

# Comparison of two small $x$ Monte Carlos with and without coherence effects.

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We compare two Monte Carlo implementations of resummation schemes for the description of parton evolution at small values of Bjorken  $x$ . One of them is based on the BFKL evolution equation and generates fully differential parton distributions in momentum space making use of reggeized gluons. The other one is based on the CCFM partonic kernel where QCD coherence effects are introduced. It has been argued that both approaches agree with each other in the  $x \rightarrow 0$  limit. We show that for azimuthal angle dependent quantities, although at high energies the BFKL approach is dominated by its zero conformal spin component, the CCFM gluon Green function receives contributions from all conformal spins even at very small  $x$ .

## 1 Introduction

An important challenge in the phenomenology of Quantum Chromodynamics (QCD) is to understand what are the dominant effective degrees of freedom underlying the strong interaction at high energies. In the limit where the center-of-mass energy  $\sqrt{s}$  in a collision is much larger than any of the relevant mass scales the Balitsky–Fadin–Kuraev–Lipatov (BFKL) approach [1, 2, 3] appears to be a very useful framework to describe the scattering. In its original formulation this approach is based on the exchange of “reggeized” gluons in the  $t$ -channel. The interaction among them takes place via a gauge invariant (reggeized gluon)-(reggeized gluon)-(gluon) vertex. This simple effective structure stems from the dominance of the so-called multi-Regge kinematics where gluon cascades are only ordered in longitudinal components.

The simplicity of the final integral equation should not shadow the strong self-consistency of the full BFKL program where tight bootstrap conditions linking the reggeization of the gluon with the pomeron wavefunction, dominant in diffractive interactions, are fulfilled. The regime of applicability of the leading order BFKL approach to phenomenology is limited since it should be valid in a certain window of center-of-mass energies and perturbative scales. To extend its range of applicability one should include either higher order corrections, going beyond the ‘multi-Regge’ kinematics, include non-linear corrections responsible for the restoration of unitarity at very small  $x^1$  or, as we are going to discuss in this paper, to include a more global treatment of collinear regions in phase space using the Catani-Ciafaloni-Fiorani-Marchesini

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<sup>1</sup>In this work we consider equivalent variables the center-of-mass energy  $s$ , the DIS Bjorken  $x$  and the rapidity  $Y$  linking them by  $Y \simeq \ln s \simeq \ln 1/x$ .

(CCFM) equation, which provides a good matching from BFKL to the  $x \rightarrow 1$  regime, at least, as we are going to show in this work, as long as anomalous dimensions and  $k_t$ -diffusion properties are concerned.

## 2 Results

We use Monte Carlo program SMALLX to calculate observables related to the CCFM Green and the Monte Carlo implementation of the BFKL equation described in [4].

We compare the diffusion picture in the  $t$ -channel transversal momentum  $k_t$  – rapidity  $y$  plane and angular angle correlation dependence of the BFKL and the CCFM Green’s functions.

In the CCFM case, instead of a symmetric diffusion like in the BFKL case, the diffusion is asymmetric with extra soft emissions, fig. 1.

We quantify the angular correlation dependence by calculating Fourier moments of Green’s functions with respect to the angle between the transversal momenta  $\mathbf{k}_a$  and  $\mathbf{k}_b$ . As one can see from figures 2 and 3 the maximum rapidity  $Y$  dependence of the moments is much stronger in the CCFM case than in the BFKL case. The stronger correlation is especially prominent for  $n > 0$  moments, where in the CCFM case the moments for  $n = 1$  and 2 grow with rapidity, in the BFKL case they show constant or decreasing tendency.

