Orbital Angular Momentum

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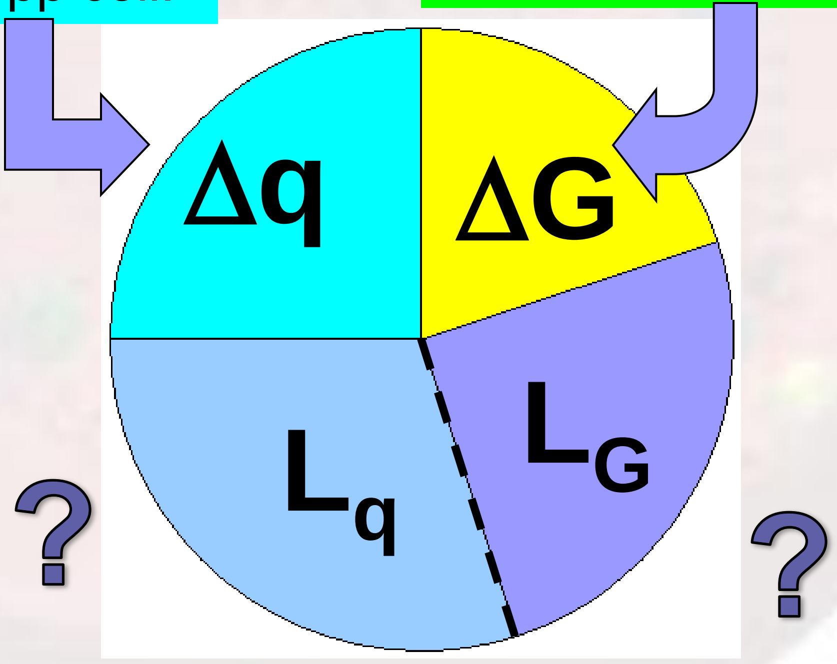
Proton Spin Sum

Quark spin ~30%
DIS, and pp coll.

Gluon spin~ 0-70%
RHIC, EIC, …



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Agenda

Talks online

Application form

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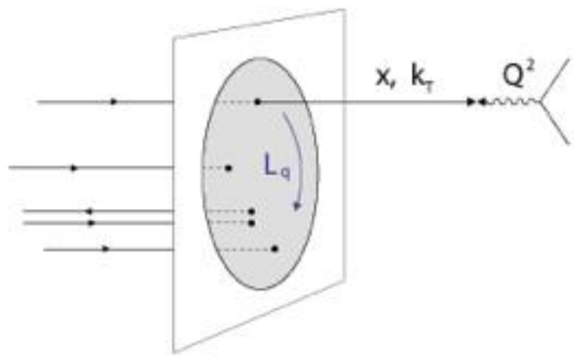
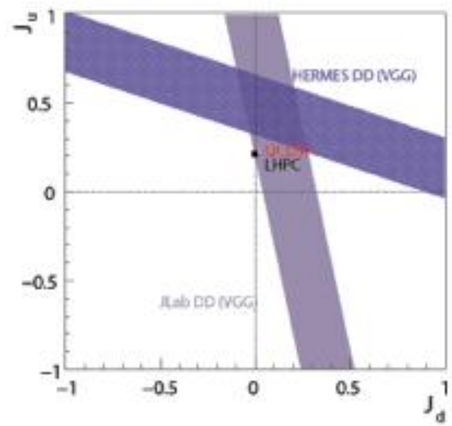
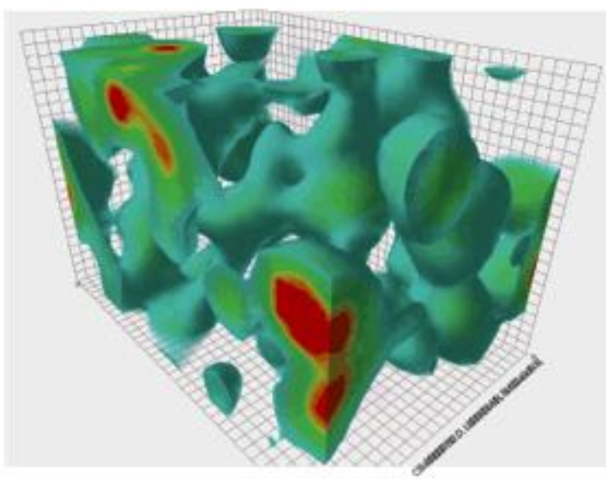
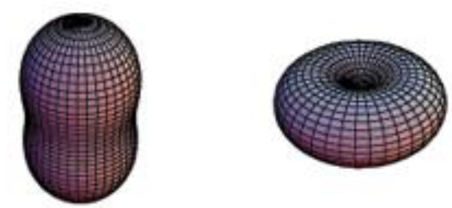
Orbital Angular Momentum in QCD

