A NEW APPROACH TO GLUON-QUARK MULTIPLICITY RATIO

1. INTRODUCTION and HISTORICAL OVERVIEW
2. DIAGONALIZATION PROPERTIES
3. EFFECTIVE-N APPROACH TO $r=N_g/N_q$
4. RESULTS and ANALYSIS
5. CONCLUSIONS

Universität Hamburg

Deutsche Forschungsgemeinschaft

Paolo Bolzoni – DIS 2012 , Bonn, 26-30 Mar. 2012

1. INTRODUCTION and HISTORICAL OVERVIEW

THE DIFFERENCE BETWEEN A GLUON AND A QUARK JET

QCD yelds results on partons and not hadrons

An assumption about hadronization always must be made to compare exp. with th.

We assume local parton-hadron duality (LPHD): parton distributions are simply renormalized in the process of hadronization without changing their shape

[Ya.I.Azimov,Yu.L.Dokshitzer,V.A.Khoze,S.I.Troyan '85]

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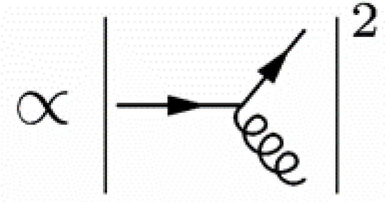
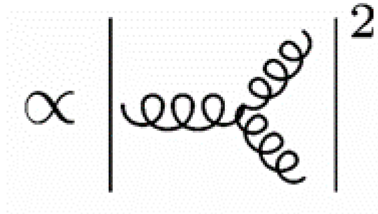
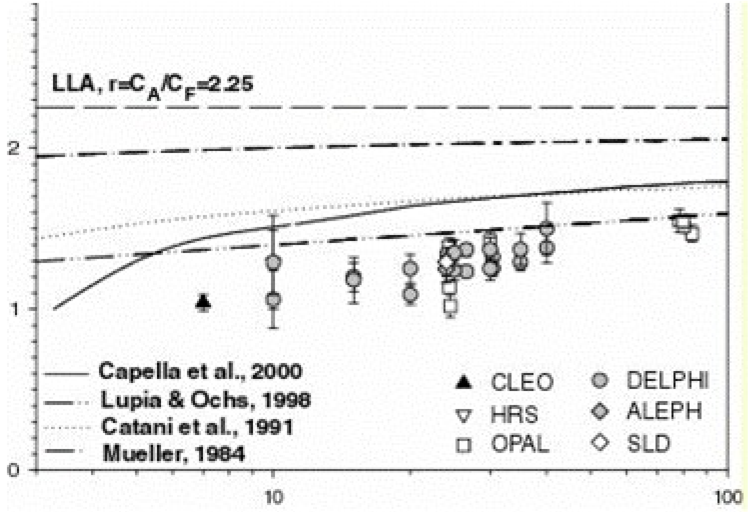
$C_F \propto$  ^{2}

