Overview of Parton Orbital Angular Momentum

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In this talk, I present the theory overview of recent developments on the parton orbital angular momentum in nucleon.

1 Introduction

Understanding the proton spin structure has been a driving motive for intense spin-physics activities in hadron physics in the last two decades. Much progress has been made both experimentally and theoretically [1, 2, 3]. Orbital angular momentum (OAM) has played a central role in all these studies. There have been many investigations from both theory and experiments [4]. In recent years, three major developments have been particularly emphasized to extract the quark OAM information: (1) GPDs measurements [5]; (2) TMD studies [6]; and (3) very recently the Wigner distributions. In this talk, we would like to present an overview for these developments based on recent publications [7]. Here, we focus on the partonic interpretation of the proton spin and the experimental measurability of the relevant distributions. We explain why a simple partonic sum rule exists only for the transverse polarization. We find that the gauge-invariant OAM contribution to the proton helicity is related to twist-two and three GPDs which are measurable in hard exclusive processes. Finally, the canonical OAM distribution in the light-cone gauge is related to a Wigner distribution [8], which is accessible through certain hard processes. Our discussions are mainly focused on quarks, but they can be easily extended to gluons.

The starting point is the matrix element of the QCD AM density $M^{\mu\alpha\beta}$ in the nucleon plane-wave state [2]

$$\langle PS|\int d^4\xi M^{\mu\alpha\beta}(\xi)|PS\rangle = J\frac{2S_\rho P_\sigma}{M^2}(2\pi)^4\delta^4(0)\left(\epsilon^{\alpha\beta\rho\sigma}P^\mu + \epsilon^{[\alpha\mu\rho\sigma}P^{\beta]} - (\text{trace})\right) + \cdots \ , \quad (1)$$

where ξ^μ is the space-time coordinates, P^μ and S^μ ($S\cdot P = 0$, $S^2 = -M^2$) are the four-momentum and polarization of the nucleon, and $J = 1/2$ and M are the spin and mass, respectively. To seek the partonic interpretation, we consider the nucleon in the Infinite Momentum Frame (IMF) along the z-direction and take μ to be $+$ component. Because of the antisymmetry between α and β, the leading component of the angular momentum density comes from $\alpha = +$ and $\beta = \perp = (1, 2)$. This is only possible if the nucleon is transversely polarized and the matrix element reduces to

$$\langle PS|\int d^4\xi M^{++\perp}|PS\rangle = J3(P^+)^2 S^{\perp'}(2\pi)^4\delta^4(0)/M^2 \ , \quad (2)$$

where $S^{\perp'} = \epsilon^{-+\perp\rho}S_\rho$ with convention of $\epsilon^{0123} = 1$. The longitudinal polarization supports the matrix element of the next-to-leading AM tensor component M^{+12},

$$\langle PS| \int d^3\vec{\xi} M^{+12} |PS\rangle = J(2S^+)(2\pi)^3\delta^3(0) \ , \tag{3}$$

which has one P^+-factor less. Thus the nucleon helicity J is a subleading light-cone quantity, and a partonic interpretation will in general involve parton transverse-momentum and correlations.

2 Transverse-polarization Sum Rule

According to Eq. (2), one expects a simple partonic interpretation of the transverse proton polarization from the leading parton distributions. Indeed, the quark AM sum-rule derived in terms of the quark distribution $q(x)$ and GPD $E(x, 0, 0)$ is exactly of this type [3],

$$J_q = \frac{1}{2}\sum_i \int dx x \left[q_i(x) + E_i(x, 0, 0)\right] \ , \tag{4}$$

where i sums over different flavor of quarks, and similarly for the gluon AM. We emphasize that this spin sum rule is frame-independent. To attribute the above sum rule with a simple parton picture, one has to justify that $(x/2)(q(x) + E(x))$ is the transverse AM density in x, i.e., it is just the contribution to the transverse nucleon spin from partons with longitudinal momentum xP^+. We can define the quark longitudinal momentum density $\rho^+(x, \xi, S^\perp)$ through

