Small x Monte Carlos with and without QCD coherence effects

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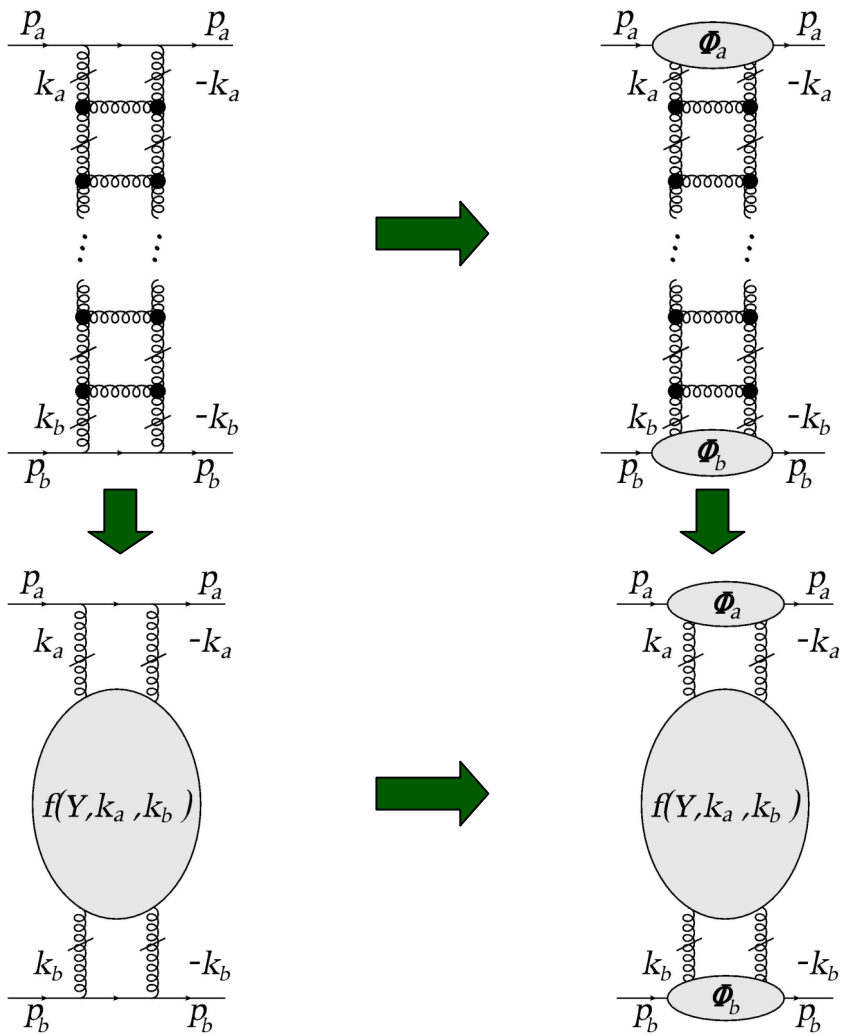
Motivation

- To describe high energy scattering cross section for $s >> t >> \Lambda_{QCD}$ $(x \to 0)$ the optimal effective picture seems to be the BFKL equation
  - Exchange of reggeised gluons in the t-channel
  - Only valid in a certain window of phase space
- Extension of applicability by
  - Higher order corrections, non-linear corrections
- Or CCFM equation
  - Coherence effects in angular ordering
  - Matching from BFKL to the $x \to 1$ region

Gluon green's function

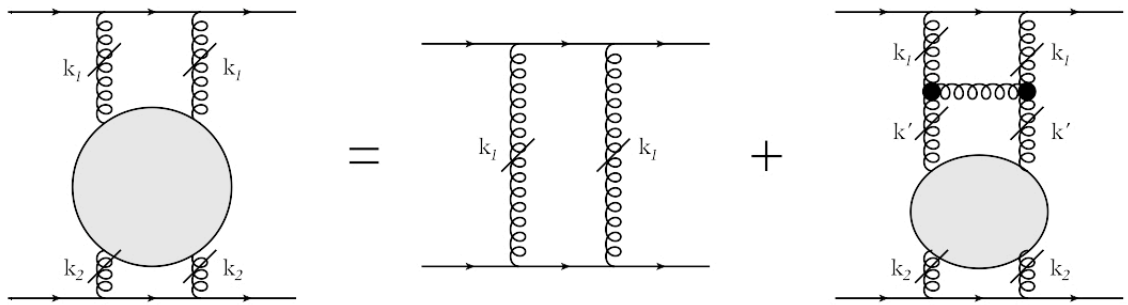
- Example

- A way how to calculate the cross section by convoluting the Green's function with the (nonpertubative) 'source'