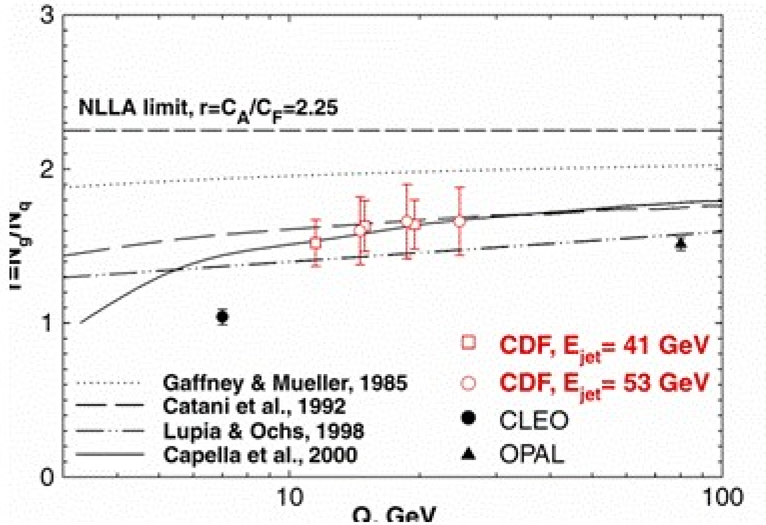
$C_A \propto$  ^{2}

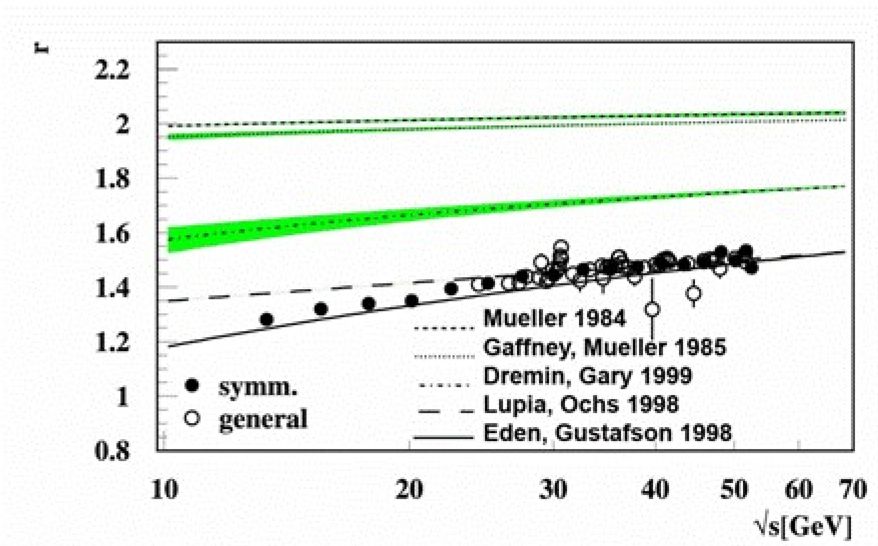
$$\frac{N_g}{N_q} = r_{naive} = \frac{C_A}{C_F} = 2.25$$

However the difference between a gluon and a quark jet is much less than that…

HISTORY OF DATA and THEORY PREDICTIONS







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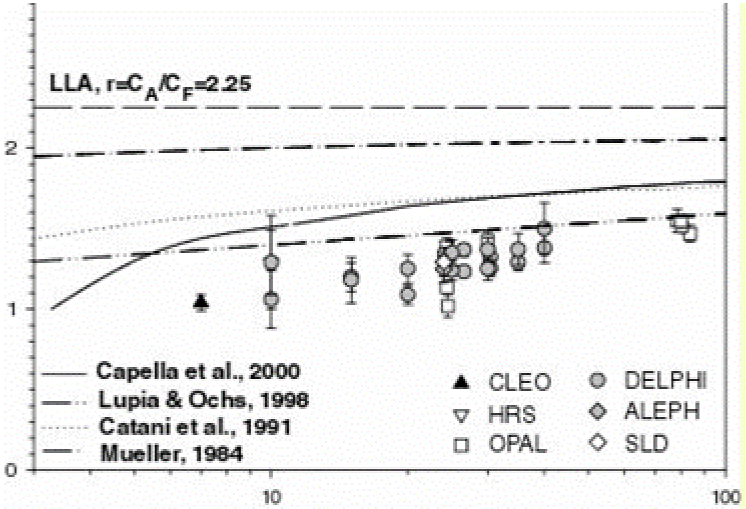
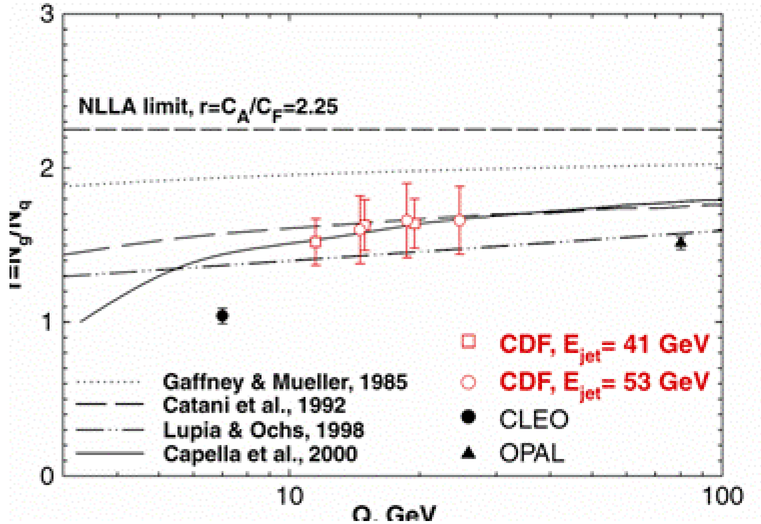
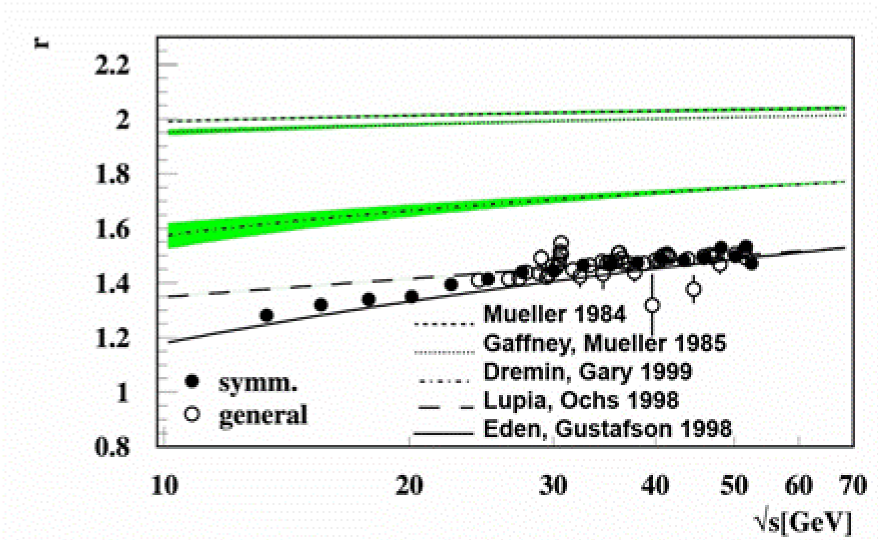