February 6 - 17, 2012









http://www.int.washington.edu/PROGRAMS/12-49w/

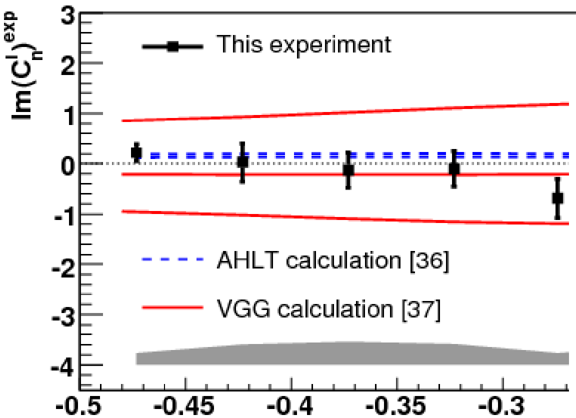
Relevance of OAM in hadron physics

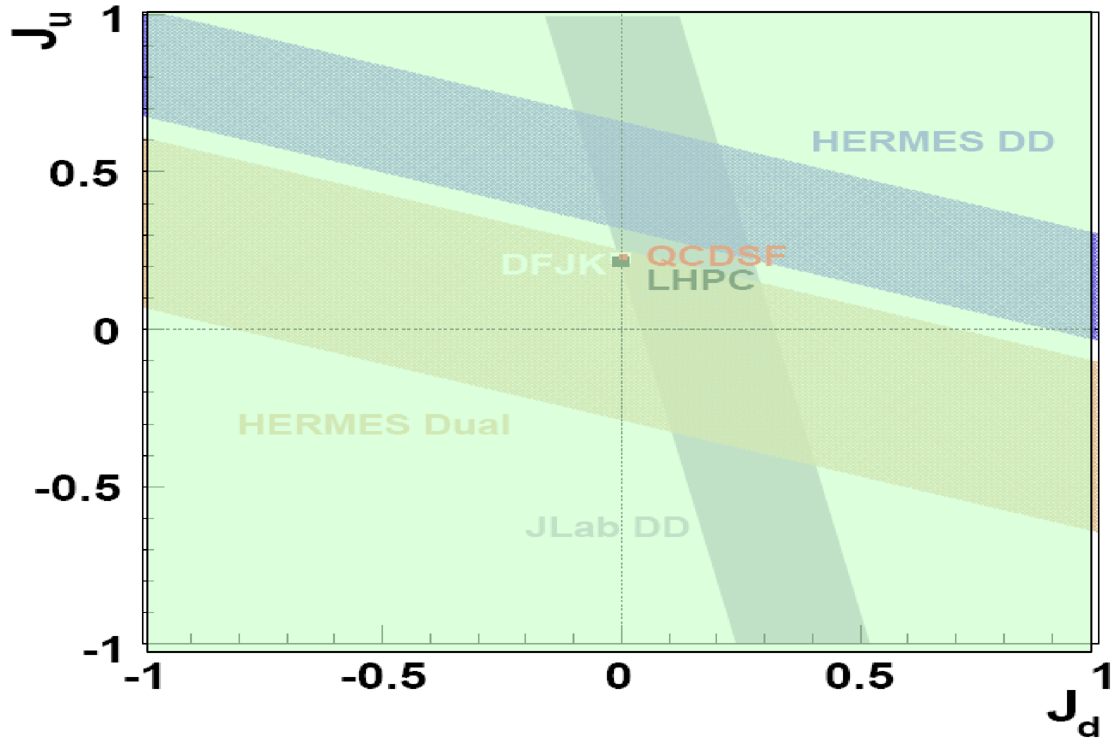
- GPD E and Nucleon Pauli Form Factor
  - Magnetic moments
- Higher-twist functions
  - g_T structure function, g_2
- Single spin asymmetries
  - T-odd TMDs
- …

