



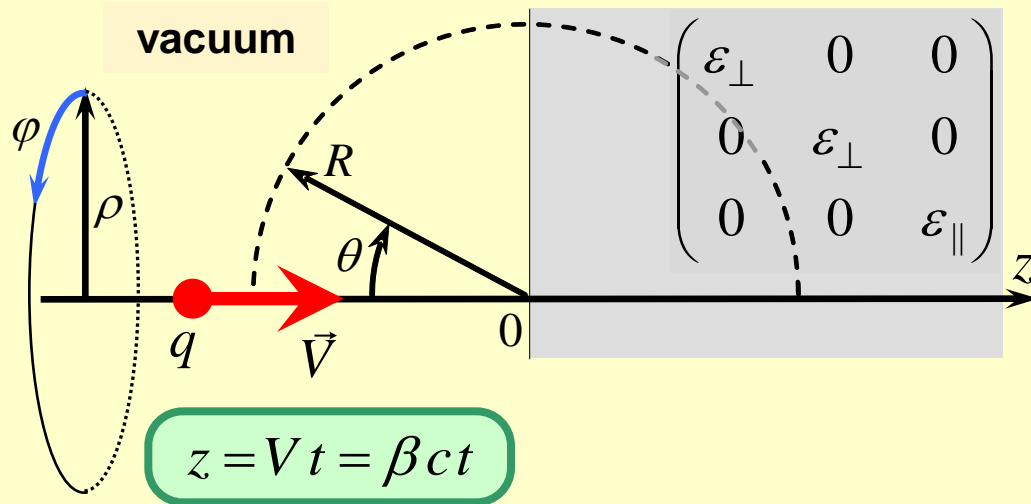
RADIATION OF A CHARGE FLYING FROM VACUUM INTO ANISOTROPIC DISPERSIVE MEDIUM



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Formulation of the problem

Vacuum – anisotropic plasma-like medium interface



$$\mu = 1$$
$$\epsilon_{\perp} = 1 - \frac{\omega_{p\perp}^2}{\omega^2 + 2i\omega\omega_{d\perp}},$$
$$\epsilon_{\parallel} = 1 - \frac{\omega_{p\parallel}^2}{\omega^2 + 2i\omega\omega_{d\parallel}}$$

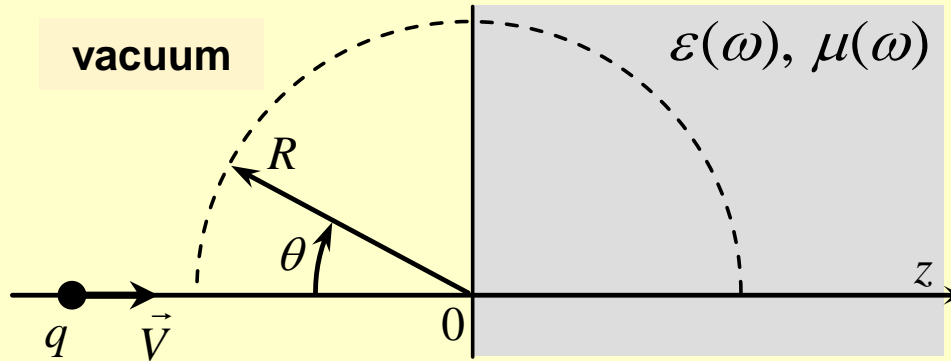
Goals:

Detailed investigation of the structure of the electromagnetic field

Taking into account the specific frequency dispersion and losses

Motivation

Vacuum – “left-handed medium” (LHM) interface



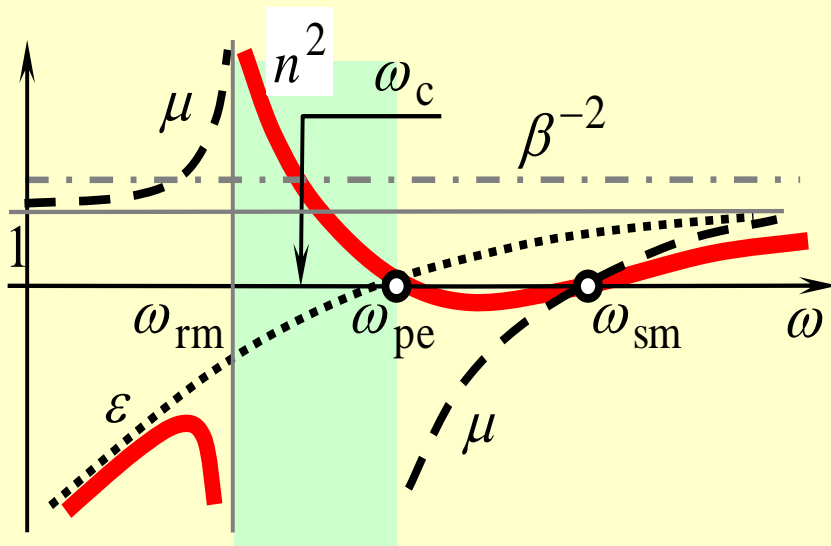
Reversed Cherenkov-transition radiation (RCTR) occurs in both vacuum and medium

Isotropic LHM:

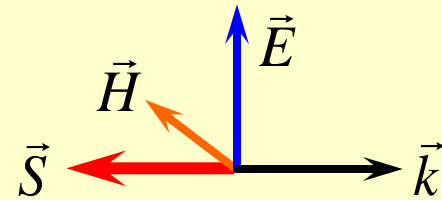
$$\epsilon(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2 + 2i\omega_{de}\omega}, \quad \mu(\omega) = 1 + \frac{\omega_{pm}^2}{\omega_{rm}^2 - 2i\omega_{dm}\omega - \omega^2}$$

$$\omega_{pe} > \omega_{rm}$$

Left-handed frequency band



reversed VCR:

