
DIS with Neutrinos: Now and When

DIS12 Workshop
University of Bonn
March, 2012

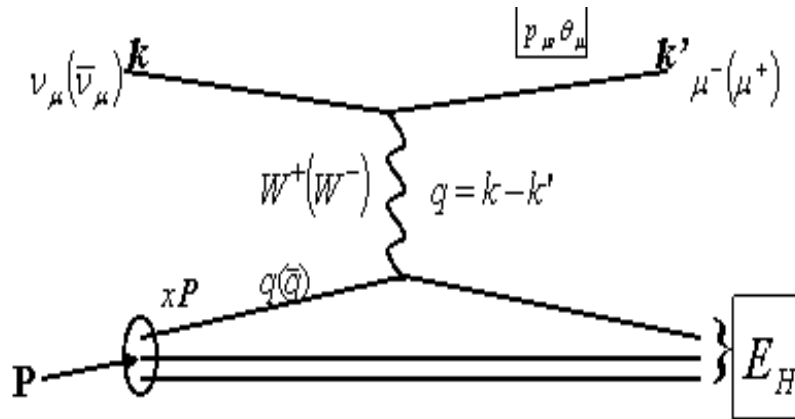
Jorge G. Morfin
Fermilab

With thanks for the contributions of Trung Li (Rutgers), Rosen Matev(Sofia) and Martin Tzanov (LSU)

Studying Deep-Inelastic Scattering with $\nu/\bar{\nu}$

- ◆ Interacting with the weak current means a **much smaller interaction rate** than e/μ scattering
 - ▼ Need huge, **higher-A** detectors and/or intense neutrino beams to get reasonable statistics
- ◆ The incoming neutrino energy is not a priori known and even the neutrino energy dependent flux is difficult to predict – solution wait until end of talk!
- ◆ However **can select which set of quarks involved in the interaction via ν or $\bar{\nu}$**
- ◆ While F_2 is measured precisely by the charge lepton scattering, **xF_3 is accessible by neutrino DIS** and yields increased sensitivity to the **valence quark distributions**.
- ◆ Measuring charm production with **ν and $\bar{\nu}$** also gives us insight into the **s and \bar{s} quark distributions within a nucleon in a nucleus**.
- ◆ **Measuring the difference between $xF_3(\nu)$ and $xF_3(\bar{\nu})$ ($\Delta xF_3 = s - c$) gives information on heavy quarks**
- ◆ **Being forced to use heavy nuclear targets presents some challenges in disentangling nuclear effects from the study of nucleON PDFs. Need to study nuclear effects with neutrinos (as compared to charged lepton scattering) or use lighter targets, like H_2/D_2 , or do both!**

The Parameters of ν DIS



$$Q^2 = 4E_\nu E_\mu \sin^2 \frac{\theta}{2},$$

Squared 4-momentum transferred to hadronic system

$$x = \frac{Q^2}{2ME_{HAD}},$$

Fraction of momentum carried by the struck quark

$$y = \frac{\nu}{E_\nu} = \frac{E_{HAD}}{E_\nu},$$

Inelasticity

Differential cross section in terms of structure functions:

$$\frac{1}{E_\nu} \frac{d^2\sigma^{\nu(\bar{\nu})}}{dx dy} = \frac{G_F^2 M}{\pi(1 + Q^2/M_W^2)} \left[\left(1 - y - \frac{Mxy}{2E_\nu} + \frac{y^2}{2} \frac{1 + 4M^2 x^2/Q^2}{1 + R(x, Q^2)} \right) F_2^{\nu(\bar{\nu})} \pm \left(y - \frac{y^2}{2} \right) x F_3^{\nu(\bar{\nu})} \right]$$

Structure Functions in terms of parton distributions (for ν -scattering)

$$F_2^{\nu(\bar{\nu})N} = \sum [xq^{\nu(\bar{\nu})N}(x) + x\bar{q}^{\nu(\bar{\nu})N}(x) + 2xk^{\nu(\bar{\nu})N}(x)]$$

$$xF_3^{\nu(\bar{\nu})N} = \sum [xq^{\nu(\bar{\nu})N}(x) - x\bar{q}^{\nu(\bar{\nu})N}(x)] = x(d_\nu(x) + u_\nu(x)) \pm 2x(s(x) - c(x))$$

$$R = \frac{\sigma_L}{\sigma_T}$$

Parton Distribution Functions:

What Can We Learn With All Six ν and $\bar{\nu}$ Structure Functions?

**Recall Neutrinos have the ability to directly resolve flavor of the nucleon's constituents:
 ν interacts with $d, s, \bar{u},$ and \bar{c} while $\bar{\nu}$ interacts with u, c, \bar{d} and \bar{s} .**

Using Leading order expressions:

$$F_2^{\bar{\nu}N}(x, Q^2) = x[u + \bar{u} + d + \bar{d} + 2s + 2c]$$

$$F_2^{\nu N}(x, Q^2) = x[u + \bar{u} + d + \bar{d} + 2s + 2\bar{c}]$$

$$xF_3^{\bar{\nu}N}(x, Q^2) = x[u + d - \bar{u} - \bar{d} - 2s + 2c]$$

$$xF_3^{\nu N}(x, Q^2) = x[u + d - \bar{u} - \bar{d} + 2s - 2\bar{c}]$$

Taking combinations of the Structure functions

$$F_2^{\nu} - xF_3^{\nu} = 2(\bar{u} + \bar{d} + 2\bar{c})$$

$$F_2^{\bar{\nu}} - xF_3^{\bar{\nu}} = 2(\bar{u} + \bar{d} + 2\bar{s})$$

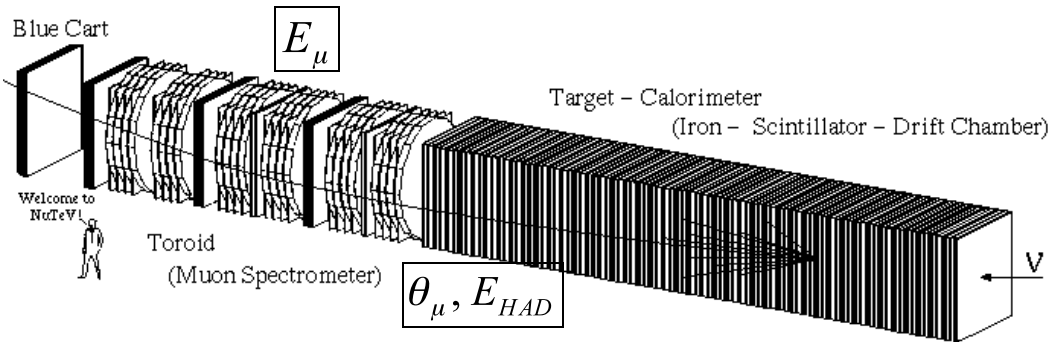
$$xF_3^{\nu} - xF_3^{\bar{\nu}} = 2[(s + \bar{s}) - (\bar{c} + c)]$$

Most “Recent” ν DIS Experiments

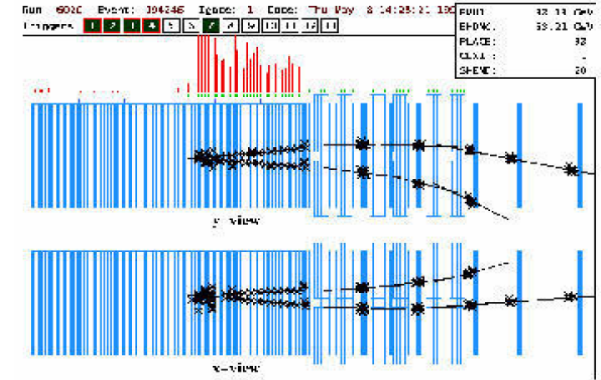
	E_ν range ($\langle E_\nu \rangle$) (GeV)	Run	Target A	E_μ scale	E_{HAD} scale	Detector
NuTeV (CCFR)	30-360(120)	96-97	Fe	0.7%	0.43%	Coarse
NOMAD	10-200(27)	95-98	Various (mainly C)	--	---	Fine- grained
CHORUS	10-200(27)	95-98	Pb	2%	5%	Fine- grained
MINOS	3-15	05-10	Fe	2.5%	5.6%	Coarse

Old Style: The NuTeV Experiment: 800 GeV Protons

> 3 million neutrino/antineutrino events with $20 \leq E_\nu \leq 400 \text{ GeV}$



Refurbished CCFR detector



Target Calorimeter:

- ◆ Steel-Scintillator Sandwich (10 cm)

$$\frac{\delta E}{E} \approx \frac{0.86}{\sqrt{E}} \text{ -resolution}$$

- ◆ Tracking chambers for muon track and vertex

◆ Muon Spectrometer:

Three toroidal iron magnets with five sets of drift chambers

$$\langle B_\phi \rangle \approx 1.7T, p_t \approx 2.4 \text{ GeV} / c$$

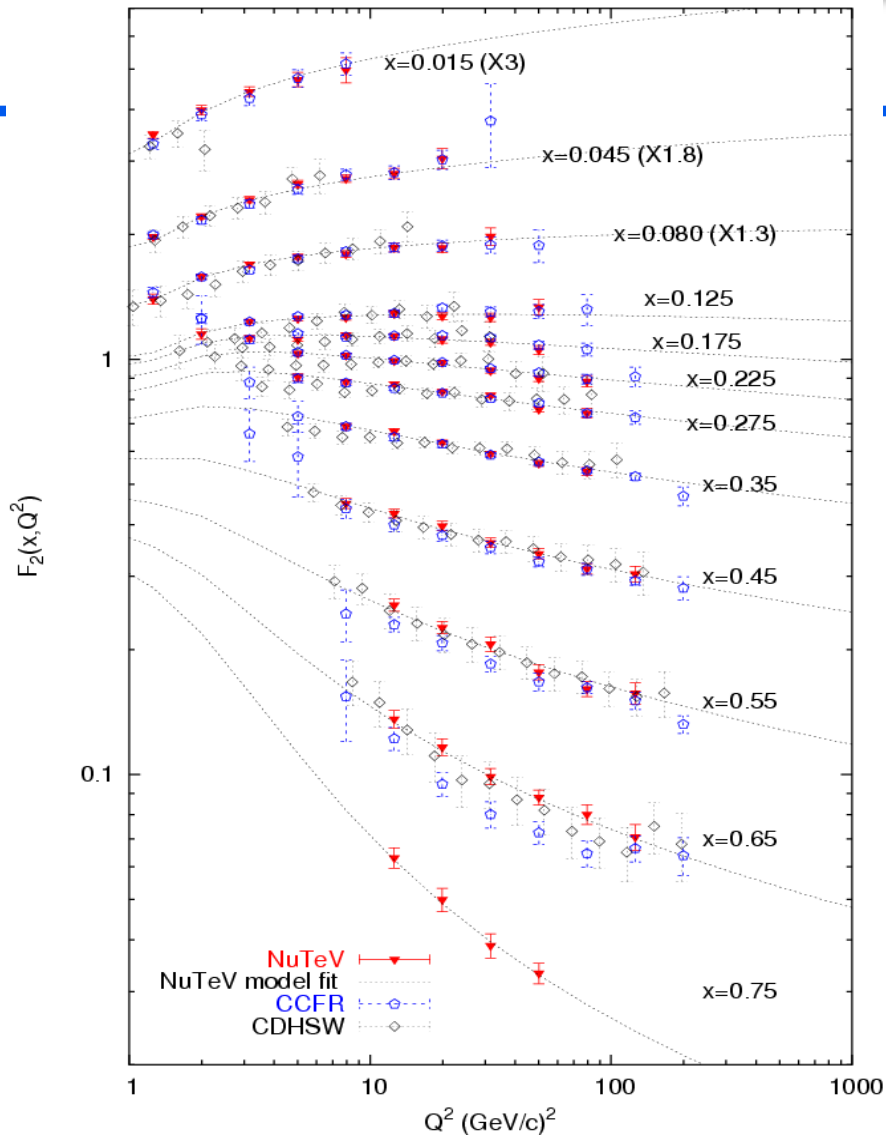
$$\delta(1/p)/(1/p) \sim 11\% \text{ MCS dominated}$$

- ◆ Always focusing for leading muon

1170 ν and 966 $\bar{\nu}$ data points with seven correlated systematic errors.