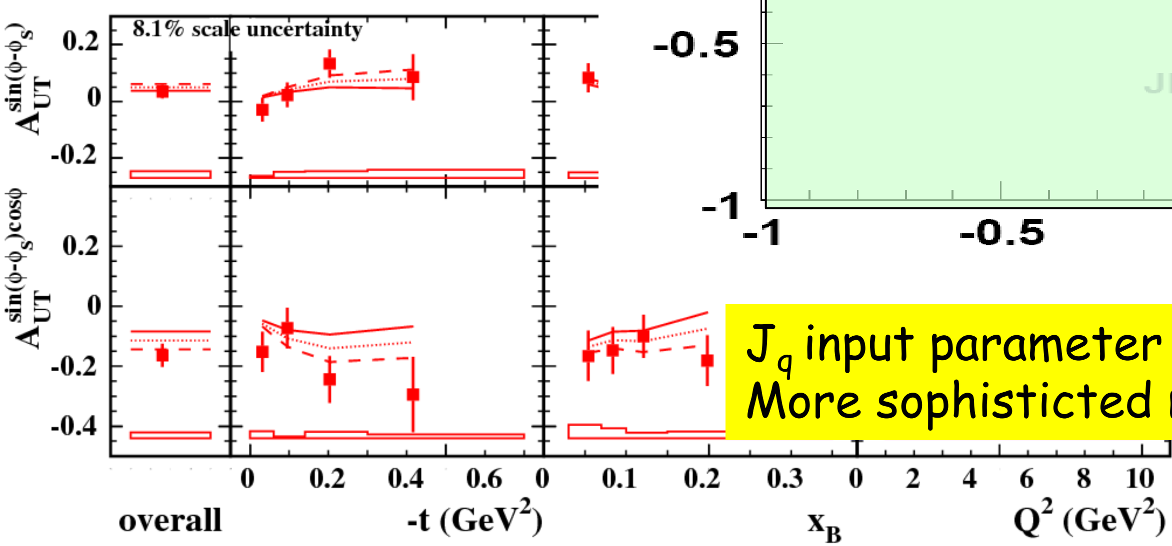
Access to the OAM

- Generalized Parton Distributions
  - Exp. And/or Lattice
- Transverse Momentum Dependent Distributions
  - Complementary
- Wigner Distributions
  - Model calculations (so far)

DVCS with transversely polarized target from HERMES & Jlab







J_q input parameter in the GPD ansatz, need More sophisticated model for GPDs

Jlab Hall A, 0709.0450; HERMES, 0802.2499

TMDs

Constraining Quark Angular Momentum through Semi-Inclusive Measurements

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The determination of quark angular momentum requires the knowledge of the generalized parton distribution E in the forward limit. We assume a connection between this function and the Sivers transverse-momentum distribution, based on model calculations and theoretical considerations. Using this assumption, we show that it is possible to fit nucleon magnetic moments and semi-inclusive single-spin asymmetries at the same time. This imposes additional constraints on the Sivers function and opens a plausible way to quantifying quark angular momentum.

$$J^u = 0.229 \pm 0.002^{+0.008}_{-0.012},$$
$$J^{\bar{u}} = 0.015 \pm 0.003^{+0.001}_{-0.000},$$
$$J^d = -0.007 \pm 0.003^{+0.020}_{-0.005},$$
$$J^{\bar{d}} = 0.022 \pm 0.005^{+0.001}_{-0.000},$$
$$J^s = 0.006^{+0.002}_{-0.006},$$
$$J^{\bar{s}} = 0.006^{+0.000}_{-0.005}.$$

