# Distribution of linearly polarized gluons inside a large nucleus

Jian Zhou
Regensburg University

Based on: **Phys.Rev. D84 (2011) 051503.** A. Metz and ZJ
**arXiv:1203.1534 [hep-ph].** A. Schäfer and ZJ

# Outline:

- Review
- Distribution of linearly polarized gluons at small x
- Matching between TMD and CGC at small x

  I. cos2Φ asymmetries for dijet in eA and γ*-jet in pA

  II. Higgs boson production in pA
- Summary

# Gluon distributions

So far, the main focus of the spin physics field has been g,Δg,

g(x): collinear unpolarized gluons in unp. hadrons/nucleus

Δg(x): circularly polarized gluons in polarized hadrons

Keeping gluon transverse momentum, many other transverse momentum dependent gluon distributions can be nonzero.

Among them, the distribution of linearly polarized gluons inside an unpolarized nucleon/nucleus is of particular interest.

# Distribution of linearly polarized gluons: definition

$$\int \frac{dr^- d^2 r_\perp}{(2\pi)^3 P^+} e^{-ix_1 P^+ r^- + i\vec{k}_{1\perp}\cdot\vec{r}_\perp} \langle A | F^{+i}(r^- + y^-, r_\perp + y_\perp)\, L^\dagger\, L\, F^{+j}(y^-, y_\perp) | A \rangle$$

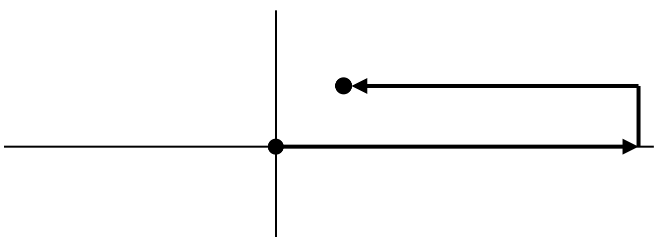
$$= \frac{\delta_\perp^{ij}}{2} x_1 G(x_1, k_{1\perp}) + \left( \hat{k}_{1\perp}^i \hat{k}_{1\perp}^j - \frac{1}{2}\delta_\perp^{ij} \right) x_1 h_1^{\perp g}(x_1, k_{1\perp}) \,, \qquad \delta_\perp^{ij} = -g^{ij} + (p^i n^j + p^j n^i)/p\cdot n$$

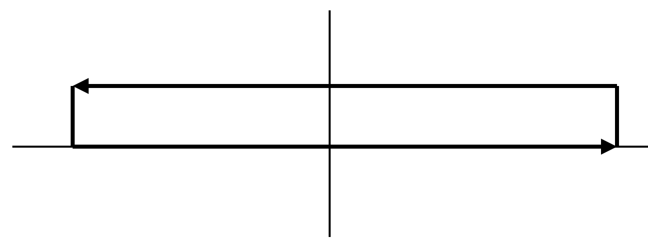
P. Mulders, J. Rodrigues, 2001

Gluon TMD distributions receive the contribution from ISI/FSI ⟶ make gauge link process dependent

in the adjoint representation

in the fundamental representation or in the adjoint representation





**Weizsäcker-Williams distribution** (probability interpretation)

**Dipole distribution**

F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan, 2011

# Distribution of linearly polarized gluons: phenomenology

Cos 2 Φ azimuthal asymmetries in the various processes:

**Φ: $k_t \wedge P_t$**

- dijet or heavy quark pair production in ep

  **D. Boer, S. J. Brodsky, P. Mulders, C. Pisano, 2011**

- heavy quark pair production in pp

  **D. Boer, S. J. Brodsky, P. Mulders, C. Pisano, 2011**

- photon pair production in pp

  **P. Nadolsky, C. Balazs, E. Berger, C. P. Yuan, 2007**
  **J. Qiu, M. Schlegel, W. Vogelsang, 2011**

- virtual photon-jet production in pA

  **A. Metz, ZJ, 2011**

It also affects the transverse momentum spectrum of Higgs produced in pp/pA

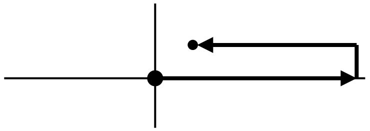
**S. Mantry, F. Petriello, 2010**
**S. Catani, M. Grazzini, 2011**
**D. Boer, W. J. Dunnen, C. Pisano, M. Schlegel, W. Vogelsang, 2011**
**P. Sun, B-W. Xiao, F. Yuan, 2011**
**A. Schafer, ZJ, 2012**

**See talk by Den Dunnen, Wilco**

# Distribution of linearly polarized gluons: dynamics I

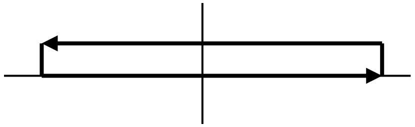
At small x, due to the presence of semi-hard scale($Q_S$), usual unp. gluon distributions can be computed using Mclerran-Venugopalan model,

$$xG^g_{WW}(x,k_\perp) = \frac{N_c^2-1}{N_c}\frac{S_\perp}{4\pi^4\alpha_s}\int d^2\xi_\perp\, e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp}\,\frac{1}{\xi_\perp^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$



