Soft-gluon resummation for high-p_T hadrons at COMPASS

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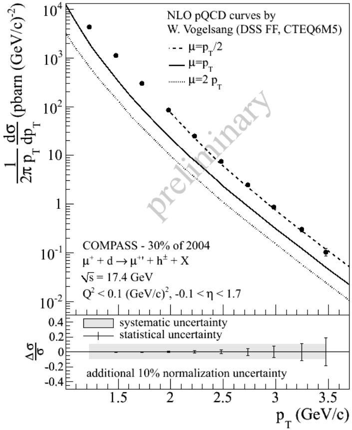
high-p_T-hadron production: $\mu^+ + d \to \mu' + h^{+-} + X$

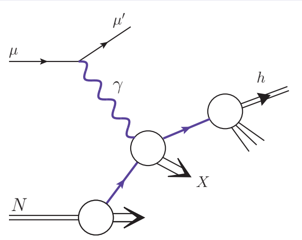
COMPASS at CERN

- $\sqrt{S}$=17.4 GeV
- $0.2 < y < 0.8$
- 10 mrad $< \theta <$ 120 mrad

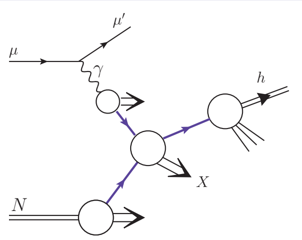


direct- and resolved-γ-contribution

direct-γ-contribution

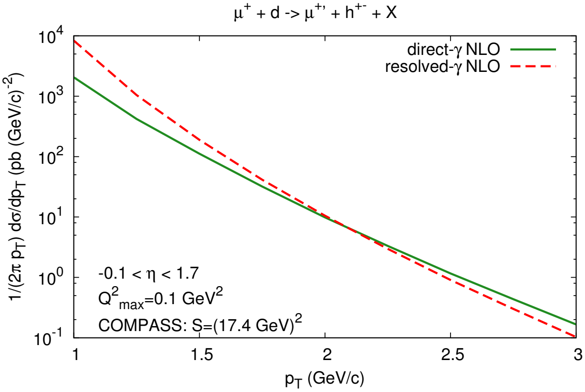


resolved-γ-contribution



Factorization theorem: Long distance effects (parton distribution functions f_a and fragmentation functions D_c) decouple from perturbatively calculable short distance effects (partonic cross section $\hat{\sigma}_{ab}$).

direct- and resolved-γ-contribution



The hadronic cross section is a convolution

$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \sum_{a,b,c} \int_{x_{P,min}}^1 dx_P \int_{x_{l,min}}^1 dx_l \int_x^1 dz \frac{\hat{x}_T^4 z^2}{8v} f_{a/l}(x_l) f_{b/H}(x_P) D_{h/c}(z) \frac{\hat{s} d\hat{\sigma}}{dv dw}$$

$$x_T = 2p_T/\sqrt{S}$$
$$\hat{s} = x_l x_P S$$
$$\hat{x}_T = \frac{x_T}{z\sqrt{x_l x_P}} = \frac{2p_T}{z\sqrt{\hat{s}}}$$
$$\eta = \hat{\eta} - \ln\sqrt{\frac{x_P}{x_l}}$$
$$v = 1 - \frac{\hat{x}_T}{2} e^{-\hat{\eta}}$$
$$w = \frac{1}{v}\frac{\hat{x}_T}{2} e^{\hat{\eta}}$$

$$x_{P,min} = e^{2\eta}/(2e^{\eta}/x_T - 1)$$
$$x_{l,min} = x_T e^{-\eta}/(2 - x_T e^{\eta}/x_P)$$
$$x := z_{min} = x_T/\sqrt{x_P x_l}\cosh\hat{\eta}$$

partonic threshold: $\hat{x}_T \to 1/\cosh(\hat{\eta}) \Rightarrow w \to 1$

$\frac{\hat{s} d\hat{\sigma}}{dv dw}$ is calculated in perturbation theory

$$\frac{\hat{s}d\hat{\sigma}}{dvdw} = \sigma_B(v)\delta(1-w) +$$

$$\frac{\alpha_s}{\pi}\left[A^{log}(v)\left(\frac{\ln(1-w)}{1-w}\right)_+ + A^{pl}(v,w)\frac{1}{(1-w)_+} + A^{del}(v)\delta(1-w) + R(v,w)\right]$$

