The NLO jet vertex for Mueller-Navelet and forward jets in the small-cone approximation

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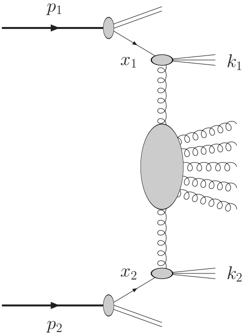
Outline

Outline

Mueller-Navelet jets

proton(p_1) + proton(p_2) $\rightarrow$ jet 1(k_1) + jet 2(k_2) + X



Sudakov decomposition: $k_1 = \alpha_1 p_1 + \frac{\vec{k}_1^{\,2}}{\alpha_1 s} p_2 + k_{1,\perp}\ , \quad k_{1,\perp}^2 = -\vec{k}_1^{\,2}\ , \quad s = 2 p_1 \cdot p_2$

$$k_2 = \alpha_2 p_2 + \frac{\vec{k}_2^{\,2}}{\alpha_2 s} p_1 + k_{2,\perp}\ , \quad k_{2,\perp}^2 = -\vec{k}_2^{\,2}$$

- large jet transverse momenta: $\vec{k}_1^{\,2} \sim \vec{k}_2^{\,2} \gg \Lambda_{\rm QCD}^2$
- large rapidity gap between jets, $\Delta y = \ln \frac{\alpha_1 \alpha_2 s}{|\vec{k}_1||\vec{k}_2|}$,
  which requires large c.m. energy of the proton collisions, $s = 2p_1 \cdot p_2 \gg \vec{k}_{1,2}^{\,2}$

- perturbative QCD description of the process:
  - hard scale $Q^2 \sim \vec{k}_{1,2}^2 \gg \Lambda_{\rm QCD}^2$
  - neglect power-suppressed contributions $\sim 1/Q$

  $\longrightarrow$ leading-twist PDFs, $f_g(x)$ and $f_q(x)$

- need to resum the QCD perturbative series
  - DGLAP: $\sum_n a_n \, \alpha_s^n \ln^n Q^2 + b_n \, \alpha_s^n \ln^{n-1} Q^2$
  - BFKL: $\sum_n c_n \, \alpha_s^n \ln^n s + d_n \, \alpha_s^n \ln^{n-1} s$

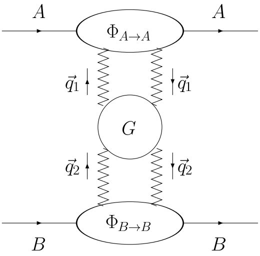
- [Mueller-Navelet, 1987]: for $\Delta y \gg 1$ BFKL approach more adequate
  - "faster" energy dependence
  - more decorrelation in the relative jet azimuthal angle $\phi = \phi_1 - \phi_2 - \pi$ expected

Outline

BFKL "survival kit"

Total cross section $A + B \to X$, via the optical theorem, $\sigma = \frac{\mathrm{Im}_s \mathcal{A}}{s}$

- Regge limit ($s \to \infty$)
  $\Rightarrow$ BFKL factorization for $\mathrm{Im}_s \mathcal{A}$:
  convolution of the Green's function of two interacting Reggeized gluons and of the impact factors of the colliding particles



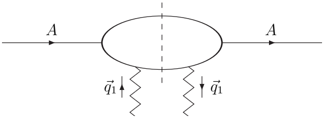
- Valid both in
  LLA (resummation of all terms $(\alpha_s \ln s)^n$)
  NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$)

$$\mathrm{Im}_s \mathcal{A} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

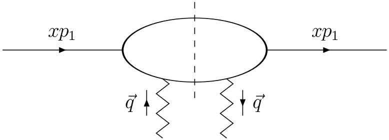
- The Green's function is process-independent and is determined through the BFKL equation [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)$$

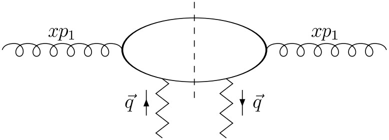
- Impact factors are process-dependent;
  only very few have been calculated in the NLA



For the process under consideration, the starting point is provided by the impact factors for colliding partons [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)] [M. Ciafaloni and G. Rodrigo (2000)]