**Y. V. Kovchegov, 96**
**J. J. Marian, A. Kovner, L. D. Mclerran, H. Weigert, 97**

$$xG^g_{DP}(x,k_\perp) = \frac{k_\perp^2 N_c}{2\pi^2\alpha_s}S_\perp\int\frac{d^2\xi_\perp}{(2\pi)^2}\, e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp}\, e^{-\frac{Q_{sq}^2\xi_\perp^2}{4}}$$



Following the similar procedure, linearly polarized gluon distribution also can be computed in the MV model.

# Distribution of linearly polarized gluons: dynamics II

with a random color source which has only a plus component reads,

$$(D_\nu F^{\nu\mu})_a(y) = \delta^{\mu +}\rho_a(y). \tag{2}$$

After a few algebraic manipulations, one has the solution,

$$F^{+i}(y_\perp) = \partial^+ A^i(y_\perp) = -U(y_\perp)\partial^i \alpha U^\dagger(y_\perp) \tag{3}$$

Where

$$U^\dagger(y_\perp) = P\exp^{ig\int_{y^-}^{\infty} d\zeta^- \alpha(\zeta^-, y_\perp)} \tag{4}$$

$\alpha$ satisfies the equation,

$$-\nabla_\perp^2 \alpha_a(y_\perp) = \tilde{\rho}_a(y_\perp). \tag{5}$$

or in the momentum space,

$$\alpha_a(k_\perp) = \frac{1}{\vec{k}_\perp^2}\tilde{\rho}_a(k_\perp). \tag{6}$$

$\tilde{\rho}_\alpha$ is the color source in the covariant gauge. Making the linear approximation$F^{+i}(r_\perp) = -\partial^i\alpha$, inserting this expression into the matrix element and contracting $\alpha$ fields with the following propagator:

$$\begin{aligned}\langle \alpha_a(x)\alpha_b(y)\rangle &= \delta_{ab}\delta(x^- - y^-)\Gamma_A(x_\perp - y_\perp)\lambda_A(x^-)\\ \Gamma_A(k_\perp) &= \frac{1}{k_\perp^4}\end{aligned} \tag{7}$$

$\lambda_A$ comes from the correlation of color source generated by a Gaussian weight function.

$$\begin{aligned}\langle \rho_a(x)\rho_b(y)\rangle_A &= \delta_{ab}\delta^2(x_\perp - y_\perp)\delta(x^- - y^-)\lambda_A(x^-)\\ W_A[\rho] &= \exp\left[-\frac{1}{2}\int d^3x \frac{\rho_a(x)\rho_a(x)}{\lambda_A(x^-)}\right]\end{aligned} \tag{8}$$

One can read UGD from this result,

$$xG(x,k_\perp) \simeq \delta^{ij}\frac{N_c^2-1}{4\pi^3}\int dy^- d^2y_\perp d^2r_\perp e^{-ik_\perp\cdot r_\perp}(-\partial_\perp^i\partial_\perp^j\Gamma_A(r_\perp))\lambda_A(y^-) = S_\perp\frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{k_\perp^2} \tag{9}$$

where $S_\perp = \pi R_A^2, \mu_A = \int_{-\infty^-}^{\infty^-} dy^-\lambda_A(y^-)$. Similarly,

$$\begin{aligned}&xh_{1,WW}^{\perp g}(x,k_\perp) \simeq \left[4\hat{k}_\perp^i\hat{k}_\perp^j - 2\delta^{ij}\right]\frac{N_c^2-1}{4\pi^3}\int dy^- d^2y_\perp d^2r_\perp e^{-ik_\perp\cdot r_\perp}(-\partial_\perp^i\partial_\perp^j\Gamma_A(r_\perp))\lambda_A(y^-)\\ &= 2S_\perp\frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{k_\perp^2} = 2xG(x,k_\perp)\end{aligned} \tag{10}$$

It is easy to see that gluon Boer-Mulders function saturates the positivity bound in the linearly perturbative theory. Now, let's turn to discuss nonlinear effect which is encoded in the Wilson line. We start with considering the contraction of field operators in all possible ways.

$$\left\langle F^{+i}(y+r)F^{+j}(y)\right\rangle_A = \left\langle [U_{ab}^\dagger\partial^i\alpha^b](y+r)[U_{ac}^\dagger\partial^j\alpha^c](y)\right\rangle_A \tag{11}$$