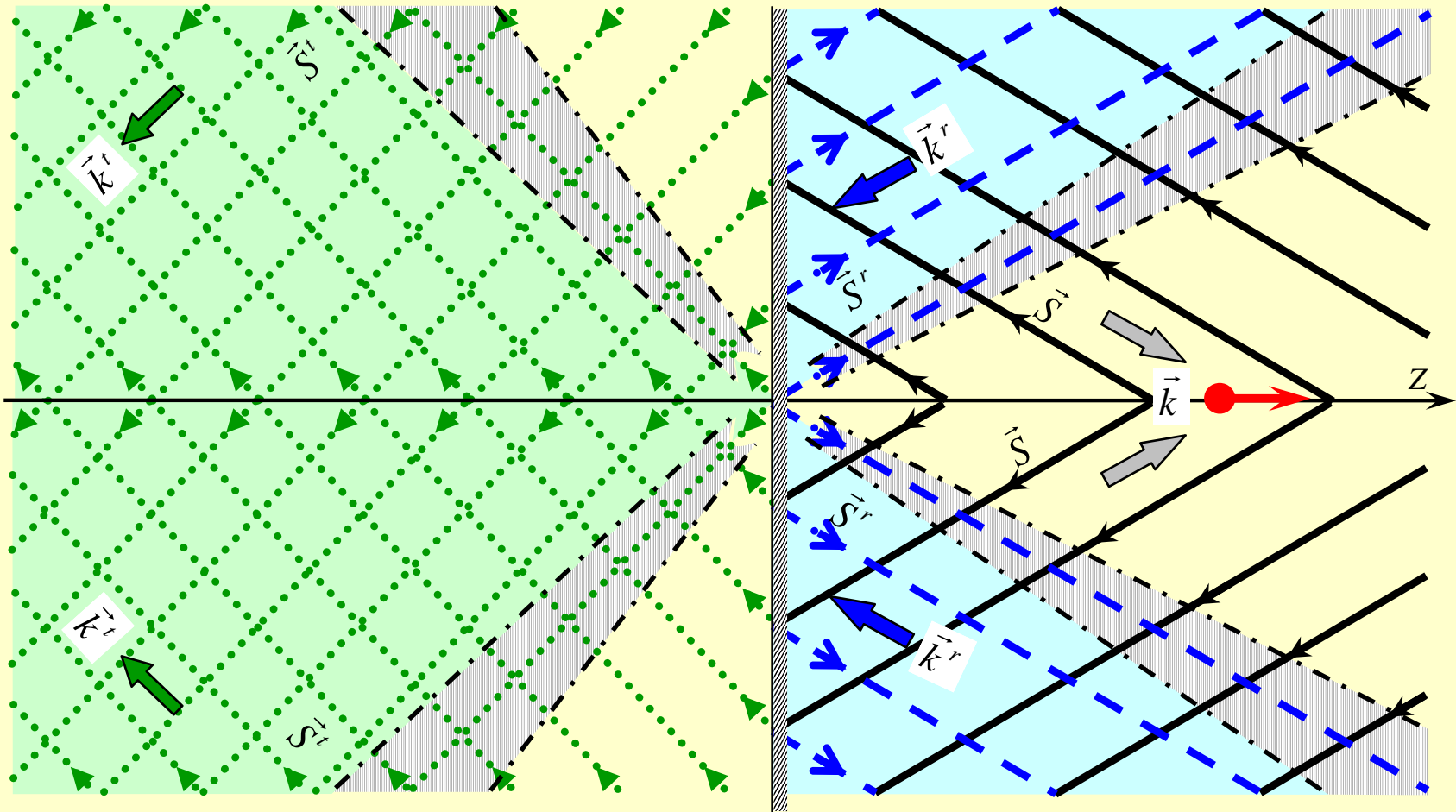


Motivation

Vacuum – “left-handed medium” (LHM) interface

Spatial distribution of the Fourier harmonic

$$\beta_{\text{CR}} < \beta < \beta_{\text{TIR}}$$

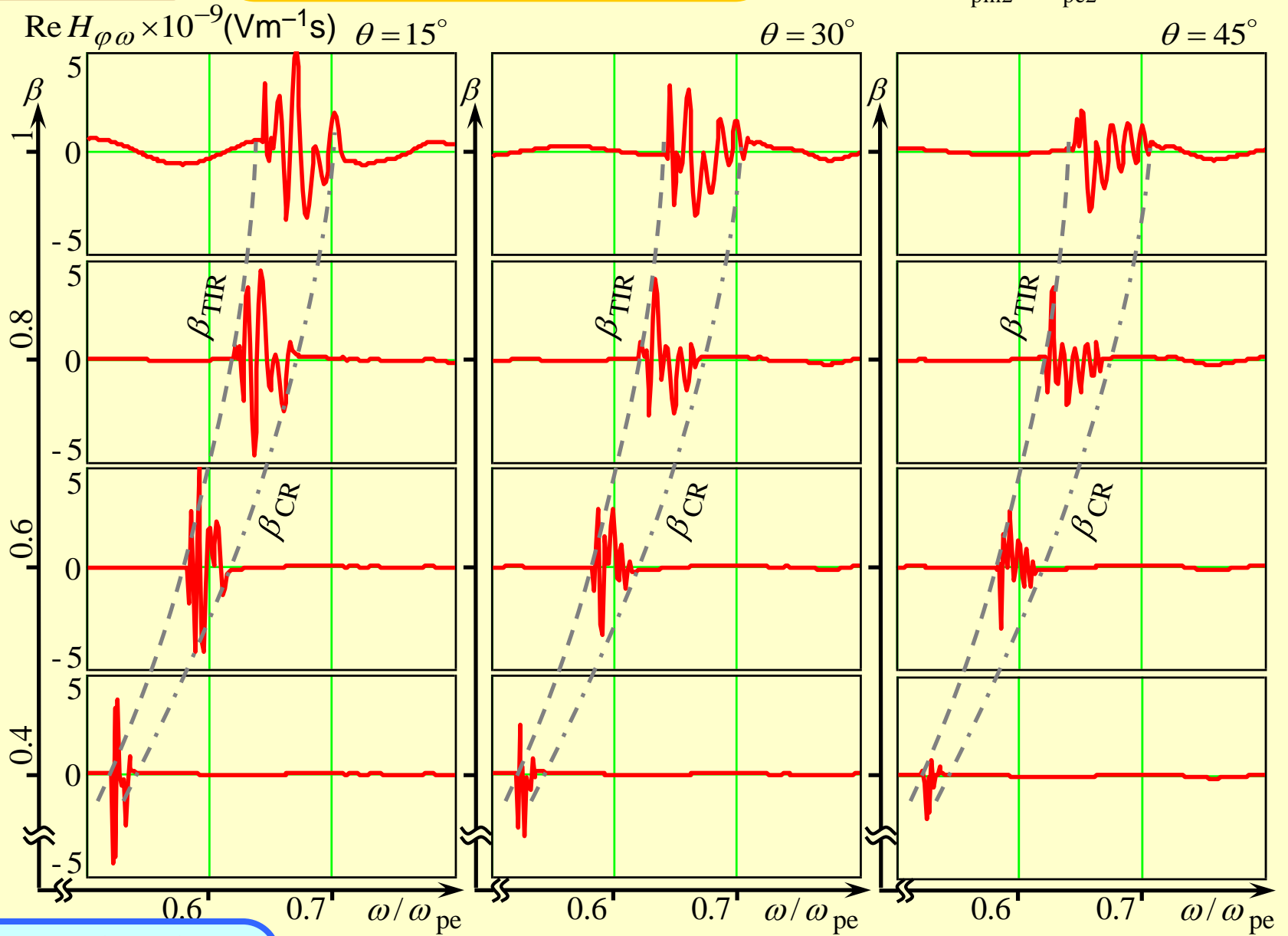


- - lines parallel to the Poyting vector of VCR
- - -→ - lines parallel to the Poyting vector of RCTR in medium
- ⋯→ - lines parallel to Poyting vector of RCTR in vacuum