$$\rho^+(x, \xi, S^\perp) = x \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS^\perp|\overline{\psi}(-\frac{\lambda n}{2}, \xi)\gamma^+\psi(\frac{\lambda n}{2}, \xi)|PS^\perp\rangle \ , \tag{5}$$

where n is the conjugation vector associated with P: $n = (0^+, n^-, 0_\perp)$ with $n \cdot P = 1$. A careful calculation shows that beside the usual momentum distribution, it has an additional term

$$\rho^+(x, \xi, S^\perp)/P^+ = xq(x) + \frac{1}{2}x\left(q(x) + E(x)\right) \lim_{\Delta_\perp \to 0} \frac{S^{\perp'}}{M^2}\partial^{\perp\xi} e^{i\xi_\perp\Delta_\perp} \tag{6}$$

where the $\xi^\perp$-dependence comes from the slightly off-forward matrix element. The parton contribution to the transverse spin is just the transverse-space moment of $\rho^+(x, \xi, S^\perp)$,

$$S_\perp^q(x) = \frac{M^2}{2P^+S^{\perp'}(2\pi)^2\delta^2(0)} \int d^2\xi \xi^\perp \rho^+(x, \xi, S^\perp) = \frac{x}{2}(q(x) + E(x)) \ . \tag{7}$$

where we have included the contribution from the energy-momentum component $T^{+\perp}$ through Lorentz symmetry.

3 Helicity Sum Rule

For a longitudinal polarized nucleon, we consider the z-component of the quark AM,

$$J^3 = \int d^3\vec{\xi} M^{+12}(\xi) = \int d^3\vec{\xi} \left[\overline{\psi}\gamma^+(\frac{\Sigma^3}{2})\psi + \overline{\psi}\gamma^+\left(\xi^1(iD^2) - \xi^2(iD^1)\right)\psi\right] \ , \tag{8}$$

where the quark helicity is well-known to have a simple parton density interpretation. However, the quark OAM involves transverse component of the gluon field, and thus is related to three-parton correlations.

Thus a partonic picture of the orbital contribution to the nucleon helicity necessarily involves parton's transverse momentum. In other words, TMD parton distributions are the right objects for physical measurements and interpretation. In recent years, TMDs and novel effects associated with them have been explored extensively in both theory and experiment [1], where it was found that the physical parton represents a gauge-invariant object with a gauge link extended from the location of parton field to infinity along the conjugating light-cone direction n^μ,

$$\Psi_{LC}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda n\cdot A(\lambda n+\xi)\right)\right]\psi(\xi)\ . \tag{9}$$

where P indicates path ordering. Therefore, in perturbative diagrams, a parton with momentum $k^+ = xP^+$ represents in fact the sum of all diagrams with longitudinal gluons involved.

When considering parton's transverse momentum, we also need appropriate gauge links formed of gauge potentials. In practical applications, two choices stand out. First one uses the same light-cone gauge link as shown in the above. The second choice is a straightline gauge link along the direction of spacetime position ξ^μ,

$$\Psi_{FS}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda \xi\cdot A(\lambda\xi)\right)\right]\psi(\xi)\ , \tag{10}$$

The link reduces to unity in Fock-Schwinger gauge, $\xi\cdot A(\xi)=0$.