$$r = N_g/N_q = 2.25 - 0.616\sqrt{\alpha_s} - 2.192\alpha_s + 0.243\alpha_s^{3/2} + \ldots$$

$n_f = 5$



NNLOr(complete),NNNLOr
[Dremin,Gary,Capella,Nechitailo,Van '99]

NLOr(MLLA), NNLOr (incomplete) [Gaffney,Mueller '85]

2. DIAGONALIZATION PROPERTIES

DIAGONALIZATION OF THE SINGLET SYSTEM

$$\frac{d}{d \ln \mu^2} \begin{pmatrix} D_s \\ D_g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix} \begin{pmatrix} D_s \\ D_g \end{pmatrix}$$

$$P_{ij} = \left(\frac{\alpha_s(\mu^2)}{4\pi}\right) P_{ij}^{LO} + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 P_{ij}^{NLO} + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^3 P_{ij}^{NNLO} + \dots$$

Splitting functions matrix known up to three loops

[M.Glueck,E.Reya,A.Vogt '93; S.Moch,A.Vogt '08; A.A.Almasy,S.Moch,A.Vogt '11]

A nice thing would be to fully diagonalize this equation at all orders

This is not possible because the splitting function matricies of different orders do not commute

One is hence forced to write a series expansion around the LO which in turn can be diagonalized

DIAGONALIZATION OF THE SINGLET SYSTEM cntd.

We move to a basis where the LO splitting function matrix is diagonal

[A.J.Buras '80]

$$P = \begin{pmatrix} P_{++} & P_{+-} \\ P_{-+} & P_{--} \end{pmatrix} \qquad P_{-+}^{LO} = P_{+-}^{LO} = 0$$

After exact LO diagonalization the sea-qaurk and gluon densities have form

$$D_a = D_a^+ + D_a^- \qquad (a = s, g)$$

With the '+' and '-' evolution taking the form

$$D_a^{\pm}(\mu^2) = \tilde{D}_a^{\pm}(\mu_0^2) \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right]^{-\frac{P_{\pm\pm}^{LO}}{2\beta_0}} H_a^{\pm}(\mu^2)$$

$H_a^{\pm}(\mu^2)$ **are perturbative functions:** $H_a^{\pm}(\mu^2) = \delta_{\pm,+} + \alpha_s(\mu^2)/4\pi H_a^{(1)\pm} + \dots$