Motivation

Field spectrum in vacuum

$q = -1$ nC $R = 14$ cm
 $\omega_{pm2} = \omega_{pe2} = 2\pi \cdot 10 \cdot 10^9$ c⁻¹



$\beta_{CR} < \beta < \beta_{TIR}$

$\omega_{rm2} = 0, \omega_{de2} = \omega_{dm2} = 10^{-3} \omega_{pe2}, \omega_{pe1} = 10^{-2} \omega_{pe2}, \omega_{de1} = 10^{-6} \omega_{pe2}$

Solution of the problem

Vacuum – anisotropic plasma-like medium interface

$$E_\rho, E_z, H_\varphi \neq 0 \quad H_\varphi^{(1,2)} = H_\varphi^{q(1,2)} + H_\varphi^{b(1,2)}$$

V.L. Ginzburg, V.N. Tsytovich.
 “Transition radiation and transition scattering”

Self-field of the charge

$$H_\varphi^{q(1,2)} = \frac{q}{2\pi\beta c} \int_{-\infty}^{+\infty} H_{\varphi\omega}^{q(1,2)} \exp(i\omega\zeta V^{-1}) d\omega, \quad \zeta = z - Vt$$

$$H_{\varphi\omega}^{q(1,2)} = i\pi\beta s_{1,2} H_1^{(1)}(s_{1,2}\rho) \quad s_1 = i\sqrt{\omega^2 V^{-2}(1-\beta^2)} \quad s_2 = \sqrt{\omega^2 V^{-2} \varepsilon_{\parallel} \varepsilon_{\perp}^{-1} (\varepsilon_{\perp} \beta^2 - 1)}$$

$\text{Im } s_{1,2} > 0$

Vavilov-Cherenkov radiation (VCR): s – Real

$$\min(\omega_{p\parallel}, \omega_{p\perp}) < \omega < \max(\omega_{p\parallel}, \omega_{p\perp})$$

Scattered field

$$H_\varphi^{b(1,2)} = \frac{q}{2\pi\beta c} \int_{-\infty}^{+\infty} H_{\varphi\omega}^{b(1,2)} \exp(-i\omega t) d\omega$$

$$H_{\varphi\omega}^{b(1,2)} = \mp \int_{-\infty}^{+\infty} dk_\rho B^{(1,2)} k_\rho^2 [k_z^{(1,2)}]^{-1} H_1^{(1)}(\rho k_\rho) \exp(ik_z^{(1,2)}|z|)$$

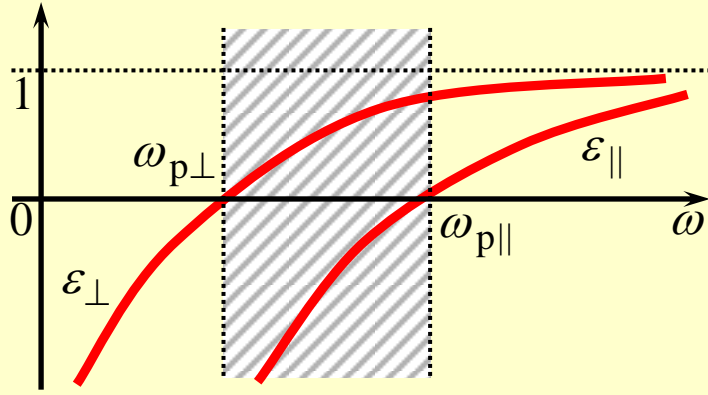
$$k_z^{(1)} = \sqrt{\omega^2 c^{-2} - k_\rho^2} \quad k_z^{(2)} = \sqrt{\varepsilon_{\perp} \varepsilon_{\parallel}^{-1} (\omega^2 c^{-2} \varepsilon_{\parallel} - k_\rho^2)} \quad \text{Im } k_z^{(1,2)} \geq 0$$

$$B^{(1)} = \frac{k_z^{(1)}}{k_z^{(2)} + \varepsilon_{\perp} k_z^{(1)}} \left(\frac{\beta k_z^{(2)} - \omega c^{-1} \varepsilon_{\perp}}{k_\rho^2 - s_1^2} + \frac{c \varepsilon_{\perp} \varepsilon_{\parallel}^{-1} \beta^2}{\omega \left(1 + \sqrt{1 - c^2 \beta^2 \omega^{-2} \varepsilon_{\perp} \varepsilon_{\parallel}^{-1} (k_\rho^2 - s_2^2)} \right)} \right)$$

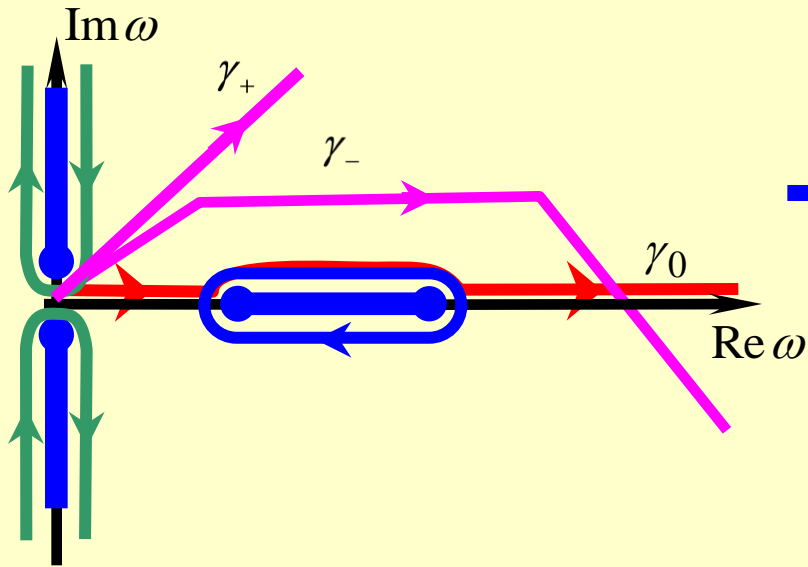
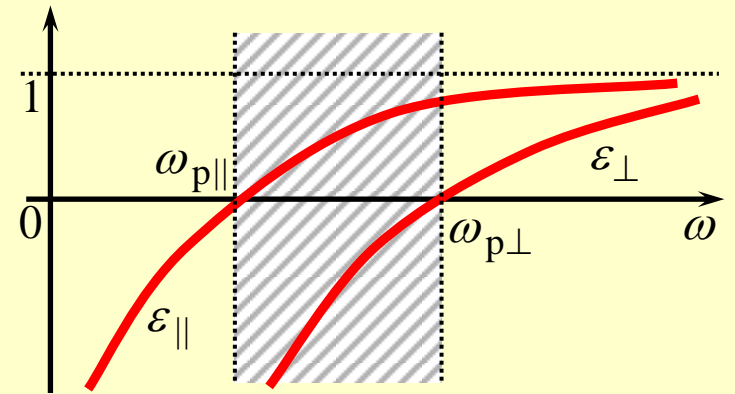
Transition radiation (TR)