By rotational symmetry and ordering of the Wilson lines in $y^-$, the only allowed contraction is,

$$\begin{aligned}&\left\langle [U_{ab}^\dagger\partial^i\alpha^b](y+r)[U_{ac}^\dagger\partial^j\alpha^c](y)\right\rangle_A = \left\langle \partial^i\alpha^b(y+r)\partial^j\alpha^c(y)\right\rangle_A\left\langle U_{ab}^\dagger(y+r)U_{ca}(y)\right\rangle_A\\ &= \delta(r^-)\left\langle \mathrm{Tr}U^\dagger(y+r)U(y)\right\rangle_A\left[-\partial_\perp^i\partial_\perp^j\Gamma_A(r_\perp)\right]\lambda_A(y^-)\end{aligned} \tag{12}$$

where $U_{ac}^\dagger = U_{ca}$ in the adjoint representation. One can calculate the contraction of two Wilson lines by expanding them in power $\alpha$. One thus finds,

$$\left\langle \mathrm{Tr}U^\dagger(y+r)U(y)\right\rangle_A = (N_c^2-1)\exp\left\{-g^2N_c\left[\Gamma_A(0_\perp)-\Gamma_A(r_\perp)\right]\int_{y^-}^{\infty^-} d\zeta^-\lambda_A(\zeta^-)\right\} \tag{13}$$

Embedding the above results into the matrix element,

$$\begin{aligned}M^{ij} &= \frac{N_c^2-1}{4\pi^3}\int dy^- d^2y_\perp d^2r_\perp e^{-ik_\perp\cdot r_\perp}\left[-\partial_\perp^i\partial_\perp^j\Gamma_A(r_\perp)\right]\lambda_A(y^-)\\ &\quad\times\exp\left\{-g^2N_c\left[\Gamma_A(0_\perp)-\Gamma_A(r_\perp)\right]\int_{y^-}^{\infty^-} d\zeta^-\lambda_A(\zeta^-)\right\}\end{aligned} \tag{14}$$

Integrating out $y_\perp$ and $y^-$,

$$M^{ij} = \frac{N_c^2-1}{4\pi^3}S_\perp\int d^2r_\perp e^{-ik_\perp\cdot r_\perp}\frac{-\partial_\perp^i\partial_\perp^j\Gamma_A(r_\perp)}{g^2N_c\left[\Gamma_A(0_\perp)-\Gamma_A(r_\perp)\right]}\left(1-e^{-g^2N_c[\Gamma_A(0_\perp)-\Gamma_A(r_\perp)]\mu_A}\right) \tag{15}$$

where

$$\Gamma_A(0_\perp)-\Gamma_A(r_\perp) = \int_{\Lambda_{QCD}}^{1/r_\perp}\frac{d^2k_\perp}{(2\pi)^2}\frac{1}{k_\perp^4}\left[1-e^{ik_\perp\cdot r_\perp}\right] \simeq \frac{r_\perp^2}{16\pi}\ln\frac{1}{r_\perp^2\Lambda_{QCD}^2} \tag{16}$$

defining $Q_s^2 = \alpha_s N_c\mu_A\ln\frac{1}{r_\perp^2\Lambda_{QCD}^2}$, $M$ is rewritten as,

$$M^{ij} = \frac{N_c^2-1}{4\pi^3}S_\perp\int d^2r_\perp e^{-ik_\perp\cdot r_\perp}\frac{-\partial_\perp^i\partial_\perp^j\Gamma_A(r_\perp)}{\frac{1}{4\mu_A}r_\perp^2Q_s^2}\left(1-e^{-\frac{r_\perp^2Q_s^2}{4}}\right) \tag{17}$$

We can calculate UGD $xG_{WW}(x,k_\perp) = \delta_{ij}M^{ij}$with $-\nabla_\perp^2\Gamma_A(r_\perp) = \frac{1}{4\pi}\ln\frac{1}{r_\perp^2\Lambda_{QCD}^2}$,

$$xG_{WW}(x,k_\perp) = \frac{N_c^2-1}{N_c}\frac{S_\perp}{4\pi^4\alpha_s}\int d^2r_\perp e^{-ik_\perp\cdot r_\perp}\frac{1}{r_\perp^2}\left(1-e^{-\frac{r_\perp^2Q_s^2}{4}}\right) \tag{18}$$

which was first obtained in []. Gluon Boer-Mulders function is,

$$xh_{1,WW}^{\perp g}(x,k_\perp) = \left[4\hat{k}_\perp^i\hat{k}_\perp^j - 2\delta^{ij}\right]M_{ij} \tag{19}$$

The core integration is,

$$\left[4\hat{k}_\perp^i\hat{k}_\perp^j - 2\delta^{ij}\right]\left[-\partial_\perp^i\partial_\perp^j\Gamma_A(r_\perp)\right] = \int\frac{d^2p_\perp}{(2\pi)^2}e^{ip_\perp\cdot r_\perp}\frac{p_\perp^ip_\perp^j}{p_\perp^4}\left[4\hat{k}_\perp^i\hat{k}_\perp^j - 2\delta^{ij}\right] \tag{20}$$

We choose x-axis along the $r_\perp$ direction.