BFKL Green's function

- The BFKL equation

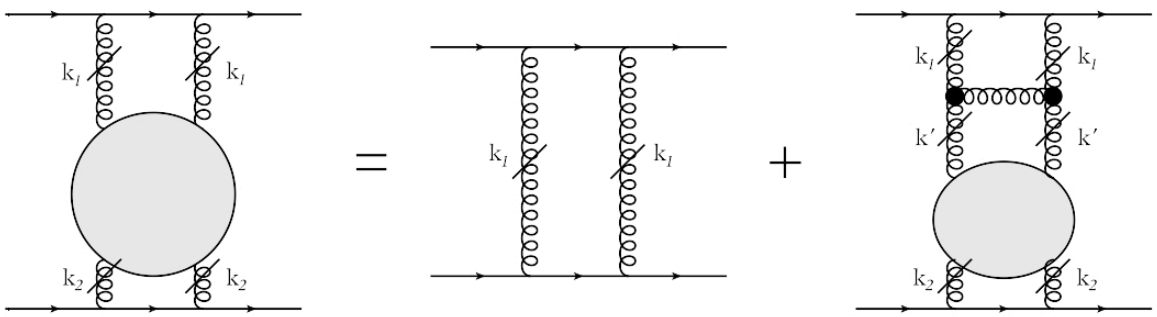


$$f^{\mathrm{BFKL}}(\mathbf{k}_a,\mathbf{k}_b,\mathrm{Y}) = e^{2\,\omega_0\left(-k_a^2\right)\mathrm{Y}}\Bigg\{\delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b) + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\bar{\alpha}_s\int\frac{d^2\mathbf{k}_i}{\pi k_i^2}\theta\left(k_i^2-\lambda^2\right)\int_0^{y_{i-1}}dy_i\,e^{2\,\omega_0^{(i,i-1)}y_i}\,\delta^{(2)}\left(\mathbf{k}_a-\mathbf{k}_b+\sum_{l=1}^{n}\mathbf{k}_l\right)\Bigg\}$$

- $\omega_0^{(i,i-1)} = \omega_0(t_i,\lambda) - \omega_0(t_{i-1},\lambda)$, $t_i = -(k_1+\Sigma^i_{j=1}\,k_j)^2$, λ an infrared regulator
- $\omega_0(t,\lambda)$ is the gluon Regge trajectory
- Infrared finite
- This iterative formulation is useful for Monte Carlo implementation
- **Conformal symmetry, symmetric diffusion in the k_T-Y plane**

CCFM Green's function

- Equation for the CCFM gluon Green's function



$$f^{\mathrm{CCFM}}(\mathbf{k}_a, \mathbf{k}_b, x) = \delta(x - x_0)\,\delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b)\,\Delta_S(\eta, q_0)\,\theta(\eta - q_0)$$
$$+ \sum_{n=1}^{\infty} \int \left[\prod_{i=1}^{n} d\mathrm{PS}_i\right] \theta(\eta - z_n l_n)\,\Delta_S(\eta, z_n l_n)\,\delta(x - x_n)\,\delta^{(2)}(\mathbf{k}_a - \mathbf{k}_n)$$

$$\frac{q_{i+1}}{1 - z_{i+1}} > \frac{z_i q_i}{1 - z_i}$$

small z limit

$$q_{i+1} > z_i q_i$$

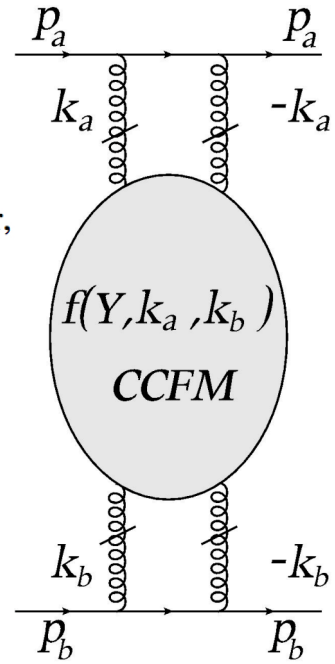
- **<u>Coherence effects - angular ordering</u>** and **1/(1-z)** in the kernel in addition to BFKL
- This iterative formulation is useful for Monte Carlo implementation