Reversed Cherenkov – transition radiation (RCTR)

Self-field of the charge



Anisotropic plasma-like medium



- - branch points
- - cuts $\text{Im } s_2 = 0$

$$H_{\varphi}^{q(2)} = H_{\varphi C}^{q(2)} + H_{\varphi W}^{q(2)}$$

wave field (VCR)

$$H_{\varphi W}^{q(2)} = \frac{2q}{c} \int_{\omega_{p\parallel}}^{\omega_{p\perp}} |s_2(\omega)| J_1(\rho |s_2(\omega)|) \sin\left(\omega \frac{|\zeta|}{V}\right) d\omega \Theta(-\zeta)$$

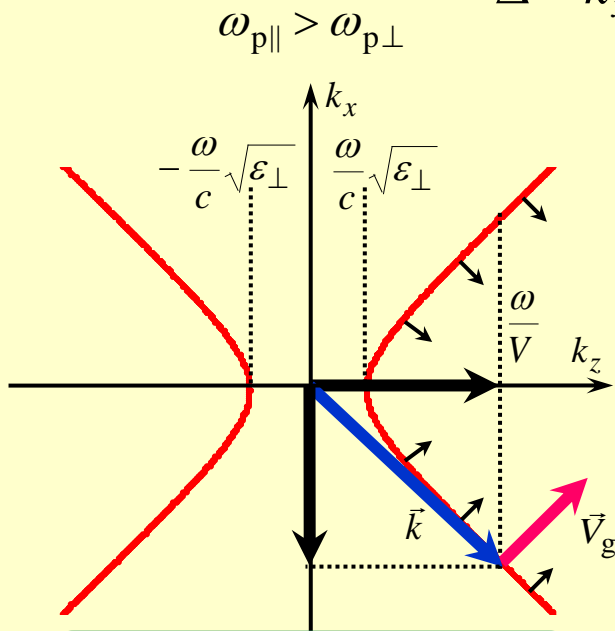
quasistatic
("quasiqulomb") field

$$H_{\varphi C}^{q(2)} = \frac{q}{c} \int_{|\omega_c|}^{\infty} |s_2(i\tilde{\omega})| J_1(\rho |s_2(i\tilde{\omega})|) \exp\left(-\tilde{\omega} \frac{|\zeta|}{V}\right) d\tilde{\omega}$$

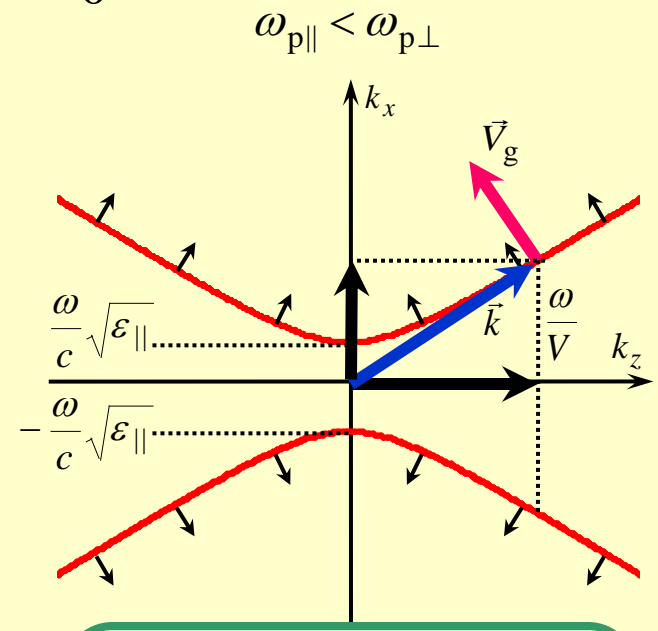
Self-field of the charge

Anisotropic plasma-like medium

$$\Delta = k_x^2 / \epsilon_{\parallel} + k_z^2 / \epsilon_{\perp} - \omega^2 / c^2 = 0$$



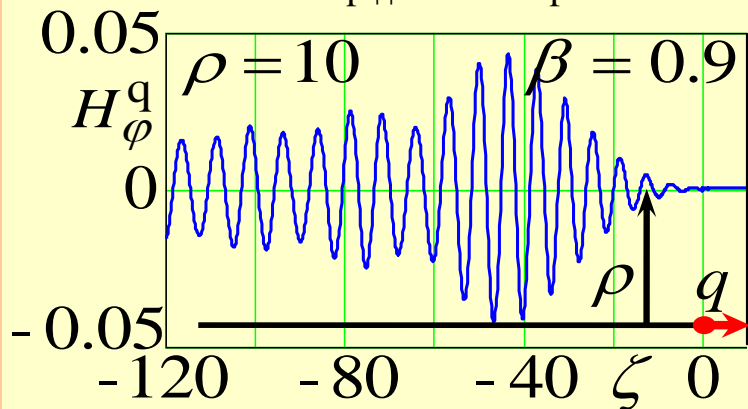
forward radiation



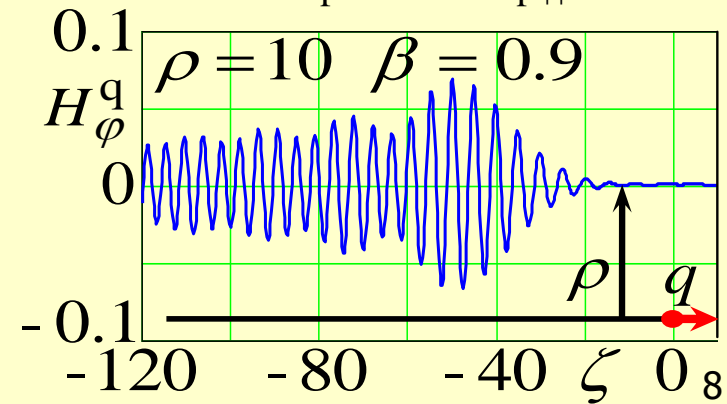
backward radiation

$$\omega_{p\parallel} = 1.5 \omega_{p\perp}$$

$$\omega_{p\perp} = 1.5 \omega_{p\parallel}$$



$\omega_{d\parallel} = \omega_{d\perp} = 10^{-3} \omega_{p\parallel}$
 ζ, ρ in units of $c\omega_{p\parallel}^{-1}$