To confront leading systematic errors, there was a continuous calibration beam

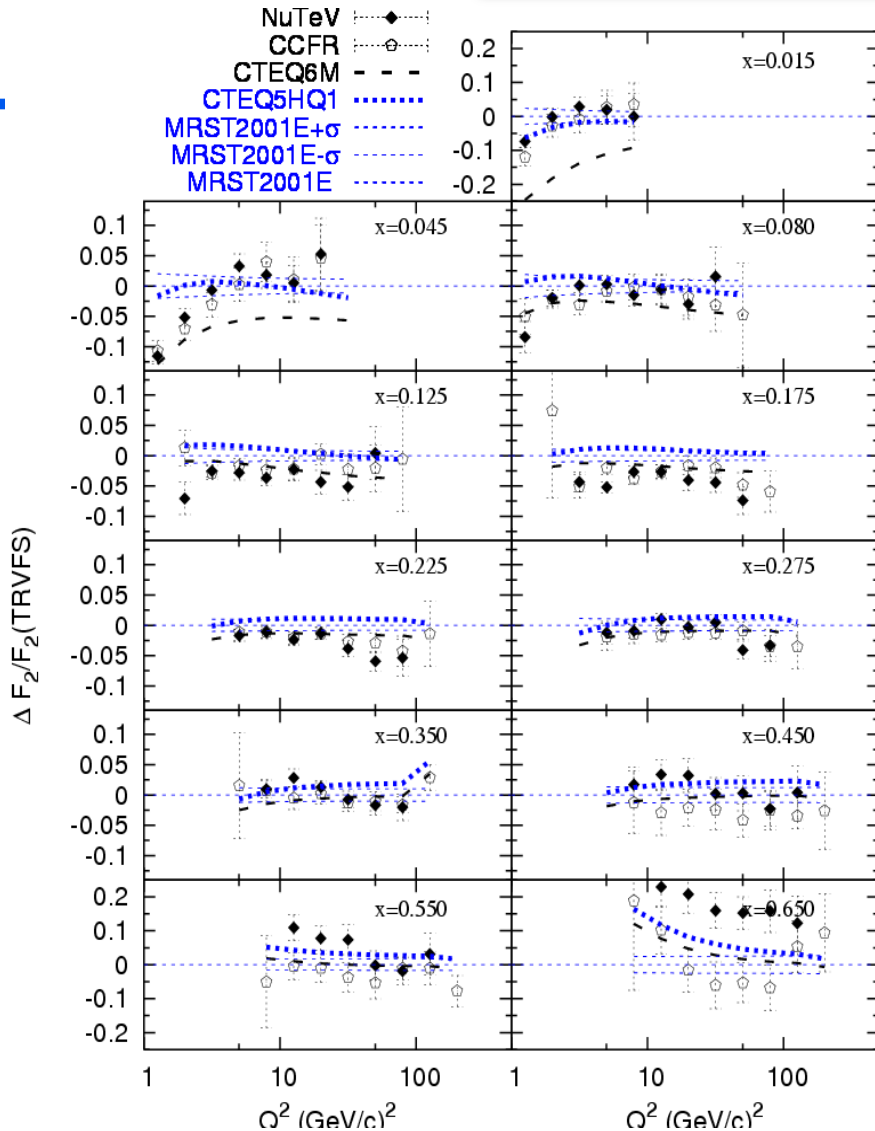
Average F_2 Measurement



Notice the Q^2 range!

- ◆ Isoscalar ν -Fe F_2
- ◆ NuTeV F_2 compared with CCFR and CDHSW results
- ◆ All systematic uncertainties are included
- ◆ All data sets agree for $x < 0.4$.
- ◆ At $x > 0.4$ NuTeV agrees with CDHSW.
- ◆ At $x > 0.4$ NuTeV is systematically above CCFR.

Comparison with Global Fits for F_2



- Baseline is TRVFS(MRST2001E)

- NuTeV and CCFR F_2 are compared to TRVFS(MRST2001E)

$$\frac{F_2^{NuTeV} - F_2^{TRVFS}}{F_2^{TRVFS}}$$

- Theoretical models shown are:

- ACOT(CTEQ6M)
- ACOT(CTEQ5HQ1)
- TRVFS (MRST2001E)

- Theory curves are corrected for:

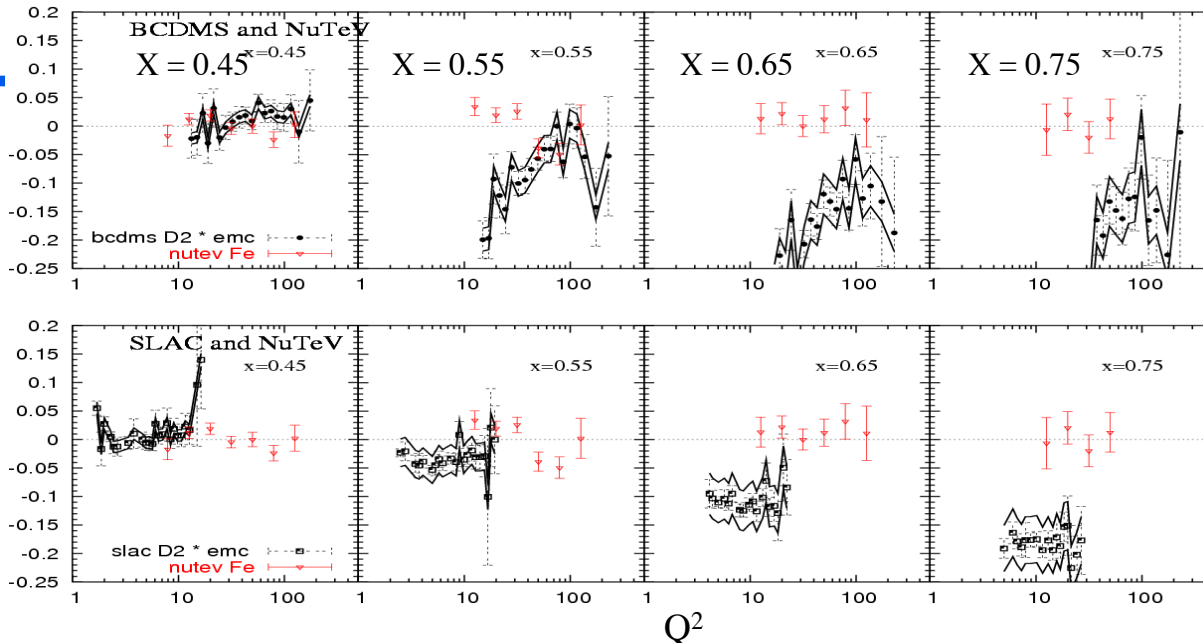
- target mass

(*H. Georgi and H. D. Politzer, Phys. Rev. D14, 1829*)

- nuclear effects – **parameterization from charge lepton data, assumed to be the same for neutrino scattering** (no Q^2 dependence added) nuclear effects parameterization is dominated by SLAC (lower Q^2 in this region) data at high-x

- NuTeV F_2 agrees with theory for medium x.
- At low x different Q^2 dependence.
- At high x ($x > 0.5$) NuTeV is systematically higher.

Comparison with Charge Lepton Data for $x > 0.4$

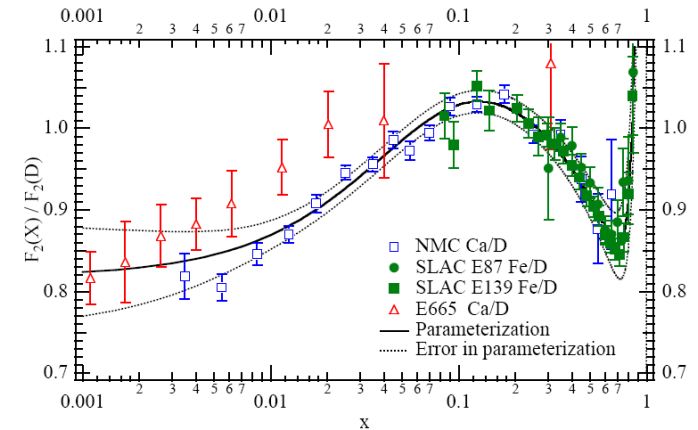


- NuTeV agrees with charge lepton data for $x=0.45$.
- NuTeV is higher than BCDMS(D_2), different Q^2 dependence
 - 7% at $x=0.55$, 12% at $x=0.65$, and 15% at $x=0.75$
- NuTeV is higher than SLAC(D_2) (bottom 4 plots)
 - 4% at $x=0.55$, 10% at $x=0.65$, and 17% at $x=0.75$

“Perhaps the nuclear correction is smaller for neutrino scattering at high x .”

Martin Tzanov

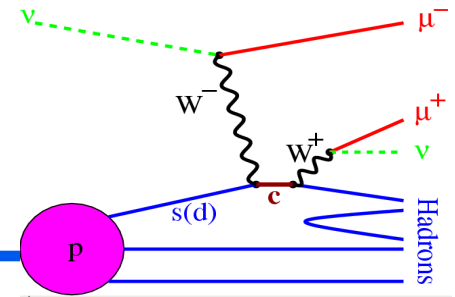
- Baseline is NuTeV model fit
- data points are $\frac{F_2^{DATA} - F_2^{BG}}{F_2^{BG}}$
- charge lepton data is corrected for:
 - $\frac{F_2^V}{F_2^I}$ using CTEQ4D
 - heavy target $\frac{F_2^N}{F_2^D}$



the nuclear correction is dominated by SLAC data, which is at lower Q^2 than NuTeV in this region

Charm Production by Neutrinos

a direct look at strange sea.



- ◆ Charm quark is produced from CC neutrino interaction with s(d) quark in the nucleon. d-quark interaction is CKM suppressed
- ◆ Detect charm via the semi-leptonic decay which yields a very clear signature – two opposite sign muons
- ◆ It is sensitive to m_c through E_ν dependence.
- ◆ With high-purity ν and $\bar{\nu}$ beams, NuTeV made high statistics separate s and \bar{s} measurements: 5163 ν and 1380 $\bar{\nu}$
- ◆ Could then make a measurement of s \rightarrow \bar{s} .

This is an analysis of strange quarks in an Fe nucleus!

Summary ν Scattering Results – NuTeV

NuTeV accumulated over 3 million neutrino / antineutrino events with $20 \leq E_\nu \leq 400$ GeV. Most accurate results available until NOMAD.

NuTeV considered multiple correlated systematic uncertainties.

NuTeV agrees with other ν experiments and theory for medium x .

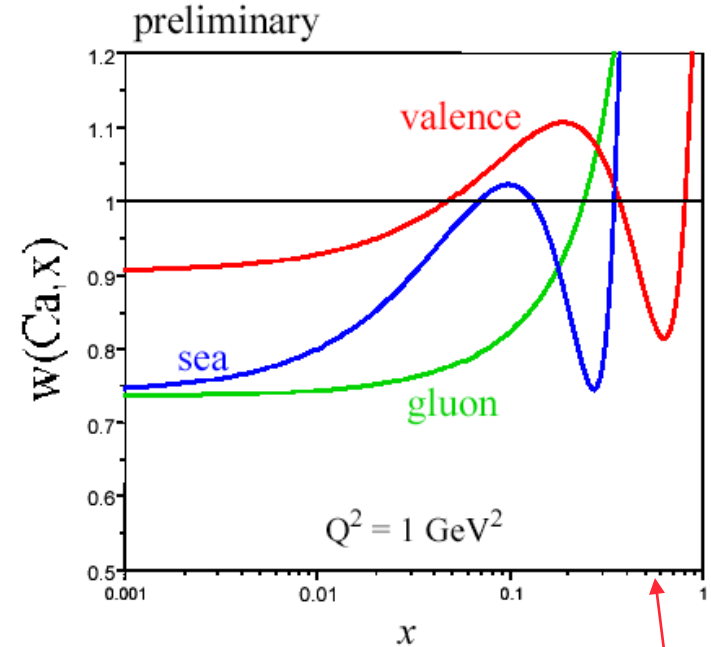
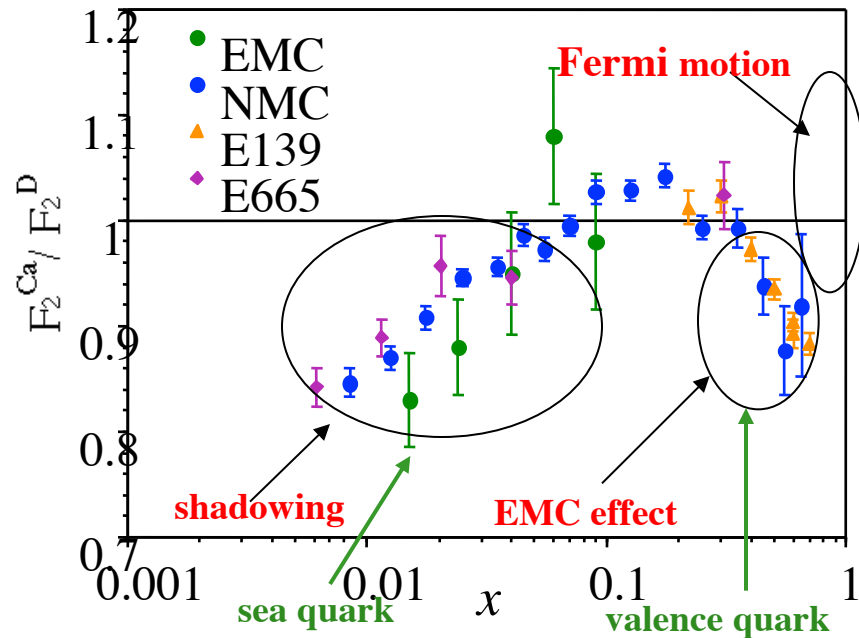
NuTeV has a **different Q^2 dependence at low x** .