$\tilde{D}_a^{\pm}(\mu_0^2)$ **are normalization coefficients carrying also non-perturbative information**

$$\tilde{D}_g^+ = \left[\left(P_{++}^{LO} - P_{qq}^{LO}\right)/P_{qg}^{LO}\right]\, \tilde{D}_q^+$$

3. EFFECTIVE-N APPROACH TO $r=N_g/N_q$

EFFECTIVE-N AT LO

Consider the bahaviour for N $\sim$ 1 of the LO and the LL resummed quantities

$$P_{++}^{LO} = \frac{\alpha_s C_A}{\pi(N-1)} + \text{reg.terms}$$

$$\begin{aligned} P_{++}^{LL} &= \frac{1}{4}[-(N-1) + \sqrt{(N-1)^2 + 8C_A\alpha_s/\pi}] \\ &= \frac{\alpha_s C_A}{\pi(N-1)} + \ldots = \\ &= \frac{\alpha_s C_A}{\pi(N-1+2P_{++}^{LL})} \Rightarrow \quad P_{++}^{LL}(1) = \frac{\alpha_s C_A}{\pi(N_{\text{eff}}^{LO}-1)} \end{aligned}$$

Algebraic relation
by Mueller 1983

$$N_{\text{eff}}^{LO} = 1 + 2P_{++}^{LL}(1) = 1 + \sqrt{\frac{2C_A\alpha_s}{\pi}} = 1 + 1.38\sqrt{\alpha_s}$$

EFFECTIVE-N APPROACH TO GLUON-QUARK MULT. RATIO

$$\widehat{r}(N,Q^2) \equiv \frac{D_g(N,Q^2)}{D_s(N,Q^2)}$$

To get the mult. ratio one has to evaluate the first Mellin moment but it is possible only after resummation. Our proposal is to achieve it by a proper choice of $N=N_{eff}$

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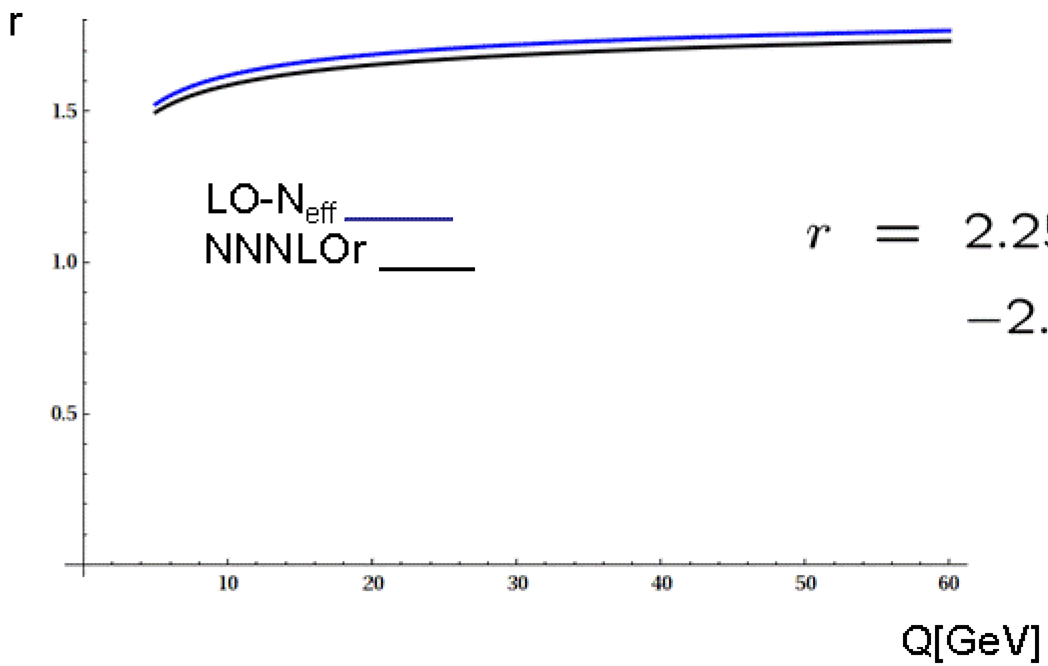
At LO the '-' component is absent for vanishing (N-1)