Mellin-N moment space:

$$\sigma(N) = \int_0^1 dx_T^2 (x_T^2)^{N-1}\frac{p_T^3 d\sigma(x_T)}{dp_T}$$

$$\sigma(N) = \sum_{a,b,c} f_{a/l}^{N+1} f_{b/H}^{N+1} D_{h/c}^{2N+3}\hat{\sigma}_{ab\to cX}(N)$$

$$\hat{\sigma}_{ab\to cX}(N) = \frac{1}{2}\int_0^1 dw \int_0^1 dv [4v(1-v)w]^{N+1}\frac{\hat{s}d\hat{\sigma}_{ab\to cX}(w,v)}{dwdv}$$

$$\frac{\hat{\sigma}^{NLO}_{ab\to cX}(N)}{\hat{\sigma}_B} = \frac{\alpha_s}{\pi}\left[\tilde{A}^{log}_{ab\to cX}\ln^2 N + \tilde{A}^{pl}_{ab\to cX}\ln N + C_{ab\to cX} + \mathcal{O}(\ln N/N)\right]$$

in Mellin moment space $\hat{\sigma}/\hat{\sigma}_B$:

$$\mathcal{O}(\alpha_s^n): \quad \alpha_s^n \ln^m N \quad m \leq 2n$$

$$\hat{\sigma}^N_{direct}/\hat{\sigma}_B =$$

$$= 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{m=0}^{2n} \tilde{c}_{n,m} L^m$$

$$= C(\alpha_s) \exp\left\{ \sum_{n=1}^{\infty} \alpha_s^n \sum_{m=1}^{n+1} c_{n,m} L^m \right\}$$

$$= C(\alpha_s) \times$$

$$\times \exp\{\underbrace{\underbrace{L h^{(1)}(\alpha_s L)}_{\text{LL}} + h^{(2)}(\alpha_s L)}_{\text{NLL}} + \alpha_s h^{(3)}(\alpha_s L)\}$$

LO	1		
NLO	$\alpha_s L^2$	$\alpha_s L$	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^{\{3,2\}}$	$\alpha_s^2 L$
$\vdots$			$\ddots$

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	LL	NLL	NNLL	...
LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$		
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^{\{3,2\}}$	$\alpha_s^2 L$	
$\vdots$	$\downarrow$	$\downarrow$		$\ddots$
	$h^{(1)}(\alpha_s L)$	$h^{(2)}(\alpha_s L)$		

resummed partonic cross section: de Florian, Vogelsang '05

$$\hat{\sigma}^{(res)}_{ab\to cd}(N) = C_{ab\to cd}\Delta^a_N\Delta^b_N\Delta^c_N J^d_N \left[\sum_I G^I_{ab\to cd}\Delta^{(int)ab\to cd}_{I\ N}\right]\hat{\sigma}^B_{ab\to cd}(N)$$

$\rightarrow$ Kidonakis, Oderda, Sterman '98

$$\ln \Delta^a_N = \int_0^1 \frac{z^{N-1}-1}{1-z}\int_{\mu_f^2}^{(1-z)^2Q^2}\frac{dq^2}{q^2}A_a(\alpha_s(q^2))$$

$$\ln J^d_N = \int_0^1 \frac{z^{N-1}-1}{1-z}\left[\int_{(1-z)^2Q^2}^{(1-z)Q^2}\frac{dq^2}{q^2}A_a(\alpha_s(q^2)) + \frac{1}{2}B_a(\alpha_s(1-z)Q^2)\right]$$