NuTeV **is systematically higher at high x ($x > 0.6$)**.

How do we now incorporate these NuTeV results in the analysis of nucleon structure?

We need to understand neutrino nuclear correction factors (NCF) to bring $\nu / \bar{\nu} - \text{Fe}$ to $\nu / \bar{\nu} - \text{N}$!

Knowledge of Nuclear Effects with Neutrinos: Very sparse



- ◆ F_2 / nucleon changes as a function of A. Measured in $\mu/e - A$ not in $\nu - A$
- ◆ Good reason to consider nuclear effects are DIFFERENT in $\nu - A$.
 - ▼ Presence of axial-vector current.
 - ▼ SPECULATION: Much stronger shadowing for $\nu - A$ but somewhat weaker “EMC” effect.
 - ▼ Different nuclear effects for valence and sea --> different shadowing for xF_3 compared to F_2 .
 - ▼ Different nuclear effects for d and u quarks.

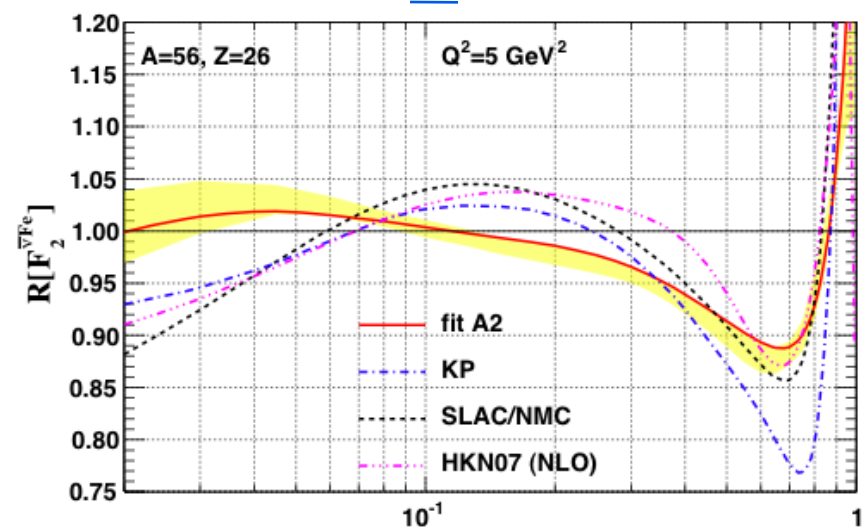
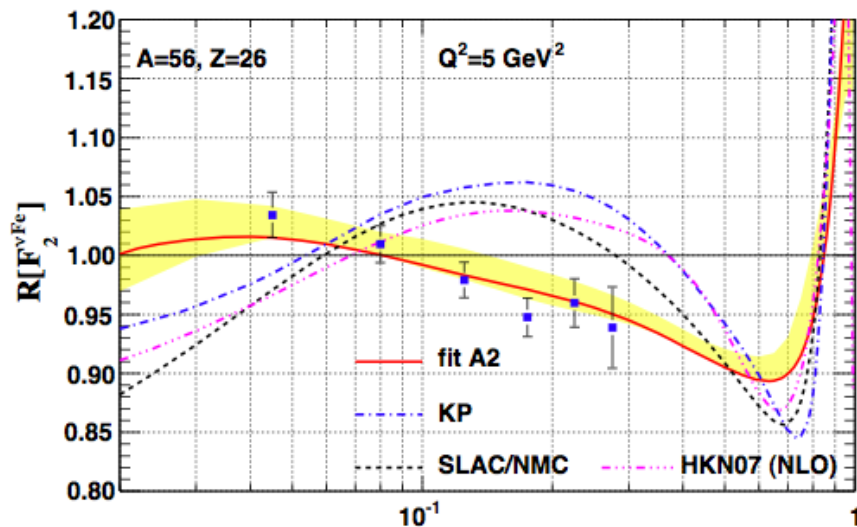
Nuclear PDFs from neutrino deep inelastic scattering

Karol Kovarik Presentation – this afternoon in SF session

I. Schienbein (SMU & LPSC-Grenoble, J-Y. Yu (SMU)
C. Keppel (Hampton & JeffersonLab) J.G.M. (Fermilab),
F. Olness (SMU), J.F. Owens (Florida State U)

F_2 Structure Function Ratios: ν -Iron

and $\bar{\nu}$ -Iron



This would suggest that the nuclear parton distribution function for ν are different than those found by ℓ

Where are we now: Conclusions

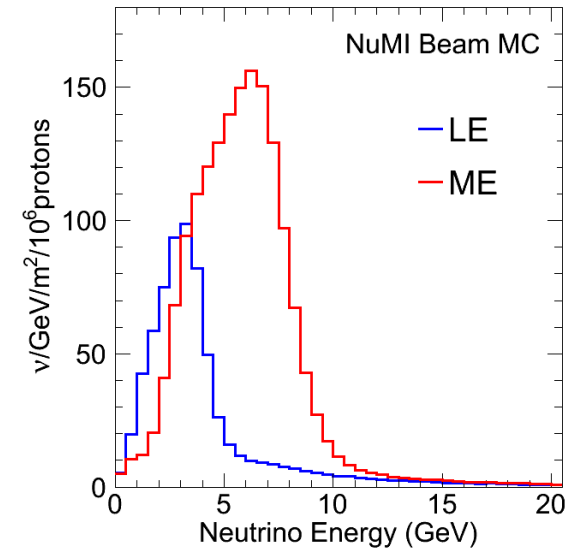
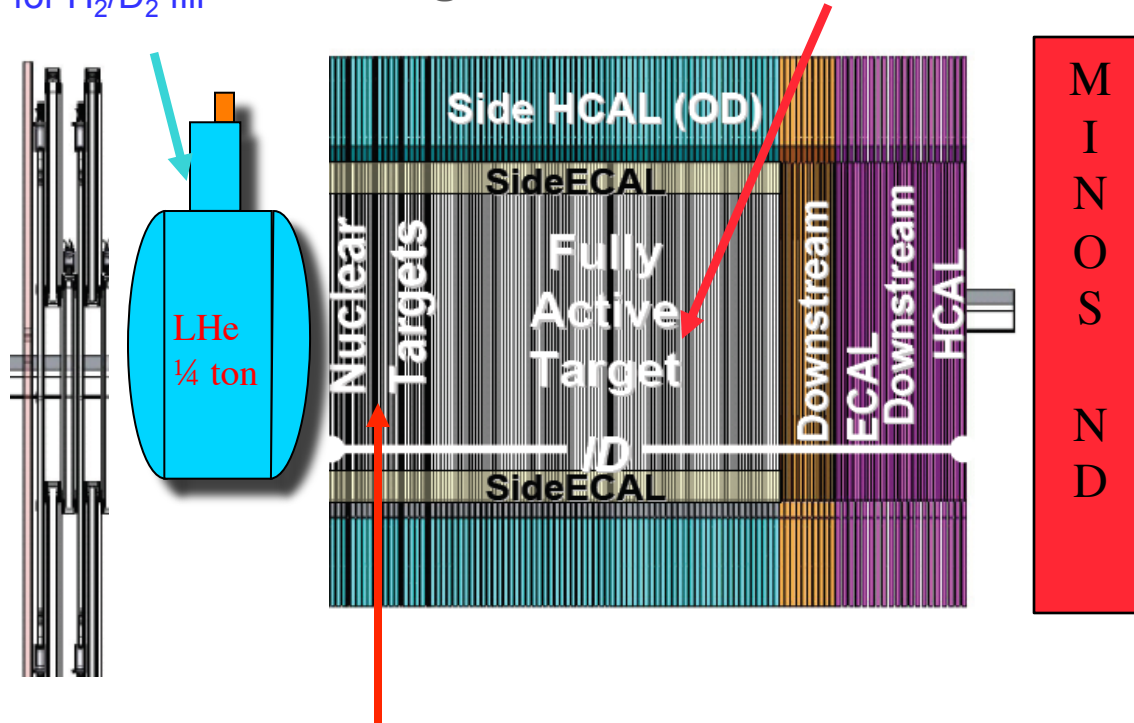
- ◆ All high-statistics neutrino data is off nuclear targets. Need nuclear correction factors to include data off nuclei in fits with nucleon data.
- ◆ Nuclear correction factors (R) seems to be different for ν -Fe scattering compared to ℓ^\pm -Fe.
 - ▼ **Results from one experiment on one nuclear target... careful.**
- ◆ We need ν -experiments to measure these nuclear correction factors!
 - ▼ For the cleanest study of nucleon structure, $\nu/\bar{\nu}$ - H₂/D₂ experiment would be excellent!
- ◆ **Aside from the question of nuclear correction factors, there are differences between the highly accurate NuTeV results and results from other neutrino experiments and theory we need to understand.**
- ◆ **How do we answer these questions?**

The MINERνA Experiment – First of the New Style High-Statistics Neutrino Detectors

Joel Mousseau (Univ. Florida) - just described this

Cryotarget: ready for H₂/D₂ fill

Fully Active Fine Segmented Scintillator
Target: 8.3 tons, 3 - 5 tons fiducial



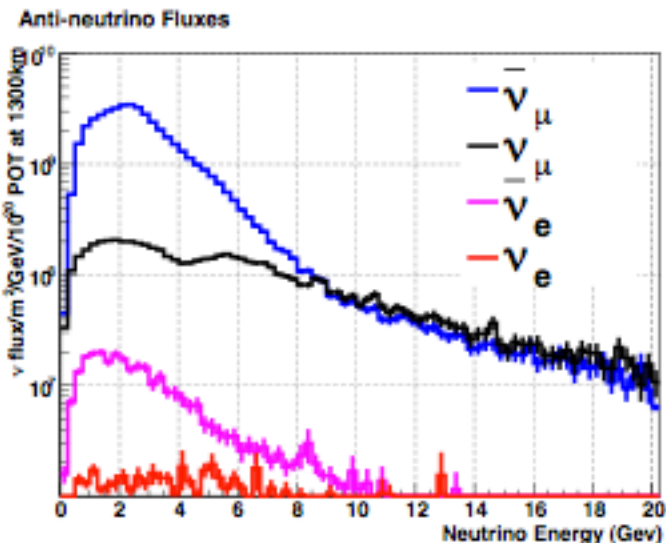
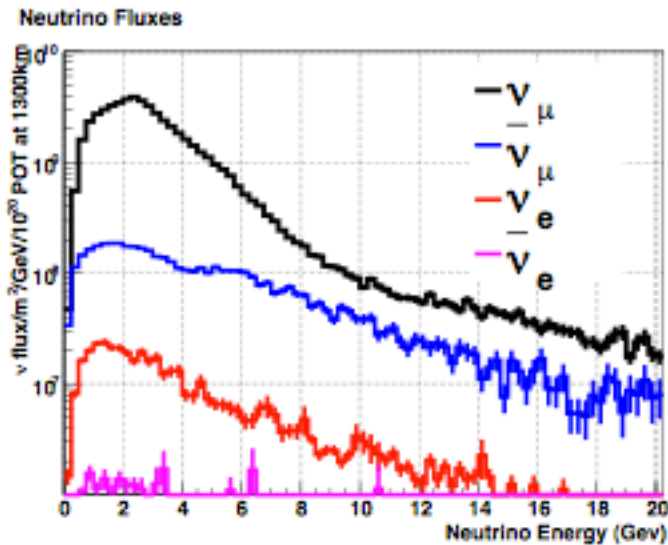
The ME beam peaks at 7 GeV rather than the LE beam peak of 3.5 GeV. Not exactly designed for DIS

Nuclear Targets with Pb, Fe, C, H₂O, CH
 Simultaneous in the same neutrino beam reduces systematic errors between nuclei

Where do we go after MINERvA?