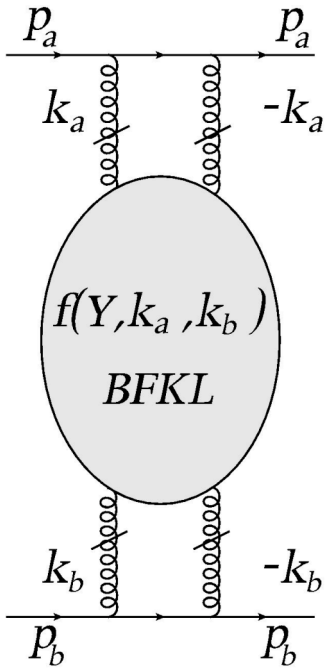
Comparing

- We will compare the gluon Green's functions of BFKL and CCFM
- <u>Leading order, constant α_S</u>

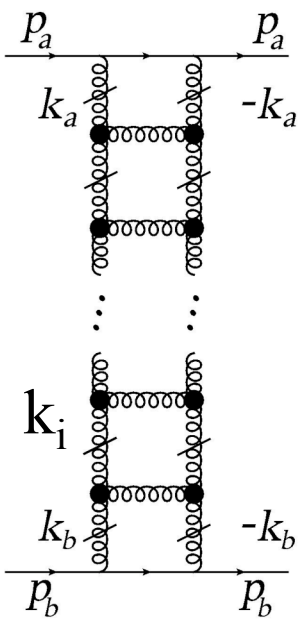
Smallx

G. Marchesini and B. R. Webber, Nucl. Phys. B **349** (1991) 617, Nucl. Phys. B **386** (1992) 215.





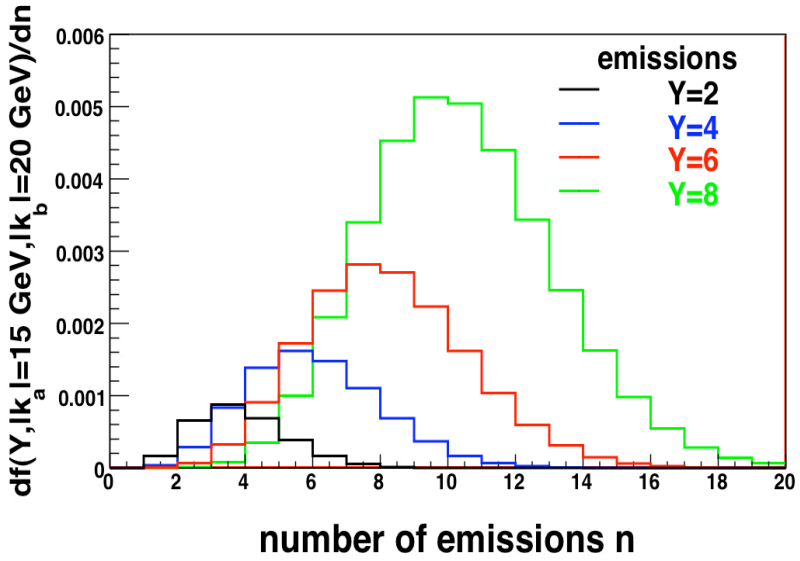
A. Sabio Vera and P. Stephens, PoS D **IFF2006** (2006) 030.