$$\begin{aligned}\int\frac{d^2p_\perp}{(2\pi)^2}e^{ip_\perp\cdot r_\perp}\frac{p_\perp^ip_\perp^j}{p_\perp^4}\left[4\hat{k}_\perp^i\hat{k}_\perp^j - 2\delta^{ij}\right] &= 2\int\frac{|p_\perp|d|p_\perp|d\theta_p}{(2\pi)^2}e^{-i|p_\perp||r_\perp|\cos\theta_p}\frac{2\cos^2(\theta_k-\theta_p)-1}{p_\perp^2}\\ &= 2\int\frac{d|p_\perp|d\theta_p}{(2\pi)^2}e^{-i|p_\perp||r_\perp|\cos\theta_p}\frac{\cos(2\theta_k-2\theta_p)}{|p_\perp|}\end{aligned} \tag{21}$$

Decomposing $\cos(2\theta_k-2\theta_p) = \cos 2\theta_k\cos 2\theta_p + \sin 2\theta_k\sin 2\theta_p$, and using the elementary integrations,

$$\int_0^{2\pi} d\theta_p e^{-iC\cos\theta_p}\cos(2\theta) = -2\pi K_2(C) \quad \int_0^{2\pi} d\theta_p e^{-iC\cos\theta_p}\sin(2\theta) = 0 \tag{22}$$

one thus has,

$$2\int\frac{d|p_\perp|d\theta_p}{(2\pi)^2}e^{-i|p_\perp||r_\perp|\cos\theta_p}\frac{\cos(2\theta_k-2\theta_p)}{|p_\perp|} = -2\int\frac{d|p_\perp|}{(2\pi)^2}\frac{2\pi K_2(|p_\perp||r_\perp|)\cos 2\theta_k}{|p_\perp|} \tag{23}$$

we further integrate out $p_\perp$ with the help of elementary integration,

$$\int_0^\infty dx\frac{K_2(Cx)}{x} = \frac{1}{2} \tag{24}$$

Finally, we work out this integration,

$$\int\frac{d^2p_\perp}{(2\pi)^2}e^{ip_\perp\cdot r_\perp}\frac{p_\perp^ip_\perp^j}{p_\perp^4}\left[4\hat{k}_\perp^i\hat{k}_\perp^j - 2\delta^{ij}\right] = -\frac{\cos 2\theta_k}{2\pi} \tag{25}$$

Gluon Boer-Mulders function is expressed as,

$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{4\pi^3}S_\perp\int d^2r_\perp e^{-ik_\perp\cdot r_\perp}\frac{-\frac{\cos 2\theta_k}{2\pi}}{\frac{1}{4\mu_A}r_\perp^2Q_s^2}\left(1-e^{-\frac{r_\perp^2Q_s^2}{4}}\right) \tag{26}$$

integrating out the azimuthal angle,

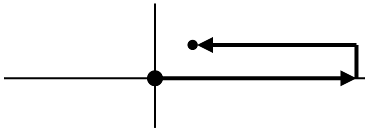
$$\begin{aligned}xh_{1,WW}^{\perp g}(x,k_\perp) &= \frac{N_c^2-1}{4\pi^3}S_\perp\int d^2r_\perp e^{-ik_\perp\cdot r_\perp}\frac{-\frac{\cos 2\theta_k}{2\pi}}{\frac{1}{4\mu_A}r_\perp^2Q_s^2}\left(1-e^{-\frac{r_\perp^2Q_s^2}{4}}\right)\\ &= \frac{N_c^2-1}{4\pi^3}S_\perp\int d|r_\perp|\frac{K_2(|k_\perp||r_\perp|)}{\frac{1}{4\mu_A}|r_\perp|Q_s^2}\left(1-e^{-\frac{r_\perp^2Q_s^2}{4}}\right)\end{aligned} \tag{27}$$

This is our main result.

Let's skip all derivation details…

# Distribution of linearly polarized gluons: dynamics III

WW type:



$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{8\pi^3} S_\perp \int d\xi_\perp \frac{K_2(k_\perp \xi_\perp)}{\frac{1}{4\mu_A}\xi_\perp Q_s^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$

**A. Metz, ZJ, 2011**

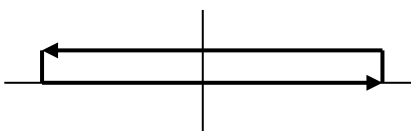
Large $k_t$, dilute limit $(k_\perp \gg Q_s)$

$$xG_{WW}^g(x,k_\perp) = xh_{1,WW}^{\perp g}(x,k_\perp) \simeq S_\perp \frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{k_\perp^2}$$ same perturbative tail

Small $k_t$, dense medium limit $(\Lambda_{QCD} \ll k_\perp \ll Q_s)$

$$xh_{1,WW}^{\perp g}(x,k_\perp) \simeq S_\perp \frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{Q_s^2} \qquad xG_{WW}^g(x,k_\perp) \simeq S_\perp \frac{N_c^2-1}{4\pi^3}\frac{1}{\alpha_s N_c}\ln\frac{Q_s^2}{k_\perp^2}$$