Scattered field

Vacuum – anisotropic plasma-like medium interface

Analytical approach

Fourier harmonics of the scattered field

$$H_{\varphi\omega}^{b(1,2)} = \mp \int_{-\infty}^{+\infty} dk_{\rho} B^{(1,2)} \frac{k_{\rho}^2}{k_z^{(1,2)}} H_1^{(1)}(\rho k_{\rho}) \exp(ik_z^{(1,2)}|z|)$$

Asymptotic representation in the far field zone
(with respect to the incident point)

$$\sim R^{-1}, \quad \sim \rho^{-1/2}$$

“Half-shadow” regions estimation

Impact of losses in medium on the radiation field

Numerical approach

Fourier harmonics of the scattered field

Analytical investigation of the integrands behavior, choosing the appropriate integration step and integration interval

Total scattered field

$$H_{\varphi}^{b(1,2)} = \frac{q}{2\pi\beta c} \int_{-\infty}^{+\infty} d\omega H_{\varphi\omega}^{b(1,2)} \exp(-i\omega t)$$

Numerical investigation of the integrands behavior, choosing the appropriate integration step and interval

Results

Vacuum – anisotropic plasma-like medium interface

Conditions of the RCTR existence in vacuum

$$\omega_{p\parallel} < \omega < \Omega(\beta)$$

$$\beta > \beta_{\text{RCTR}}(\omega)$$

$$\Omega^2 = \frac{\omega_{p\parallel}^2(1-\beta^2)}{2} + \sqrt{\frac{\omega_{p\parallel}^4(1-\beta^2)^2}{4} + \omega_{p\parallel}^2\omega_{p\perp}^2\beta^2}$$

$$\beta_{\text{RCTR}}(\omega) = \sqrt{\omega^2(\omega^2 - \omega_{p\parallel}^2)\omega_{p\parallel}^{-2}(\omega_{p\perp}^2 - \omega^2)^{-1}}$$

Field asymptotic in vacuum, $\sim R^{-1} \sim \rho^{-1/2}$

$$H_{\varphi\omega}^{b(1)} \approx H_{\varphi\omega}^{b(1)S} + H_{\varphi\omega}^{b(1)P}$$

Spherical wave of TR:

$$H_{\varphi\omega}^{b(1)S} \sim \frac{\exp(ik_1 R)}{R} \quad k_1 R \gg 1$$

Cylindrical wave of RCTR:

$$H_{\varphi\omega}^{b(1)P} = \frac{iq}{\beta c^2} \frac{-s_2 \omega}{g_3^*(\omega)} H_1^{(1)}(\rho s_2) \exp(ik_z^{(1)}(s_2)|z|) \Theta(\theta - \theta_1),$$

$$g_3^*(\omega) = -\omega/c\beta + \varepsilon_{\perp} k_z^{(1)}(s_2) \quad \sin \theta_{10} = \sqrt{\varepsilon'_{\parallel}} \sqrt{1 + \frac{1}{|\varepsilon'_{\perp}| \beta^2}}$$

$$\theta > \theta_1 = \theta_{10} + \delta \theta_1$$

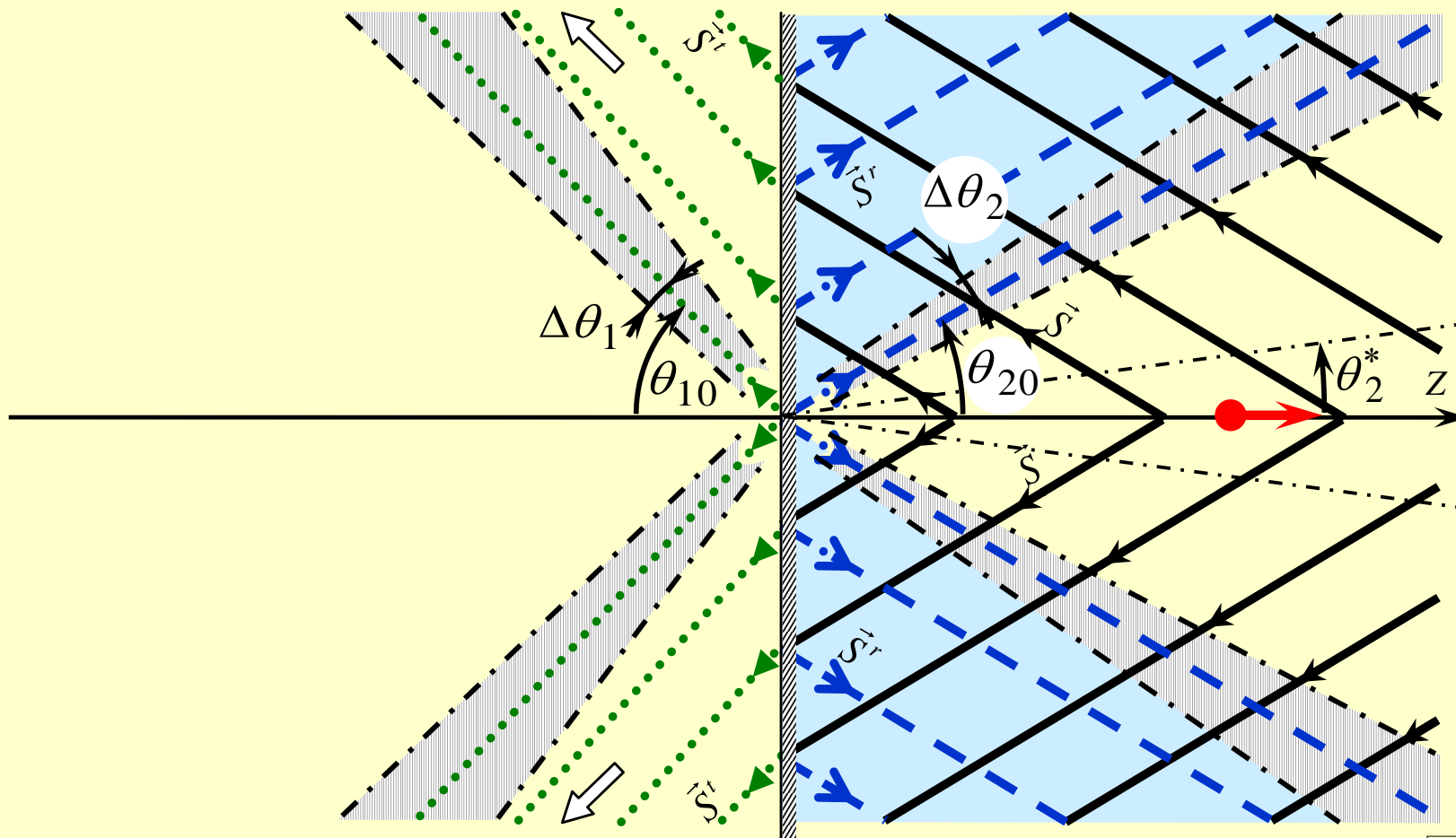
$$\delta \theta_1 = \sqrt{\frac{2c}{\omega R_1}} \quad R_1 = 8 \frac{c}{\omega} \left[\text{tg} \theta_{10} \left(\frac{\varepsilon''_{\parallel}}{\varepsilon'_{\parallel}} + \frac{\varepsilon''_{\perp}}{|\varepsilon'_{\perp}|} \frac{1}{1 + |\varepsilon'_{\perp}| \beta^2} \right) \right]^{-2}$$

