Subtraction method for parton shower to $2 \rightarrow 2$ matrix element matching in $k_{\perp}$-factorisation

Michal Deák
Universidade de Santiago de Compostela

M. D., F. Hautmann, H. Jung, and K. Kutak, JHEP 09, 121 . 0908.0538; M. D., F. Hautmann, H. Jung, and K. Kutak . 1012.6037
2 \rightarrow 2 \text{ process with uPDF}

- 2\rightarrow 2 \text{ jet production process} - qg^* \rightarrow qg \\
  \text{in } pp \text{ collisions}
2 $\rightarrow$ 2 process with uPDF

- 2$\rightarrow$2 jet production process - \( qg^* \rightarrow qg \) in \( pp \) collisions
- In \( k_\perp \)-factorisation: extra initial transversal momentum
- Important for resummation of small-x logarithms
A problem

Transversal momenta cross sections example

\[
\frac{d^2\sigma(p_f)}{dp_f^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \frac{d^2\hat{\sigma}(\xi_1, \xi_2, p_f)}{dp_f^2} H(\xi_1, \xi_2, \mu^2(p_f))
\]

\[
\frac{d^2\sigma(p_{f1})}{dp_{f1}^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \int_{p_{cut\perp}} d^2 p_{f2} \frac{d^2\hat{\sigma}(\xi_1, \xi_2, p_{f1}, p_{f2})}{dp_{f1}^2 dp_{f2}^2}
\]

\[
\times \frac{\bar{H}(\xi_1, \xi_2, p_{f1} + p_{f2}, \mu^2(p_{f1}, p_{f2}))}{p_{cut\perp}}
\]
A problem

Transversal momenta cross sections example

\[
\frac{d^2\sigma(p_f)}{dp_f^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \frac{d^2\hat{\sigma}(\xi_1, \xi_2, p_f)}{dp_f^2} H(\xi_1, \xi_2, \mu^2(p_f))
\]

\[
\frac{d^2\sigma(p_{f1})}{dp_{f1}^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \int d^2p_{f2} \frac{d^2\hat{\sigma}(\xi_1, \xi_2, p_{f1}, p_{f2})}{dp_{f1}^2 dp_{f2}^2}
\]

\[
\times \tilde{H}(\xi_1, \xi_2, p_{f1} + p_{f2}, \mu^2(p_{f1}, p_{f2}))
\]

Cross section function of cutoff \(p_{cut\perp}\)

3/27/12
A solution

- Remove the reason for the cut-off = regulate the cross section
  - If cut-off necessary: reduction of the previous cut-off dependence

- Remove the cut-off
A solution

• Remove the reason for the cut-off = regulate the cross section
  – If cut-off necessary: reduction of the previous cut-off dependence

• Remove the cut-off
Regularisation

\[ d\sigma_{qq^*\rightarrow qq}^{\text{exact}} \leftrightarrow |\text{leading order exact ME}|^2 \times \text{PhSp} \]

Includes collinear singularities +
finite terms

\[ d\sigma_{qq^*\rightarrow qq}^{0} \leftrightarrow \sum_i |\alpha_s|^1 \text{parton } i \text{ emission}|^2 \times \text{PhSp}_i \]

Includes collinear singularities

\[ d\sigma_{qq^*\rightarrow qq}^{\text{shower}} \leftrightarrow \sum_i |\text{LL parton } i \text{ emission}|^2 \times \text{PhSp}_i \]

Includes collinear singularities +
resummed loop corrections - regular
Regularisation

• First we remove the singular terms

\[ d\sigma_{qg^*\to qg}^{\text{finite}} = d\sigma_{qg^*\to qg}^{\text{exact}} - d\sigma_{qg^*\to qg}^0 \]

• We add the finite terms to the finite shower

\[ d\sigma_{qg^*\to qg}^{\text{corrected}} = d\sigma_{qg^*\to qg}^{\text{shower}} + d\sigma_{qg^*\to qg}^{\text{finite}} \]

• Using Monte Carlo implementation
CCFM equation

- The CCFM equation
  - Includes BFKL kernel
  - Coherence effects
  - Angular ordering
- The equation

\[
\mathcal{F}(x, k, q'^2) = \mathcal{F}(x, k, q_0'^2) + \int_{q_0'^2}^{q^2} \frac{d^2q'}{q'^2} \frac{N_C \alpha_s}{\pi} \left( 1 - \frac{q_0'}{|q'|} \right) \int_x^{\frac{1 - \frac{q_0'}{|q'|}}{z}} \frac{dz}{z} \mathcal{F}(x/z, k', q'^2) \left( \frac{\Delta_{NS}(k'^2, (zq')^2)}{z} + \frac{1}{1 - z} \right) \Delta_S(q'^2, (zq')^2)
\]

M. Ciafaloni, Nucl. Phys. B296, 49 (1988);
S. Catani, F. Fiorani, and G. Marchesini, Nucl. Phys. B336, 18 (1990);
CASCADE

  - Backward evolution algorithm for initial state parton showers for
    - Exact kinematics in each step of the parton shower
    - No difference between parton shower evolved uPDF and CCFM evolved uPDF
    - Gluon chains
    - Valence quarks/Non-singlet uPDFs from one-loop CCFM equation
  - Final state parton showers by Pythia algorithm
  - Hadronisation of partons by the Lund String Model
  - Gluon uPDFs obtained from fits to HERA data
Results

• Transversal momentum of the quark for cut-off 1 and 2 GeV

• Good agreement of different cross sections
Results

- Transversal momentum of the gluon for cut-off 1 and 2 GeV

- Disagreement in the magnitude of the cross sections
Results

- Ratios of the cross sections with cut-offs 1 and 2 GeV

- black dashed - the leading order ME
- red - the result of the subtraction method
Summary and Outlook

• We identified an important problem of cut-off dependence in $kt$-factorisation $2 \rightarrow 2$ process

• We suggest a solution to the problem by regularisation by ME to PS matching

• We make a successful “proof of concept” calculation

• Extension to phenomenology necessary

• Event by event implementation needed to remove the cut-off
Back-Up Slides
Other point of view

\[ d\sigma^{\text{virtual}}_{qq^* \rightarrow qg} = d\sigma^{\text{shower}}_{qq^* \rightarrow qg} - d\sigma^0_{qq^* \rightarrow qg} \]

\[ d\sigma^{\text{corrected}}_{qq^* \rightarrow qg} = d\sigma^{\text{exact}}_{qq^* \rightarrow qg} + d\sigma^{\text{virtual}}_{qq^* \rightarrow qg} \]
Phenomenology results

• Only the $2\to 2$ matrix element