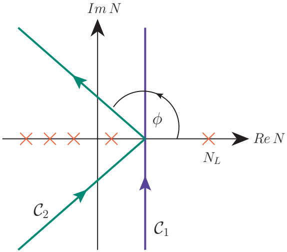
$$\ln \Delta^{(int)ab\to cd}_{I\ N} = \int_0^1 \frac{z^{N-1}-1}{1-z}D_{I ab\to cd}(\alpha_s((1-z)^2Q^2))$$

Inverse Mellin transform

$$f^N = \int_0^1 dx x^{N-1} f(x)$$

$$\updownarrow$$

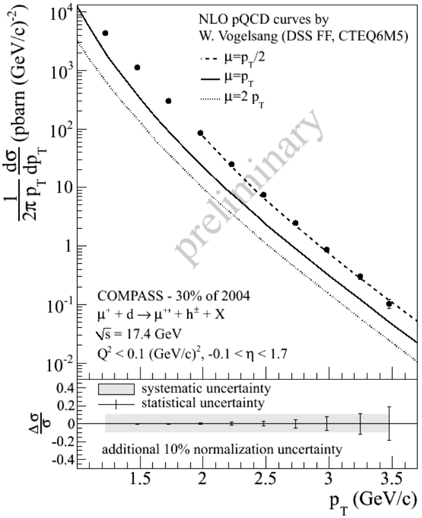
$$f(x) = \frac{1}{2\pi i} \int_{\mathcal{C}} dN x^{-N} f^N$$

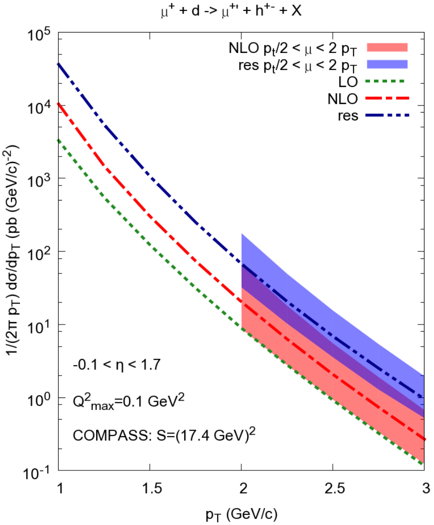


Minimal Prescription

Catani, Mangano, Nason, Trentadue (1996)

- define inverse Mellin contour as the left of the Landau pole at $N_L = \exp(1/2b_0\alpha_s(\mu^2))$
- all other singularities are to the left of the contour


NLO pQCD curves by
W. Vogelsang (DSS FF, CTEQ6M5)
$\mu = p_T/2$
$\mu = p_T$
$\mu = 2\,p_T$
preliminary
COMPASS - 30% of 2004
$\mu^+ + d \to \mu^{+\prime} + h^\pm + X$
$\sqrt{s} = 17.4$ GeV
$Q^2 < 0.1\ (\mathrm{GeV}/c)^2$, $-0.1 < \eta < 1.7$
$\frac{1}{2\pi p_T}\frac{d\sigma}{dp_T}$ (pbarn (GeV/c)$^{-2}$)
systematic uncertainty
statistical uncertainty
additional 10% normalization uncertainty
$\frac{\Delta\sigma}{\sigma}$
p_T (GeV/c)


$\mu^+ + d \to \mu^{+\prime} + h^{+-} + X$
NLO $p_t/2 < \mu < 2\,p_T$
res $p_t/2 < \mu < 2\,p_T$
LO
NLO
res
$1/(2\pi\, p_T)\, d\sigma/dp_T$ (pb (GeV/c)$^{-2}$)
$-0.1 < \eta < 1.7$
$Q^2_{max} = 0.1$ GeV^2
COMPASS: $S = (17.4\ \mathrm{GeV})^2$
p_T (GeV/c)

rapidity differential resummed cross section

$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \sum_{abc} \int_0^1 dx_P \int_0^1 dx_l f_{a/l}(x_l) f_{b/H}(x_P) \times$$
$$\times \int_{-i\infty}^{i\infty} \frac{dN}{2\pi i} (x^2)^{-N} \int_0^1 d(x^2)(x^2)^{N-1} \int_x^1 dz \frac{\hat{x}_T^4 z^2}{8v} D_{h/c}(z) \frac{\hat{s} d\hat{\sigma}}{dv dw}$$

