Subleading N_c Improved Parton Showers

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DESY Theory Group

– in collaboration with Malin Sjödahl –

Physics: SP & M. Sjödahl, `arXiv:1201.0260`

Some technicalities: SP & M. Sjödahl, EPJ Plus **127** (2012) 26 (`arXiv:1108.6180`)

Motivation and general remarks.

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A more precise input to fixed-order matching.

- NLO matching condition often only satisfied in large-N_c limit.

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Consider a simple approach, fitting well into existing MC generators.

[Matchbox and DipoleShower modules in Herwig++, SP & S. Gieseke arXiv:1109.6256]

We can't (yet) do a full evolution at the level of amplitudes.

Outline.

- A brief recap of dipole factorization and showering.
- Color matrix element corrections for dipole showers.
- The Sudakov veto algorithm and other technicalities.
- Results and conclusions.

From dipole factorization to dipole showering.

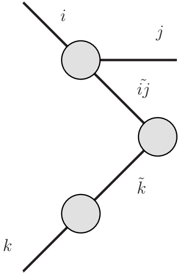
Singly unresolved limits, i, j collinear or one of them soft:

[S. Catani & M. H. Seymour, Nucl. Phys. B485 (1997) 291]

$$
|\mathcal{M}_{n+1}(..., p_i, ..., p_j, ..., p_k, ...)|^2 \approx
\sum_{k \neq i,j} \frac{1}{2 p_i \cdot p_j} \langle \mathcal{M}_n(..., p_{\tilde{ij}}, ..., p_{\tilde{k}}, ...) | \mathbf{V}_{ij,k}(p_i, p_j, p_k) | \mathcal{M}_n(..., p_{\tilde{ij}}, ..., p_{\tilde{k}}, ...) \rangle
$$

Well established subtraction scheme for NLO calculations.

Use dipole factorization to define a shower algorithm $\rightarrow$



From dipole factorization to dipole showering.

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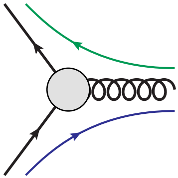
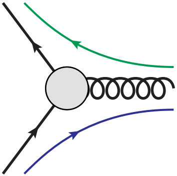
$$\mathbf{V}_{ij,k} = -V_{ij,k} \frac{\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_k}{\mathbf{T}^2_{\tilde{ij}}}$$
$$\to \frac{V_{ij,k}}{1+\delta_{\tilde{ij}}} \delta(\tilde{ij}, k \text{ colour connected})$$



From dipole factorization to dipole showering.

$$|\mathcal{M}_{n+1}(..., p_i, ..., p_j, ..., p_k, ...)|^2 \approx \sum_{k\neq i,j} \frac{1}{2p_i \cdot p_j} \langle \mathcal{M}_n(..., p_{\tilde{ij}}, ..., p_{\tilde{k}}, ...)|\mathbf{V}_{ij,k}(p_i, p_j, p_k)|\mathcal{M}_n(..., p_{\tilde{ij}}, ..., p_{\tilde{k}}, ...)\rangle$$

$$dP_{ij,k}(p_\perp^2, z) = \frac{V_{ij,k}(p_\perp^2, z)}{1+\delta_{\tilde{ij}}} \frac{d\phi_{n+1}(p_\perp^2, z)}{d\phi_n} \times\delta(\tilde{ij}, k \text{ colour connected})$$



Sensible for $p_\perp$ ordering and improvements to initial state radiation.

[SP & S. Gieseke, JHEP 1101 (2011) 024]

Taking dipoles serious: Color matrix element corrections.

$$|\mathcal{M}_{n+1}(..., p_i, ..., p_j, ..., p_k, ...)|^2 \approx \sum_{k\neq i,j} \frac{1}{2p_i \cdot p_j} \langle \mathcal{M}_n(..., p_{\tilde{ij}}, ..., p_{\tilde{k}}, ...)|\mathbf{V}_{ij,k}(p_i, p_j, p_k)|\mathcal{M}_n(..., p_{\tilde{ij}}, ..., p_{\tilde{k}}, ...)\rangle$$

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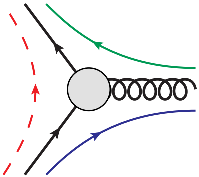

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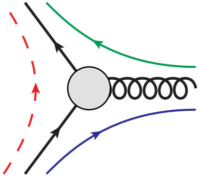

Similar to matrix element corrections: 'Color matrix element corrections'.