- ◆ With MINERvA and the ME exposure, we will begin to resolve the question of neutrino nuclear correction factors... however:
 - ▼ Will know the neutrino flux to within (5 – 10)%
 - ▼ Was designed to understand the low- E_ν , low multiplicity QE \rightarrow transition resonance region.
 - ▼ Even with the ME beam cannot investigate the full x range at high- Q^2
- ◆ To make significant advances in DIS studies with neutrinos we have to work on lowering the **systematics** (there's that word again!)
 - ▼ Beam: Need a higher energy beam and to understand the neutrino flux to within 1% or so.
 - ▼ Detector: Need a detector with excellent acceptance over full Q^2 range.
 - ▼ Detector: Need a detector with improved measurement accuracy of E_μ and θ_μ .
 - ▼ Detector: Need to reduce the error on the E_{hadron} shower measurement.

What's Next.... LBNE (but we have to wait awhile!)



- ◆ Beam energy lower than ME!
- ◆ Uses same double-horn, pion-decay source neutrino beam – not much help there same flux errors!
- ◆ However has the beam power (0.7 MW) and the time is right to employ a new type of neutrino detector to reduce systematics.
 - ▼ 7 ton interaction volume
- ◆ DIS event rates: 5 year ν and 5 year $\bar{\nu}$ yields 20 M ν and 4 M $\bar{\nu}$.
 - ▼ **Statistics fine – systematics!**

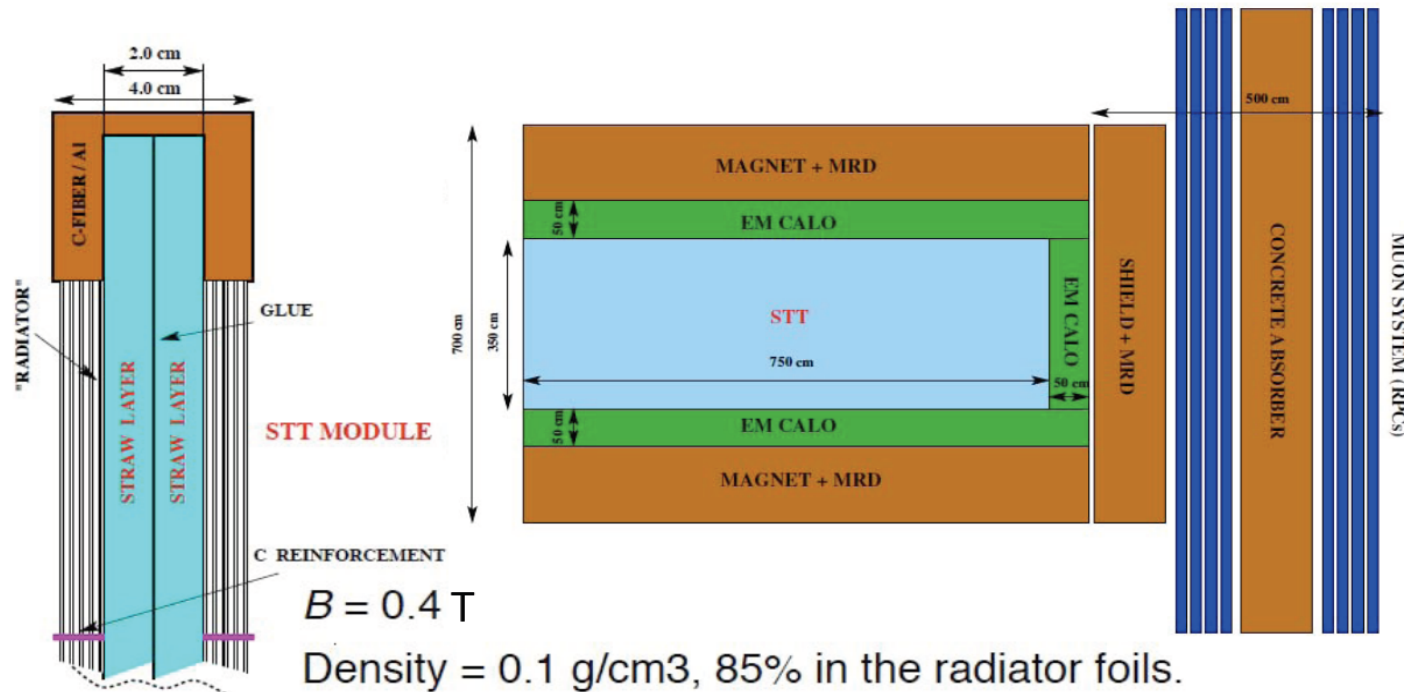
Straw-tube Tracker Design

S. Mishra – Univ. S. Carolina

Electronic Bubble Chamber with 10^9 events

High resolution magnetised detector (*HiResMv*) – LBNE Standard Near detector

Builds on NOMAD experience, ATLAS TRT and COMPASS detector designs



Transition Radiation $\Rightarrow e^-/e^+ \text{ ID} \Rightarrow \gamma$ (w. Kinematics)
 $dE/dx \Rightarrow \text{Proton, } \pi^{+/-}, \text{K}^{+/-} \text{ ID}$
 Magnet/Muon Detector $\Rightarrow \mu^{+/-}$

HiResMv design parameters

◆ *Space point resolution better than 200 μm (in ATLAS 130 μm).*

◆ *Momentum resolution for $\rho = 0.1\text{g}/\text{cm}^3$ and $B = 0.4\text{T}$:*

- *Multiple scattering contribution 0.05 for $L = 1\text{m}$ ($B = 0.4\text{T}$, default radiator)*
- *Measurement error ($B = 0.4\text{T}$)*

$$\frac{\sigma(p)}{p} = \frac{\sigma(x)p}{0.3BL^2} \sqrt{\frac{720}{N+4}}$$

which gives 0.006 for $L = 1\text{m}$ and $p = 1\text{ GeV}/c$ ($N = 50$ if along beam direction)

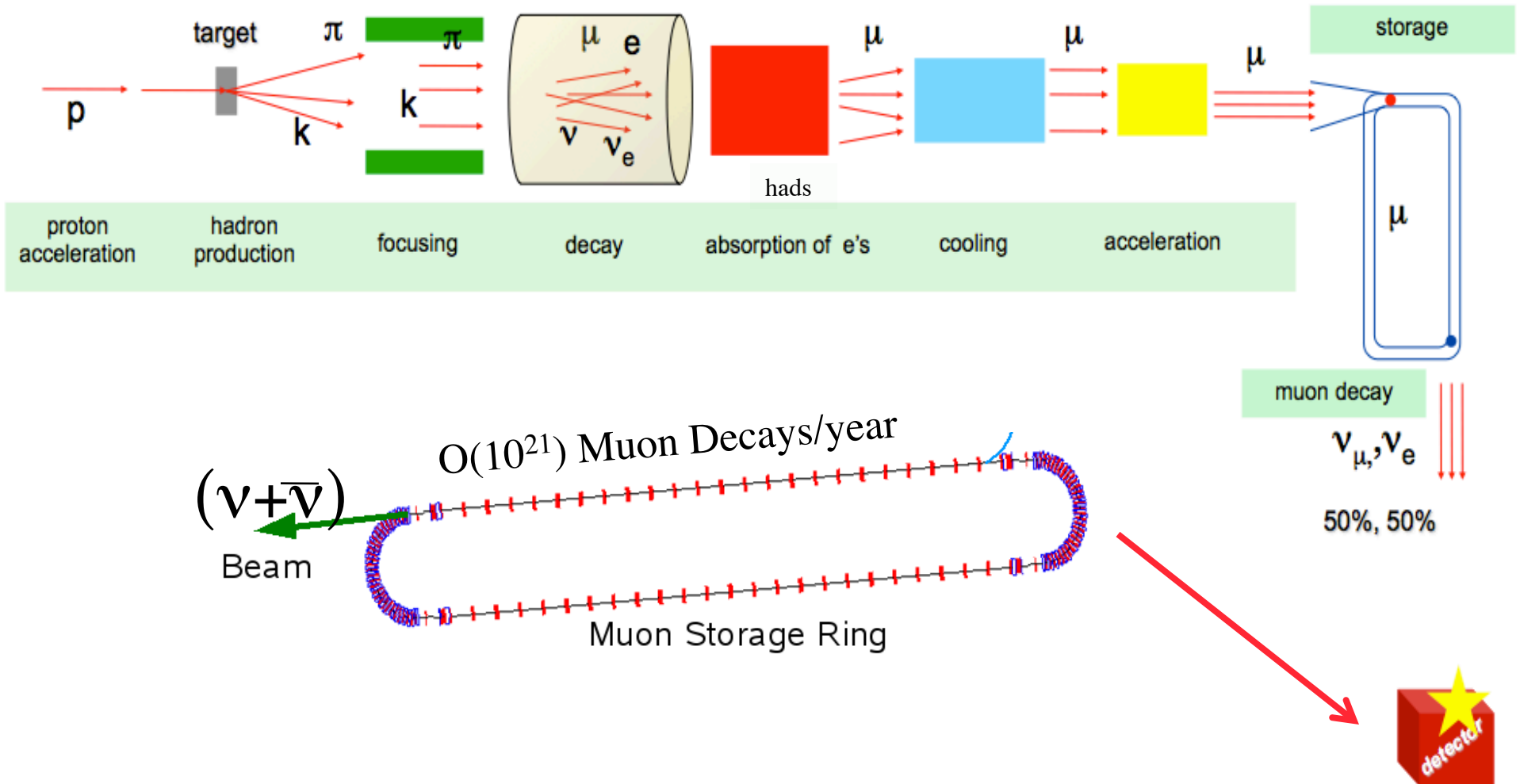
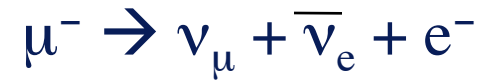
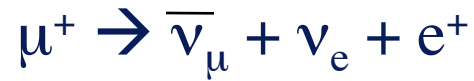
◆ *Full reconstruction of charged particles and γ 's*

◆ *Identify e, π, K, p from dE/dx . Use *Transition Radiation* for electron identification in the whole fiducial volume with Xe filling.*

◆ *Reconstruction of electrons down to 80 MeV from curvature in magnetic field ($B = 0.4\text{T}$)*

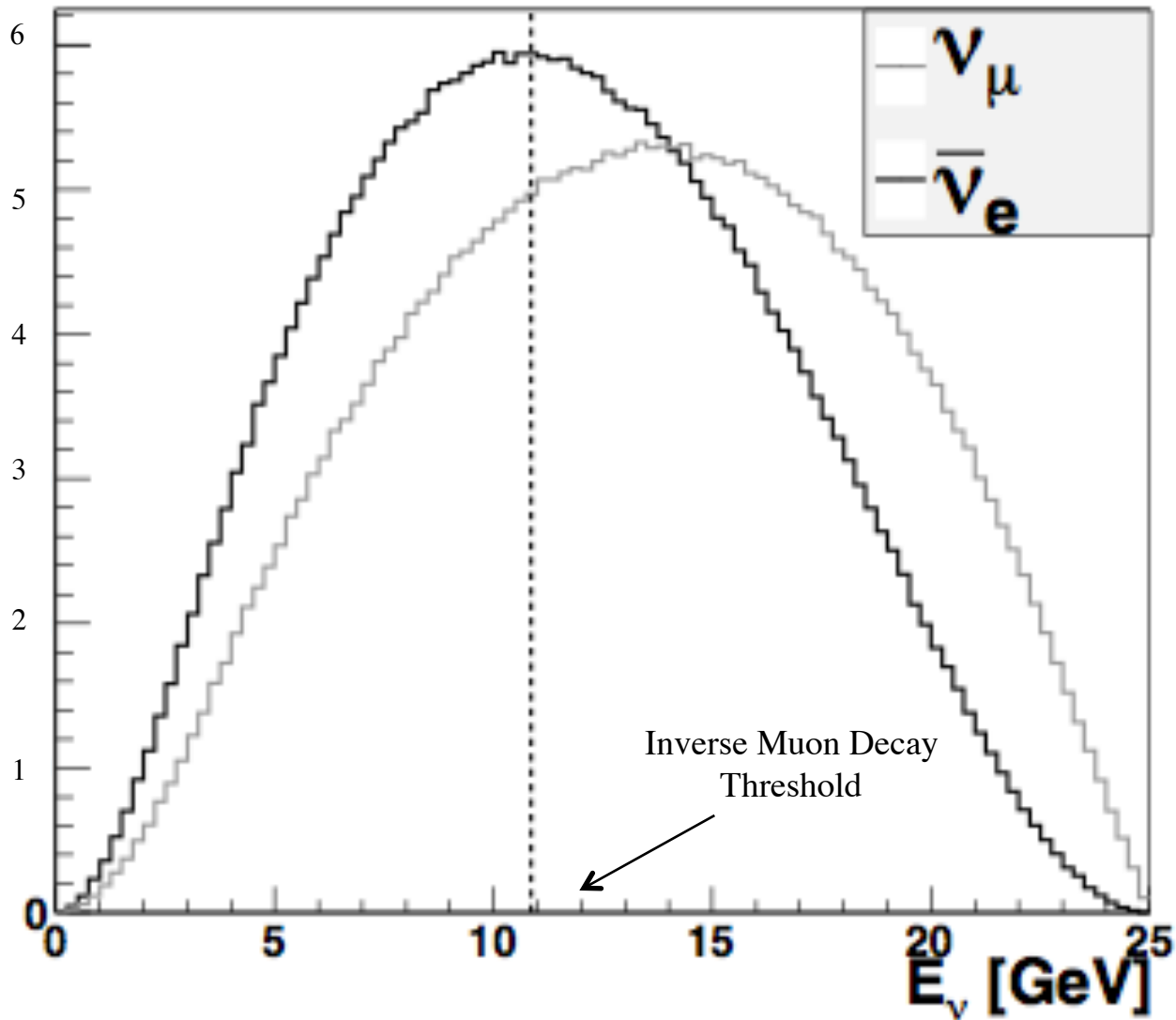
Now - How do we improve the BEAM?