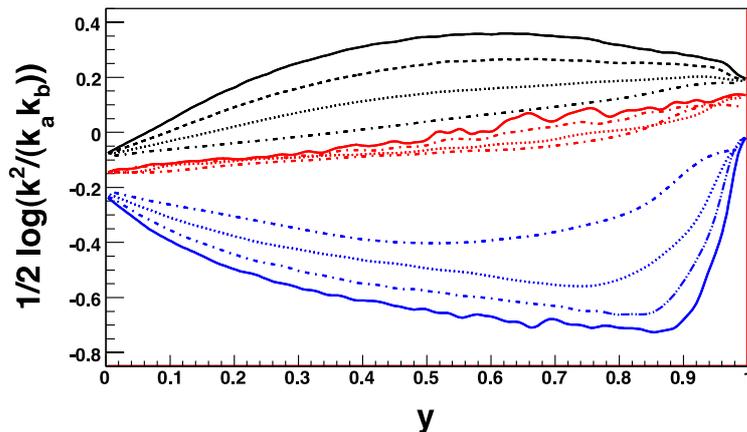


Figure 1: Distribution of transverse scales in the evolution with  $x$  of the CCFM equation.

## 3 Conclusions and scope

In this letter we have compared two Monte Carlo implementations of the CCFM and BFKL formalisms for the description of small  $x$  observables. The main difference between them from the theoretical point of view is the introduction of QCD coherence effects in the CCFM equation. We have found that the symmetric diffusion into infrared and ultraviolet regions of phase space characteristic of the BFKL parton evolution is broken in the CCFM case, where the

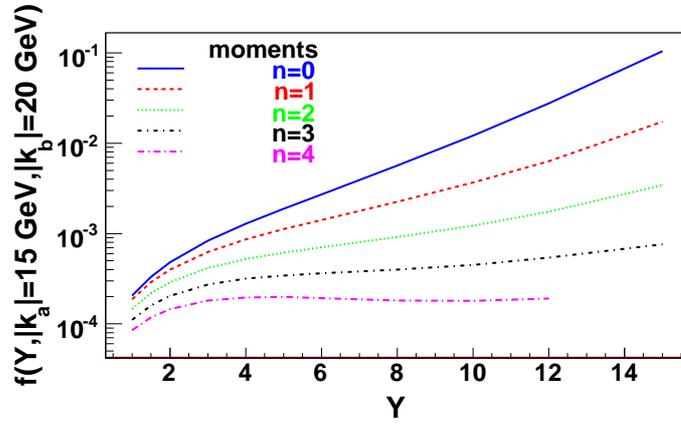


Figure 2: Variation with rapidity of the different components of the Fourier expansion on the azimuthal angle of the CCFM gluon Green function.

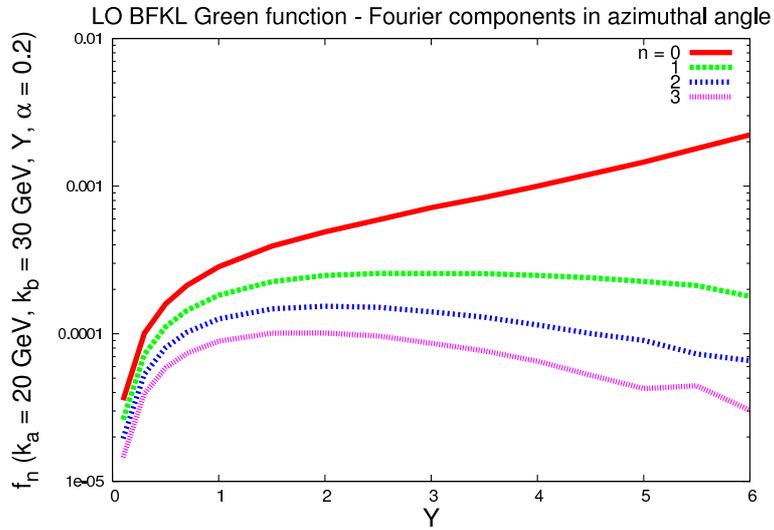


Figure 3: Variation with rapidity of the different components of the Fourier expansion on the azimuthal angle of the BFKL gluon Green function.

infrared scales play a dominant role. As our main result we have found that the higher Fourier components in the gluon Green function have a very different behaviour in both theories, rising with energy in the CCFM case and decreasing in the BFKL one. It will be very interesting to trace these differences at an observable level [5, 6, 7, 8] and to implement higher order corrections [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 9] to evaluate their effects on them. These lines of research will be the subject of our future investigations.

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