAM by S. Jadach)
  - as a check of the code we have recovered $F_{2\,c}$ at NLO in the ACOT scheme

    $\Rightarrow$ this proves that the constructed dipoles, their integrals and quasi-collinear subtraction terms are correct
  - the work is in progress...



- the method can be extended to hadron-hadron collisions at NLO