[SP & M. Sjödahl, arXiv:1201.0260]

What amplitudes?

Choose a definite basis for color space:

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha}|\alpha_n\rangle \quad \leftrightarrow \quad \mathcal{M}_n = (c_{n,1}, ..., c_{n,d_n})^T$$

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How to get $|\mathcal{M}_{n+1}\rangle$ to enter the next correction after emission?

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Observe that

$$|\mathcal{M}_n|^2 = \mathcal{M}_n^\dagger S_n \mathcal{M}_n = \mathrm{Tr}\left(S_n \times \mathcal{M}_n \mathcal{M}_n^\dagger\right)$$

and

$$\langle\mathcal{M}_n|\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}}|\mathcal{M}_n\rangle = \mathrm{Tr}\left(S_{n+1} \times T_{\tilde{k},n}\mathcal{M}_n\mathcal{M}_n^\dagger T_{\tilde{ij},n}^\dagger\right)$$

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Use an 'amplitude matrix' M_n as basic object:

$$M_{n+1} = -\sum_{i \neq j} \sum_{k \neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}_{\tilde{ij}}^2} \; T_{\tilde{k},n} M_n T_{\tilde{ij},n}^\dagger$$

where

$$M_{n_{\text{hard}}} = \mathcal{M}_{n_{\text{hard}}} \mathcal{M}_{n_{\text{hard}}}^\dagger$$

Technicalities.

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Two things to tackle:

– An efficient treatment of the color basis $\{|\alpha\rangle\}$.

[M. Sjödahl, in preparation]

– Splitting kernels which can become negative.

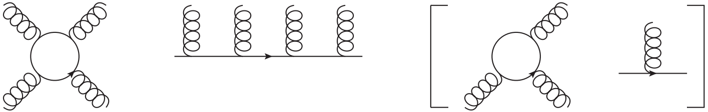
[SP & M. Sjödahl, EPJ Plus 127 (2012) 26]

Then hook into existing shower and sampling machinery.

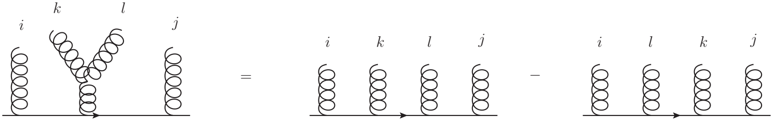
[SP & S. Gieseke, arXiv:1109.6256], [SP, EPJ C72 (2012) 1929]

Technicalities: The color basis.

We use a trace basis.
The framework is general enough to cope with other choices.



Mapping to the basis after emission is simple:



more details in [M. Sjödahl, JHEP 0812 (2008) 083]

Technicalities: The Sudakov veto algorithm.

Evolve from $Q \to q$ in presence of an infrared cutoff μ. Driven by:

$$\frac{dS_P(\mu; q|Q)}{dq} = \Delta_P(\mu|Q)\delta(q-\mu) + P(q)\Delta_P(q|Q)\theta(Q-q)\theta(q-\mu)$$

with the Sudakov form factor

$$\Delta_P(q|Q) = \exp\left(-\int_q^Q P(k)dk\right)$$

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This is truly a probability density. General version of the Sudakov veto algorithm proven to work for this case. [SP & M. Sjödahl, EPJ Plus 127 (2012) 26]

Adaptive sampling methods are available: ExSample C++ library.

[SP, EPJ C72 (2012) 1929]

Also know how to combine several channels, $P = \sum_i P_i$
$\to$ 'competing processes'.

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What if $P < 0$?

Possible to solve, up to overall normalization.

Much simpler, if $\sum_i P_i > 0$ with some $P_i < 0$.
$\to$ Interleave vetoing and competition.

- For each i propose candidates q_i driven by $P_i\ \theta(P_i)$.
- Select largest q_i.
- Accept with probability $\sum_i P_i / \sum_i P_i\ \theta(P_i)$.

Results: Warmup.

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Proof of concept: $e^+e^- \to$ jets.

No hadronization: check effects on the level of the shower.
Would anyway require rethinking, as manifestly a large-N_c picture.

Only gluon emission, as no colour correlations for $g \to q\bar{q}$.

Three types of approximations:

- Full: C_F and correlations exact.
- Shower: C_F exact, correlations large N_c.
- Strict large-N_c: Everything approximated, including $C_F \approx C_A/2$

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What to expect, and what to check.

