A new approach to gluon-quark multiplicity ratio

Paolo Bolzoni[1]

[1]II. Institut für Theoretische Physik, Luruper Chaussee 149, 22761 Hamburg, Germany

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We present a new approach in considering and including both the perturbative and the nonperturbative contributions to the multiplicity ratio r of gluon and quark jets. The new method is motivated by recent developments in timelike small-x resummation obtained in the $\overline{\mathrm{MS}}$ factorization scheme. A global analysis to fit the available data is also presented.

1 Introduction

The gluon-quark multiplicity ratio is defined as $r = N_g/N_q$, where $N_{g(q)}$ is the number of hadrons produced in a gluon(quark) jet. A purely perturbative and analytical prediction has been achieved by a solution to the equations for the generating functionals in the modified leading logarithmic approximation (MLLA) in Ref.[1] up to the so called N^{3}LOr in the expansion parameter $\gamma_0 = \sqrt{2N_c\alpha_s/\pi}$ *i.e.* γ_0^3. Here the theoretical prediction is about 10% higher than the data at the scale of the Z^0 vector boson and the difference with the data becomes even larger at lower scales. Among the many attempts to predict r numerically, the most successfull refers to numerical sulutions to the coupled system of equations of the generating functionals for the quark (Z_F) and the gluon (Z_G) in the MLLA framework (see *e.g.* [2]). These numerical solutions describe well the data only above at relatively high energies [3, 4, 5]. This shows that the slope of the multiplicity ratio predicted by this approach tends to be smaller than its experimental value. An alternative approach was given in Ref. [6] where equations for the derivative of the ratio of the multiplicities are obtained in the MLLA within the framework of the colour dipole model. There a constant of integration which encodes nonperturbative contributions is fixed by the data. Here a new approach is presented.

2 The multiplicity ratio in the effective-ω approach

We consider the standard Mellin-space moments of the coupled gluon-singlet system whose evolution in the scale μ^2 is governed in QCD by the DGLAP equations:

$$\mu^2\frac{d}{d\mu^2}\begin{pmatrix} D_g \\ D_s \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix}\begin{pmatrix} D_g \\ D_s \end{pmatrix}. \tag{1}$$

The timelike splitting functions P_{ij} can be computed perturbatively in the strong coupling constant:

$$P_{ij}(\omega,\mu^2) = \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)P_{ij}^{(0)}(\omega)+\left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 P_{ij}^{(1)}(\omega)+\left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^3 P_{ij}^{(2)}(\omega)+O(\alpha_s^4), \quad i,j = g,q, \tag{2}$$

where $\omega = N - 1$ with N the usual Mellin conjugate variable to the fraction of longitudinal momentum x. The functions $P_{ij}^{(k)}(\omega)$ with $k = 0, 1, 2$ appearing in Eq.(2) in the $\overline{MS}$ scheme can be found in Ref.[7, 8, 9] up to NNLO and in Ref.[10] the NNLL contributions up to $O(\alpha_s^{16})$ in the same scheme. Fully analytical resummed results in a closed form in the $\overline{MS}$ scheme are known at NLL for the eigenvalues of the siglet-gluon matrix [10, 11].

It would be desiderable to fully diagonalize Eq.(1). However in general this is not possible because the contributions to the splitting function matrix do not commute at different orders. One is hence enforced to write a series expansion about the LO which in turn can be diagonalized. Therefore we start choosing a basis where the LO is diagonal (see *e.g.* [12]) with the timelike splitting function matrix taking the form:

$$P(\omega) = \begin{pmatrix} P_{++}(\omega) & P_{+-}(\omega) \\ P_{-+}(\omega) & P_{--}(\omega) \end{pmatrix}, \tag{3}$$

where by definition

$$P_{+-}^{(0)}(\omega) = P_{-+}^{(0)}(\omega) = 0, \tag{4}$$

and where $P_{\pm\pm}^{(0)}(\omega)$ are the eigenvalues of the LO splitting matrix.

Now relating the (g, s) basis to this new $(+, -)$ basis, we can decompose the singlet and the gluon fragmentation function symbolically in the following way:

$$D_a(\omega, \mu^2) = D_a^+(\omega, \mu^2) + D_a^-(\omega, \mu^2); \qquad a = s, g. \tag{5}$$

According to Eq.(1) the plus and minus components have the form

$$D_a^\pm(\omega, \mu^2) = \tilde{D}_a^\pm(\omega, \mu_0^2) \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right]^{-\frac{P_{\pm\pm}^{(0)}}{2\beta_0}} H_a^\pm(\omega, \mu^2), \tag{6}$$

where the normalization factors $\tilde{D}_a^\pm(\omega, \mu_0^2)$ satisfy

$$\tilde{D}_g^+(\omega, \mu_0^2) = -\frac{\alpha_\omega}{\epsilon_\omega}\, \tilde{D}_s^+(\omega, \mu_0^2)\,; \qquad \tilde{D}_g^-(\omega, \mu_0^2) = \frac{1 - \alpha_\omega}{\epsilon_\omega}\, \tilde{D}_s^-(\omega, \mu_0^2), \tag{7}$$

with

$$\alpha_\omega = \frac{P_{qq}^{(0)}(\omega) - P_{++}^{(0)}(\omega)}{P_{--}^{(0)}(\omega) - P_{++}^{(0)}(\omega)}, \quad \epsilon_\omega = \frac{P_{gq}^{(0)}(\omega)}{P_{--}^{(0)}(\omega) - P_{++}^{(0)}(\omega)}. \tag{8}$$