Dipole type:



$$xh_{1,DP}^{\perp g}(x,k_\perp) = xG_{DP}^g(x,k_\perp)$$ positivity bound is saturated for any value of $k_t$

**A. Metz, ZJ, 2011**

Remarks: I. two gluon distributions become identical in the dilute region,

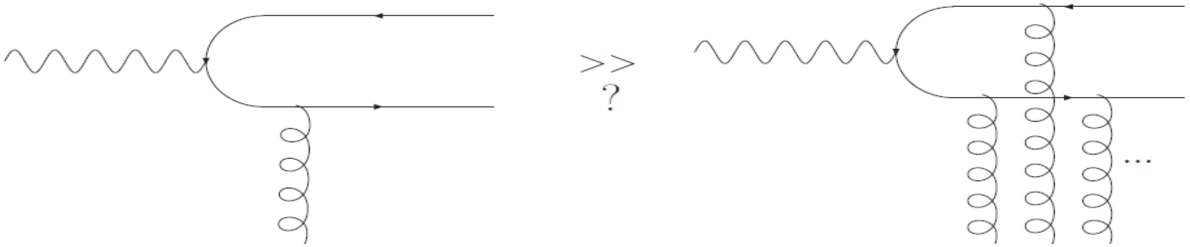
II. governed by the same BFKL evolution equation.

**F. Dominguez, J. W. Qiu, B. W. Xiao, F. Yuan 2011**

# How to probe them?

# Matching between TMD and CGC at small x

## Dijet production in eA scattering




- high gluon density ⟶higher twist contribution is not necessarily much smaller than leading twist contribution (one can not take TMD factorization for granted)
- resume gluon rescattering to all orders ⟶multiple Wilson lines
- employ power expansion in the correlation limit ($k_{t,gluon} << p_{t,jet}$)
- multiple Wilson lines ⟶ gluon TMD distributions

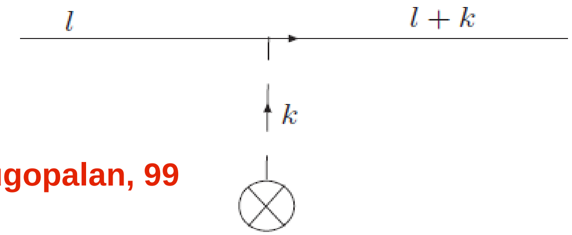
Effective TMD factorization at small x

**F. Dominguez, B-W. Xiao, F. Yuan 2011**
**F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan 2011**

◆ Extend to azimuthal dependent cross sections…

# Cos2Φ asymmetry for dijet production in eA

Eikonal scattering $2\pi\delta(k^-)\not{n}\,[U-1](k_\perp)$

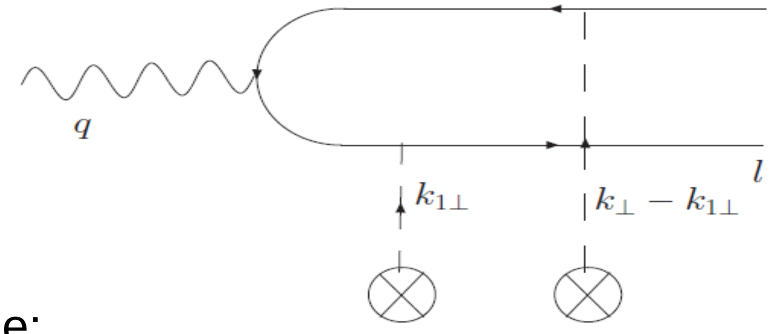



**I. Balitsky, 96**
**L. D. Mclerran, R. Venugopalan, 99**

Where

$$[U-1](k_\perp) = \int d^2x_\perp e^{i\vec{k}_\perp\cdot\vec{x}_\perp}[U(x_\perp)-1]$$

$$U(x_\perp) = \langle \mathrm{P} e^{ig\int_{-\infty}^{+\infty} dx^- A^+(x^-,x_\perp)}\rangle_A$$




Scattering amplitude:

$$\mathrm{M}^\mu = \int \frac{d^2k_{1\perp}}{(2\pi)^2} H^\mu(q,k,l,k_{1\perp})\left[U(k_{1\perp})U^\dagger(k_\perp - k_{1\perp}) - 1\right]$$

Integration over $k_{1t}$ is dominated by the kinematical region $k_{1t} \sim Q_s$
($Q_s$ typical small x gluon transverse momentum)

