GPDs from present and future measurements

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• GPDs and perturbative framework
• deeply virtual photon leptoproduction
• GPD modeling and data descriptions
• predictions for COMPASS and EIC

in collaboration with K. Kumerički (DVCS)
two codes in Phyton (+ Minuit + GeParD) & Mathematica + GeParD

contribution to EIC white paper [M. Diehl, F. Sabatie]
with E. Aschenauer, S. Fazio, K. Kumerički
**Partonic interpretation of GPDs**

\[ F(x, \eta, t, Q^2) \]

\[ F \in \{ H, E, \tilde{H}, \tilde{E}, \ldots \} \]

(intricate distributions)

GPDs simultaneously carry information on **longitudinal** and **transverse** distributions of partons in a proton

probabilistic interpretation for \( \eta = 0 \) (infinite momentum frame) [Burkhardt (00)]

\[
b_\perp = \sqrt{4 \frac{d}{dt} \ln H(x, 0, t)} \bigg|_{t=0}
\]

GPDs contain also information on partonic angular momentum [X. Ji (96)]

\[
\frac{1}{2} = \sum_{a=q,G} J^z_a
\]

\[
J^z_a = \lim_{\Delta \to 0} \frac{1}{2} \int_{-1}^{1} dx \ x \left( H_a + E_a \right)(x, \eta, \Delta^2)
\]
GPD phenomenology

- Deeply virtual Compton scattering (clean probe) (factorization holds)

\[
ep \rightarrow e'p'\gamma \\
ep \rightarrow e'p'\mu^+\mu^- \\
\gamma p \rightarrow p'e^-e^+ \\
p \rightarrow p'
\]

- Deeply virtual meson production (flavor filter) (factorization holds for longitudinal photon)

\[
ep \rightarrow e'p'\rho \\
ep \rightarrow e'n\rho^+ \\
ep \rightarrow e'p'\pi \\
ep \rightarrow e'n\pi^+ \\
p \rightarrow N
\]

- hand bag models for exclusive photo- and electro production ect.

scanned area of the surface as a function of lepton energy

twist-two observables:
longitudinal cross sections
transverse target spin asymmetries
factorization proof
[Collins, Frankfurt, Strikman (96)]
**Status of perturbative framework**

The NLO framework has been essentially set up for some time:

- **next-to-leading** order anomalous dimensions and evolution kernels

- **next-to-leading** order DVCS coefficients (twist-two)

• even for time-like and double DVCS (just use prescription from)

- **next-to-next-to-leading** order in a specific conformal subtraction scheme

- **twist-three** DVCS coefficients at leading order (off-forward LO evolution is known)

- partial **twist-three** DVCS coefficients at **next-to-leading** order

- **twist-four** for DVCS and a previous study

- **next-to-leading** DVMP coefficients (non-singlet) (singlet)
Photon leptoproduction $e^\pm N \rightarrow e^\pm N \gamma$

measured by **H1, ZEUS, HERMES, CLAS, HALL A** collaborations

planned at **COMPASS II, JLAB@12GeV**, perhaps at ?? EIC,

\[
\frac{d\sigma}{dx_{Bj}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{Bj}y}{16 \pi^2 Q^2} \left(1 + \frac{4M^2x_{Bj}^2}{Q^2}\right)^{-1/2} \left|\frac{T}{e^3}\right|^2,
\]

\[
x_{Bj} = \frac{Q^2}{2P_1 \cdot q_1} \approx \frac{2\xi}{1 + \xi},
\]

\[
y = \frac{P_1 \cdot q_1}{P_1 \cdot k},
\]

\[
\Delta^2 = t \text{ (fixed, small)},
\]

\[
Q^2 = -q_1^2 (> 1\text{GeV}^2),
\]

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\[
\Delta^2 = t \text{ (fixed, small)},
\]

\[
Q^2 = -q_1^2 (> 1\text{GeV}^2),
\]
interference of **DVCS** and **Bethe-Heitler** processes

\[ |T_{\text{BH}}|^2 = \frac{e^6 (1 + \epsilon^2)^{-2}}{x_B^2 y^2 t \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ c_0^{\text{BH}} + \sum_{n=1}^{2} c_n^{\text{BH}} \cos(n\phi) \right\} (F_1, F_2), \quad \text{(LO, QED)} \]

\[ |T_{\text{DVCS}}|^2 = \frac{e^6}{y^2 Q^2} \left\{ c_0^{\text{DVCS}} + \sum_{n=1}^{2} \left[ c_n^{\text{DVCS}} \cos(n\phi) + s_n^{\text{DVCS}} \sin(n\phi) \right] \right\} (\mathcal{F}), \]

\[ I = \frac{\pm e^6}{x_B y^3 t \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ c_0^\mathcal{I} + \sum_{n=1}^{3} \left[ c_n^\mathcal{I} \cos(n\phi) + s_n^\mathcal{I} \sin(n\phi) \right] \right\} (F_1, F_2, \mathcal{F}). \]
Can one `measure’ GPDs?