“half-shadow” regions

$$|\theta - \theta_{10}| \leq \Delta \theta_1 \quad R \leq R_1$$

$$\Delta \theta_1 = \sqrt{2c\omega^{-1} R^{-1} (1 - R/R_1)}$$

$$\omega_{p||} < \omega < \Omega(\beta)$$

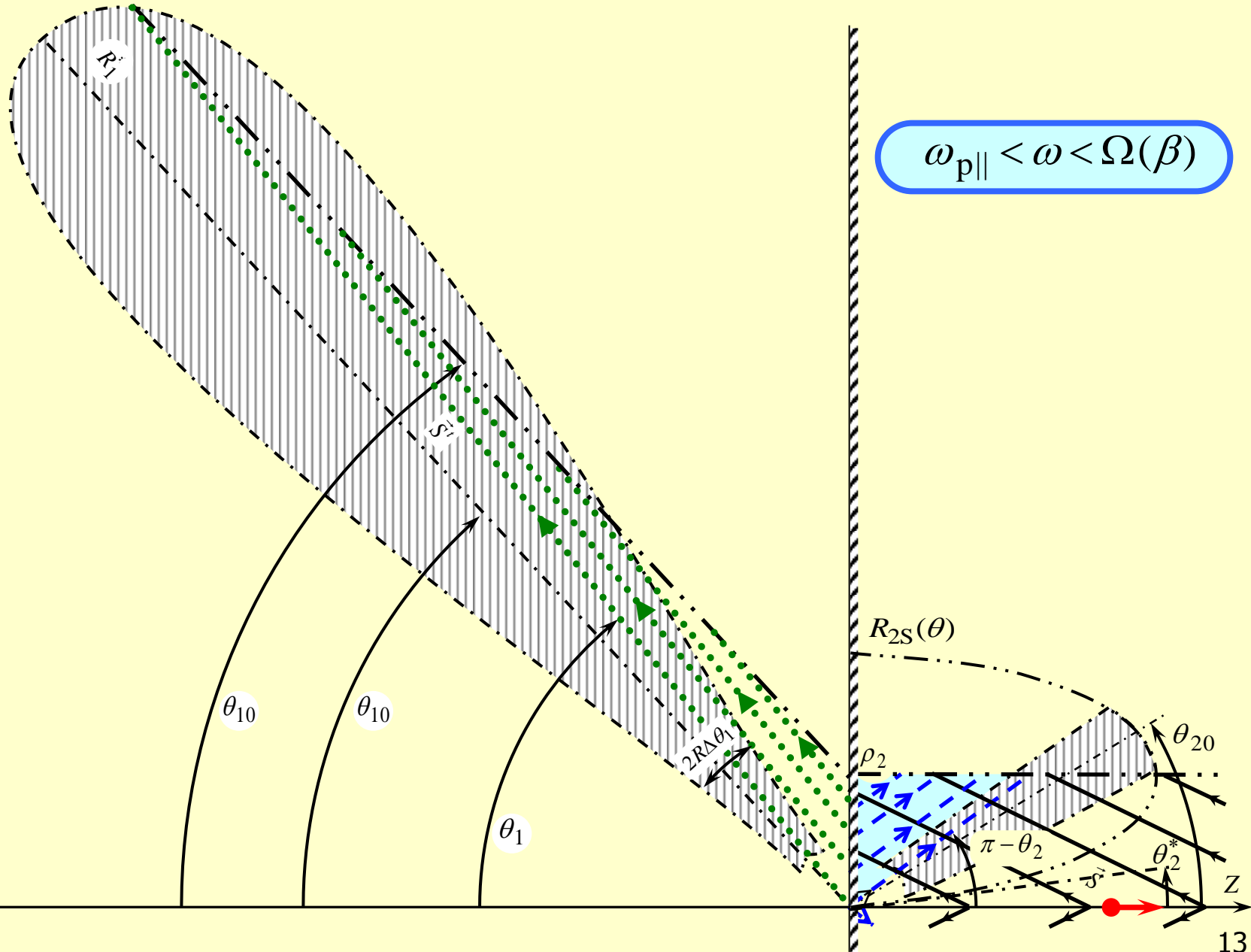


$$\text{tg}\theta_2^* = \sqrt{|\epsilon'_\perp|/\epsilon'_\parallel}$$

- lines parallel to the Poyting vector of VCR
- lines parallel to the Poyting vector of RCTR in medium
- lines parallel to Poyting vector of RCTR in vacuum