# Cos2Φ asymmetry for dijet production in eA I

In the correlation limit $k_{1t} \sim Q_s << l_t$, employ the power expansion

$$
\begin{aligned}
M^\mu &= \int \frac{d^2k_{1\perp}}{(2\pi)^2} H^\mu(q,k,l,k_{1\perp}=0)\left[U(k_{1\perp})U^\dagger(k_\perp - k_{1\perp}) - 1\right] \\
&+ \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{H^\mu(q,l,k_\perp,k_{1\perp})}{\partial k^i_\perp}\Big|_{k_\perp=0,k_{1\perp}=0} k^i_\perp \left[U(k_\perp)U^\dagger(k_\perp - k_{1\perp}) - 1\right] \\
&+ \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{H^\mu(q,l,k_\perp,k_{1\perp})}{\partial k^i_{1\perp}}\Big|_{k_\perp=0,k_{1\perp}=0} k^i_{1\perp} \left[U(k_{1\perp})U^\dagger(k_\perp - k_{1\perp}) - 1\right] + \ldots \\
&\approx \frac{H^\mu(q,l,k_\perp,k_{1\perp})}{\partial k^i_{1\perp}}\Big|_{k_\perp=0,k_{1\perp}=0} \int \frac{d^2k_{1\perp}}{(2\pi)^2} k^i_{1\perp} \left[U(k_{1\perp})U^\dagger(k_\perp - k_{1\perp}) - 1\right]
\end{aligned}
$$

Neglected terms are suppressed by a power of $Q_s/l_t$

Taking a partial integration,

$$
\int \frac{d^2k_{1\perp}}{(2\pi)^2} k^i_{1\perp} \left[U(k_{1\perp})U^\dagger(k_\perp - k_{1\perp}) - 1\right] = -i \int d^2x_\perp e^{i\vec{k}_\perp\cdot\vec{x}_\perp} \left[(\partial^i U(x_\perp))U^\dagger(x_\perp)\right]
$$

# Cos2Φ asymmetry for dijet production in eA II

The cross section reads

$$\begin{aligned} d\sigma &\propto M_\mu M_\nu^* \varepsilon_{L,T}^{\mu} \varepsilon_{L,T}^{\nu *} \\ &= \frac{H_\mu(q,l,k_\perp,k_{1\perp})}{\partial k_{1\perp}^i}\Big|_{k_\perp=0,k_{1\perp}=0} \frac{H_\nu^*(q,l,k_\perp,k'_{1\perp})}{\partial k'^{\,j}_{1\perp}}\Big|_{k_\perp=0,k'_{1\perp}=0} \varepsilon_{L,T}^{\mu} \varepsilon_{L,T}^{\nu *} \\ &\times (-1) \int d^2x_\perp d^2x'_\perp e^{i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{x}'_\perp)} \langle \mathrm{Tr}[\partial^i U(x_\perp)] U^\dagger(x'_\perp)[\partial^j U(x'_\perp)] U^\dagger(x_\perp)\rangle_x \end{aligned}$$

with the help of the formula

$$\partial^i U(x_\perp) = ig \int_{-\infty}^{\infty} dx^- U[-\infty,x^-,x_\perp] \partial^i A^+(x^-,x_\perp) U[x^-,\infty,x_\perp]$$

One can identify,

$$\begin{aligned} M_{WW}^{ij} &= -\frac{2}{\alpha_s} \int \frac{d^2x_\perp}{(2\pi)^2} \frac{d^2x'_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot(\vec{x}_\perp-\vec{x}'_\perp)} \langle \mathrm{Tr}[\partial^i U(x_\perp)] U^\dagger(x'_\perp)[\partial^j U(x'_\perp)] U^\dagger(x_\perp)\rangle_x \\ &= \frac{\delta_\perp^{ij}}{2} x f_{1,WW}^g(x,k_\perp) + \left(\frac{1}{2}\hat{k}_\perp^i \hat{k}_\perp^j - \frac{1}{4}\delta_\perp^{ij}\right) x h_{1,WW}^{\perp g}(x,k_\perp)\,. \end{aligned}$$

# Cos2Φ asymmetry for dijet production in eA III

We arrive at,

$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{dP.S.} = \delta(x_{\gamma^*}-1)H_{\gamma_T^* g\to q\bar{q}}\Big\{xG_{WW}^g(x,k_\perp) -2\frac{[z_q^2+(1-z_q)^2]\epsilon_f^2 P_\perp^2 - m_q^2 P_\perp^2}{[z_q^2+(1-z_q)^2](\epsilon_f^4+P_\perp^4)+2m_q^2P_\perp^2}\cos(2\phi)xh_{1,WW}^{\perp g}(x,k_\perp)\Big\}$$

$$\frac{d\sigma^{\gamma_L^* A\to q\bar{q}+X}}{dP.S.} = \delta(x_{\gamma^*}-1)H_{\gamma_L^* g\to q\bar{q}}\Big\{xG_{WW}^g(x,k_\perp)+\cos(2\phi)xh_{1,WW}^{\perp g}(x,k_\perp)\Big\},$$

in full agreement with TMD factorization

Matching also found in γ*- jet production in pA

$$\frac{d\sigma^{pA\to\gamma^* q+X}}{dP.S.} = \sum_q x_p f_1^q(x_p)\, xG_{DP}^g(x,k_\perp)H_{qg\to\gamma^* q}\Big\{1+\cos(2\phi)\frac{4Q^2\hat{t}}{\hat{s}^2+\hat{u}^2+2Q^2\hat{t}}\Big\}$$