- **CFF** given as **GPD** convolution:

\[
H(\xi, t, Q^2) \overset{\text{LO}}{=} \int_{-1}^{1} dx \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \eta = \xi, t, Q^2)
\]

\[
\overset{\text{LO}}{=} i\pi H^-(x = \xi, \eta = \xi, t, Q^2) + \text{PV} \int_{0}^{1} dx \frac{2x}{\xi^2 - x^2} H^-(x, \eta = \xi, t, Q^2)
\]

- **H(x,x,t,Q^2)** viewed as ”spectral function” (s- and u-channel cuts):

\[
H^-(x, x, t, Q^2) \equiv H(x, x, t, Q^2) - H(-x, x, t, Q^2) \overset{\text{LO}}{=} \frac{1}{\pi} \Im m F(\xi = x, t, Q^2)
\]

- **CFFs** satisfy `dispersion relations’ (not the physical ones, threshold $\xi_0$ set to 1)

\[
\Re F(\xi, t, Q^2) = \frac{1}{\pi} \text{PV} \int_{0}^{1} d\xi' \left( \frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'} \right) \Im m F(\xi', t, Q^2) + C(t, Q^2)
\]

- **access** to the **GPD** on the cross-over line $\eta = x$ (at LO)

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

[Terayev (05)]
**GPD modeling & evolution**

Outer region governs the evolution at the cross-over trajectory

\[
\mu^2 \frac{d}{d\mu^2} H(x, x, t, \mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) H(y, x, \mu^2)
\]

GPD at $\eta = x$ is `measurable' (LO)

Central region follows (polynomiality of moments)

Net contribution of outer + central region is governed by a sum rule:

\[
\text{PV} \int_0^1 dx \frac{2x}{\eta^2 - x^2} H^-(x, \eta, t) = \text{PV} \int_0^1 dx \frac{2x}{\eta^2 - x^2} H^-(x, x, t) + C(t)
\]
**GPD models for valence quarks (using dispersion relation)**

- model of GPD $H(x,x,t)$ within DD motivated ansatz at $Q^2=2$ GeV$^2$

**fixed:**
- PDF normalization
- eff. Reage trajectory
- large $t$-counting rules

Fixed parameters:

$$n = 1.35, \quad \alpha(t) = 0.43 + 0.85t/\text{GeV}^2, \quad p = 1$$

$$H(x,x,t) = \frac{n r 2^\alpha}{1 + x} \left( \frac{2x}{1 + x} \right)^{-\alpha(t)} \left( \frac{1 - x}{1 + x} \right)^b \frac{1}{\left(1 - \frac{1-x}{1+x} \frac{t}{M^2}\right)^p}.$$  

**free:**
- $r$-ratio at small $x$
- large $x$-behavior
- monopole mass

Flexible subtraction constant

($D$-term content of GPD $H$ and $E$)

$$D(t) = \frac{-C}{(1-t/M_c^2)^2}$$

contains pion-pole contribution
Model based on SL(2,R) and SO(3) PWE

- SL(2,R) GPD moments:
  \[ F_j(\eta, t) = \sum_{J=J_{\text{min}}}^{j+1} f_j^J(t) \eta^{j+1-J} \hat{d}_J(\eta) \]
  partial wave amplitudes depending on \( j \) and \( J \)
  reduced Wigner rotation matrices

- taking 2 better 3 SO(3) PWs:
  (two parameters \( s_2 \) and \( s_4 \))
  \[ f_j^{-1}(t) = s_2 f_j^{j+1}(t), \]
  \[ f_j^{-3}(t) = s_4 f_j^{j+1}(t), \]

- resulting CFF easy to handle:
  \[ F = \frac{1}{2i} \sum_{k=0}^{4} \sum_{\text{even}}^{c-i\infty} d_j \xi^{-j-1} \frac{2j+1+k}{\Gamma(3/2)\Gamma(3+j+k)} \left( i - \frac{\cos(\pi j) \mp 1}{\sin(\pi j)} \right) \]
  \[ \times s_k E_{j+k}(Q^2) f_j^{j+1}(t) \hat{d}_j(\xi), \quad s_0 = 1 \]