$$r^{LO}(Q^2) = \hat{r}^{LO}(N_{\text{eff}}^{LO},Q^2) = \frac{\tilde{D}_g^{+}(N_{\text{eff}}^{LO},Q^2)}{\tilde{D}_q^{+}(N_{\text{eff}}^{LO},Q^2)} = \left[\left(P_{++}^{LO} - P_{qq}^{LO}\right)/P_{qg}^{LO}\right]_{N_{\text{eff}}^{LO}}$$



$$r = 2.25 - 0.616\sqrt{\alpha_s} + -2.460\alpha_s - 0.940\alpha_s^{3/2} + \ldots$$

EFFECTIVE-N APPROACH ABOVE LO

Above LO the '-' component appears but :

[I.Balitsky,E.Kuraev,L.Lipatov,V.Fadin '78; A.Kotikov,G.Parente '99;]

1. **It is free of N=1 singularities and for it we can put N_{eff}=1**
2. **We expect the Q^2 dependnece to be very slow (vanishing split. fc. up to NLL)**
3. **Appearing at higher orders it is also expected to be small**
4. **In the asymptotic $Q^2 \to \infty$ limit impose that:** $r(\infty) = D_g^+(\infty)/D_s^+(\infty) = C_A/C_F$

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$$r(Q^2) = \hat{r}(N_{eff}, Q^2) = K\, r_{pert} + \frac{C_A}{C_F}(1-K)$$

$$K = \frac{D_s^+(Q^2)}{D_s^+(Q^2) + D_s^-(Q^2)};$$

$$r_{pert} = \frac{D_g^+(Q^2)}{D_s^+(Q^2)}$$

K is assumed to a good approximation to be a const. close to 1; non-perturbative info is inside here!

r_{pert} can be computed in perturbation theory from the LO and the perturbative functions $H_a^{\pm}$

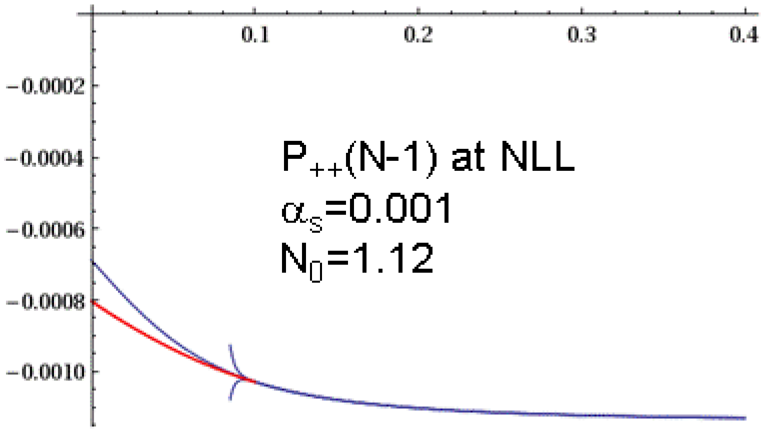
EFFECTIVE-N ABOVE LO

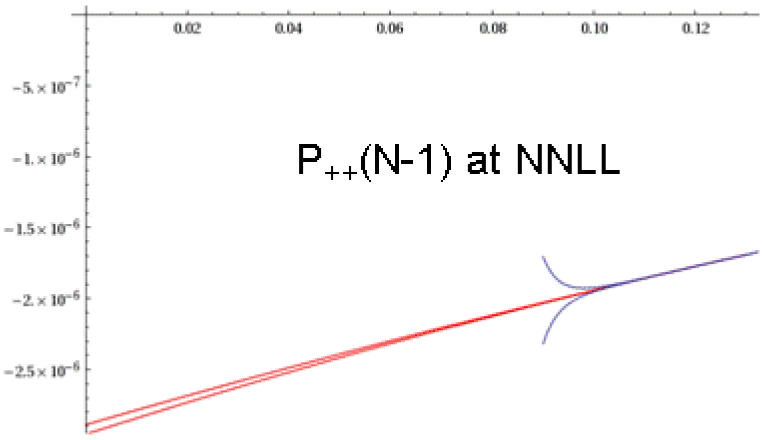
At NLO-N_{eff} we obtain the expression for N_{eff}^{NLO} according to the NLL expression of P_{++} in N=1 which is known in a closed analytical form in MSbar scheme