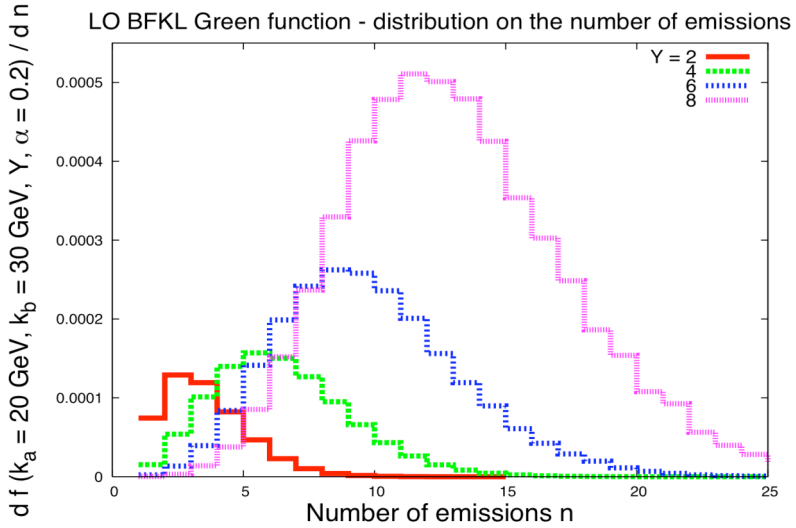


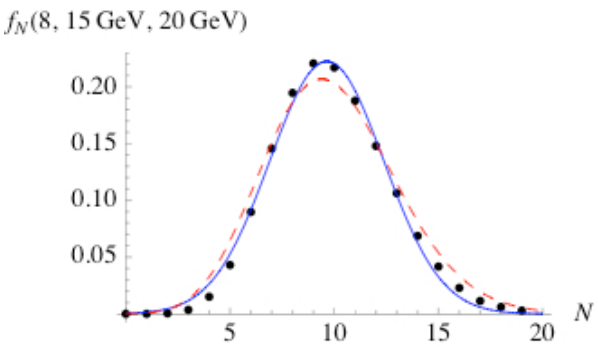
- Dependence on angle between k_a and k_b, rapidity Y, magnitude of k_a and k_b, diffusion in k along the chain
- Effect of the angular ordering and the $1/(1-z)$ term (soft emissions)

Distribution of number of emissions

- Contributions of chains of n emissions depending on maximum rapidity Y - number of steps in a ladder





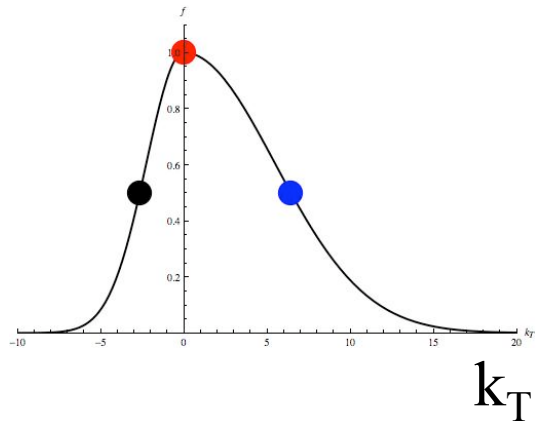


- Fitted better by Gaussian than by Poisson distribution

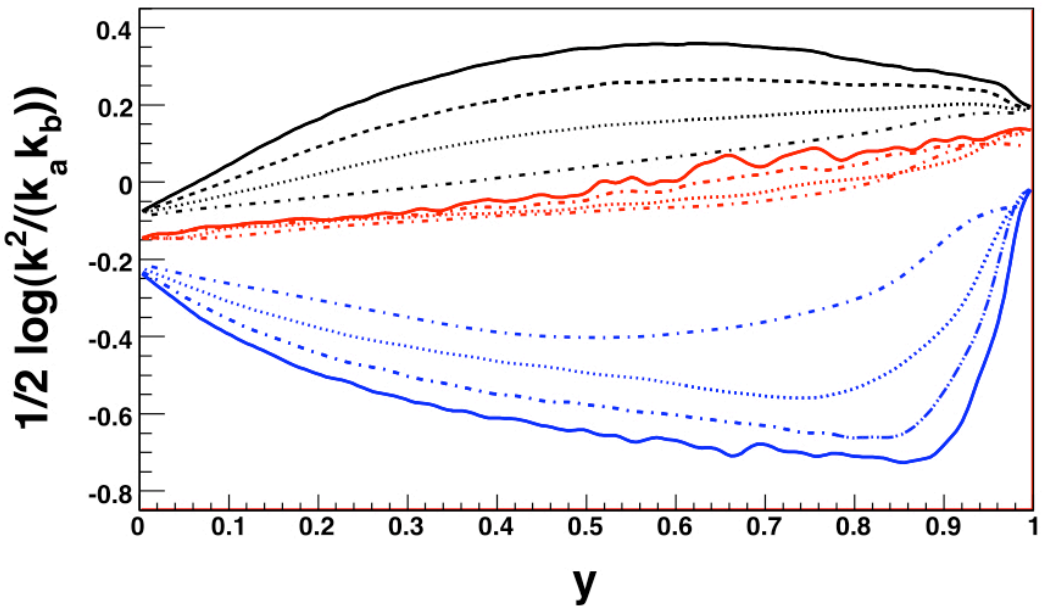
Diffusion in k_T along the parton ladder