- zero-skewness GPD:
  \[ h_j^{j+1} = q_j \frac{j + 1 - \alpha(0)}{j + 1 - \alpha(0) - \alpha' t} \left( 1 - \frac{t}{M_j^2} \right)^{-p} \]

2x(2, 3, or 4) parameters:

- PDF `pomerion intercept' (build in PDF)
- residual \( t \) dependence

\( s_2, s_4, M \) or \( b \), (perhaps \( \alpha' \))
DVCS measurements & perspectives

existing data
including longitudinal and transverse polarized proton data

new data
HERMES (recoil detector data)
JLAB (longitudinal TSA, cross sections)

planned
COMPASS II, JLAB 12

proposed
EIC
Strategies to analyze DVCS data

(ad hoc) modeling:

VGG code [Goeke et. al (01) based on Radyuskin’s DDA]
BMK model [Belitsky, DM, Kirchner, (01) based on RDDA]
‘aligned jet’ model [Freund, McDermott, Strikman (02)]
Goloskokov/Kroll (05) based on RDDA (pinned down by DVMP)
‘dual’ model [Polyakov, Shuvaev 02; Guzey, Teckentrup 06; Polyakov 07]

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KMP-K (07) in MBs-representation
polynomials [Belitski et al. (98), Liuti et. al (07), Moutarde (09)]

dynamical models: not applied [Radyuskin et.al (02); Tiburzi et.al (04); Hwang DM (07)]...
[Goldstein et. al (10)] (without sea, polynomiality issue)

flexible models: any representation by including unconstrained degrees of freedom
(for fits)
KMP-K (07/08) for H1/ZEUS in MBs-integral-representation

Extracting CFFs from data: real and imaginary part

i. CFF extraction with formulae [BMK (01), HALL-A (06)]
fits [Guidal, Moutarde (08...)]
neural networks [KMS (11)]

ii. ‘dispersion integral’ fits [KMP-K (08), KM (08...)]

iii. flexible GPD modeling [KM (08...)]

vi. model comparisons [KM (11...)]: Goloskokov/Kroll (07) model based on RDDA
Global fits with hybrid models KM10...

- 150-200 unpolarized proton data from CLAS, HALL A, HERMES, H1 & ZEUS (projected on twist-two dominated observables)
- fitted with hybrid models (‘dispersion relation’ + flexible sea quark/gluon models)
  (good fits with $\chi^2$/d.o.f. $\approx 1$)

- good description of small $x_B$ data from H1 and ZEUS
- KM10... cross sections are published as code [http://calculon.phy.hr/gpd/]
- GK07 model from DVMP hand-bag description overshoots $A_{LU}$ [Goloskokov, Kroll (07)]

see talk of M. Murray
HALL A electroproduction cross sections

Φ description of unpolarized cross section @ $x_B=0.36$ requires:

- large DVCS amplitude (model independent statement)
- KM10 model with large $\hat{H}$ and/or $\hat{E}$ (pion pole) induce a large DVCS amplitude (poor man approach: an effective parameterization which has not to be taken literally)
KM10... versus CFF fits & large-\(x\) “model” fit

GUIDAL
- 7 parameter CFF fit to all harmonics with twist-two dominance hypothesis propagated errors + “theoretical“ error estimate
- same + longitudinal TSA

Moutarde
- H dominance hypothesis within a smeared polynomial expansion propagated errors + “theoretical“ error estimate

KMS
- neural network within H dominance hypothesis

KM10 (KM10a) [KM10b] curves with (without) [ratios of] HALL A data

GK07
- model based on RDDA pinned down by DVMP data via handbag approach

- reasonable agreement for HERMES and CLAS kinematics
- large \(x_B\)-region and real part remains unsettled
What about other CFFs (from fixed target)?

