Transverse Single Spin Asymmetry in $e + p^\uparrow \rightarrow e + J/\psi + X$

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In collaboration with R. Godbole (IISc, Bangalore), A. Misra (Mumbai U.), V. Rawoot (Mumbai U.) (arXiv: 1201.1066 [hep-ph])
Aim is to study the asymmetry in photoproduction (i.e. low virtuality electroproduction) of charmonium by scattering unpolarized electrons off transversely polarized proton.

Transverse momentum dependent Sivers function describes the probability of finding an unpolarized parton inside a transversely polarized hadron: correlation between transverse momentum of the unpolarized quarks and gluons and the nucleon spin related to orbital angular momentum: Sivers asymmetry gives access to the orbital angular momentum of the partons.

SSAs involving the transverse momentum dependent pdfs and fragmentation functions: very often two or more of these functions contribute to the same physical observable.

In this process that we are considering, at LO, there is contribution only from a single partonic subprocess $\gamma g \rightarrow c\bar{c}$: can be used as a clean probe of gluon Sivers function.

May throw some light on the charmonium production mechanism as well.
**Model for charmonium production that we consider**

Color evaporation model: first proposed by Halzen and Matsuda (1978), H. Fritsch (1977)

CEM predicts a cross section for the $J/\psi$ production from the cross section of the $c\bar{c}$ pair

Statistical treatment of color: color can 'evaporate' by multiple soft gluon emission

Cross section for charmonium production is proportional to the rate of production of $c\bar{c}$ pair integrated over the mass range $2m_c$ to $2m_D$

$$\sigma = \frac{1}{9} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{d\sigma_{c\bar{c}}}{dM_{c\bar{c}}}$$

where $m_c$ is the charm quark mass and $2m_D$ is the $D\bar{D}$ threshold; $1/9$ is the statistical probability for the production of a color singlet state; $M_{c\bar{c}}^2$ is the squared invariant mass of $c\bar{c}$

In later versions data are better fitted by the inclusion of a phenomenological factor in differential cross section formula, which depends on a Gaussian distribution of the transverse momentum of the charmonium

Cross section for charmonium production

Cross section for low-virtuality electroproduction of $J/\psi$:

$$\sigma_{ep \rightarrow e+J/\psi+X} = \int_{4m_c^2}^{4m_D^2} dM_{c\bar{c}}^2 \int dy \, dx \, f_{\gamma/e}(y) \, f_{g/p}(x) \frac{d\hat{\sigma}_{\gamma g \rightarrow c\bar{c}}}{dM_{c\bar{c}}^2}.$$ The photon flux in the electron is approximated by the distribution (Weizsacker-Williams approximation)

$$f_{\gamma/e}(y, E) = \frac{\alpha}{\pi} \left\{ \frac{1 + (1 - y)^2}{y} \left( \ln \frac{E}{m} - \frac{1}{2} \right) + \frac{y}{2} \left[ \ln \left( \frac{2}{y} - 2 \right) + 1 \right] \right\} + \frac{(2 - y)^2}{2y} \ln \left( \frac{2 - 2y}{2 - y} \right).$$

Kniehl (1991)

where $y$ is the energy fraction of electron carried by the photon, $E$ is the energy of the electron and $m$ is the mass.
Single Spin Asymmetry

Generalization of CEM expression by taking into account the transverse momentum dependence

\[
d\sigma^{e+p^{\uparrow}\rightarrow e+J/\psi+X}_{dM^2} = \int dx_{\gamma} dx_g \left[ d^2 k_{\perp\gamma} d^2 k_{\perp g} \right] f_{g/p^{\uparrow}}(x_g, k_{\perp g}) f_{e/g}(x_\gamma, k_{\perp\gamma}) \frac{d\hat{\sigma}^{\gamma g\rightarrow c\bar{c}}}{dM^2}
\]

where \( M^2 \equiv M_{c\bar{c}}^2 \)