$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \sum_{abc} \int_0^1 dx_P \int_0^1 dx_l f_{a/l}(x_l) f_{b/H}(x_P) \int_{-i\infty}^{i\infty} \frac{dN}{2\pi i} (x^2)^{-N} D_{h/c}^{2N+3} \tilde{w}(2N, \hat{\eta})$$

with

$$\tilde{w}(2N, \hat{\eta}) = \frac{1}{4 \cosh^4 \hat{\eta}} \int_0^1 dm\, m^{2N+3} \frac{1}{v} \frac{\hat{s} d\hat{\sigma}}{dv dw} \qquad m := \hat{x}_T \cosh \hat{\eta}$$

rapidity differential resummed cross section

$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \sum_{abc} \int_0^1 dx_P \int_0^1 dx_l f_{a/l}(x_l) f_{b/H}(x_P) \int_{-i\infty}^{i\infty} \frac{dN}{2\pi i} x^{-2N} D_{h/c}^{2N+3} \tilde{w}_{ab\to cd}^{(res)}(2N,\hat{\eta})$$

→ Laenen, Oderda, Sterman '98

$$\tilde{w}_{ab\to cd}^{(res)}(2N,\hat{\eta}) = C_{ab\to cd} \Delta_{N_a}^a \Delta_{N_b}^b \Delta_N^c J_N^d \mathrm{Tr}\left\{ H \mathcal{S}_N^\dagger S \mathcal{S}_N \right\} \hat{\sigma}_{ab\to cd}^B(N)$$

$$N_a = N\,(-\frac{u}{s}) = N\left(-\frac{\hat{x}_T e^{\hat{\eta}}}{2}\right)$$

$$N_b = N\,(-\frac{t}{s}) = N\left(-\frac{\hat{x}_T e^{-\hat{\eta}}}{2}\right)$$

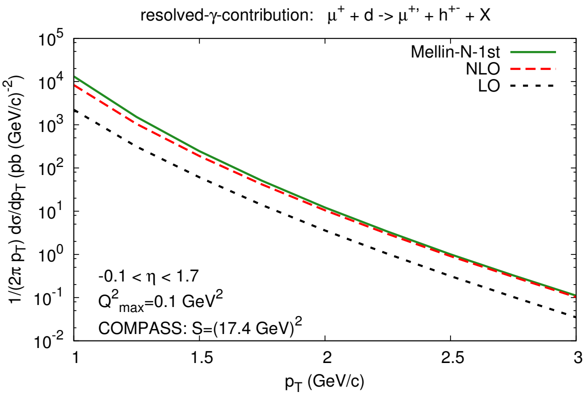
$H_{ab\to cd}$: hard scattering functions

$S_{ab\to cd}$: soft functions

$\mathcal{S}_N$: wide-angle soft gluons

$$\mathcal{S}_N = \mathcal{P} \exp\left[\frac{1}{2}\int_{\mu_r^2}^{\hat{s}/N^2} \frac{dq^2}{q^2} \Gamma_{ab\to cd}(\hat{\eta}, \alpha_s(q^2))\right]$$

rapidity differential cross section



Conclusion

- higher order calculations are needed in the threshold region due to the large logarithmic corrections
- Resummation allows for the calculation of those contributions to all orders
- rapidity differential resummed cross section
- further investigation: include $\ln(N)/N$-terms

far infrared Cutoff

Shimizu, Sterman, Vogelsang, Yokoya '05

$$\Delta_N^a = \int_0^1 dz \frac{z^N - 1}{1-z} \int_{\mu_f^2}^{(1-z)^2 M_a^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp)\right) \qquad \Rightarrow \text{hold } k_\perp > \mu_0$$