Longitudinal proton spin asymmetry is sensitive to GPD $\hat{H}$

Transverse proton spin asymmetry is sensitive to GPD $E$, however:
(beam charge flip allows to disentangles interference and DVCS terms)
Predictions for COMPASS II

fixed target, polarized muon beam (~200 GeV)
cross sections (t-dependence), transverse polarized target (access to E GPD)
NN (ideal) tool for error propagation and quantifying model uncertainties
• used to access real and imaginary part of $\mathcal{H}$ CFF from HERMES
• results are compatible with dispersion relation fits

Beam Charge-Spin Asymmetry @ COMPASS II

$$BCSA = \frac{d\sigma^{++} - d\sigma^{--}}{d\sigma^{++} + d\sigma^{--}}$$

(dominated by real part)
DVCS+DVMP fits to H1/ZEUS data

strategies:  
- pure DVCS fit $\chi^2$/d.o.f. = 130/(126-3)  
- DVCS + H1 DVMP fit $\chi^2$/d.o.f. = 342/(230-6)  
- DVCS + H1/ZEUS DVMP fit - very soft gluon $\chi^2$/d.o.f. = 618/(304-7)

confronting GK07 model with DVCS ($\chi^2$/n.o.p. = 226/126)

$R$ and normalization errors are not taken into account, cut $Q^2 > 4$ GeV$^2$ for DVMP data

DVCS data dominated by quark GPD  
gluon GPD is to some extent not pinned down

[M. Meškauskas, DM arXiv:1112.2597]
H1 and ZEUS DVMP data are compatible however, there are differences

\[
\begin{align*}
\left\{ \begin{array}{l}
b^{H1}(Q^2 = 11.5 \text{ GeV}^2) \\
b^{ZEUS}(Q^2 = 11 \text{ GeV}^2)
\end{array} \right. \\
= \left\{ \begin{array}{l}
6.72 ± 0.53 \quad +0.23
\quad -0.25 \\
5.7 \quad +0.2 \quad -0.2
\end{array} \right\}/\text{GeV}^2,
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{l}
\delta^{H1}(Q^2 = 6.6 \text{ GeV}^2) \\
\delta^{ZEUS}(Q^2 = 6 \text{ GeV}^2)
\end{array} \right. \\
= \left\{ \begin{array}{l}
0.57 ± 0.10 \quad +0.05 \\
0.4 \quad +0.048 \quad -0.045
\end{array} \right\},
\end{align*}
\]
different partonic interpretations

- $r_{\text{sea}} \sim 1$ (small skewness effect at LO) of sea quarks are driven by DVCS data
- $r^G < 1$ gluon GPD is suppressed at LO
- very soft gluon GPD is disfavored by DIS fit
- GK07 model is based on NLO PDFs [(very) good small $x_B$ DVCS description at LO]
  interchange of skewing and evolution provides a very desired GPD behavior
- remember Freund/McDermott could not reach DVCS description @LO with similar model
EIC I and II predictions from hybrid models

- various asymmetries might be sizeable (there might be sensitivity even to $\hat{H}$ and $\hat{E}$)
- predictions from KM10... models and our one (sea quark+gluons, $e^{-bt}$ dependence) (statistical errors from MILOU + 5% $d\sigma$ systematic errors in quadrature) (see talk of S. Fazio)
- beam charge asymmetry (even unpolarized beam) would be necessary
  - to clarify the status of twist-3 admixtures
  - to disentangle interference and DVCS contributions for single spin flip experiments
Impact of EIC II data to extract GPD H

two simulations from S. Fazio (see his talk) for DVCS cross section (~ 650 data)

- $t < \sim 0.8$ GeV$^2$ for ~ 10/fb for $E_e \times E_p = 20 \times 250$ GeV$^2$

1 GeV$^2 < t < 2$ GeV$^2$ for ~ 100/fb (cut: $-t < 1.5$ GeV$^2$, 4 GeV$^2 < Q^2$ to ensure $-t < Q^2$)

mock data are regenerated with GeParD (rescaled) statistical errors from MILOU + 5% systematical errors + 3% Bethe-Heitler cross section uncertainties, added in quadrature

![Graphs showing data and fits for DVCS cross section](image)
Imaging (probabilistic interpretation)

\[ q(x, \bar{b}, \mu^2) = \frac{1}{4\pi} \int_0^\infty d|t| \ J_0(|\bar{b}|\sqrt{|t|}) H(x, \eta = 0, t, \mu^2) \]

\[ \text{A}_\text{UT study to get } E \]

see talk of M. Diehl

skewness effect vanishes \((s_2, s_4 \to 0)\)

- reduce fit uncertainties
- increase model uncertainties

extrapolation errors for \(-t \to 0\)

(large \(b\) uncertainties – small effect)

extrapolation errors into \(-t > 1.5 \text{ GeV}^2\)

(small \(b\) uncertainties)
Summary

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons
- to address the spin content of the nucleon
- providing a bridge to LCWFs modeling & non-perturbative methods (lattice)

Hard exclusive leptoproduction

- DVCS is widely considered as a theoretical clean process
- it is elaborated in NLO (NNLO) and offers a new insight in QCD
- possesses a rich structure, allowing to access various CFFs/GPDs
- new experiments (high luminosity machines and dedicated detectors) are desired to quantify exclusive (and inclusive) QCD phenomena

Technology

Software tools for global GPD fits have been developed for demonstration
Back Up Slides
Why are GPDs needed?