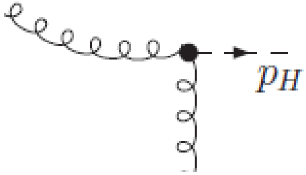
**parameter free!**

also verified by the calculation performed in the position space

**F. Dominguez, J. W. Qiu, B. W. Xiao, F. Yuan 2011**

# Higgs production in pp collisions

◆ Dominant channel: $\mathcal{L}_{eff} = -\frac{1}{4} g_\phi \Phi F^a_{\mu\nu} F^{a\mu\nu}$




➢ In TMD factorzation (Collins-Soper-Sterman) $p_{HT} << M_H$

$$\frac{d^3\sigma}{d^2 p_{H\perp} dy} = \sigma_0 \int d^2 k_{1\perp} x_2 x_1 \left\{ G_{WW}(x_2, k_{2\perp}) G_{WW}(x_1, k_{1\perp}) + \left(2(\hat{k}_{1\perp} \cdot \hat{k}_{2\perp})^2 - 1\right) h^{\perp g}_{1,WW}(x_2, k_{2\perp}) h^{\perp g}_{1,WW}(x_1, k_{1\perp}) \right.$$

**D. Boer, W. J. Dunnen, C. Pisano, M. Schlegel, W. Vogelsang, 2011**

See talk by Den Dunnen, Wilco

**P. Sun, B-W. Xiao, F. Yuan, 2011**

The effect of linearly polarized gluons on Higgs $p_t$ spectrum studied in SCET and collinear factorization

**S. Mantry, F. Petriello, 2010**

**S. Catani, M. Grazzini, 2011**

➢ In $k_t$ factorization (Kuraev-Lipatov-Fadin, Gribov-Levin-Ryskin) $p_{HT} << M_H$

$$\frac{d^3\sigma}{d^2 p_{H\perp} dy} = \sigma_0 \int d^2 k_{1\perp} x_2 G_{WW}(x_2, k_{2\perp}) x_1 G_{WW}(x_1, k_{1\perp}) 2(\hat{k}_{1\perp} \cdot \hat{k}_{2\perp})^2$$

**F. Hautmann, 2002 & A. V. Lipatov, N. P. Zotov, 2005**

In the dilute limit, they agree with each other

**P. Sun, B-W. Xiao, F. Yuan, 2011**

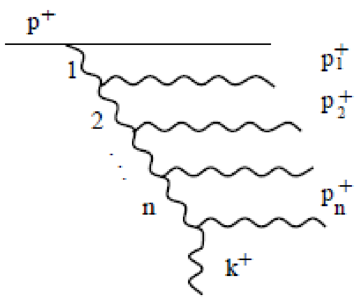
How about the dense medium?

# Lipatov approximation

Hybrid approach: proton ← Lipatov approximation

nucleus ← CGC

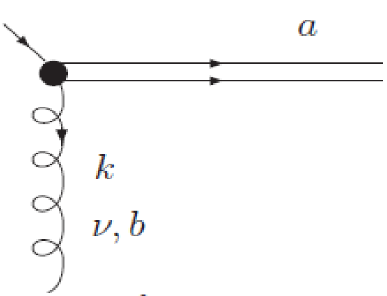
Gluon cascade:



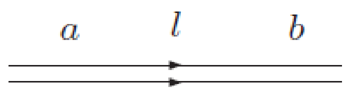

Strongly ordered in longitudinal momenta

$$p^+ \gg p_1^+ \gg p_2^+ \gg \cdots \gg p_n^+ \gg k^+$$

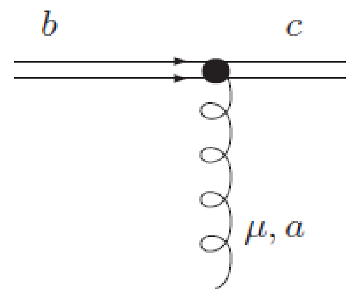
Color soucre can be treated as a eikonal line in the strong rapidity ordering region (Lipatov approximation)