Results

Spatial distribution of the Fourier harmonic

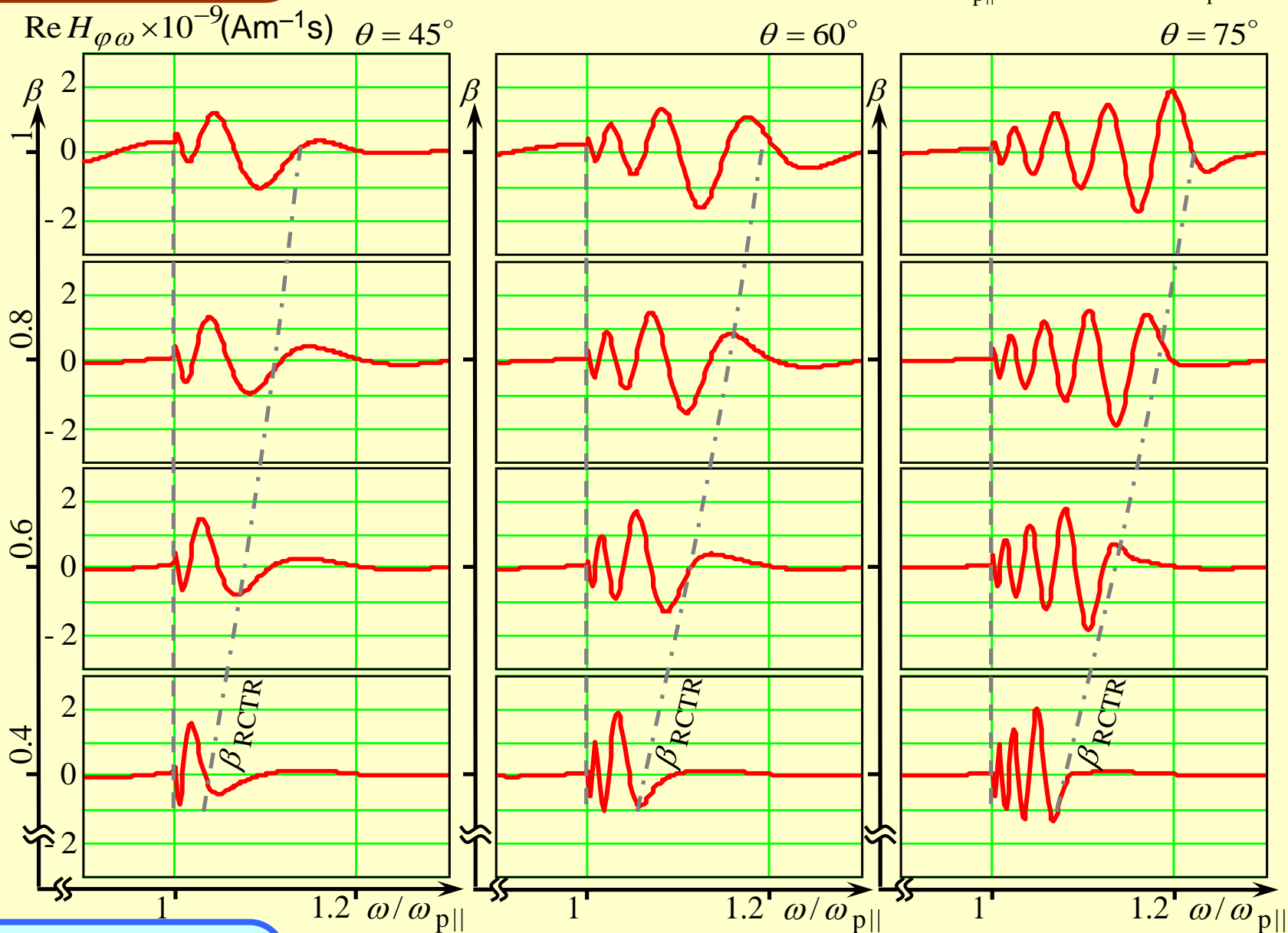


Results

Field spectrum in vacuum

$$R = 14 \text{ cm} \quad q = -1 \text{ nC}$$

$$\omega_{p\parallel} = 2\pi \cdot 10^{10} \text{ c}^{-1}, \quad \omega_{p\perp} = 1.5\omega_{p\parallel}$$