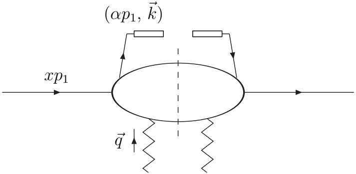
quark impact factor



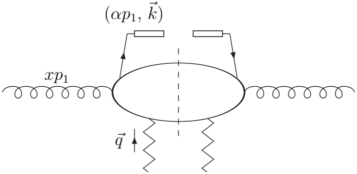
gluon impact factor

- LLA $\longrightarrow$ leading-order (LO) impact factor $\longrightarrow$ one-particle intermediate state
- NLA $\longrightarrow$ next-to-LO (NLO) impact factor:
  virtual corrections $\longrightarrow$ one-particle intermediate state
  real particle production $\longrightarrow$ two-particle intermediate state

- Step 1: "open" one of the integrations over the phase space of the intermediate state to "allow" one parton to generate the jet



quark jet vertex



gluon jet vertex

- Step 2: take the convolution with PDFs

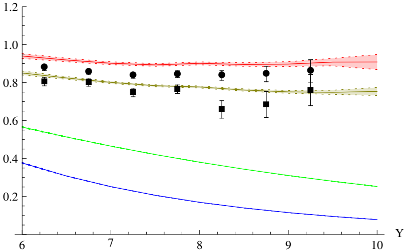
$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark jet vertex}) \qquad + \qquad f_g \otimes (\text{gluon jet vertex})$$

- Step 3: project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer to the (ν, n)-representation

$$\Phi(\nu, n) = \int d^2\vec{q}\, \frac{\Phi(\vec{q})}{\vec{q}^{\,2}} \frac{1}{\pi\sqrt{2}} \left(\vec{q}^{\,2}\right)^{\gamma - \frac{n}{2}} \left(\vec{q}\cdot\vec{l}\,\right)^n , \qquad \gamma = i\nu - \frac{1}{2} , \qquad \vec{l}^{\,2} = 0$$

NLO jet vertices

- Calculated in the transverse momentum space (no "Step 3") and cross-checked
  [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D. Yu. Ivanov, B. Murdaca, A.P., A. Perri (2011)]
- Complicated expression, to be transferred numerically to the (ν, n)-representation
- Used to study Mueller-Navelet jets in the NLA with LHC kinematics
  [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]



$\langle \cos(\phi_{J,1} - \phi_{J,2} - \pi) \rangle$ *vs* $Y \equiv \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$

(LLA, NLA, LO-NLO$_+$, NLO-NLO$_+$)

Data from [S. Cerci, D. d'Enterria (2009)] with PYTHIA (●) and HERWIG (■)

$|\vec{k}_{J,1}| = |\vec{k}_{J,2}| \equiv Q = 35$ GeV

"Mean number" of undetected gluons radiated in the final state:

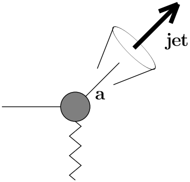
$\frac{\alpha_s(Q) N_c}{\pi} \ln \frac{s}{Q^2} \sim 0.85 \div 1.5$

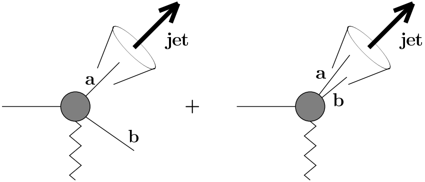
- repeat study from smaller $\vec{k}_J$
- optimize choice of s_0 and μ_R
  [D.Yu. Ivanov, A.P. (2006)-(2007)]

Jet definition and small-cone approximation (SCA)

LO: one-particle intermediate state

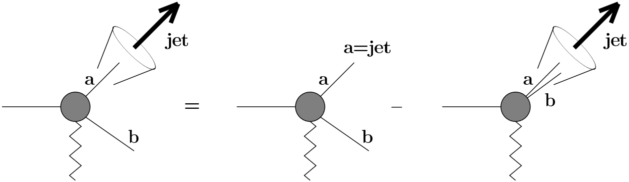
The kinematics of the produced parton *a* is completely fixed by the jet kinematics





NLO, virtual corrections: one-particle intermediate state (see above)