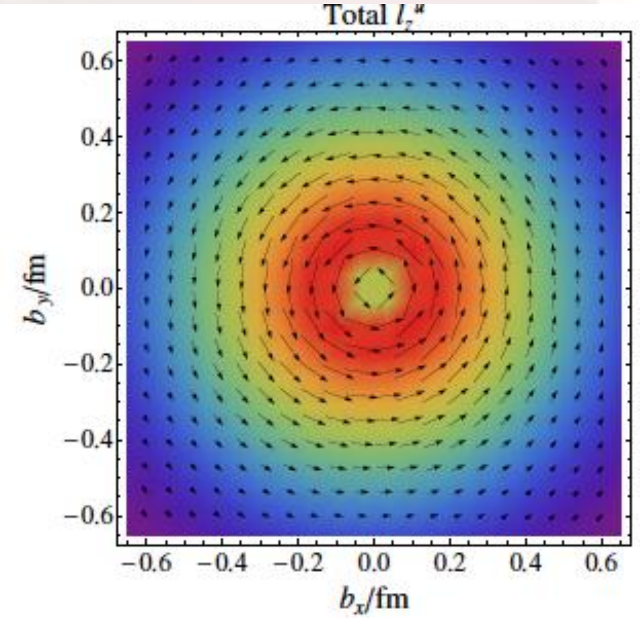
Wigner Distributions

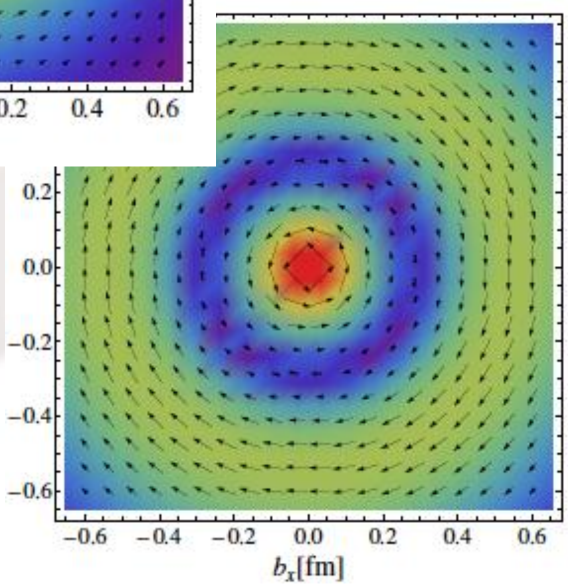
Define the net momentum projection

$$\mathcal{K}(\vec{r}_\perp) = \int d^2k_\perp \vec{k}_\perp \mathcal{H}(\vec{r}_\perp, \vec{k}_\perp)$$

Quark oribital angular momentum

$$L_q = \int d^2r_\perp d^2k_\perp \vec{r}_\perp \times \vec{k}_\perp \mathcal{H}(\vec{r}_\perp, \vec{k}_\perp)$$





Lorce, Pasquini, arXiv:1106.0139
Lorce, Pasquini, Xiong, Yuan, arXiv:1111.482
Hatta, arXiv:1111.3547

Outline

- Angular momentum tensor and parton interpretation
- Transverse polarized nucleon
  - Ji sum rule via GPDs
- Longitudinal polarized nucleon
  - Helicity, OAM via twist-three GPDs
  - Wigner distributions

Ji, Xiong, Yuan, arXive: 1202.2843; to be published

Angular momentum density

Jaffe-Manohar
Ji

- From energy-momentum tensor

$$
\begin{aligned}
M^{\alpha\mu\nu} &= T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu} \\
T_q^{\mu\nu} &= \frac{1}{2}\left[\bar{\psi}\gamma^{(\mu}i\overrightarrow{D^{\nu)}}\psi + \bar{\psi}\gamma^{(\mu}i\overleftarrow{D^{\nu)}}\psi\right] \\
T_g^{\mu\nu} &= \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F_{\alpha}{}^{\nu}
\end{aligned}
$$