- exclusive processes @ large t
- hard exclusive processes
- form factors
- GPDs
- lattice simulations
- QCD-models
  - Regge-phenom. "amplitudes"
- LC wave functions
- unintegrated PDs
- parton densities (PDs)
- 2+1D-picture
  - spin content
access of CFFs from measurements:

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<th>twist</th>
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<th>C’s</th>
<th>harmonics in $I$</th>
<th>extraction of CFFs</th>
<th>$P$ of $Q^{-P}$</th>
<th>$\Delta^T_{\perp}$ behavior</th>
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<tr>
<td>two</td>
<td>$\Re C(F), \Delta C(F)$</td>
<td>$c_1, c_0$</td>
<td>LP, TP, TP, TP</td>
<td>over compl.</td>
<td>1,2</td>
<td>1,0</td>
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<td></td>
<td>$\Im C(F), \Delta C(F)$</td>
<td>$s_1, -$</td>
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<td>over compl.</td>
<td>1,2</td>
<td>0,1</td>
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<tr>
<td>three</td>
<td>$\Re C(F^{\text{eff}})$</td>
<td>$c_2$</td>
<td>LP, TP, TP</td>
<td>complete</td>
<td>2</td>
<td>2, 1</td>
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<tr>
<td>two</td>
<td>$\Re C_T(F_T)$</td>
<td>$c_3$</td>
<td>-</td>
<td>$1 \times \Re$ of 4</td>
<td>1</td>
<td>3, 2</td>
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<td>$\Im C_T(F_T)$</td>
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<td>$s_3$</td>
<td>$3 \times \Im$ of 4</td>
<td>1</td>
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three possible nucleon polarization + electron/positron beam + neglecting transversity allows to access imaginary and real part of

$$\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}}\}$$

$$\mathcal{F}^3 = \{\mathcal{H}^3, \mathcal{E}^3, \tilde{\mathcal{H}}^3, \tilde{\mathcal{E}}^3\}$$

twist-three offers access to quark-gluon-quark correlations

transversity arises at NLO from gluons at twist-two or at LO as a twist-four effect

$$\mathcal{F}_T = \mathcal{O}(\alpha_s, 1/Q^2)$$
**effective model parameterization (small x)**

PDF:
\[ q_{\text{sea}}(\xi, Q_0) = n\xi^{-\alpha}, \quad \alpha \gtrsim 1, \quad F_{1,\text{sea}}(0) = 1 \]

GPD \( H \):
\[ H_{\text{sea}}(\xi, \xi, t, Q_0) = r(\eta/x = 1|s_2, s_4)F_{1,\text{sea}}(t)\xi^{\alpha'(t)}q_{\text{sea}}(\xi) \]

- PDF is assumed to be known (from some fit with to old HERA data)
- \( t \)-dependence of residue is taken to be exponential with slope \( B \)
- free parameters: two sets \( \{\alpha', B, s_2, s_4\} \) for sea quarks and gluons
- momentum sum rule is implemented

GPD \( E \):
\[ E_{\text{sea}}(\xi, Q_0) = nF_{2,\text{sea}}(0)\xi^{-\alpha}, \quad \alpha \gtrsim 1, \quad F_{2,\text{sea}}(0) = \kappa_{\text{sea}} \]
\[ E_{\text{sea}}(\xi, \xi, t, Q_0) = r(\eta/x = 1|s_2, s_4)F_{2,\text{sea}}(t)\xi^{\alpha'(t)}E_{\text{sea}}(\xi, \eta = 0) \]

- PDF analog is unknown
- \( t \)-dependence of residue is taken to be exponential with slope \( B \)
- free parameters: \( \kappa_{\text{sea}} \) + two sets \( \{\alpha, \alpha', B, s_2, s_4\} \) for sea quarks and gluons
- \( \kappa^G \) is constrained by Ji's sum rule

**real part of Compton form factors is determined by their imaginary parts**
PDF analog: \( E^{\text{sea}}(\xi, \eta = 0, Q_0) = nF_2^{\text{sea}}(0)\xi^{-\alpha}, \quad \alpha \gtrsim 1, \quad F_2^{\text{sea}}(0) = \kappa^{\text{sea}} \)