NLO, real corrections: two-particle intermediate state



parton a: transverse momentum $\vec{k}$, longitudinal fraction ζ
parton b: transverse momentum $\vec{q} - \vec{k}$, longitudinal fraction $\bar{\zeta} \equiv 1 - \zeta$

- Jet generated by one parton (say parton a)
  - define $\vec{\Delta}$ such that $\vec{q} = \frac{\vec{k}}{\zeta} + \vec{\Delta}$
  - $\vec{\Delta} \to 0$: $\Delta\phi^2 = \frac{\zeta^2}{\bar{\zeta}^2}\left(\frac{\vec{\Delta}^2}{\vec{k}^2} - \frac{(\vec{k}\cdot\vec{\Delta})^2}{\vec{k}^4}\right)$, $\Delta y = \frac{\zeta}{\bar{\zeta}}\frac{(\vec{k}\cdot\vec{\Delta})}{\vec{k}^2}$
  - SCA: $\Delta\phi^2 + \Delta y^2 = \frac{\zeta^2}{\bar{\zeta}^2}\frac{\vec{\Delta}^2}{\vec{k}^2} \leq R^2 \longrightarrow |\vec{\Delta}| \leq \frac{\bar{\zeta}}{\zeta}|\vec{k}|R$
- Jet generated by two partons

  jet transverse momentum $\vec{k} = \vec{k}_1 + \vec{k}_2$; jet longitudinal fraction $1 = \zeta + \bar{\zeta}$
  - jet-parton relative rapidities and azimuthal angles:

$$\Delta y_1 = \frac{1}{2}\ln\frac{\vec{k}_1^2}{\zeta^2\vec{k}^2}\,, \qquad \Delta\phi_1 = \arccos\frac{\vec{k}\cdot\vec{k}_1}{|\vec{k}_1||\vec{k}|}$$

$$\Delta y_2 = \frac{1}{2}\ln\frac{(\vec{k}_1 - \vec{k})^2}{\bar{\zeta}^2\vec{k}^2}\,, \qquad \Delta\phi_2 = \arccos\frac{\vec{k}\cdot(\vec{k} - \vec{k}_1)}{|\vec{k}||\vec{k} - \vec{k}_1|}$$

  - define $\vec{\Delta}$ as $\vec{k}_1 = \zeta\vec{k} + \vec{\Delta}$: $\Delta y_1^2 + \Delta\phi_1^2 = \frac{\vec{\Delta}^2}{\zeta^2\vec{k}^2}$, $\Delta y_2^2 + \Delta\phi_2^2 = \frac{\vec{\Delta}^2}{\bar{\zeta}^2\vec{k}^2}$
  - SCA: $|\vec{\Delta}| \leq R\,|\vec{k}|\,\min(\zeta, \bar{\zeta})$

At $s \sim Q^2$, very good agreement between SCA and Monte Carlo calculations, for cone sizes up to $R = 0.7$ [B. Jager, M. Stratmann, W. Vogelsang (2004)]

The jet vertex in the LLA

The starting point is given by “inclusive” LO parton impact factors:

$$\Phi_q = g^2 \frac{\sqrt{N^2-1}}{2N} \,, \qquad \Phi_g = \frac{C_A}{C_F} \Phi_q$$

- Step 2: take the convolution with PDFs

$$d\Phi = \mathcal{C}\, dx \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x) \right) \,, \quad \mathcal{C} \equiv g^2 \frac{\sqrt{N^2-1}}{2N} = 2\pi\alpha_s \sqrt{\frac{2\,C_F}{C_A}}$$

- Step 1: “open” the integration over the one-particle intermediate state, i.e. introduce suitable delta functions
  (jet transverse momentum $\vec{k}$, jet longitudinal fraction α)

$$\frac{d\Phi^J}{\vec{q}^{\,2}} = \mathcal{C} \int d\alpha \frac{d^2\vec{k}}{\vec{k}^{\,2}}\, dx\, \delta^{(2)}\left(\vec{k}-\vec{q}\right) \delta(\alpha - x) \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x) \right)$$