The perturbative functions $H_a^\pm(\omega, \mu^2)$ in Eq.(6) up to NNLO may be represented as

$$\begin{aligned} H_a^\pm(\omega, \mu^2) \quad &= \quad 1 + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)\left(Z_{\pm\pm,a}^{(1)}(\omega) - Z_{\pm\mp,a}^{(1)}(\omega)\right) \\ &\quad + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 \left(\tilde{Z}_{\pm\pm,a}^{(2)}(\omega) - \tilde{Z}_{\pm\mp,a}^{(2)}(\omega)\right), \end{aligned} \tag{9}$$

where the functions $Z_{\pm\pm,a}^{(1)}$, $Z_{\pm\mp,a}^{(1)}$, $\tilde{Z}_{\pm\pm,a}^{(2)}$ and $\tilde{Z}_{\pm\mp,a}^{(2)}$ with $a = g, s$ in terms of the timelike splitting functions up to NNLO in the $(+, -)$ basis are given by:

$$Z_{\pm\pm,s}^{(1)}(\omega) \quad = \quad Z_{\pm\pm,g}^{(1)}(\omega) = \frac{1}{2\beta_0}\left[P_{\pm\pm}^{(1)}(\omega) - P_{\pm\pm}^{(0)}(\omega)\frac{\beta_1}{\beta_0}\right], \tag{10}$$

$$Z_{\pm\mp,s}^{(1)}(\omega) \quad = \quad \frac{P_{\pm\mp}^{(1)}(\omega)}{2\beta_0 + P_{\pm\pm}^{(0)}(\omega) - P_{\mp\mp}^{(0)}(\omega)}, \quad Z_{\pm\mp,g}^{(1)}(\omega) = Z_{\pm\mp,s}^{(1)}(\omega)\, \frac{P_{qq}^{(0)}(\omega) - P_{\mp\mp}^{(0)}(\omega)}{P_{qq}^{(0)}(\omega) - P_{\pm\pm}^{(0)}(\omega)},$$

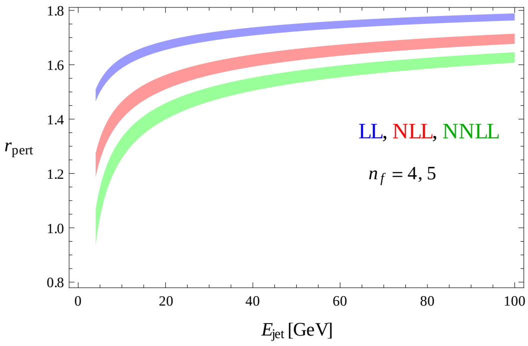

Figure 1: $r_{pert}^{N^kLL}(Q^2)$ with $k = 0,1,2$ (blue,red,green) according to Eq.(15). The bands correspond to $n_f = 4,5$.

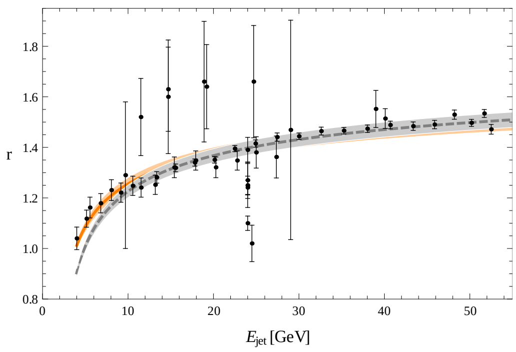

Figure 2: The multilicity ratio r according to Eq.(14) with $K(n_f = 4) = 0.941 \pm 0.019$ (orange) and $K(n_f = 5) = 0.978 \pm 0.020$ (dashed grey).