Ultimate ν – DIS Experiment: Neutrino Factory



Example Event Energy Distribution

25 GeV μ^-



Near Detector Design Requirements

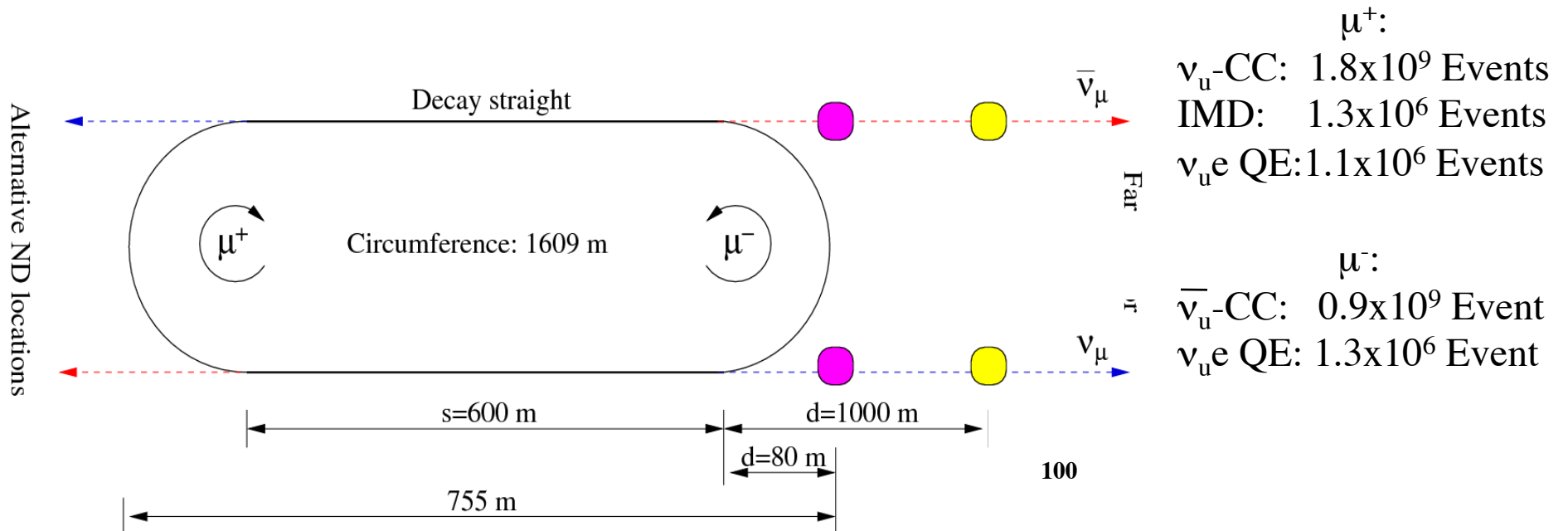
- **Determination of the neutrino flux (through the measurement of neutrino-electron scattering) to $< 1\%$!**
- Magnetic field for muon momentum ($\delta p/p \sim 1\%$)
- Muon catcher and capability for e^+/e^- identification
- **Good resolution on neutrino energy – goal $\delta E/E \sim 1\%$**

Neutrino Factory Near Detector(s)

$E_\mu = 25 \text{ GeV} \pm 80 \text{ MeV}$
 Straight section length = 600 m
 Muon angular spread 0.5 mrad

Event Rates

2.5×10^{20} μ -decays/year
 3 Years of μ^+ and μ^-
 Fid. Mass 3.5 Tons



Summary: Neutrino DIS Now and Soon

- ◆ Currently the most accurate measurement of neutrino DIS scattering is the NuTeV $\nu/\bar{\nu}$ – Fe results.
- ◆ There are inconsistencies between NuTeV and other $\nu/\bar{\nu}$ – Fe results.
- ◆ Absolute rates limited by neutrino flux determinations.
- ◆ Attempts to extract neutrino nuclear correction factors yield a different NCF for $\nu/\bar{\nu}$ – Fe than for ℓ^\pm - Fe.
- ◆ Current generation experiments (MINERvA) can measure NCF off different A. Pb / C could be a few %
- ◆ Next generation detectors can reduce detector systematics to the per cent level.
- ◆ Need Neutrino Factory beams to reach $< 1\%$ errors on the flux.
- ◆ **It could be a bit of a wait for the next great ν DIS experiment.**

Additional Details

Formalism

- ◆ PDF Parameterized at $Q_0 = 1.3$ GeV as

$$xf_i(x, Q_0) = \begin{cases} A_0 x^{A_1} (1-x)^{A_2} e^{A_3 x} (1+e^{A_4 x})^{A_5} & : i = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}, \\ A_0 x^{A_1} (1-x)^{A_2} + (1+A_3 x)(1-x)^{A_4} & : i = \bar{d}/\bar{u}, \end{cases}$$

- ◆ PDFs for a nucleus are constructed as:

$$f_i^A(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{(A-Z)}{A} f_i^{n/A}(x, Q)$$

- ◆ Resulting in nuclear structure functions:

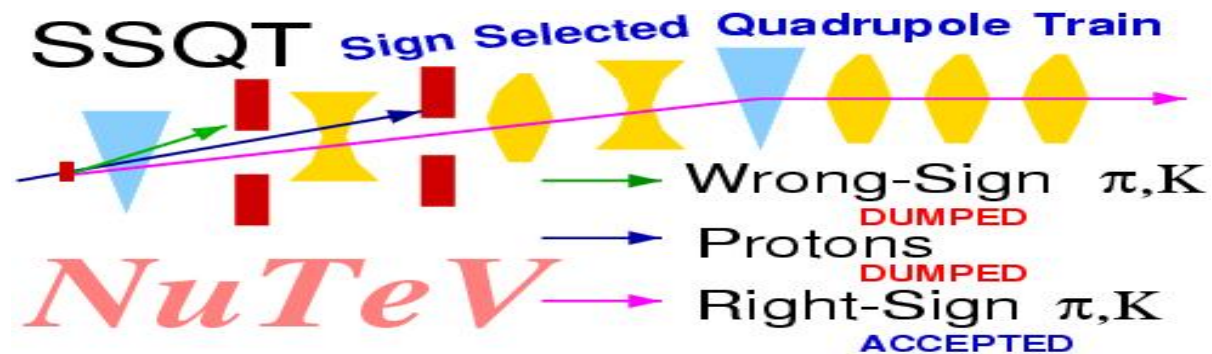
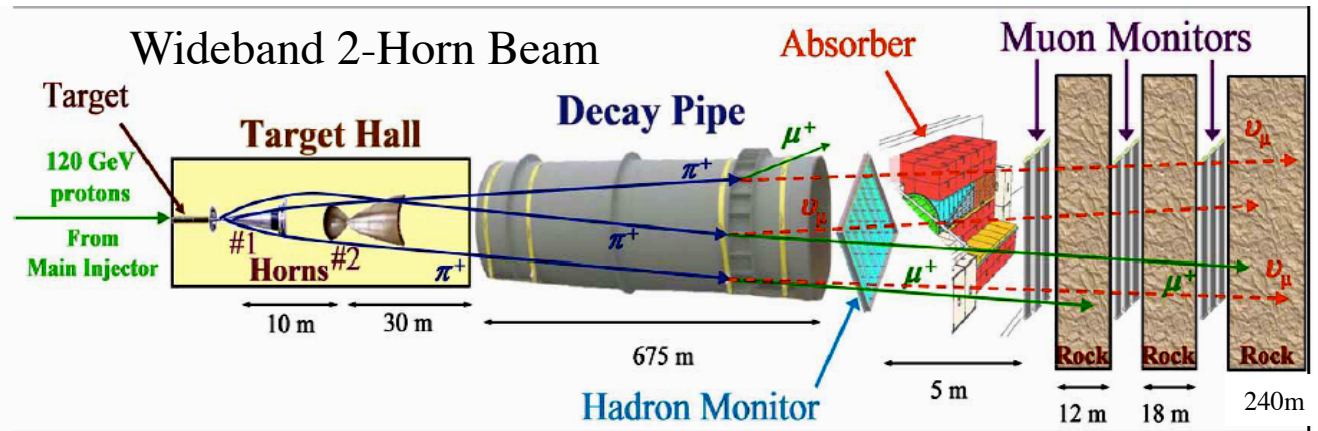
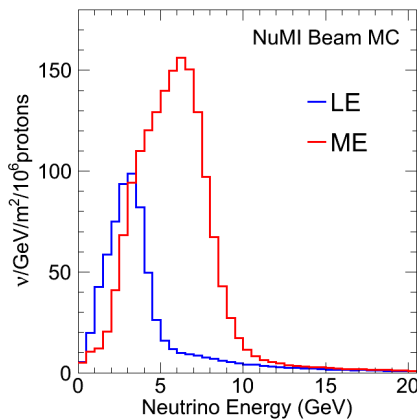
$$F_i^A(x, Q) = \frac{Z}{A} F_i^{p/A}(x, Q) + \frac{(A-Z)}{A} F_i^{n/A}(x, Q)$$

- ◆ The differential cross sections for CC scattering off a nucleus::

$$\begin{aligned} \frac{d^2\sigma}{dx dy} {}^{(\bar{\nu})A} &= \frac{G^2 ME}{\pi} \left[\left(1 - y - \frac{Mxy}{2E}\right) F_2 {}^{(\bar{\nu})A} \right. \\ &\quad \left. + \frac{y^2}{2} 2xF_1 {}^{(\bar{\nu})A} \pm y\left(1 - \frac{y}{2}\right) xF_3 {}^{(\bar{\nu})A} \right] \end{aligned}$$

Neutrino Beamlines

- ◆ Intense proton beam on a target and collect π and K and focus into a decay space.
- ◆ Absorb hadrons and muons leaving only neutrinos.
- ◆ Do not know individual E_ν a priori and absolute flux known to 5-10%.