Outline

Collinear and QCD coupling counterterms

NLA collinear singularities to be removed by the renormalization of PDFs ($\overline{\text{MS}}$ scheme):

$$f_q(x) = f_q(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\left[P_{qq}(z) f_q(\tfrac{x}{z},\mu_F) + P_{qg}(z) f_g(\tfrac{x}{z},\mu_F)\right]$$

$$f_g(x) = f_g(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\left[P_{gq}(z) f_q(\tfrac{x}{z},\mu_F) + P_{gg}(z) f_g(\tfrac{x}{z},\mu_F)\right]$$

$$\frac{1}{\hat\epsilon} = \frac{1}{\epsilon} + \gamma_E - \ln(4\pi) \approx \frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^\epsilon}$$

- Collinear counterterm (in the (ν, n)-representation):

$$\frac{\pi\sqrt{2}\,\vec{k}^{\,2}}{\mathcal{C}}\frac{d\Phi^J(\nu,n)|_{\text{collinear c.t.}}}{d\alpha\, d^{2+2\epsilon}\vec{k}} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon} + \ln\frac{\mu_F^2}{\mu^2}\right)\left(\vec{k}^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n \int_\alpha^1 \frac{dz}{z}$$

$$\times\left[\sum_{a=q,\bar q}\left(P_{qq}(z) f_a\left(\frac{\alpha}{z}\right) + P_{qg}(z) f_g\left(\frac{\alpha}{z}\right)\right) + \frac{C_A}{C_F}\left(P_{gg}(z) f_g\left(\frac{\alpha}{z}\right) + P_{gq}(z)\sum_{a=q,\bar q} f_a\left(\frac{\alpha}{z}\right)\right)\right]$$

QCD coupling renormalization ($\overline{\mathrm{MS}}$ scheme):

$$\alpha_s = \alpha_s(\mu_R)\left[1 + \frac{\alpha_s(\mu_R)}{4\pi}\beta_0\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_R^2}{\mu^2}\right)\right], \quad \beta_0 = \frac{11C_A}{3} - \frac{2n_f}{3}$$

- QCD renormalization counterterm (in the (ν, n)-representation):

$$\frac{\pi\sqrt{2}\,\vec{k}^{\,2}}{\mathcal{C}}\frac{d\Phi^J(\nu,n)|_{\text{charge c.t.}}}{d\alpha d^{2+2\epsilon}\vec{k}} = \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_R^2}{\mu^2}\right)\left(\vec{k}^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n\int_\alpha^1\frac{dz}{z}\,\delta(1-z)$$

$$\times\left(\sum_{a=q,\bar{q}}f_a\left(\frac{\alpha}{z}\right) + \frac{C_A}{C_F}f_g\left(\frac{\alpha}{z}\right)\right)\left(\frac{11C_A}{6} - \frac{n_f}{3}\right)$$

In the following

$$\frac{\pi\sqrt{2}\,\vec{k}^{\,2}}{\mathcal{C}}\frac{d\Phi^J(\nu,n)}{d\alpha d^{2+2\epsilon}\vec{k}} \equiv I$$

Outline

Virtual corrections

$$I_q^V = -\frac{\alpha_s}{2\pi} \frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon} \frac{1}{\epsilon} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}} \left(\vec{k}\cdot\vec{l}\,\right)^n \sum_{a=q,\bar{q}} f_a(\alpha)$$

$$\times \left\{ C_F \left(\frac{2}{\epsilon} - 3 \right) - \frac{n_f}{3} + C_A \left(\ln \frac{s_0}{\vec{k}^2} + \frac{11}{6} \right) \right\} + \text{finite terms}$$

“Real” corrections: quark-gluon intermediate state

Starting point

$$\Phi^{\{QG\}} = \Phi_q g^2 \vec{q}^{\,2} \frac{d^{2+2\epsilon}\vec{k}_1}{(2\pi)^{3+2\epsilon}} \frac{d\beta_1}{\beta_1} \frac{[1+\beta_2^2+\epsilon\beta_1^2]}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}(\vec{k}_2\beta_1-\vec{k}_1\beta_2)^2} \left\{C_F\beta_1^2\vec{k}_2^2 + C_A\beta_2\left(\vec{k}_1^2-\beta_1\vec{k}_1\cdot\vec{q}\right)\right\}$$