- The BFKL predicts symmetric diffusion pattern
- In the CCFM contribution of the $1/(1-z)$ term in the kernel

- Maximum rapidity Y
  - $Y = 2, 4, 6, 8$



for fixed rapidity



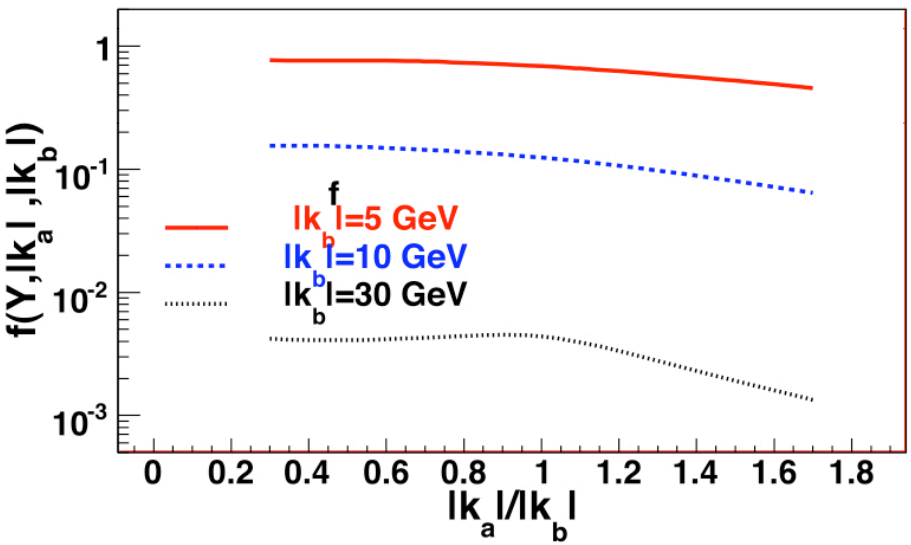
- for increasing maximum rapidity Y the ratio of the areas S_{UV}/S_{IR} is increasing

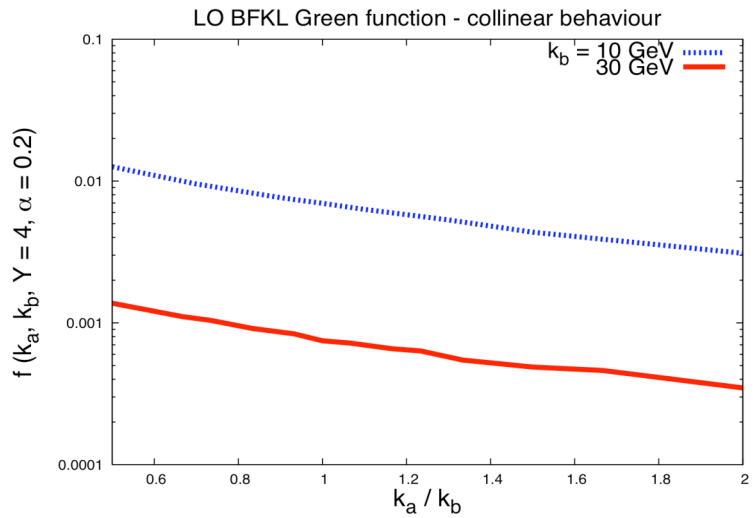
Collinear behavior

- Dependence on the relative and absolute magnitudes of momenta
- In BFKL simple scaling by a power of ratio of the momenta

$$f^{\mathrm{BFKL}}(\mathbf{k}_a, \mathbf{k}_b, \mathrm{Y}) = \frac{1}{\pi k_a k_b} \sum_{n=-\infty}^{\infty} \int \frac{d\omega}{2\pi i} e^{\omega Y} \int \frac{d\gamma}{2\pi i} \left(\frac{k_a^2}{k_b^2}\right)^{\gamma - \frac{1}{2}} \frac{e^{in\theta}}{\omega - \bar{\alpha}_s \chi(n,\gamma)}$$