- Off-forward matrix

$$
\mathcal{M}^{\mu\nu\lambda}(k) = \int d^4x e^{ik\cdot x}\langle P'S|M^{\mu\nu\lambda}(x)|PS\rangle
$$

  - Take p=p' in the end

Angular momentum density

$$\langle PS|\int d^4\xi M^{\mu\alpha\beta}(\xi)|PS\rangle = J\frac{2S_\rho P_\sigma}{M^2}(2\pi)^4\delta^4(0)$$
$$\left(\epsilon^{\alpha\beta\rho\sigma}P^\mu + \epsilon^{[\alpha\mu\rho\sigma}P^{\beta]} - (\text{trace})\right) + \cdots \ ,$$

- Partonic interpretation works in the infinite momentum frame (light-front)
- In this frame, the leading component is P^+,S^+
- Next-to-leading component, S^T

Leading component M^{++T}

$$\langle PS|\int d^4\xi M^{++\perp}|PS\rangle = J\left[\frac{3(P^+)^2 S^{\perp'}}{M^2}\right](2\pi)^4\delta^4(0)$$

- Because of anti-symmetric α,β, the leading term is α=+,β=T, which related to the transverse spin of the nucleon
- Transverse spin of nucleon has leading-twist interpretation in parton language
- However, individual spin is obscure

Next-to-leading: M^{+TT}

$$\langle PS| \int d^3\vec{\xi} M^{+12} |PS\rangle = J(2S^+)(2\pi)^3\delta^3(0)$$

- Because of two transverse indices, it inevitably involves twist-three operators
- However, it does lead to the individual spin contribution, e.g., from the quark
  - Jaffe-Manohar spin decomposition

Spin sum is frame-independent

$$J_\perp = J_z = \frac{A(0)+B(0)}{2}$$

Ji,96

- A and B are form factors

$$\langle P'S|T^{\mu\nu}(0)|PS\rangle = \bar{U}(P')\left[A(\Delta^2)\gamma^{(\mu}\bar{P}^{\nu)} + B(\Delta^2)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M_N}\right]U(P)$$

- Works for both longitudinal and transverse spin of nucleon
- Partonic interpretation is, …

Transversely polarized nucleon

- Ji sum rule

$$J_q = \frac{1}{2}\sum_i \int dx x\,[q_i(x) + E_i(x,0,0)]\ ,$$

- Burkardt picture: rest frame (2005)
  - Angular momentum obtained by impact parameter dependent PDF
  - Wave packet determines the center of the proton

Angular Momentum density (T)

- Define the momentum density (T^{++})

$$\rho^+(x,\xi,S^\perp) = x\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle PS^\perp|\overline{\psi}(-\frac{\lambda n}{2},\xi)\gamma^+\psi(\frac{\lambda n}{2},\xi)|PS^\perp\rangle$$

- AM depending momentum fraction x,

$$J_q(x) = \frac{M^2}{2(P^+)^2 S^{\perp\prime}(2\pi)^2\delta^2(0)}\int d^2\xi\,\xi^\perp\rho^+(x,\xi,S^\perp) = \frac{x}{2}(q(x)+E(x))$$

Which gives the angular momentum density for quark with longitudinal momentum x

In more detail

- Calculate $\rho^+(x,\xi,S^T)$

$$\rho^+(x,\xi,S^\perp)/P^+ = xq(x) + \frac{1}{2}x\left(q(x)+E(x)\right)\lim_{\Delta_\perp\to 0}\frac{S^{\perp\prime}}{M^2}\partial^{\perp_\xi} e^{i\xi_\perp\Delta_\perp}$$

- Integrate out ξ, second term drops out, we obtain the momentum density
- Integral with weight ξ_T', the first term drops out, → Angular Momentum density

Where is the quark transverse spin?