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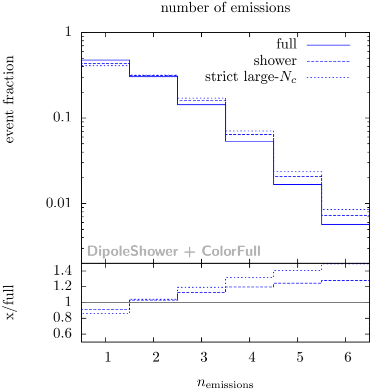
Look at first few color matrix element corrections, exact vs. large-N_c:

- No difference for two jets.
- Large-N_c reproduces shower implementation exactly for three jets.
- From four jets on, large-N_c sensitive to history:
  match shower implementation if one sequence of dipoles dominates.

Check how well large-N_c reproduces the shower implementation:
differences at per-mille level.

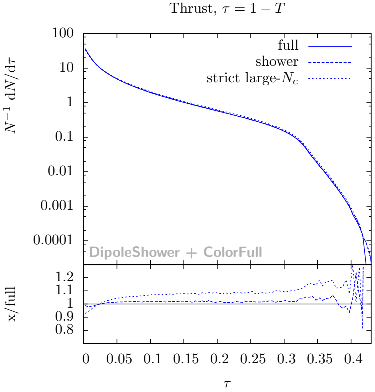
Results: Warmup.

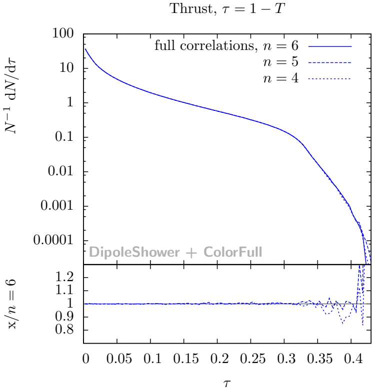


Results stable when going up to six emissions.

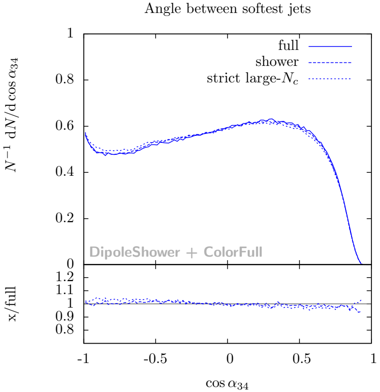
Full correlations suppress radiation: destructive interferences.

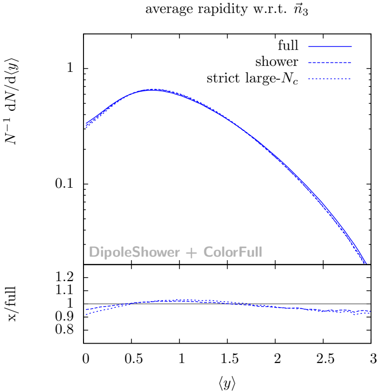
Results.


Thrust, $\tau = 1 - T$
N^{-1} dN/dτ
full
shower
strict large-N_c
DipoleShower + ColorFull
x/full
τ
100
10
1
0.1
0.01
0.001
0.0001
1.2
1.1
1
0.9
0.8
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4


Thrust, $\tau = 1 - T$
N^{-1} dN/dτ
full correlations, $n = 6$
$n = 5$
$n = 4$
DipoleShower + ColorFull
x/$n = 6$
τ
100
10
1
0.1
0.01
0.001
0.0001
1.2
1.1
1
0.9
0.8
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

Results.


Angle between softest jets
full
shower
strict large-N_c
DipoleShower + ColorFull
N^{-1} dN/d cos α_{34}
x/full
cos α_{34}
1
0.8
0.6
0.4
0.2
0
1.2
1.1
1
0.9
0.8
-1
-0.5
0
0.5
1


average rapidity w.r.t. $\vec{n}_3$
full
shower
strict large-N_c
DipoleShower + ColorFull
N^{-1} dN/d$\langle y \rangle$
x/full
$\langle y \rangle$
1
0.1
1.2
1.1
1
0.9
0.8
0
0.5
1
1.5
2
2.5
3

Conclusions and outlook.

First implementation of subleading N_c contributions in parton showers.

Small effects seen in $e^+e^- \to$ jets, except very special observables.

Solutions to technical issues, particularly negative splitting kernels.
$\to$ Will serve as input for future work.

Larger effects expected for e.g. $pp \to$ jets.

Backup.

Large-N_c vs. shower implementation.