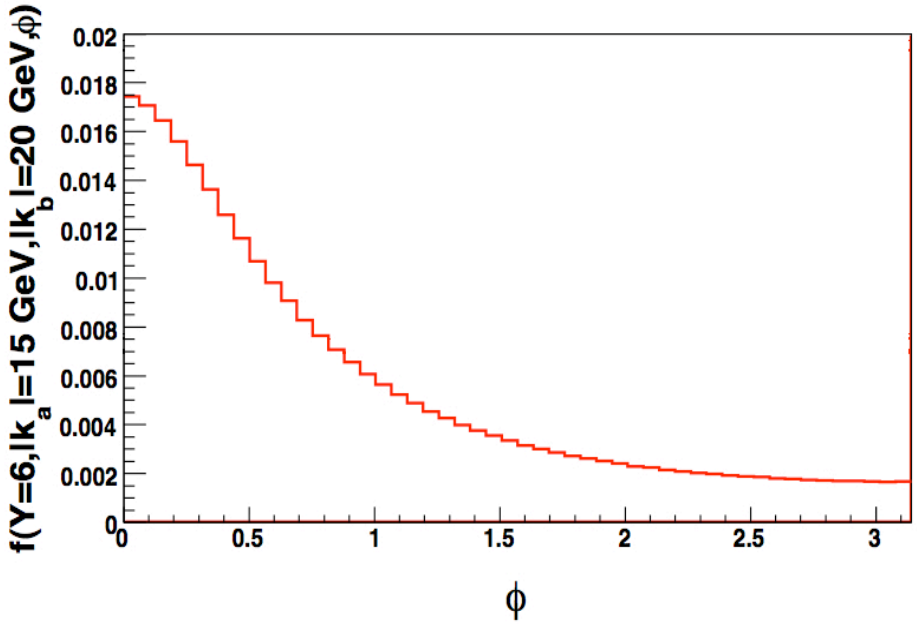
- Flattening of the behaviour for small values of momenta in CCFM
  - Restoration of the conformal symmetry?

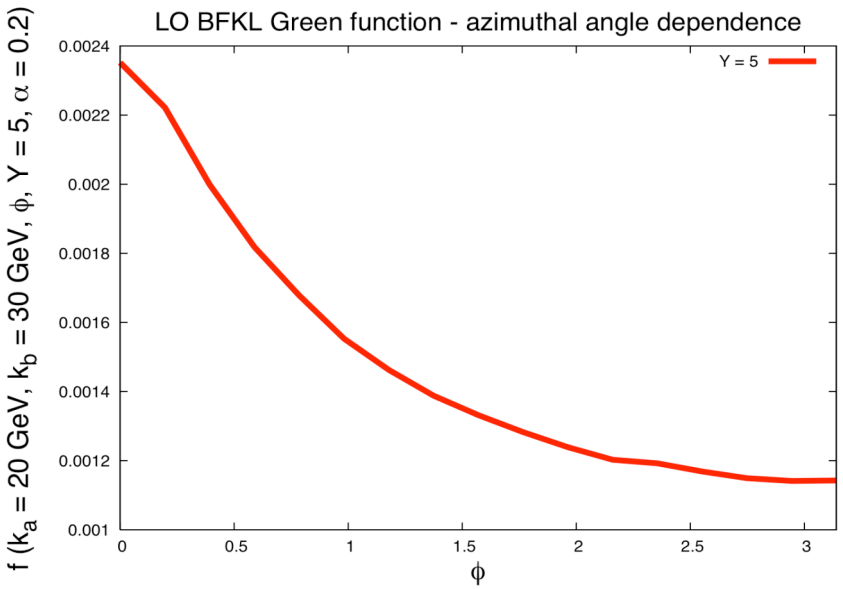




Angular correlation

- Angle ϕ between k_a and k_b at fixed rapidity
- Sharper peak and stronger dependence on ϕ for CCFM - hints to a different rapidity dependence of observables sensitive to angular correlation