Single spin asymmetry for a transversely polarized target is defined as

\[
A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}
\]

Number density of partons inside a proton with transverse polarization \( S \) and momentum \( P \) is parameterized as

\[
f_{a/p^{\uparrow}}(x_a, k_{\perp a}, S) \equiv f(x_a, k_{\perp a}) + \frac{1}{2} \Delta^N f_{a/p^{\uparrow}}(x_a, k_{\perp a}) \hat{S} \cdot (\hat{P} \times \hat{k}_{\perp a})
\]

\( \Delta^N f_{a/p^{\uparrow}}(x, k_{\perp a}) \) is the Sivers function
Single Spin Asymmetry

\[ \frac{d^4 \sigma^\uparrow}{dydM^2d^2q_T} - \frac{d^4 \sigma^\downarrow}{dydM^2d^2q_T} = \frac{1}{2} \int \left[dx\gamma d^2k_{\perp\gamma} dx_g d^2k_{\perp g}\right] \Delta^N f_{g/p}^\uparrow (x_g, k_{\perp g}) \times f_{\gamma/e}(x_\gamma, k_{\perp\gamma}) \delta^4(p_g + p_\gamma - q) \hat{\sigma}^{\gamma g \rightarrow c\bar{c}} (M^2) \]

where \( q = p_c + p_\bar{c} \) and the partonic cross section

\[ \hat{\sigma}_0^{\gamma g \rightarrow c\bar{c}} (M^2) = \frac{1}{2} e_c^2 \frac{4\pi\alpha\alpha_s}{M^2} [(1 + \gamma - \frac{1}{2}\gamma^2) \ln \frac{1 + \sqrt{1-\gamma}}{1 - \sqrt{1-\gamma}} - (1 + \gamma)\sqrt{1-\gamma}] \]

\( \gamma = \frac{4m_c^2}{M^2} \) and \( M^2 \equiv \hat{s} = xS \), \( S \) is the CM energy squared of the \( \gamma - p \) system.

\( \Delta^N f_{g/p}^\uparrow (x_g, k_{\perp g}) \) is related to the gluon Sivers function \( \Delta^N f_{g/p}^\uparrow (x, k_{\perp g}) \) by

\[ \Delta^N f_{g/p}^\uparrow (x_g, k_{\perp g}) = \Delta^N f_{g/p}^\uparrow (x_g, k_{\perp}) \hat{S} \cdot (\hat{P} \times \hat{k}_{\perp g}) \].
Models for WW Function

For \( k_{\perp g} \) dependence of the unpolarized pdf’s, we use a simple factorized and Gaussian form

\[
f_{g/p}(x_g, k_{\perp}) = f_{g/p}(x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} e^{-k_{\perp g}^2 / \langle k_{\perp g}^2 \rangle}.
\]

we have used two choices for WW function for the photon

1) A simple Gaussian form as above:

\[
f_{\gamma/e}(x_\gamma, k_{\perp \gamma}) = f_{\gamma/e}(x_\gamma) \frac{1}{\pi \langle k_{\perp \gamma}^2 \rangle} e^{-k_{\perp \gamma}^2 / \langle k_{\perp \gamma}^2 \rangle}
\]

2) Second form:

\[
f_{\gamma/e}(x_\gamma, k_{\perp \gamma}) = f_{\gamma/e}(x_\gamma) \frac{1}{2\pi} \frac{N}{k_{\perp \gamma}^2 + k_0^2}
\]
Models for Sivers Function

For the Sivers function we use two models


Model I:

\[ \Delta^N f_{g/p} (x_g, k_{\perp g}) = \Delta^N f_{g/p} (x_g) \frac{1}{\pi \langle k_{\perp g}^2 \rangle} h(k_{\perp g}) e^{-k_{\perp g}^2 / \langle k_{\perp g}^2 \rangle} \cos(\phi_{k_{\perp g}}) \]

where the gluon Sivers function, \( \Delta^N f_{g/p} (x_g) \) is defined as

\[ \Delta^N f_{g/p} (x_g) = 2 N_g(x_g) f_{g/p}(x_g) \]

\[ h(k_{\perp g}) = \sqrt{2} e^{-\frac{k_{\perp g}^2}{M_1}} e^{-\frac{k_{\perp g}^2}{M_1^2}} \]
Models for Sivers Function