[A.Vogt; S.Albino,P.B.,B.Kniehl,A.Kotikov 2011]

At NNLO-N_{eff} the N_{eff}^{NNLO} can only be numerically estimated from the available NNLL terms (16th order) of P_{++} for small (N-1)

[A.Vogt 2011]

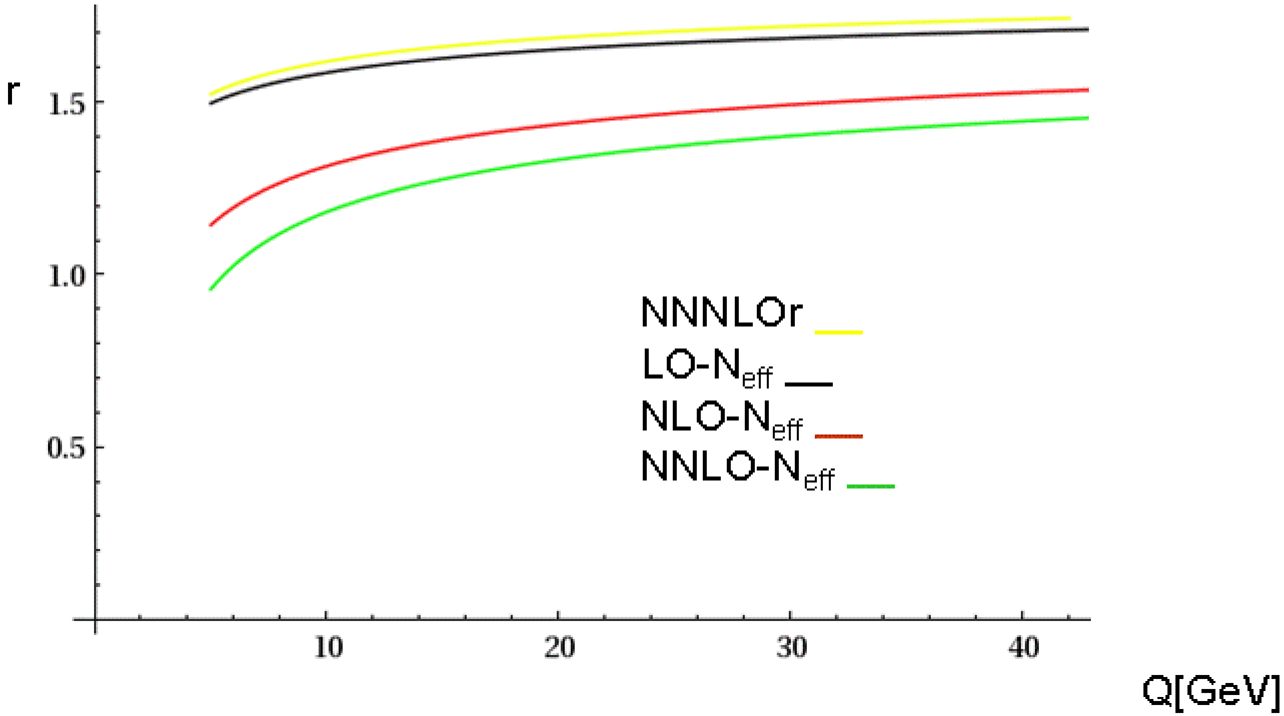




$$N_{eff}^{NNLO} = 1 + 1.382\sqrt{\alpha_s} + 0.905\,\alpha_s + 0.608\,\alpha_s^{3/2}; \qquad n_f = 5$$