F_2 and xF_3 Measurement

F_2

$$\left[\frac{d^2\sigma^v}{dx dy} + \frac{d^2\sigma^{\bar{v}}}{dx dy} \right] \frac{\pi}{G_F^2 ME} =$$

$$= 2 \bar{F}_2 \left(1 - y - \frac{Mxy}{2E} + \frac{y^2}{2} \frac{1 + 4M^2 x^2 / Q^2}{1 + R} \right) + y \left(1 - \frac{y}{2} \right) \Delta x F_3$$

- ◆ Perform 1-parameter fit for F_2
- ◆ $\Delta x F_3$ model
- ◆ R_L model

$x F_3$

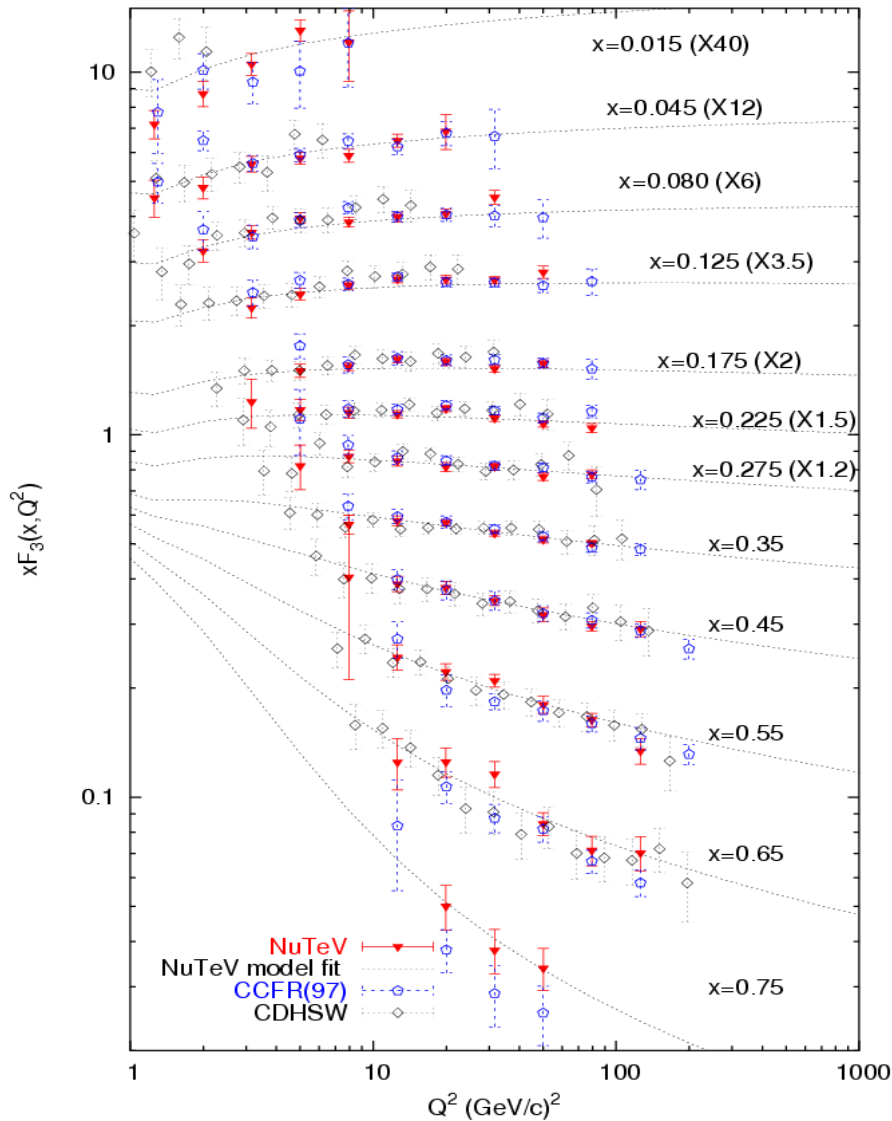
$$\left[\frac{d^2\sigma^v}{dx dy} - \frac{d^2\sigma^{\bar{v}}}{dx dy} \right] \frac{\pi}{G_F^2 ME} =$$

$$= \Delta F_2 \left(1 - y - \frac{Mxy}{2E} + \frac{y^2}{2} \frac{1 + 4M^2 x^2 / Q^2}{1 + R} \right) + 2 y \left(1 - \frac{y}{2} \right) x \bar{F}_3$$

- ◆ Perform 1-parameter fit for $x F_3$
- ◆ ΔF_2 is very small and is neglected

- ◆ Radiative corrections applied
- ◆ Isoscalar correction applied

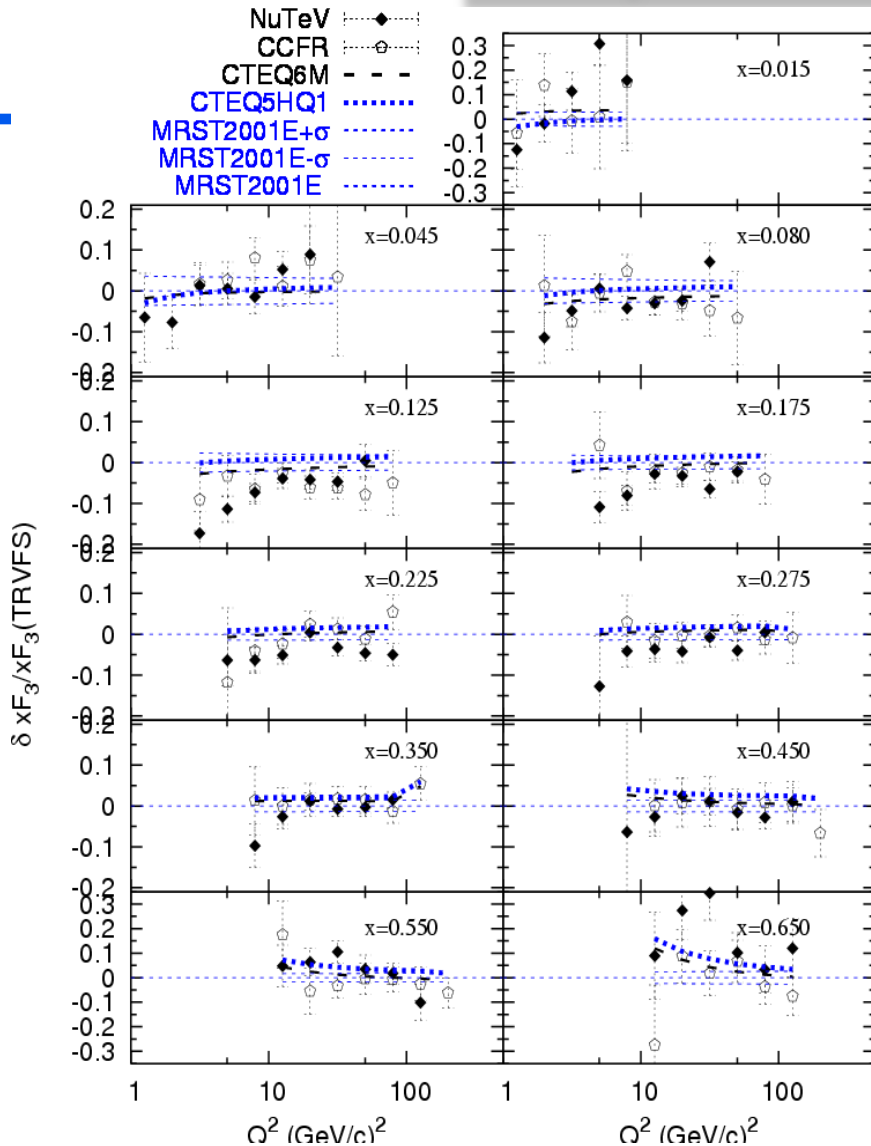
xF_3 Measurement



Martin Tzanov

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- ◆ All systematic uncertainties are included
- ◆ All data sets agree for $x < 0.4$.
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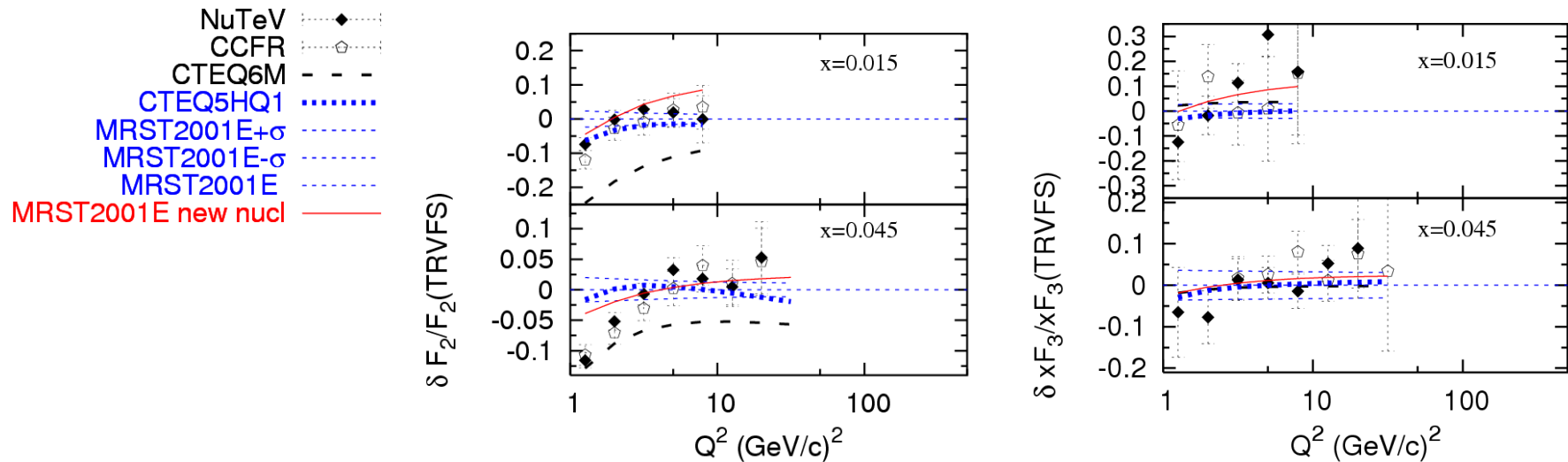
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- target mass
(*H. Georgi and H. D. Politzer, Phys. Rev. D14, 1829*)
- nuclear effects – parameterization from charge lepton data, assumed to be the same for neutrino scattering (no Q^2 dependence added) nuclear effects parameterization is dominated by SLAC (lower Q^2 in this region) data at high-x

- NuTeV xF_3 agrees with theory for medium x.
- At low x different Q^2 dependence.
- At high x ($x > 0.6$) NuTeV is systematically higher.

Comparison with Theory at Low x



- both NuTeV and CCFR agree in level with theory in the shadowing region (except CTEQ6M)

- the red curve is TRVFS(MRST) using the following model for nuclear correction:

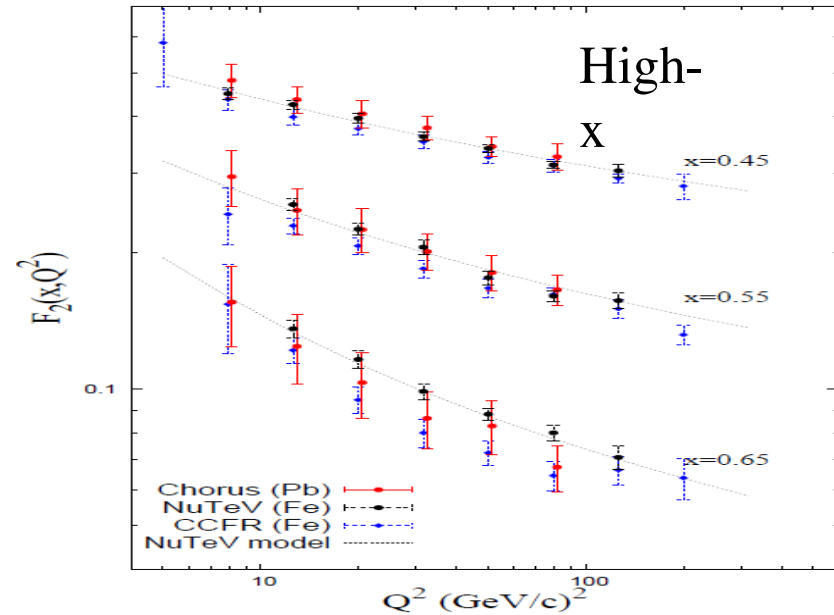
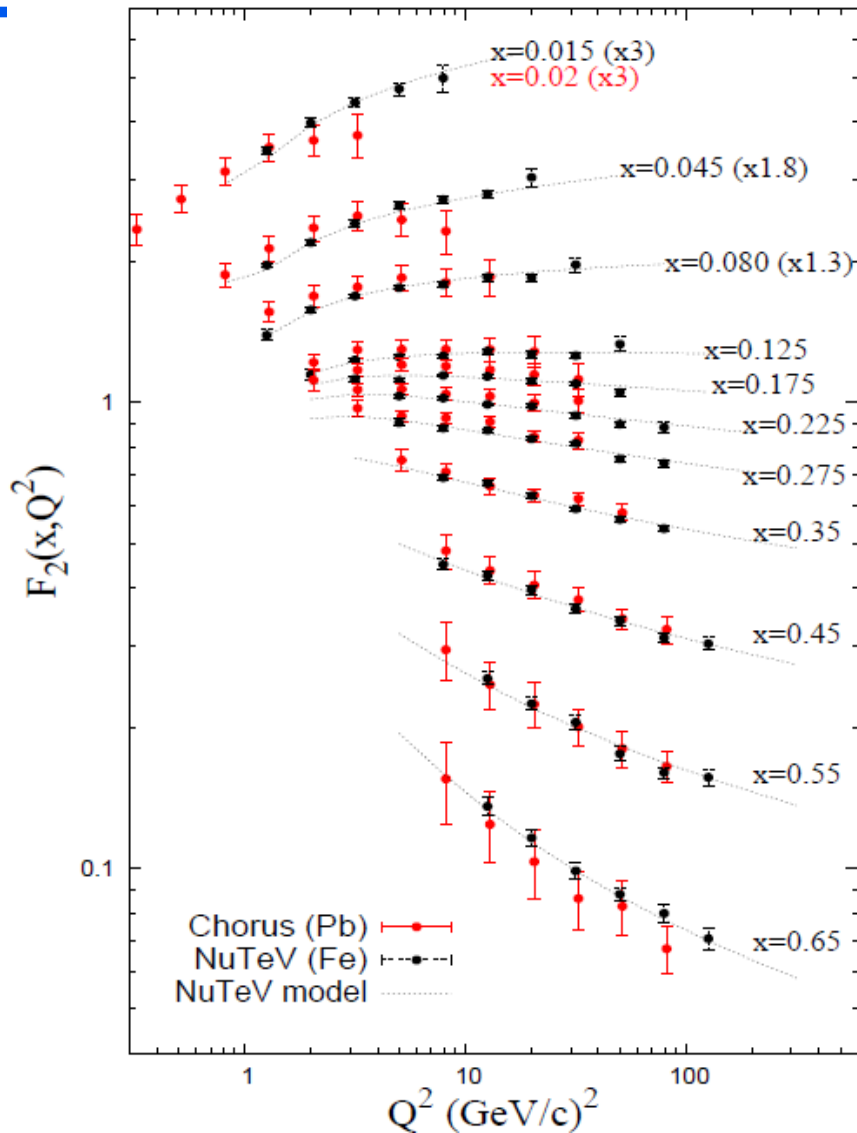
NUCLEAR SHADOWING IN NEUTRINO NUCLEUS DEEPLY INELASTIC SCATTERING.