- Higher-twist in natural (M^{+-T})

$$M_q^{+-\sigma} = \frac{i}{4}\left\{x^-\bar{\psi}\left(\gamma^+\overleftrightarrow{D}^\sigma + \gamma^\sigma\overleftrightarrow{D}^+\right)\psi - (\sigma \leftrightarrow -)\right\} + h.c.$$

$$\rightarrow \frac{1}{2}\epsilon^{-+\lambda\sigma}\bar{\psi}\gamma_\lambda\gamma_5\psi + \frac{i}{2}\left\{\bar{\psi}\gamma^+\left(x^-\overleftrightarrow{D}^\sigma - x^\sigma\overleftrightarrow{D}^-\right)\psi\right\} + h.c.$$

  - g_T structure function
- Has nothing to do with the quark transversity

Longitudinal (helicity) spin sum

$$J^3 = \int d^3\vec{\xi} M^{+12}(\xi)$$

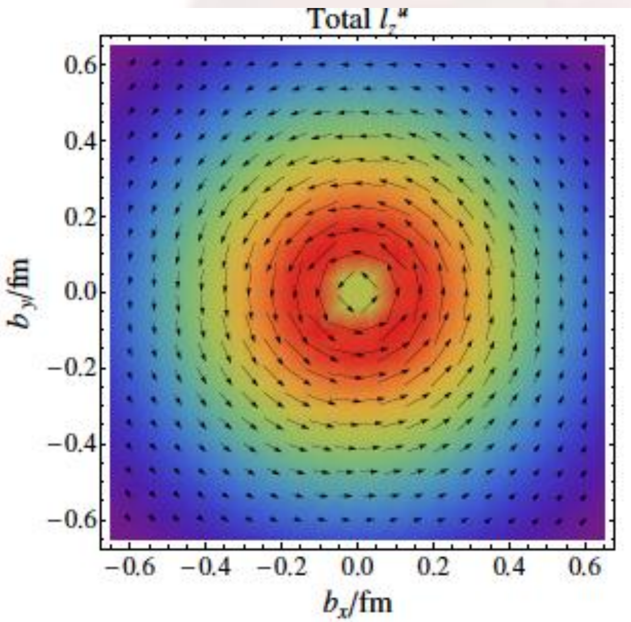
$$= \int d^3\vec{\xi} \left[\overline{\psi}\gamma^+ (\frac{\Sigma^3}{2})\psi + \overline{\psi}\gamma^+ \left(\xi^1(iD^2) - \xi^2(iD^1)\right)\psi \right]$$

- Quark spin explicitly
- OAM, twist-three nature
  - Direct extraction from twist-three GPDs
- Of course, we still have

$$J_\perp = J_z = \frac{A(0) + B(0)}{2}$$

OAM Related to the Wigner Distribution

- Lorce-Pasquini, 2011
- Lorce-Pasquini-Xiong-Yuan, 2011
- Hatta 2011



Wigner distribution for the quark

- The quark operator

Ji: PRL91,062001(2003)

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta$$

- Wigner distributions

$$W_\Gamma(\vec{r},k) = \frac{1}{2M_N}\int\frac{d^3\vec{q}}{(2\pi)^3}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}_\Gamma(\vec{r},k)\right|-\vec{q}/2\right\rangle$$

$$= \frac{1}{2M_N}\int\frac{d^3\vec{q}}{(2\pi)^3}\mathrm{e}^{-i\vec{q}\cdot\vec{r}}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}_\Gamma(0,k)\right|-\vec{q}/2\right\rangle$$

After integrating over r, one gets TMD

After integrating over k, one gets Fourier transform of GPDs

Importance of the gauge links

- Gauge invariance
- Depends on processes
- Comes from the QCD factorization

$$\Psi_{LC}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda n\cdot A(\lambda n+\xi)\right)\right]\psi(\xi)$$

- And partonic interpretation as well

Fixed point gauge link

$$\Psi_{FS}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda\xi\cdot A(\lambda\xi)\right)\right]\psi(\xi)$$

- Becomes unit in ξ.A=0 gauge
- Moment gives the quark OAM

$$L(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$

- OPE

$$\int x^{n-1} L_{FP}(x)dx = \langle PS| \int d^3\vec{r} \sum_{i=0}^{n-1} \frac{1}{n} \overline{\psi}(\vec{r})(in\cdot D)^i \times (\vec{r}_\perp \times i\vec{D}_\perp)(in\cdot D)^{n-1-i}\psi(\vec{r})|PS\rangle \ . \quad (16)$$