$\beta_{1,2}$ and $\vec{k}_{1,2}$: relative longitudinal and transverse momenta of the gluon(quark)

$$\beta_1+\beta_2=1, \quad \vec{k}_1+\vec{k}_2=\vec{q}$$

- both quark and gluon generate the jet (SCA)
  (jet variables: $\vec{k}=\vec{k}_1+\vec{k}_2$, $1=\zeta+\bar{\zeta}$)

$$I^R_{q;q+g} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n$$
$$\times\sum_{a=q,\bar{q}} f_a(\alpha)\,R^{2\epsilon}C_F\left(\frac{1}{\epsilon}-\frac{3}{2}\right)+\text{finite terms}$$

- gluon "inclusive" jet generation (jet variables: $\vec{k} = \vec{k}_1,\ \zeta = \beta_1$)

$$I^R_{q;g} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n\int_\alpha^1\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}}f_a\left(\frac{\alpha}{\zeta}\right)$$

$$\times P_{gq}(\zeta)\left[\frac{C_A}{C_F}+\zeta^{-2\gamma}\right] + \text{finite terms}$$

- gluon "inclusive" jet generation with the quark in the jet cone (SCA)

$$I^R_{q;g,-q} = -\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n R^{2\epsilon}\int_\alpha^1\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}}f_a\left(\frac{\alpha}{\zeta}\right)$$

$$\times P_{gq}(\zeta)\zeta^{-2\gamma} + \text{finite terms}$$

- quark “inclusive” jet generation (jet variables: $\vec{k} = \vec{k}_2$, $\zeta = \beta_2$)

$$\left(I^R_{q;q}\right)^{C_F} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n\int_\alpha^1\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}} f_a\left(\frac{\alpha}{\zeta}\right)$$

$$\times\left[C_F\left(\frac{2}{\epsilon}-3\right)\delta(1-\zeta)+P_{qq}(\zeta)\left(1+\zeta^{-2\gamma}\right)\right]+\text{finite terms}$$

$$\left(I^R_{q;q}\right)^{C_A} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n\int_\alpha^1\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}} f_a\left(\frac{\alpha}{\zeta}\right)$$

$$\times C_A\delta(1-\zeta)\ln\frac{s_0}{\vec{k}^{\,2}}+\text{finite terms}$$

- quark “inclusive” jet generation with the gluon in the jet cone (SCA)

$$I^R_{q;q,-q} = -\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n R^{2\epsilon}\int_\alpha^1\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}} f_a\left(\frac{\alpha}{\zeta}\right)$$

$$\times\left[P_{qq}(\zeta)\zeta^{-2\gamma}+C_F\delta(1-\zeta)\left(\frac{1}{\epsilon}-\frac{3}{2}\right)\right]+\text{finite terms}$$

Outline

Virtual corrections

$$I_g^V = -\frac{\alpha_s}{2\pi}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{1}{\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n f_g(\alpha)\,\frac{C_A}{C_F}$$
$$\times\left\{C_A\left(\ln\frac{s_0}{\vec{k}^2}+\frac{2}{\epsilon}-\frac{11}{6}\right)+\frac{n_f}{3}\right\}+\text{finite terms}$$

“Real” corrections: quark-antiquark intermediate state

Starting point ($T_R = 1/2$)

$$\Phi^{\{Q\bar{Q}\}} = \Phi_g g^2 \vec{q}^{\,2} \frac{d^{2+2\epsilon}\vec{k}_1}{(2\pi)^{3+2\epsilon}} d\beta_1 \, T_R \left(1 - \frac{2\beta_1\beta_2}{1+\epsilon}\right) \left\{ \frac{C_F}{C_A} \frac{1}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}} + \beta_1\beta_2 \frac{\vec{k}_1 \cdot \vec{k}_2}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}(\vec{k}_2\beta_1 - \vec{k}_1\beta_2)^2} \right\}$$

$\beta_{1,2}$ and $\vec{k}_{1,2}$: relative longitudinal and transverse momenta of the quark(antiquark)

$$\beta_1 + \beta_2 = 1, \quad \vec{k}_1 + \vec{k}_2 = \vec{q}$$

- both quark and antiquark generate the jet (SCA)
  (jet variables: $\vec{k} = \vec{k}_1 + \vec{k}_2$, $1 = \zeta + \bar{\zeta}$)

$$I^R_{g;q+\bar{q}} = \frac{\alpha_s}{2\pi} \frac{1}{\epsilon} \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}} \left(\vec{k}\cdot\vec{l}\,\right)^n f_g(\alpha)\, R^{2\epsilon} \frac{C_A}{C_F} \frac{n_f}{3}$$