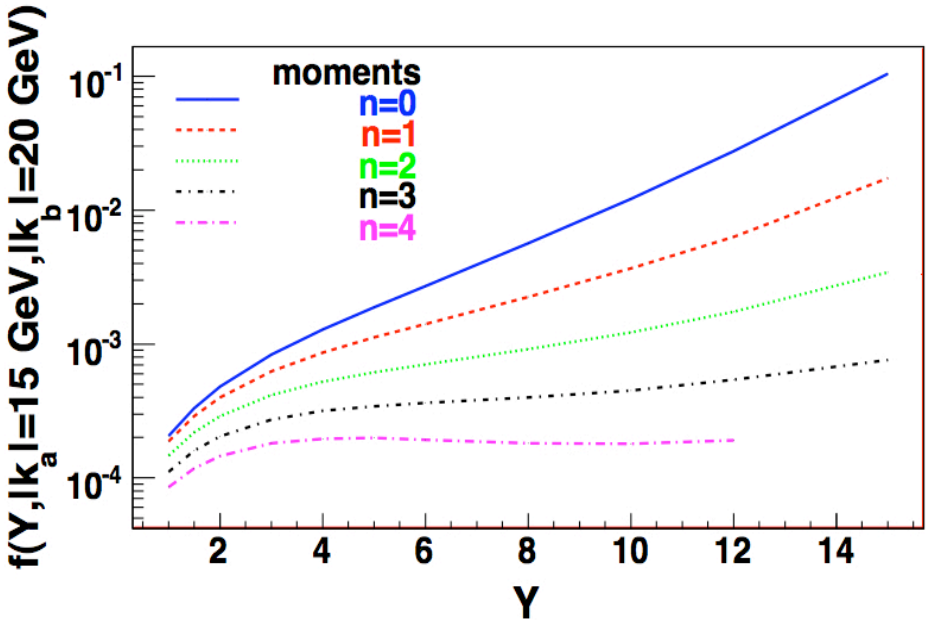


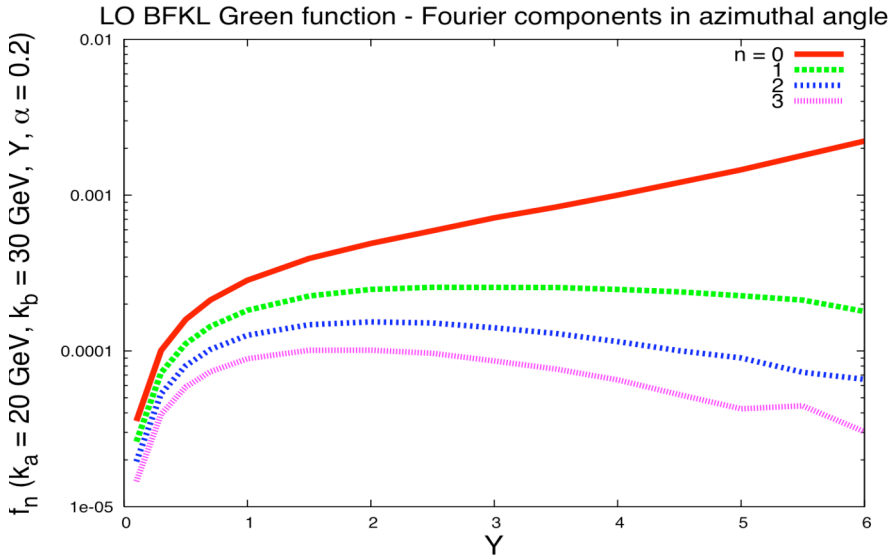


Fourier moments

- Rapidity Y dependence of the angular moments of the gluon Green's function
- Very different for n>0 - reflects the difference in the angular dependence

$$f_n^{\mathrm{CCFM}}(k_a, k_b, x) = \int_0^{2\pi} \frac{d\theta}{2\pi} f^{\mathrm{CCFM}}(\mathbf{k}_a, \mathbf{k}_b, x) \cos(n\,\theta)$$