Quark OAM

- Any smooth gauge link results the same OAM for the partons

$$\frac{\langle PS|\int d^3\vec{r}\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp\times i\vec{D}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle}$$

$$=\int(\vec{b}_\perp\times\vec{k}_\perp)W_{FS}(x,\vec{b}_\perp,\vec{k}_\perp)dxd^2\vec{b}_\perp d^2\vec{k}_\perp$$

Light-cone gauge link

$$\Psi_{LC}(\xi) = P\left[\exp\left(-ig\int_0^\infty d\lambda n \cdot A(\lambda n+\xi)\right)\right]\psi(\xi)$$

- it comes from the physical processes
  - DIS: future pointing
  - Drell-Yan: to -∞
- Cautious: have light-cone singularities, and need to regulate
- Moments related to twist-three PDFs, and GPDs

Light-cone decomposition

$$J^3 = \int d^3\vec{\xi}\left[\overline{\psi}\gamma^+(\vec{\xi}\times i\vec{\partial})^3\psi + \frac{1}{2}\overline{\psi}\gamma^+\Sigma^3\psi + E^i(\vec{\xi}\times\vec{\partial})^3A^i + (\vec{E}\times\vec{A})^3\right],$$

Bashinsky-Jaffe

- Quark OAM only contains the partial derivative

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \tilde{L}_q + \Delta G + \tilde{L}_g .$$

$$\tilde{L}^q(x) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} d^2\xi \langle PS|\overline{\psi}(-\frac{\lambda n}{2},\xi)\gamma^+ \times(\xi^1 i\partial^2 - \xi^2 i\partial^1)\psi(\frac{\lambda n}{2},\xi)|PS\rangle$$

Gauge Invariant Extension

- GIE is not unique

$$i\partial_\xi^\perp = iD_\xi^\perp + \int^{\xi^-} d\eta^- L_{[\xi^-,\eta^-]} g F^{+\perp}(\eta^-,\xi_\perp) L_{[\eta^-,\xi^-]}$$

- Canonical OAM can be calculated

$$\tilde{L}_q = \frac{\langle PS| \int d^3\vec{r}\, \overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{\partial}_\perp)\psi(\vec{r})|PS\rangle}{\langle PS|PS\rangle}$$

$$= \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x,\vec{b}_\perp,\vec{k}_\perp) dx d^2\vec{b}_\perp d^2\vec{k}_\perp \ .$$

OAM from Wigner distribution

$$\tilde{L}_q(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$

- Can be measured from hard processes
- Moments relate to the canonical OAM
- In the end of day, depends on twist-3 GPDs
  - Might be studied in many processes