- (anti)quark "inclusive" jet generation (jet variables: $\vec{k} = \vec{k}_1$, $\zeta = \beta_1$)

$$I^R_{g;q} = \frac{\alpha_s}{2\pi} \frac{1}{\epsilon} \frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon} \left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}} \left(\vec{k}\cdot\vec{l}\,\right)^n \int_\alpha^1 \frac{d\zeta}{\zeta} f_g\left(\frac{\alpha}{\zeta}\right) \frac{C_A}{C_F} n_f$$

$$\times \left[P_{qg}(\zeta)\left(\frac{C_F}{C_A} + \zeta^{-2\gamma}\right)\right] + \text{finite terms}$$

- (anti)quark "inclusive" jet generation with the antiquark in the jet cone (SCA)

$$I^R_{q;q,-\bar{q}} = -\frac{\alpha_s}{2\pi} \frac{1}{\epsilon} \frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon} \frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}} \left(\vec{k}\cdot\vec{l}\,\right)^n R^{2\epsilon} \int_\alpha^1 \frac{d\zeta}{\zeta} f_g\left(\frac{\alpha}{\zeta}\right) n_f \frac{C_A}{C_F}$$

$$\times P_{qg}(\zeta)\zeta^{-2\gamma} + \text{finite terms}$$

“Real” corrections: two-gluon intermediate state

Starting point

$$\Phi^{\{GG\}} = \Phi_g g^2 \vec{q}^{\,2} \frac{d^{2+2\epsilon}\vec{k}_1}{(2\pi)^{3+2\epsilon}} d\beta_1 \frac{C_A}{2} \left[\frac{1}{\beta_1} + \frac{1}{\beta_2} - 2 + \beta_1\beta_2\right] \left\{\frac{1}{\vec{k}_1^{\,2}\vec{k}_2^{\,2}} + \frac{\beta_1^2}{\vec{k}_1^{\,2}(\vec{k}_2\beta_1 - \vec{k}_1\beta_2)^2} + \frac{\beta_2^2}{\vec{k}_2^{\,2}(\vec{k}_2\beta_1 - \vec{k}_1\beta_2)^2}\right\}$$

$\beta_{1,2}$ and $\vec{k}_{1,2}$: relative longitudinal and transverse momenta of the two gluons

$$\beta_1 + \beta_2 = 1, \quad \vec{k}_1 + \vec{k}_2 = \vec{q}$$

- both gluons generate the jet (SCA)
  (jet variables: $\vec{k} = \vec{k}_1$, $\zeta = \beta_1$)

$$I^R_{g;g+g} = \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n f_g(\alpha)\,\frac{C_A}{C_F}\,R^{2\epsilon}$$

$$\times C_A\left(\frac{1}{\epsilon} - \frac{11}{6}\right) + \text{finite terms}$$

- gluon "inclusive" jet generation

$$I^R_{g;g} = \frac{1}{\epsilon}\frac{\alpha_s}{2\pi}\frac{\Gamma[1-\epsilon]}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n\int_\alpha^1\frac{d\zeta}{\zeta}f_g\left(\frac{\alpha}{\zeta}\right)\frac{C_A}{C_F}$$

$$\times\left\{P_{gg}(\zeta)\left(1+\zeta^{-2\gamma}\right)+\delta(1-\zeta)\left[C_A\left(\ln\frac{s_0}{\vec{k}^{\,2}}+\frac{2}{\epsilon}-\frac{11}{3}\right)+\frac{2n_f}{3}\right]\right\}$$

$$+\text{finite terms}$$

- gluon "inclusive" jet generation with the other gluon in the jet cone (SCA)

$$I^R_{g;g,-g} = -\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left(\vec{k}^{\,2}\right)^{\gamma+\epsilon-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n\int_\alpha^1\frac{d\zeta}{\zeta}f_g\left(\frac{\alpha}{\zeta}\right)\frac{C_A}{C_F}$$

$$\times R^{2\epsilon}\left\{P_{gg}(\zeta)\zeta^{-2\gamma}+\delta(1-\zeta)\left[C_A\left(\frac{1}{\epsilon}-\frac{11}{6}\right)+\frac{n_f}{3}\right]\right\}+\text{finite terms}$$