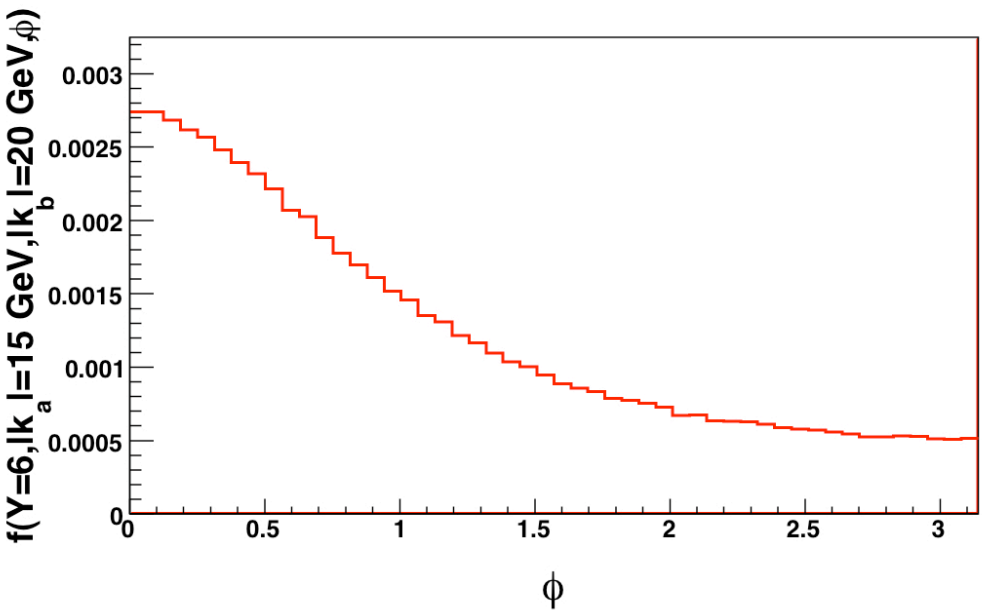
Summary and conclusions

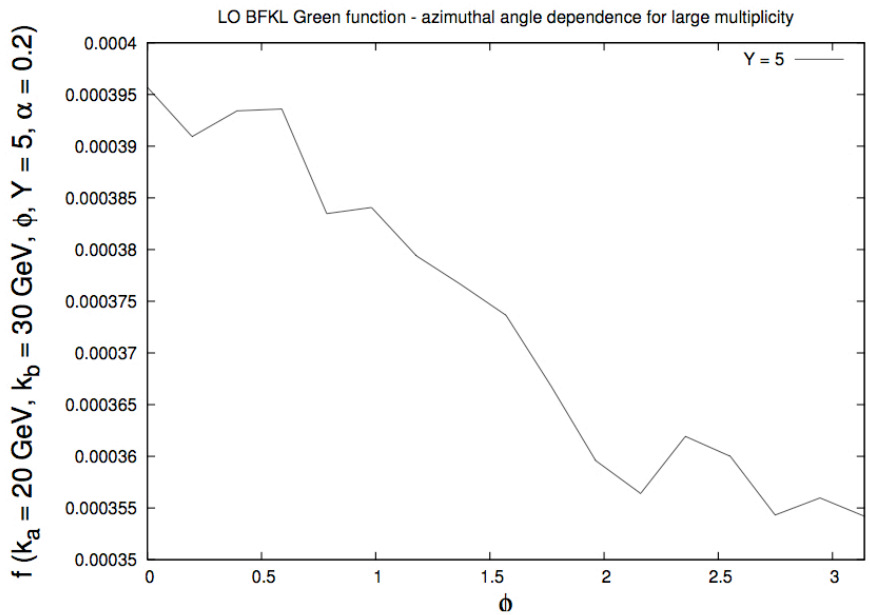
- Comparison of BFKL and CCFM gluon Green's functions
- Similar distribution of emissions
- Differences in diffusion pattern - CCFM diffusion 'leaking' into IR region
- Flattening of the collinear behavior of CCFM for small momenta - hint for symmetry restoration
- Stronger angular dependence in CCFM reflected in rapidity dependence of the Fourier components

Spare slides

Angular correlation

- Angle between k_a and k_b
- Flatter after the cut on number of emissions >10





Rapidity-angle correlations

- Correlation between to emitted gluons in the Y-ϕ plane