Road map to OAM

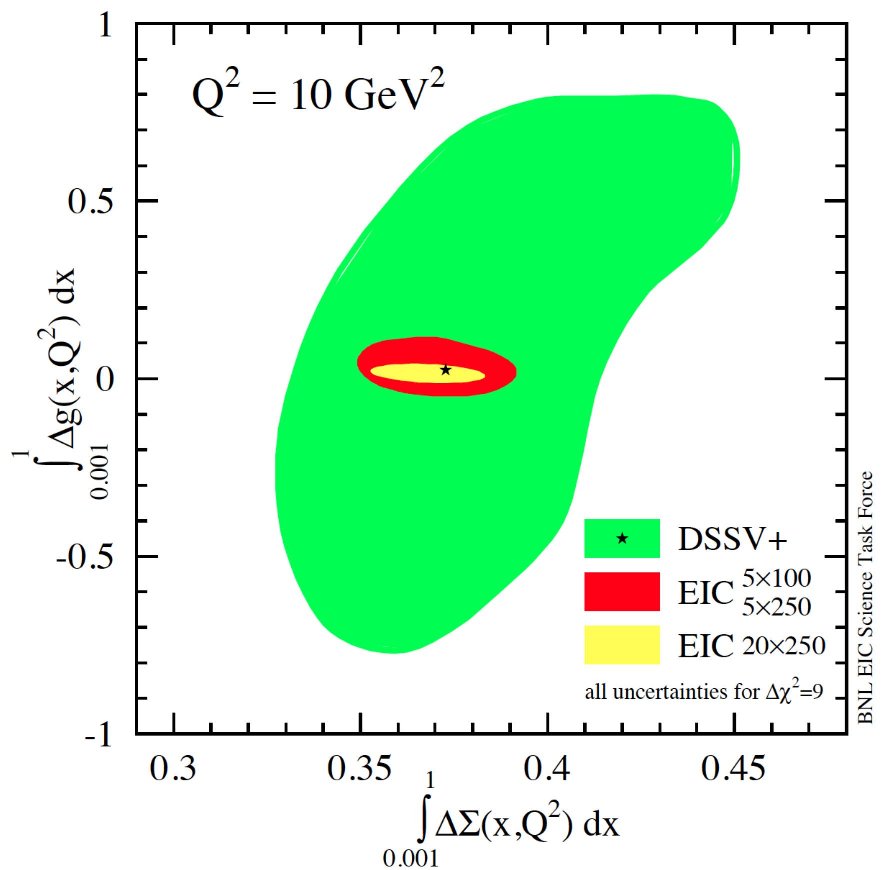
- Model dependent determinations
  - GPDs fit to the DVCS and other processes
  - TMDs study in the SIDIS
  - Wigner distributions, and many others,…
- Toward a model-independence?
  - With great efforts, we may converge to a model-independent constraint
  - + the helicity→Spin sum rule

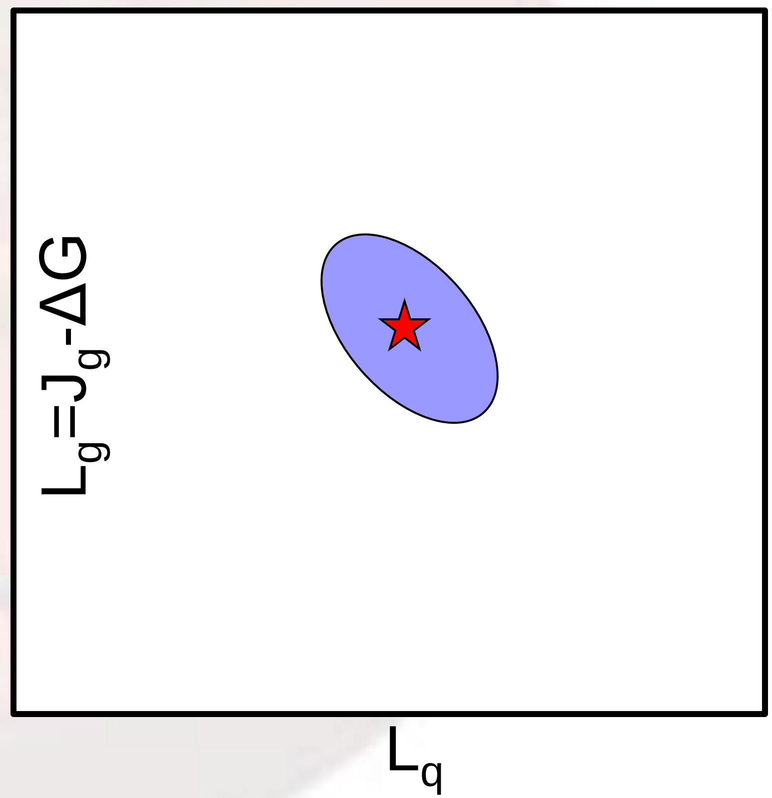
Conclusions

- Partonic interpretation of the proton spin can be understood through quantum phase space Wigner distribution
- Transverse polarized nucleon induces the angular momentum density (x)
- Longitudinal polarized nucleon depends on twist-2 and 3 GPDs
- Canonical OAM can be measured from experiments

My wish

- In 5 (10) years?





Comments on Chen et al. decomposition

- Start with Coulomb gauge results
- Extend to other gauge by using the gauge invariant extension
- Not associated with the physical observable, can not be evaluated in lattice QCD
- Not gauge invariant