and

$$\begin{aligned}
\tilde{Z}^{(2)}_{\pm\pm,s}(\omega) &= \tilde{Z}^{(2)}_{\pm\pm,g}(\omega) = \frac{1}{4\beta_0}\Bigg[P^{(2)}_{\pm\pm}(\omega) - \left(P^{(1)}_{\pm\pm}(\omega) - P^{(0)}_{\pm\pm}(\omega)Z^{(1)}_{\pm\pm,s}(\omega)\right)\frac{\beta_1}{\beta_0} \\
&\quad + P^{(0)}_{\pm\pm}(\omega)\left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right) - \sum_{i=\pm} P^{(1)}_{\pm i}(\omega)Z^{(1)}_{i\pm,s}(\omega)\Bigg] + \sum_{i=\pm} Z^{(1)}_{\pm i,s}(\omega)Z^{(1)}_{i\pm,s}(\omega), \\
\tilde{Z}^{(2)}_{\pm\mp,s}(\omega) &= \frac{1}{4\beta_0 + P^{(0)}_{\pm\pm}(\omega) - P^{(2)}_{\mp\mp}(\omega)}\Bigg[P^{(2)}_{\pm\mp}(\omega) - \left(P^{(1)}_{\pm\mp}(\omega) - P^{(0)}_{\pm\pm}(\omega)Z^{(1)}_{\pm\mp,s}(\omega)\right)\frac{\beta_1}{\beta_0} \\
&\quad - \sum_{i=\pm} P^{(1)}_{\pm i}(\omega)Z^{(1)}_{i\mp,s}(\omega)\Bigg] + \sum_{i=\pm} Z^{(1)}_{\pm i,s}(\omega)Z^{(1)}_{i\mp,s}(\omega), \\
\tilde{Z}^{(2)}_{\pm\mp,g}(\omega) &= \tilde{Z}^{(2)}_{\pm\mp,s}(\omega)\,\frac{P^{(0)}_{qq}(\omega) - P^{(0)}_{\mp\mp}(\omega)}{P^{(0)}_{qq}(\omega) - P^{(0)}_{\pm\pm}(\omega)}.
\end{aligned} \tag{11}$$

It is a well known fact that the multiplicity can be obtained from the DGLAP evolution equations Eq.(1) once one is able to take its first Mellin moment $\omega = N - 1 = 0$. This is not possible using a fixed order computation because of the presence of singularitues at $\omega = 0$ due to multiple soft emissions. Resummation of these divergences has been shown to be the appropriate thing to do to avoid this problem. This has been shown a long time ago in [13] at leading logarithmic accuracy (LL). The algebraic relations in Ref. [14] show that the first Mellin moment of the resummed leading logarithmic splitting function $P^{LL}_{++}(\omega)$ can be obtained by taking the LO leading singular term and assign an effective value to ω:

$$P^{LL}_{++}(\omega = 0) = \frac{\alpha_s C_A}{\pi \omega^{LL}_{eff}}; \quad \omega^{LL}_{eff} = 2\,P^{LL}_{++}(\omega = 0) = \sqrt{\frac{2C_A\alpha_s}{\pi}} = 1.382\,\sqrt{\alpha_s} \tag{12}$$

Our approach consists in adopting the same procedure also to fix $P^{NLL}_{++}(\omega = 0)$ and $P^{NNLL}_{++}(\omega = 0)$. The former quantity is analytically known [10, 11], while the last one is known up to the 16th order [10]. In this case we have obtained a numerical estimation of the first Mellin moment of $P_{++}^{NNLL}(\omega)$ performing a numerical extrapolation. Our result is:

$$\omega_{eff}^{NNLL} = 1.3820\sqrt{\alpha_s} + (0.0059\,n_f + 0.8754)\alpha_s + (0.0300\,n_f + 1.0881)\alpha_s^{3/2}, \qquad (13)$$

which is valid for $n_f = 4, 5$ number of active flavors.

Neglecting the evolution of the minus component in Eq.(5) and using Eq.(7) we arrive at our definition of the gluon-quark multiplocity ratio which is given by

$$r^{N^kLL}(Q^2) \equiv \frac{D_g(\omega_{eff}^{N^kLL}, Q^2)}{D_s(\omega_{eff}^{N^kLL}, Q^2)} = K\, r_{pert}^{N^kLL}(Q^2), \qquad (14)$$

where

$$r_{pert}^{N^kLL}(Q^2) = \frac{D_g^+(\omega_{eff}^{N^kLL}, Q^2)}{D_s^+(\omega_{eff}^{N^kLL}, Q^2)} = -\frac{\alpha_\omega}{\epsilon_\omega}\frac{H_g^+(\omega_{eff}^{N^kLL}, Q^2)}{H_s^+(\omega_{eff}^{N^kLL}, Q^2)}; \quad K = \frac{D_s^+(\omega_{eff}^{N^kLL}, Q^2)}{D_s^+(\omega_{eff}^{N^kLL}, Q^2) + \bar{D}_s}, \qquad (15)$$

by use of Eq.(6). Fig.1 shows our results for $r_{pert}^{LL}(Q^2)$,$r_{pert}^{NLL}(Q^2)$ and $r_{pert}^{NNLL}(Q^2)$ for $n_f = 4, 5$ and Fig.2 shows our 90% C.L. fit of K in Eqs.(14,15) using the NNLL result for r_{pert}. In our analysis we have used the first three terms of the ω expansion for the splitting functions and the double counted terms due to resummation have been subtracted. The running of α_s has been evaluated at NNLO with $n_f = 5$ and with $\alpha_s(M_Z) = 0.118$. The data are taken from the summary tables of [15] and references therein and from [16].

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