4. RESULTS AND ANALYSIS

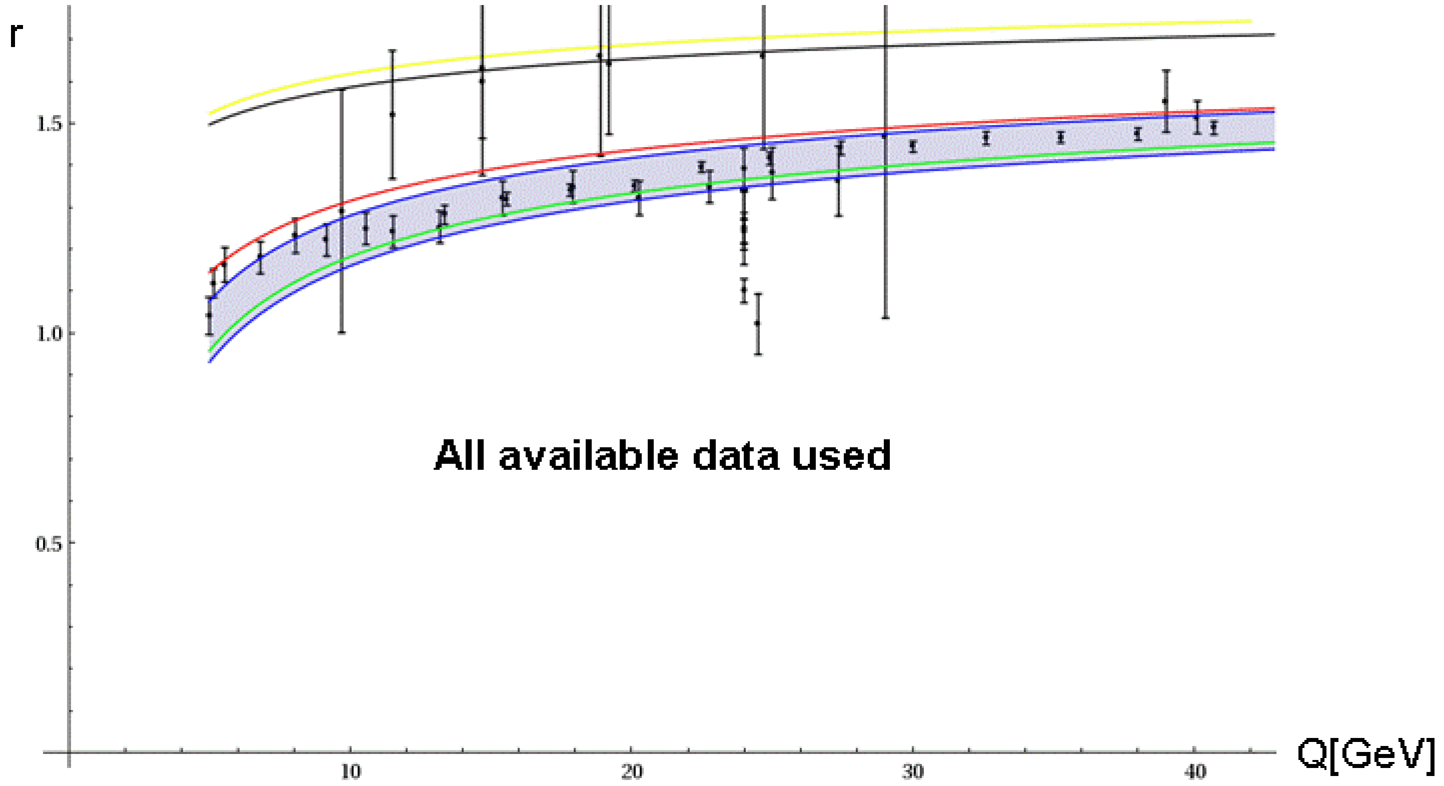
CONVERGENCE OF THE PERTURBATIVE SERIES



Double counted terms due to resummation have been removed

At around 40 GeV: NLO-N_{eff} correction $\sim$ 10%; NNLO-N_{eff} correction $\sim$ 5%

FIT OF K AT 90% CONFIDENCE LEVEL



$$r(Q^2) = K\, r_{pert} + \frac{C_A}{C_F}(1 - K);$$

$$K = 0.965 \pm 0.056 \quad 90\%\text{C.L.}$$

CONCLUSIONS

1. We have presented a new approach to the gluon-quark mult. Ratio
2. We have theoretical input as never before (NNLO, NNLL anom. dim.)
3. Under certain reasonable assumptions we have essentially control over the relative size of nonperturbative contributions
4. The perturbative series in our method shows a good convergence
5. The global fit with the data agrees with our assumptions
6. However we still do not have control over the single jet multiplicity
7. There are systematic disagreements beween data of different experiments not yet clarified