By Jianwei Qiu, Ivan Vitev (Iowa State U.), Jan 2004. 7pp.

Published in Phys.Lett.B587:52-61,2004

e-Print Archive: hep-ph/0401062

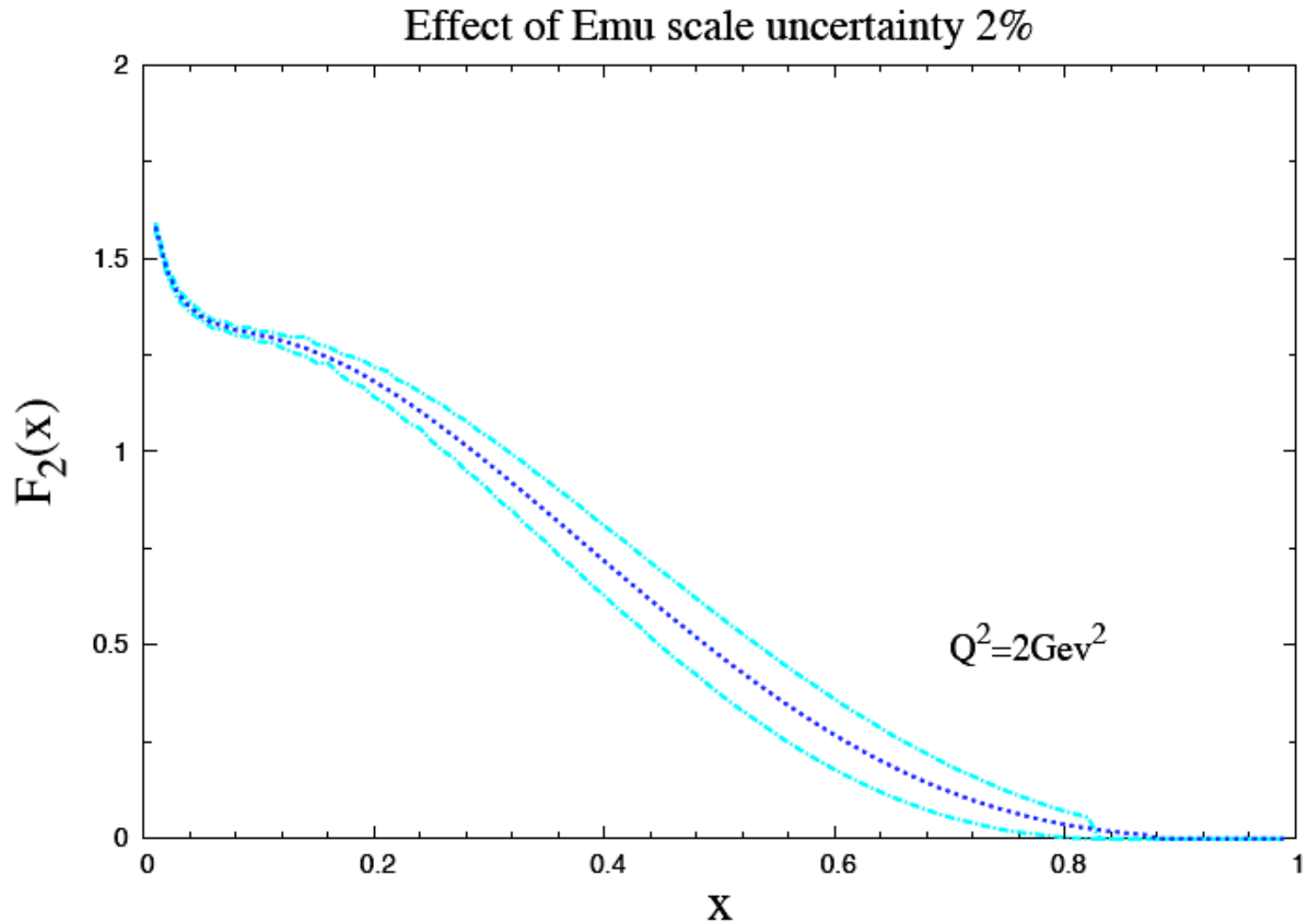
CHORUS (using Pb targets and nuclear emulsions), NuTeV and CCFR F_2 Comparison



- ◆ CHORUS is not as precise,
- ◆ CHORUS agrees well with NuTeV **and** CCFR over the whole range,
- ◆ hint of a different Q^2 shape at low-x
- ◆ This comparison assumes nuclear corrections similar for Fe and Pb.

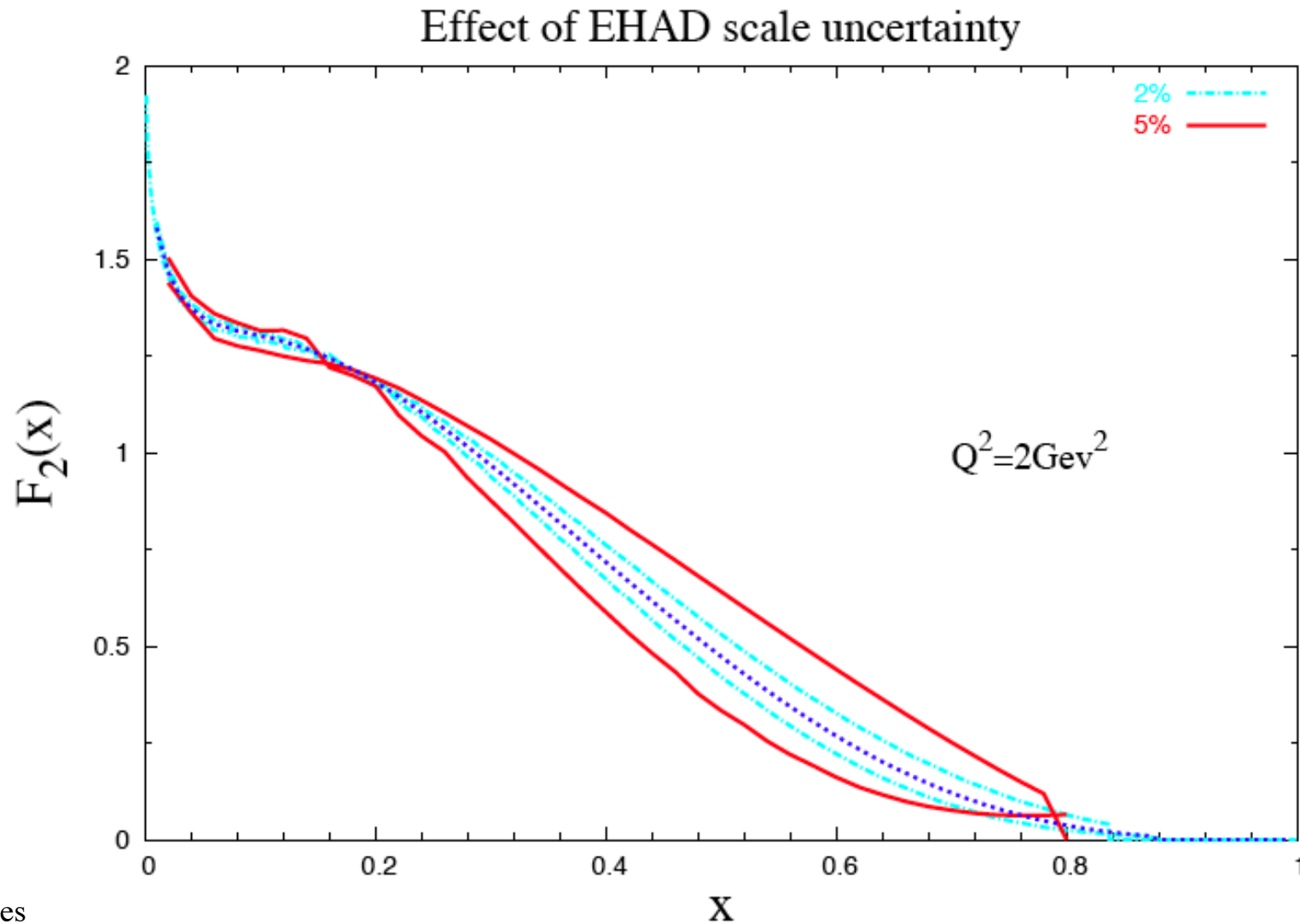
Estimated systematic error: E_μ scale

NuTeV achieved 0.7%

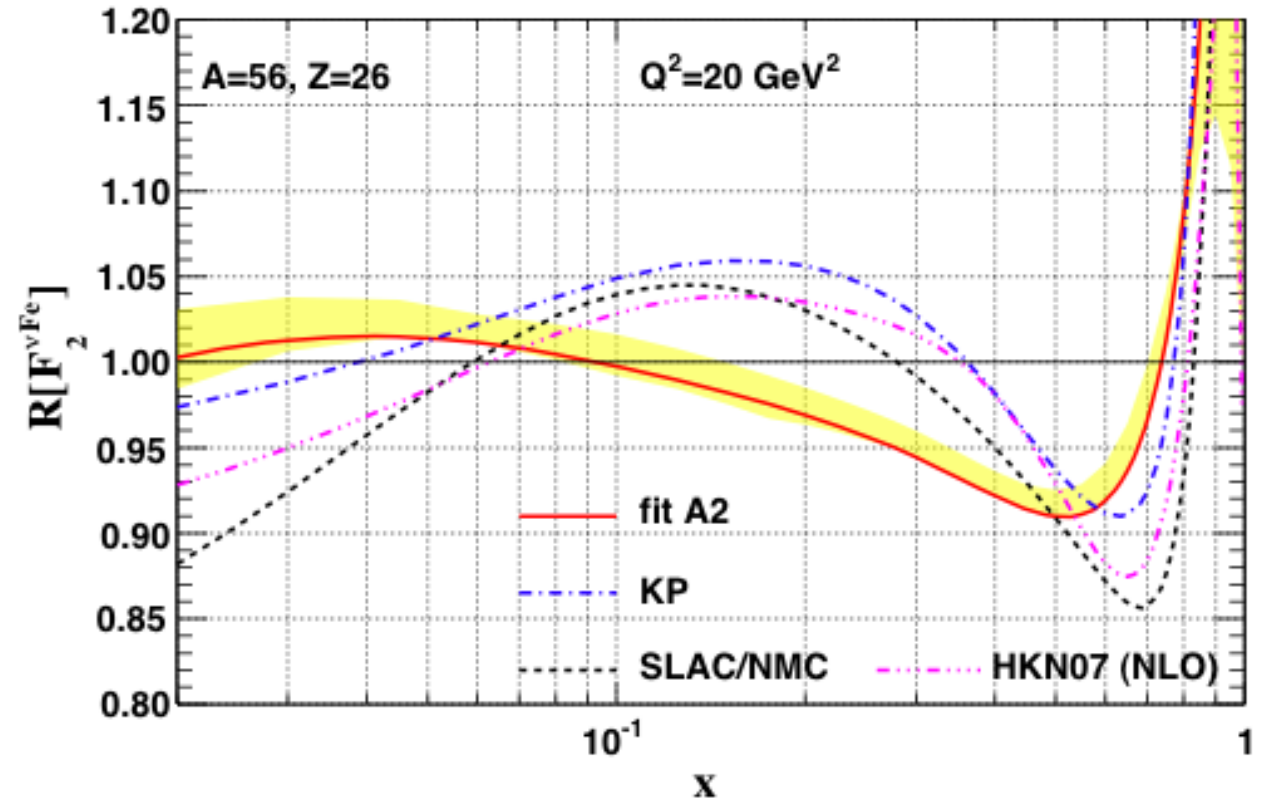
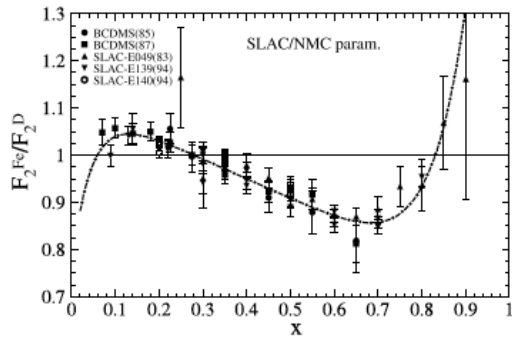


