Small x Monte Carlos with and without QCD coherence effects

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Motivation

• To describe high energy scattering cross section for \( s \gg t \gg \Lambda_{QCD} (x \to 0) \) the optimal effective picture seems to be the BFKL equation
  – Exchange of reggeised gluons in the t-channel
  – Only valid in a certain window of phase space
• Extension of applicability by
  – Higher order corrections, non-linear corrections
• Or CCFM equation
  – Coherence effects in angular ordering
  – Matching from BFKL to the \( x \to 1 \) region
Gluon green’s function

- Example

- A way how to calculate the cross section by convoluting the Green’s function with the (nonpertubative) ‘source’
BFKL Green’s function

- The BFKL equation

\[ f_{\text{BFKL}}(k_a, k_b, Y) = e^{2\omega_0(-k_a^2)} Y \left\{ \delta^{(2)}(k_a - k_b) \right\} \]

\[ + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \tilde{\alpha}_s \int \frac{d^2k_i}{\pi k_i^2} \theta(k_i^2 - \lambda^2) \int_{0}^{Y_i-1} dy_i e^{2\omega_0(-k_{i-1}^2) y_i} \delta^{(2)}(k_a - k_b + \sum_{l=1}^{n} k_l) \]  

- \( \omega_0^{(i,i-1)} = \omega_0(t, \lambda) - \omega_0(t_{i-1}, \lambda), \ t_i = -(k_1 + \sum_{j=1}^{i} k_j)^2, \ \lambda \) an infrared regulator
- \( \omega_0(t, \lambda) \) is the gluon Regge trajectory
- Infrared finite
- This iterative formulation is useful for Monte Carlo implementation
- **Conformal symmetry, symmetric diffusion in the \( k_T - Y \) plane**
CCFM Green’s function

- Equation for the CCFM gluon Green’s function

\[
f^{\text{CCFM}}(k_a, k_b, x) = \delta(x - x_0) \delta^{(2)}(k_a - k_b) \Delta_S(\eta, q_0) \theta(\eta - q_0) \\
+ \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} dPS_i \theta(\eta - z_n l_n) \Delta_S(\eta, z_n l_n) \delta(x - x_n) \delta^{(2)}(k_a - k_n)
\]

- Coherence effects - angular ordering and \(1/(1-z)\) in the kernel in addition to BFKL
- This iterative formulation is useful for Monte Carlo implementation

\[
\frac{q_{i+1}}{1 - z_{i+1}} > \frac{z_i q_i}{1 - z_i}
\]

small \(z\) limit

\[
q_{i+1} > z_i q_i
\]
Comparing

- We will compare the gluon Green’s functions of BFKL and CCFM
- Leading order, constant $\alpha_s$

Smallx

G. Marchesini and B. R. Webber,
Nucl. Phys. B 349 (1991) 617,

- Dependence on angle between $k_a$ and $k_b$, rapidity $Y$, magnitude of $k_a$ and $k_b$, diffusion in $k$ along the chain
- Effect of the angular ordering and the $1/(1-z)$ term (soft emissions)

A. Sabio Vera and P. Stephens,
Distribution of number of emissions

- Contributions of chains of $n$ emissions depending on maximum rapidity $Y$ - number of steps in a ladder

- Fitted better by Gaussian than by Poisson distribution
Diffusion in $k_T$ along the parton ladder

• The BFKL predicts symmetric diffusion pattern
• In the CCFM contribution of the $1/(1-\zeta)$ term in the kernel

• Maximum rapidity $Y$
  – $Y = 2, 4, 6, 8$

• for increasing maximum rapidity $Y$ the ratio of the areas $S_{UV}/S_{IR}$ is increasing

for fixed rapidity
Collinear behavior

- Dependence on the relative and absolute magnitudes of momenta
- In BFKL simple scaling by a power of ratio of the momenta

\[ f_{\text{BFKL}}(k_a, k_b, Y) = \frac{1}{\pi k_a k_b} \sum_{n=-\infty}^{\infty} \int \frac{d\omega}{2\pi i} e^{\omega Y} \int \frac{d\gamma}{2\pi i} \left( \frac{k_a^2}{k_b^2} \right)^{\gamma - \frac{1}{2}} \frac{e^{i n \theta}}{\omega - \tilde{\alpha}_s \chi(n, \gamma)} \]

- Flattening of the behaviour for small values of momenta in CCFM
  - Restoration of the conformal symmetry?
Angular correlation

- Angle $\phi$ between $k_a$ and $k_b$ at fixed rapidity
- Sharper peak and stronger dependence on $\phi$ for CCFM - hints to a different rapidity dependence of observables sensitive to angular correlation
Fourier moments

- Rapidity $Y$ dependence of the angular moments of the gluon Green’s function
- Very different for $n>0$ - reflects the difference in the angular dependence

\[ f_{n}^{\text{CCFM}} (k_a, k_b, x) = \int_{0}^{2\pi} \frac{d\theta}{2\pi} f_{n}^{\text{CCFM}} (k_a, k_b, x) \cos (n \theta) \]
Summary and conclusions

- Comparison of BFKL and CCFM gluon Green’s functions
- Similar distribution of emissions
- Differences in diffusion pattern - CCFM diffusion ‘leaking’ into IR region
- Flattening of the collinear behavior of CCFM for small momenta - hint for symmetry restoration
- Stronger angular dependence in CCFM reflected in rapidity dependence of the Fourier components
Spare slides
Angular correlation

- Angle between $k_a$ and $k_b$
- Flatter after the cut on number of emissions >10
Rapidity-angle correlations

- Correlation between to emitted gluons in the $Y-\phi$ plane