Final result

All the IR and UV divergences canceled!

$$I_q = \frac{\alpha_s}{2\pi}\left(\vec{k}^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n \int_\alpha^1 \frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}} f_a\left(\frac{\alpha}{\zeta}\right)$$

$$\times\Bigg[\left\{P_{qq}(\zeta)+\frac{C_A}{C_F}P_{gq}(\zeta)\right\}\ln\frac{\vec{k}^{\,2}}{\mu_F^2} - 2\zeta^{-2\gamma}\ln R\,\{P_{qq}(\zeta)+P_{gq}(\zeta)\} - \frac{\beta_0}{2}\ln\frac{\vec{k}^{\,2}}{\mu_R^2}\delta(1-\zeta)$$

$$+C_A\delta(1-\zeta)\left\{\chi(n,\gamma)\ln\frac{s_0}{\vec{k}^{\,2}}+\frac{85}{18}+\frac{\pi^2}{2}+\frac{1}{2}\left(\psi'\left(1+\gamma+\frac{n}{2}\right)-\psi'\left(\frac{n}{2}-\gamma\right)-\chi^2(n,\gamma)\right)\right\}$$

$$+(1+\zeta^2)\left\{C_A\left(\frac{(1+\zeta^{-2\gamma})\,\chi(n,\gamma)}{2(1-\zeta)_+}-\zeta^{-2\gamma}\left(\frac{\ln(1-\zeta)}{1-\zeta}\right)_+\right)+\left(C_F-\frac{C_A}{2}\right)\left[\frac{\bar{\zeta}}{\zeta^2}I_2-\frac{2\ln\zeta}{1-\zeta}\right.\right.$$

$$\left.\left.+2\left(\frac{\ln(1-\zeta)}{1-\zeta}\right)_+\right]\right\}+\delta(1-\zeta)\left(C_F\left(3\ln 2-\frac{\pi^2}{3}-\frac{9}{2}\right)-\frac{5n_f}{9}\right)$$

$$+C_A\zeta+C_F\bar{\zeta}+\frac{1+\bar{\zeta}^2}{\zeta}\left\{C_A\frac{\bar{\zeta}}{\zeta}I_1+2C_A\ln\frac{\bar{\zeta}}{\zeta}+C_F\zeta^{-2\gamma}(\chi(n,\gamma)-2\ln\bar{\zeta})\right\}\Bigg]\ .$$

$$
I_g = \frac{\alpha_s}{2\pi}\left(\vec{k}^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^{n}\int\limits_{\alpha}^{1}\frac{d\zeta}{\zeta}f_g\left(\frac{\alpha}{\zeta}\right)\frac{C_A}{C_F}
$$

$$
\times\left\{\left\{P_{gg}(\zeta)+2\,n_f\frac{C_F}{C_A}P_{qg}(\zeta)\right\}\ln\frac{\vec{k}^{\,2}}{\mu_F^2}-2\zeta^{-2\gamma}\ln R\,\{P_{gg}(\zeta)+2\,n_f P_{qg}(\zeta)\}-\frac{\beta_0}{2}\ln\frac{\vec{k}^{\,2}}{4\mu_R^2}\delta(1-\zeta)\right.
$$

$$
+C_A\delta(1-\zeta)\left\{\chi(n,\gamma)\ln\frac{s_0}{\vec{k}^{\,2}}+\frac{1}{12}+\frac{\pi^2}{6}+\frac{1}{2}\left(\psi'\left(1+\gamma+\frac{n}{2}\right)-\psi'\left(\frac{n}{2}-\gamma\right)-\chi^2(n,\gamma)\right)\right\}
$$

$$
+2C_A(1-\zeta^{-2\gamma})\left(\left(\frac{1}{\zeta}-2+\zeta\bar{\zeta}\right)\ln\bar{\zeta}+\frac{\ln(1-\zeta)}{1-\zeta}\right)
$$