Parametrization for quark Sivers function

\[ N_f(x) = N_f x^{a_f} (1 - x)^{b_f} \frac{(a_f + b_f)^{(a_f + b_f)}}{a_f a_f b_f b_f} \, . \]

For gluons we use (Boer and Vogelsang (2004))

(a) \( N_g(x) = (N_u(x) + N_d(x)) / 2 \),

(b) \( N_g(x) = N_d(x) \).

Model II :

\[ \Delta^N f_{g/p \uparrow} (x_g, \mathbf{k}_\perp g) = \Delta^N f_{g/p \uparrow} (x_g) \frac{1}{\pi \langle k^2_{\perp g} \rangle} e^{-k^2_{\perp g} / \langle k^2_{\perp g} \rangle} \frac{2 k^2_{\perp g} M_0}{k^2_{\perp g} + M^2_0} \cos(\phi k_{\perp}) , \]

and \( M_0 = \sqrt{\langle k^2_{\perp g} \rangle} \) where the gluon Sivers function is given as in Model I.
SSA for $J/\psi$ production at JLab

Best fit parameters of Sivers functions (Anselmino et al (2011))

$$N_u = 0.40, \ a_u = 0.35, \ b_u = 2.6, \ N_d = -0.97, \ a_d = 0.44, \ b_d = 0.90,$$

$$M_1^2 = 0.19 \ GeV^2.$$  

Plot for JLab ($\sqrt{s} = 4.7 \ GeV$)
SSA for $J/\psi$ production at HERMES

\[ A_N \sin(\phi_{q_T} - \phi_s) \]

\[ \langle k_{\perp g}^2 \rangle = \langle k_{\perp \gamma}^2 \rangle = 0.25 \text{ GeV}^2. \]

Integration ranges are (0 ≤ $q_T$ ≤ 1) GeV and (0 ≤ $y$ ≤ 0.6)

Plot for HERMES ($\sqrt{s} = 7.2$ GeV)
Integration ranges are \((0 \leq q_T \leq 1)\) GeV and \((0 \leq y \leq 1)\)

Plot for COMPASS \((\sqrt{s} = 17.33\) GeV\)
Integration ranges are \((0 \leq q_T \leq 1)\) GeV and \((0 \leq y \leq 1)\)

Plot for eRHIC \((\sqrt{s} = 31.6\) GeV)
Integration ranges are \((0 \leq q_T \leq 1)\) GeV and \((0 \leq y \leq 1)\)

Plot for COMPASS \((\sqrt{s} = 17.33\text{ GeV})\) : Comparison of two models
Asymmetry in model I with parameterization (a) compared for JLab ($\sqrt{s} = 4.7$ GeV) (solid red line), HERMES ($\sqrt{s} = 7.2$ GeV) (dashed green line), COMPASS ($\sqrt{s} = 17.33$ GeV) (dotted blue line), eRHIC-1 ($\sqrt{s} = 31.6$ GeV) (long dashed pink line) and eRHIC-2 ($\sqrt{s} = 158.1$ GeV) (dot-dashed black line)
Conclusion

- Sizable SSA for $J/\psi$ electroproduction: can give information on gluon Sivers function
- SSA does not depend much on the choice of the $k_\perp$ dependence of the photon distribution
- Photoproduction: have to consider higher order and resolved photon contributions
- Would be interesting to see how sensitive the SSA is on the charmonium production model
- Also need to investigate how much the scale evolution of the $k_\perp$ dependent distributions affect the SSA