A leading systematic error: E_{had} scale

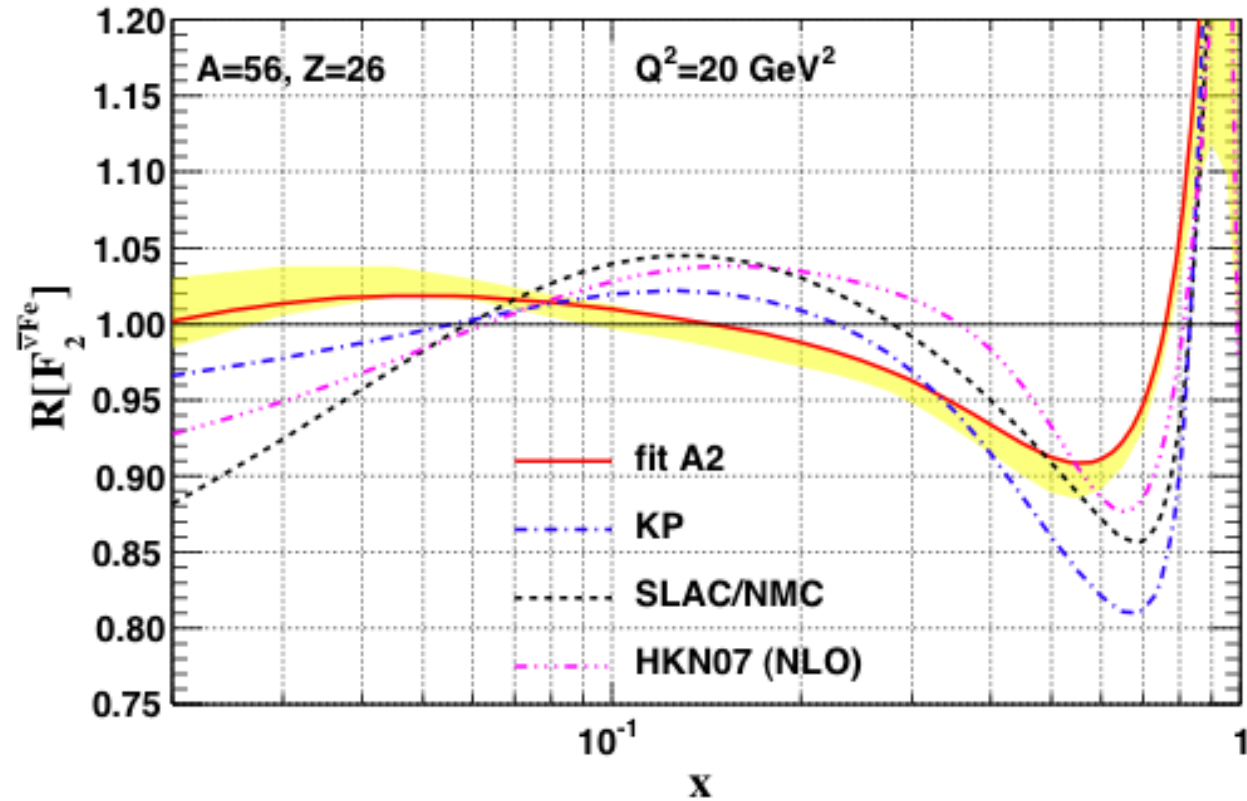
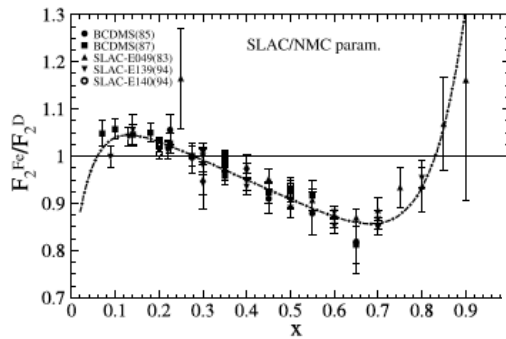
NuTeV achieved 0.43%



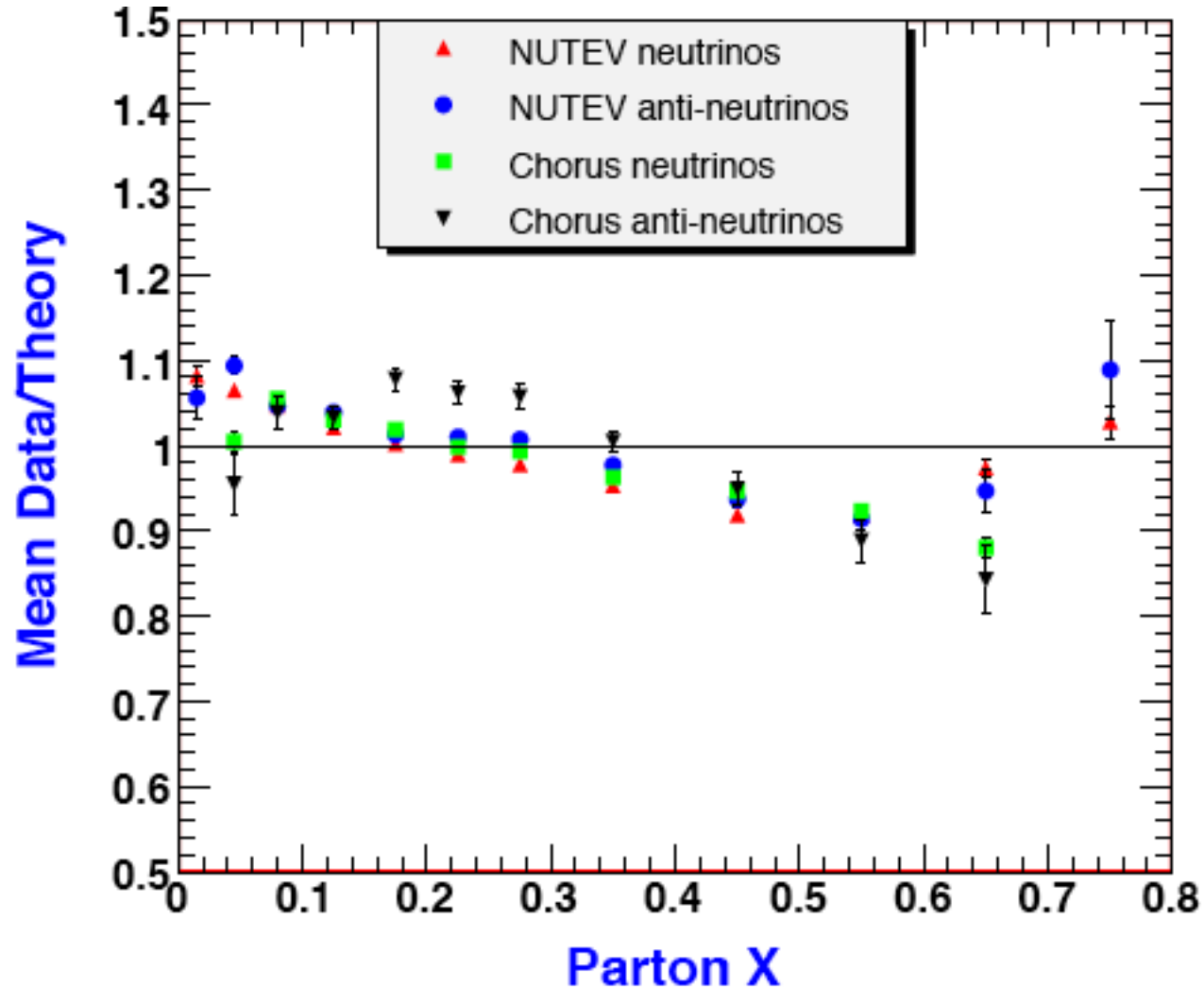
F_2 Structure Function Ratios: ν -Iron



F_2 Structure Function Ratios: $\bar{\nu}$ -Iron



NuTeV(Fe) and CHORUS (Pb) ν scattering (unshifted) results compared to reference fit **no nuclear corrections**

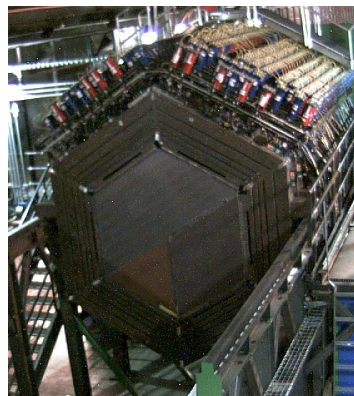


Broad Range of Nuclear Targets

Acceptance for μ 's in MINOS from the nuclear targets...complicated!

- ◆ 5 nuclear targets + water target
- ◆ He target upstream of detector
- ◆ Near million-event samples
(4×10^{20} POT LE beam + 12×10^{20} POT in ME beam)

Target	Mass in tons	CC Produced Events (Million)
Scintillator	3	9
He	0.2	0.6
C (graphite)	0.15	0.4
Fe	0.7	2.0
Pb	0.85	2.5
Water	0.3	0.9

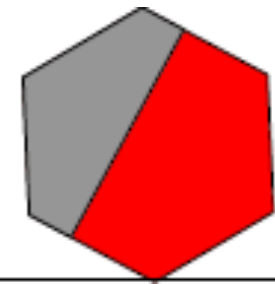
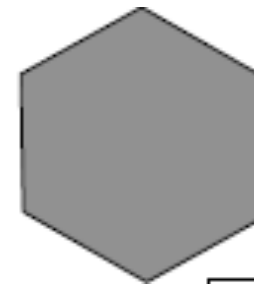
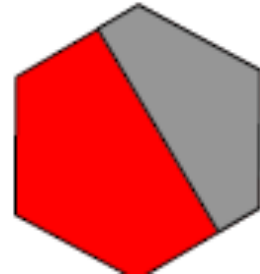
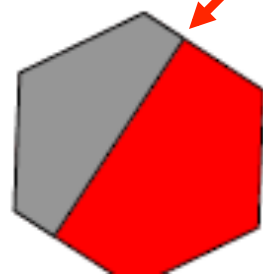


Water target



5 Nuclear Targets

Fe Pb C



High- x Structure Functions & PDFs

ν - p Scattering

$$\left. \begin{aligned} F_2^{\nu p} &= 2x (d + \bar{u} + s) \\ F_2^{\bar{\nu} p} &= 2x (\bar{d} + u + \bar{s}) \end{aligned} \right\} \xrightarrow{\text{At high } x} \boxed{\begin{aligned} \frac{F_2^{\nu p}}{F_2^{\bar{\nu} p}} &\approx \frac{d}{u} \end{aligned}}$$

Add in...

$$\left. \begin{aligned} xF_3^{\nu p} &= 2x (d - \bar{u} + s) \\ xF_3^{\bar{\nu} p} &= 2x (-\bar{d} + u - \bar{s}) \end{aligned} \right\} \longrightarrow \begin{aligned} F_2^{\nu p} - xF_3^{\nu p} &= 4x\bar{u} \\ F_2^{\bar{\nu} p} + xF_3^{\bar{\nu} p} &= 4xu \end{aligned}$$