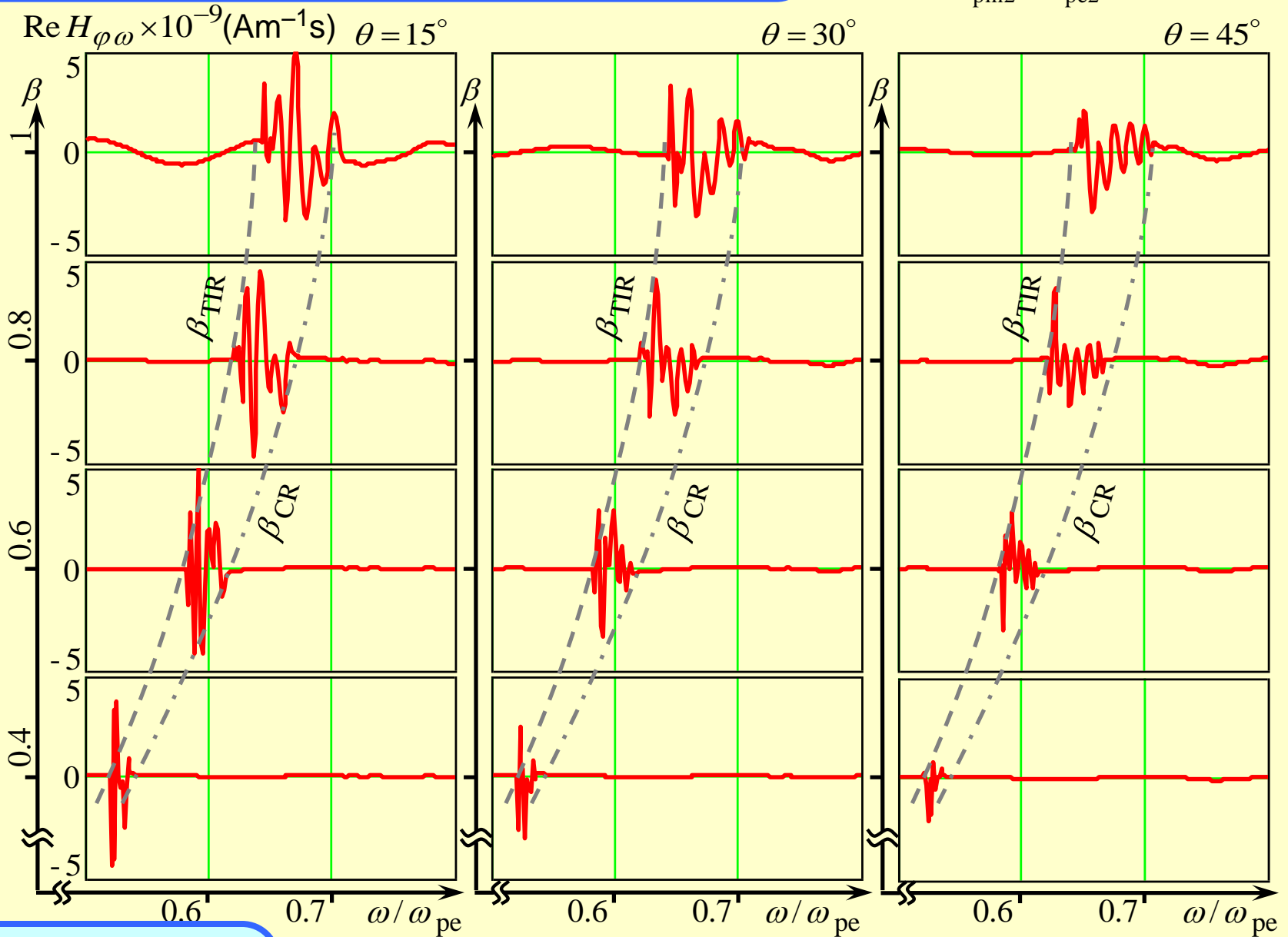


$$\omega_{p\parallel} < \omega < \Omega(\beta)$$

$$\omega_{d\parallel} = \omega_{d\perp} = 10^{-3} \omega_{p\parallel}, \quad \omega_{pe\parallel} = 10^{-2} \omega_{p\parallel}, \quad \omega_{de\parallel} = 10^{-6} \omega_{p\parallel}$$

Vacuum – “left-handed medium” (LHM) interface

$q = -1 \text{ nC}$ $R = 14 \text{ cm}$
 $\omega_{\text{pm}2} = \omega_{\text{pe}2} = 2\pi \cdot 10 \cdot 10^9 \text{ c}^{-1}$

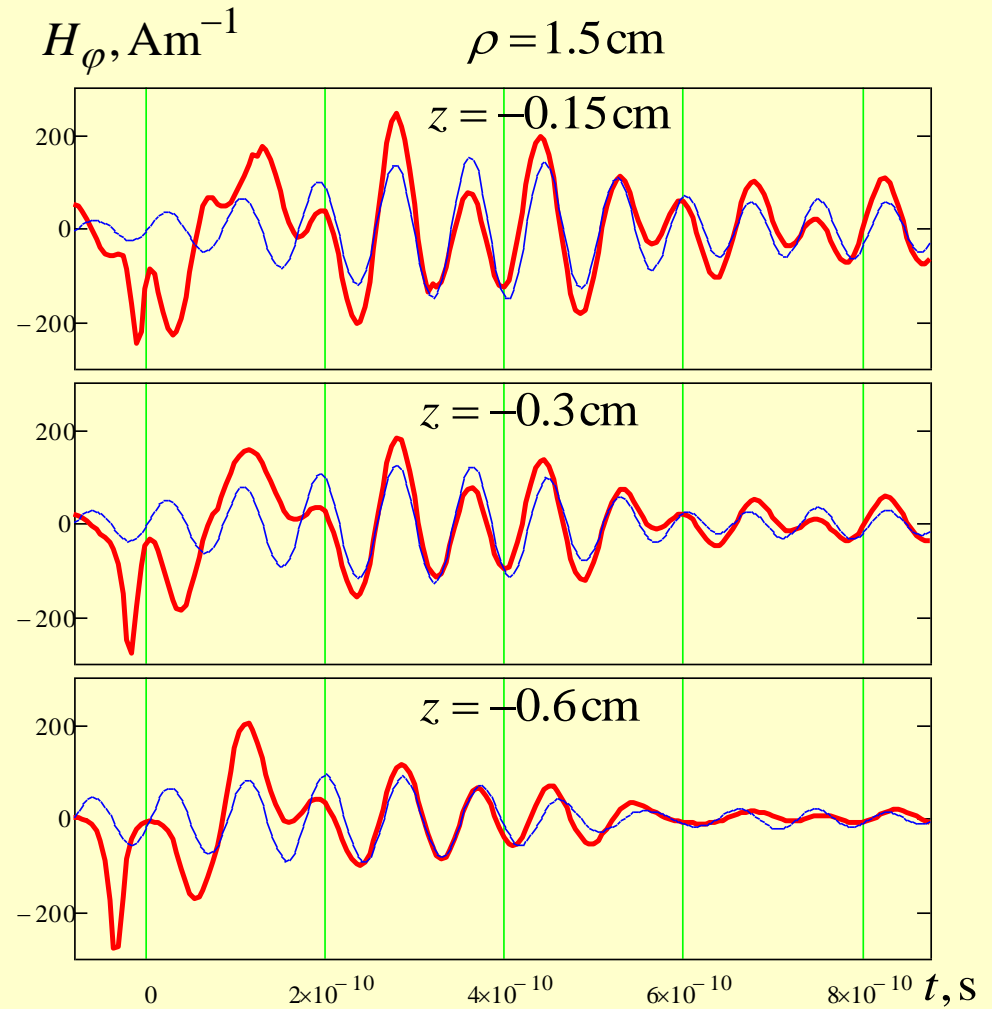
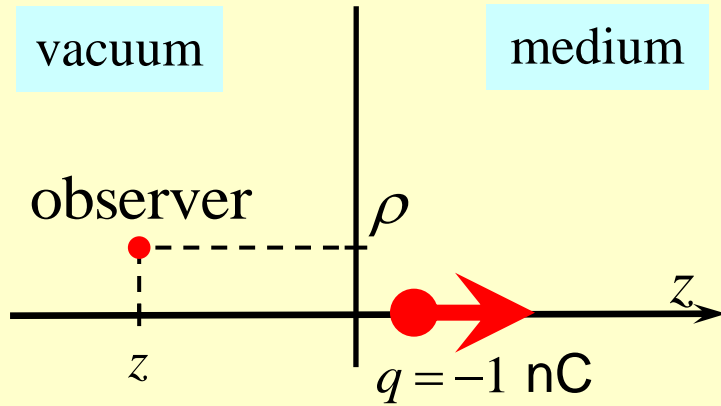


$\beta_{\text{CR}} < \beta < \beta_{\text{TIR}}$

$\omega_{\text{rm}2} = 0, \omega_{\text{de}2} = \omega_{\text{dm}2} = 10^{-3} \omega_{\text{pe}2}, \omega_{\text{pe}1} = 10^{-2} \omega_{\text{pe}2}, \omega_{\text{de}1} = 10^{-6} \omega_{\text{pe}2}$

Results

Time evolution of the total field



Conclusion

Investigation of the electromagnetic field generated at a charge flight from vacuum into anisotropic plasma-like medium

Reversed Cherenkov-Transition Radiation (RCTR)

Analytical approach:

- **Rigorous condition of the RCTR presence**
- **Spatial structure of the far-field Fourier harmonics**
- **“Half-shadow” areas**
- **Impact of losses**

Numerical approach:

- **Field spectrum results**
- **Time evolution of the total field**
- **Possibility of the RCTR dominance in the total field**

Thank you for your attention!