To investigate parton's OAM contribution to the proton helicity, one also needs their transverse coordinates. The most natural concept is a phase-space Wigner distribution, which was first introduced in Ref. [9]. A Wigner distribution operator for quarks is defined as

$$\hat{\mathcal{W}}_{\mathcal{P}}(\vec r, k) = \int \overline{\Psi}_{\mathcal{P}}(\vec r-\xi/2)\gamma^+\Psi_{\mathcal{P}}(\vec r+\xi/2)e^{ik\cdot\xi}d^4\xi\ , \tag{11}$$

where $\mathcal{P}$ denotes the path choice of LC or FS, $\vec r$ is the quark phase-space position and k the phase-space four-momentum. The above quantity is gauge invariant although it does depend on the choices of the gauge link. The Wigner distribution can be define as the expectation value of $\hat{\mathcal{W}}$ in the nucleon state,

$$W_{\mathcal{P}}(k^+=xP^+,\vec b_\perp,\vec k_\perp) = \frac{1}{2}\int\frac{d^2\vec q_\perp}{(2\pi)^3}\int\frac{dk^-}{(2\pi)^3}e^{-i\vec q_\perp\cdot\vec b_\perp}\left\langle\frac{\vec q_\perp}{2}\left|\hat{\mathcal{W}}_{\mathcal{P}}(0,k)\right|-\frac{\vec q_\perp}{2}\right\rangle\ . \tag{12}$$

where the nucleon has definite helicity $1/2$.

The total OAM sum rule in term of parton's Wigner distribution,

$$\frac{\langle PS|\int d^3\vec r\ \overline{\psi}(\vec r)\gamma^+(\vec r_\perp\times i\vec D_\perp)\psi(\vec r)|PS\rangle}{\langle PS|PS\rangle} = \int(\vec b_\perp\times\vec k_\perp)W_{FS}(x,\vec b_\perp,\vec k_\perp)dxd^2\vec b_\perp d^2\vec k_\perp \tag{13}$$

which gives a parton picture for the gauge-invariant OAM [3], although the straightline gauge link destroys the straightforward parton density interpretation.

4 Canonical Orbital Angular Momentum

The quark contribution to the canonical orbital angular momentum was explored in Ref. [10],

$$l^q(x) = \frac{1}{(2\pi)^2 2P^+\delta^2(0)} \int \frac{d\lambda}{2\pi} e^{ix\lambda} d^2\xi \langle PS|\overline{\psi}(-\frac{\lambda n}{2},\xi)\gamma^+(\xi^1 i\partial^2 - \xi^2 i\partial^1)\psi(\frac{\lambda n}{2},\xi)|PS\rangle \ . \quad (14)$$

This definition represents the canonical OAM in the light-cone gauge $A^+ = 0$. Its matrix element is in principle related to the twist-three GPDs, and an infinite number of moments are involved due to non-locality. Therefore, we arrive at the interesting conclusion that $l_q(x)$ in light-cone gauge is actually accessible through twist-two and three GPDs.

A clear parton picture emerges through connections between $l_q(x)$ and TMDs and Wigner distributions [7, 8]. One can introduce a Wigner distribution with the gauge link in the light-cone direction, $W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp)$. Integration over the impact parameter space $\int d^2\vec{b}_\perp W_{LC}$ generates quark-spin independent TMDs. It can be shown that the canonical AM distribution in $A^+ = 0$ gauge as defined in [10] can be obtained from the simple moment of a gauge-invariant Wigner distribution,

$$l_q(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp \ . \quad (15)$$

From the discussion of the previous paragraph, this also implies constraints on the moments of Wigner distributions from the GPDs. Finally, the canonical OAM in light-cone gauge acquires the simple but gauge-dependent parton sum rule in the quantum phase space [7, 8],

$$l_q = \frac{\langle PS| \int d^3\vec{r}\ \overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{\partial}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp) dx d^2\vec{b}_\perp d^2\vec{k}_\perp \ . \quad (16)$$

The measurability of this Wigner distribution will be studied in a future publication.

To summarize, we reviewed parton pictures for the proton spin: For the transverse polarization, we found that it is simple to interpret in terms of parton AM density measurable through twist-two GPDs; For the nucleon helicity, the gauge-invariant parton picture can be probed through twist-two and three GPDs, and also calculable in lattice QCD. A simpler parton picture in the light-cone gauge can be established through the quantum phase space Wigner distribution, and can be measured through either twist-two and three GPDs or directly from Wigner distribution.

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