GPDs: \( E^{\text{sea}}(\xi, \xi, t, Q_0) = r(\eta/x = 1|s_2, s_4)F_2^{\text{sea}}(t)\xi^{\alpha'}(t)E^{\text{sea}}(\xi, \eta = 0) \)

\( \kappa^{\text{sea}} \) or \( J^{\text{sea}} \textbf{skewness} \textbf{transverse distribution} \)

- mostly not seen in Regge phenomenology
- might be sizeable in instanton models
- reggeized spectator quark models
- pQCD suggests ‘pomeron’ intercept
- however, \( B \) vanishes asymptotically
- so far \( E \) is “not clearly seen” in DVCS

qualitative understanding of \( E \) is needed (not only for Ji’s spin sum rule)

\[
B = \int_0^1 dx \, xE(x, \eta, t, Q)
\]

transverse target spin asymmetry is \textbf{sensitive to } \textbf{E and sizeable} at EIC
**Observables for e\(\rightarrow\)p \(\rightarrow\)e\(\rightarrow\)p\(\gamma\) at small \(x_B\)**

**DVCS cross section** (dominated by \(H\) and slightly dependent on \(E\))

\[
\frac{d\sigma_{\text{DVCS}}}{dt}(W, t, Q^2) \approx \frac{\pi \alpha^2 x_B^2}{Q^4} \left[ |\mathcal{H}|^2 - \frac{t}{4M_p^2} |\mathcal{E}|^2 \right] (x_B, t, Q^2) \bigg|_{x_B \approx \frac{Q^2}{W^2+Q^2}}
\]

(electron) beam spin asymmetry (dominated by \(H\) and slightly dependent on \(E\))

\[
A^{(1)}_{\text{BS}} \propto y \left[ F_1(t) H(\xi, \xi, t, Q^2) - \frac{t}{4M^2} F_2(t) E(\xi, \xi, t, Q^2) + \cdots \right]
\]

\(\sin(\psi)\) transverse target spin asymmetry (governed by \(E\) and \(H\))

\[
A^{\uparrow(1)}_{\text{TS}} \propto \sqrt{-\frac{t}{4M^2}} \left[ F_2(t) H(\xi, \xi, t, Q^2) - F_1(t) E(\xi, \xi, t, Q^2) + \cdots \right]
\]

\(\cos(\psi)\) transverse and longitudinal target spin asymmetries are sensitive to parity odd GPDs \(\tilde{H}\) and \(\tilde{E}\) – expected to be suppressed at small \(x_B\)

\[
A^{\downarrow(1)}_{\text{TS}} \propto \sqrt{-\frac{t}{4M^2}} \left[ F_2(t) \tilde{H}(\xi, \xi, t, Q^2) - F_1(t) \xi \tilde{E}(\xi, \xi, t, Q^2) + \cdots \right]
\]

\[
A^{\Rightarrow(1)}_{\text{TS}} \propto \left[ F_1(t) \tilde{H}(\xi, \xi, t, Q^2) - \frac{t}{4M^2} F_2(t) \xi \tilde{E}(\xi, \xi, t, Q^2) + \cdots \right]
\]
Single transverse proton spin asymmetry

20x250 2x5/fb mock data
(~1200 data points with statistical errors + 5% systematics at cross section level)

flexible GPD model for $E_{\text{sea}}$ and $E^G$ normalization (and $t$-dependency) of $E_{\text{sea}}$
is reasonable constraint

$E^G$ is essentially unconstrained
Imaging (probabilistic interpretation)

density for a transverse polarized proton in impact space

\[ q^\uparrow(x, \vec{b}, \mu^2) = q(x, \vec{b}, \mu^2) - \frac{1}{2M} \frac{\partial}{\partial b_y} E(x, \vec{b}, \mu^2) \]

\[ = \frac{1}{4\pi} \int_0^\infty d|t| \left[ J_0(|\vec{b}|\sqrt{|t|}) H + \frac{b_y \sqrt{|t|}}{|\vec{b}| 2M} J_1(|\vec{b}|\sqrt{|t|}) E \right] (x, \eta = 0, t, \mu^2) \]

already assumed that E is constrained for \(-t < 1.5 \text{ GeV}^2\)

extrapolation errors into \(-t > 1.5 \text{ GeV}^2\) are taken

NOTE:

normalization and t-dependency of E are now extracted while normalization of H is fixed by unpolarized PDF