$$
+C_A\left[\frac{1}{\zeta}+\frac{1}{(1-\zeta)_+}-2+\zeta\bar{\zeta}\right]\left((1+\zeta^{-2\gamma})\chi(n,\gamma)-2\ln\zeta+\frac{\bar{\zeta}^2}{\zeta^2}I_2\right)
$$

$$
\left.+n_f\left[2\zeta\bar{\zeta}\frac{C_F}{C_A}+(\zeta^2+\bar{\zeta}^2)\left(\frac{C_F}{C_A}\chi(n,\gamma)+\frac{\bar{\zeta}}{\zeta}I_3\right)-\frac{1}{12}\delta(1-\zeta)\right]\right\}\,.
$$

The dependence on the cone size: $A\ln R+B+\mathcal{O}(R^2)$ [M. Furman (1982)]

$$I_2 = \frac{\zeta^2}{\bar{\zeta}^2}\Bigg[\zeta\left(\frac{{}_2F_1(1,1+\gamma-\frac{n}{2},2+\gamma-\frac{n}{2},\zeta)}{\frac{n}{2}-\gamma-1} - \frac{{}_2F_1(1,1+\gamma+\frac{n}{2},2+\gamma+\frac{n}{2},\zeta)}{\frac{n}{2}+\gamma+1}\right)$$

$$+\zeta^{-2\gamma}\left(\frac{{}_2F_1(1,-\gamma-\frac{n}{2},1-\gamma-\frac{n}{2},\zeta)}{\frac{n}{2}+\gamma} - \frac{{}_2F_1(1,-\gamma+\frac{n}{2},1-\gamma+\frac{n}{2},\zeta)}{\frac{n}{2}-\gamma}\right)$$

$$+\left(1+\zeta^{-2\gamma}\right)\left(\chi(n,\gamma)-2\ln\bar{\zeta}\right)+2\ln\zeta\Bigg]$$

$$I_1 = \frac{\bar{\zeta}}{2\zeta}I_2 + \frac{\zeta}{\bar{\bar{\zeta}}}\left[\ln\zeta + \frac{1-\zeta^{-2\gamma}}{2}\left(\chi(n,\gamma)-2\ln\bar{\zeta}\right)\right]$$

$$I_3 = \frac{\bar{\zeta}}{2\zeta}I_2 - \frac{\zeta}{\bar{\bar{\zeta}}}\left[\ln\zeta + \frac{1-\zeta^{-2\gamma}}{2}\left(\chi(n,\gamma)-2\ln\bar{\zeta}\right)\right]$$

The following property of the hypergeometric function

$${}_2F_1(1,a,a+1,\zeta) = a(\psi(1)-\psi(a)-\ln\bar{\zeta}) + \mathcal{O}(\bar{\zeta}\ln\bar{\zeta})$$

implies that, for $\zeta \to 1$,

$$I_2 = \mathcal{O}(\ln\bar{\zeta})\,, \quad I_1 = \mathcal{O}(\ln\bar{\zeta})\,, \quad I_3 = \mathcal{O}(\ln\bar{\zeta})$$

guaranteeing the convergence of the integrals in the above results at the upper limit.

Summary and Outlook

- The NLO vertex (impact factor) for the forward production of high-p_T jet from an incoming quark or gluon, emitted by a proton, has been calculated in the "small-cone" approximation.

- At the basis of the calculation of the hard part of the vertex were NLO BFKL parton impact factors; then the collinear factorization with the PDFs of the incoming partons was suitably considered.

- The result has been presented in the so called (ν, n)-representation, very convenient for numerical implementation. [D.Yu. Ivanov, A.P. (2006)-(2007)] [F. Caporale, D.Yu. Ivanov, A.P. (2008)]

- Besides Mueller-Navelet jets, the vertex can be used also for forward-jet electroproduction, $\gamma^* p \to \text{Jet} + X$.

- The related vertex for the NLO forward production of an identified hadron was recently calculated [D.Yu. Ivanov, A.P. (2011)]; it involves also the collinear fragmentation functions of the detected hadron; it can be used to study the processes

$$\gamma^* p \to \text{hadron} + X \,, \qquad pp \to \text{hadron 1} + \text{hadron 2} + X \,.$$