$\frac{k\cdot p}{p\cdot n} n^\nu \delta^{ab}$




$\frac{i}{l\cdot n - i\epsilon}\delta^{ab}$



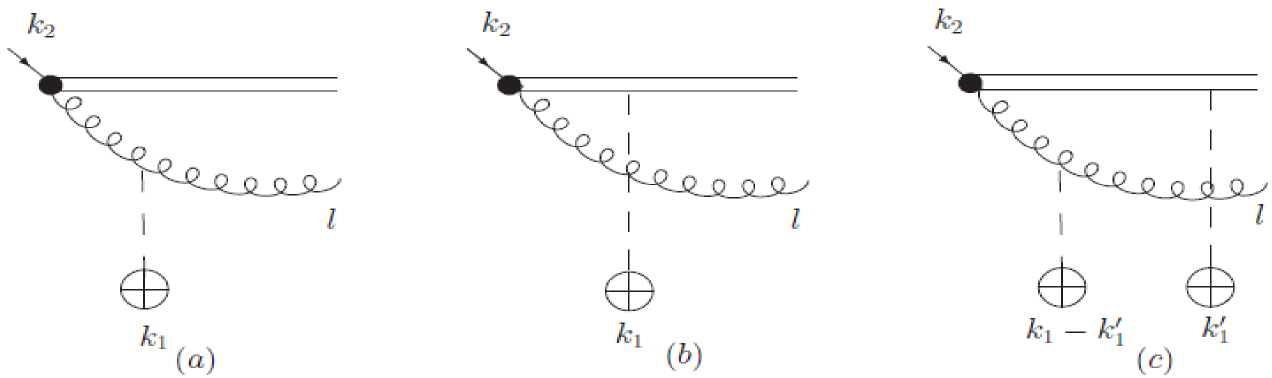

$-ig(T^a)_{cb} n^\mu$

S. Catani, M. Ciafaloni, F. Hautmann, 91

J. C. Collins, R. K. Ellis, 91

# Soft gluon production in pA collisions

As a test of the method, soft gluon production in pA

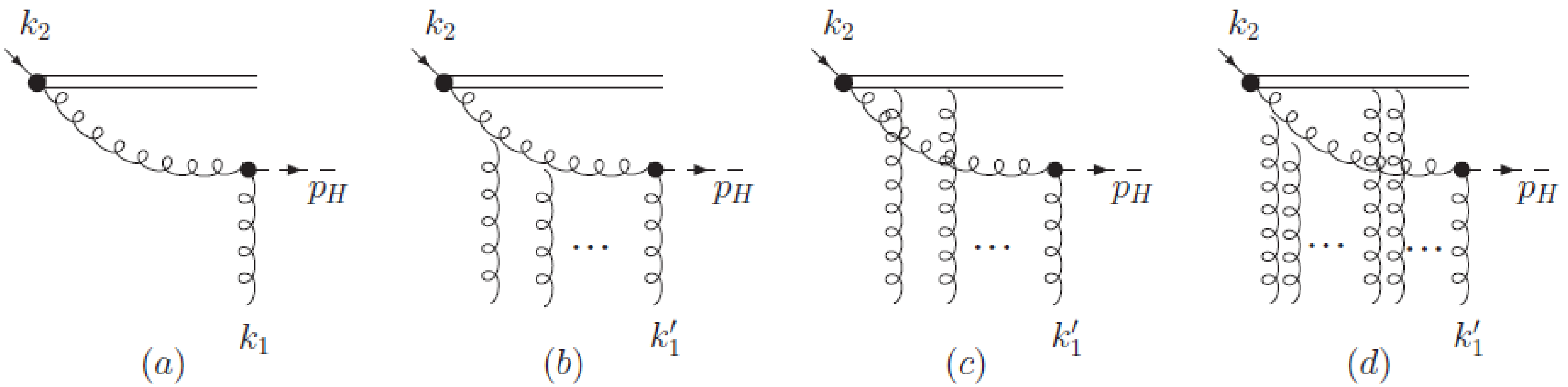



$$\frac{d\sigma}{d^2 l dy} = \frac{4\pi^2 \alpha_s N_c}{(N_c^2 - 1) l_\perp^2} \int d^2 k_{1\perp} x_2 g(x_2, k_{2\perp}) x_1 G_{DP}(x_1, k_{1\perp})$$

**Y. V. Kovchegov, A. H. Muller 98**
**A. Dumitru, L. D. Mclerran, 2002**
**J. P. Blaizot, F. Gelis, R. Venugopalan, 2004**

➢ Remark: the hybrid approach only works in the light cone gauge of the proton.

# Higgs production in pA collisions




$$\frac{d^3\sigma}{d^2p_{H\perp}dy} = \sigma_0 \int d^2k_{1\perp} x_2 g(x_2, k_{2\perp}) x_1 \left[ G_{WW}(x_1, k_{1\perp}) + \left( 2(\hat{k}_{1\perp} \cdot \hat{k}_{2\perp})^2 - 1 \right) h_{1,WW}^{\perp g}(x_1, k_{1\perp}) \right]$$

**A. Schäfer and ZJ**

In the dilute limit: $x_2 g(x_2, k_{2\perp}) = xG_{WW}^g(x, k_\perp) = xh_{1,WW}^{\perp g}(x, k_\perp) \simeq S_\perp \frac{N_c^2 - 1}{4\pi^3} \frac{\mu_A}{k_\perp^2}$

consistent with TMD result

# Summary:

- The distribution of linearly polarized gluon using MV model

  **Dipole type: positivity bound saturated**

  **WW type: positivity bound saturated at large $k_t$, suppressed at small $k_t$**

- cos2Φ asymmetries for dijet in eA and γ*-jet in pA in CGC

  **Effective TMD factorization holds.**

- Higgs boson production in pA

  **$k_t$ factorization breaks down in the dense medium region**

  **$k_t$ factorization and TMD approach consistent in the dilute limit**

## Outlook: cos2Φ asymmetry for heavy quark pair production in pA