The Combined Effects for PXR at Channeling

K.B. Korotchenko[1], Yu.L. Pivovarov[1], Y. Takabayashi[2], T.A. Tukhfatullin[1]

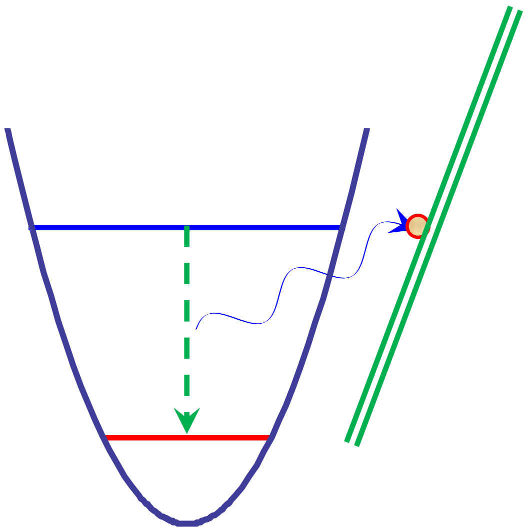
[1]Tomsk Polytechnic University,Tomsk, Russia

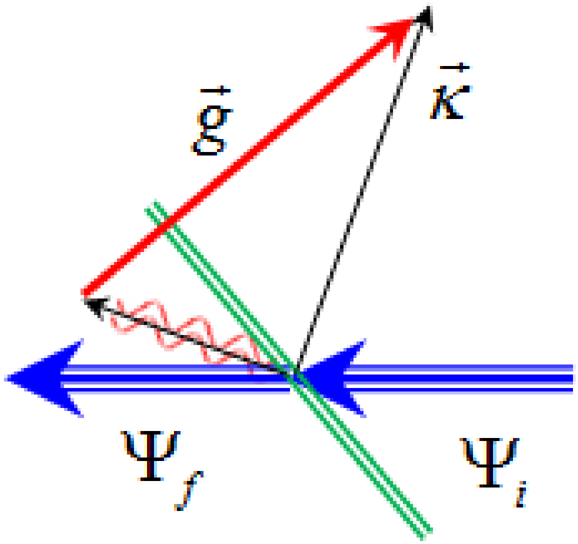
[2]Saga Light Sourse, Japan

- **Combined effects for PXR: PXRC and DCR**
- **PXRC:** PXR from channeled electrons (positrons) – arises as very special kind of DCR (diffracted channeling radiation).
- **PXRC:** quantum effect connected with “transverse” form-factor of channeled electron (positron) → modification (quantum correction) of angular distribution of emitted X-ray photons compared to ordinary PXR.
- **PXRC:** orientation dependence on angle of incidence into a crystal (relative to the channeling planes)
- **PXRC:** dependence on initial beam energy (number of quantum sub-barrier channeled states)
- **SAGA-LS:** first experiment devoted to PXRC observation

The formation of DCR

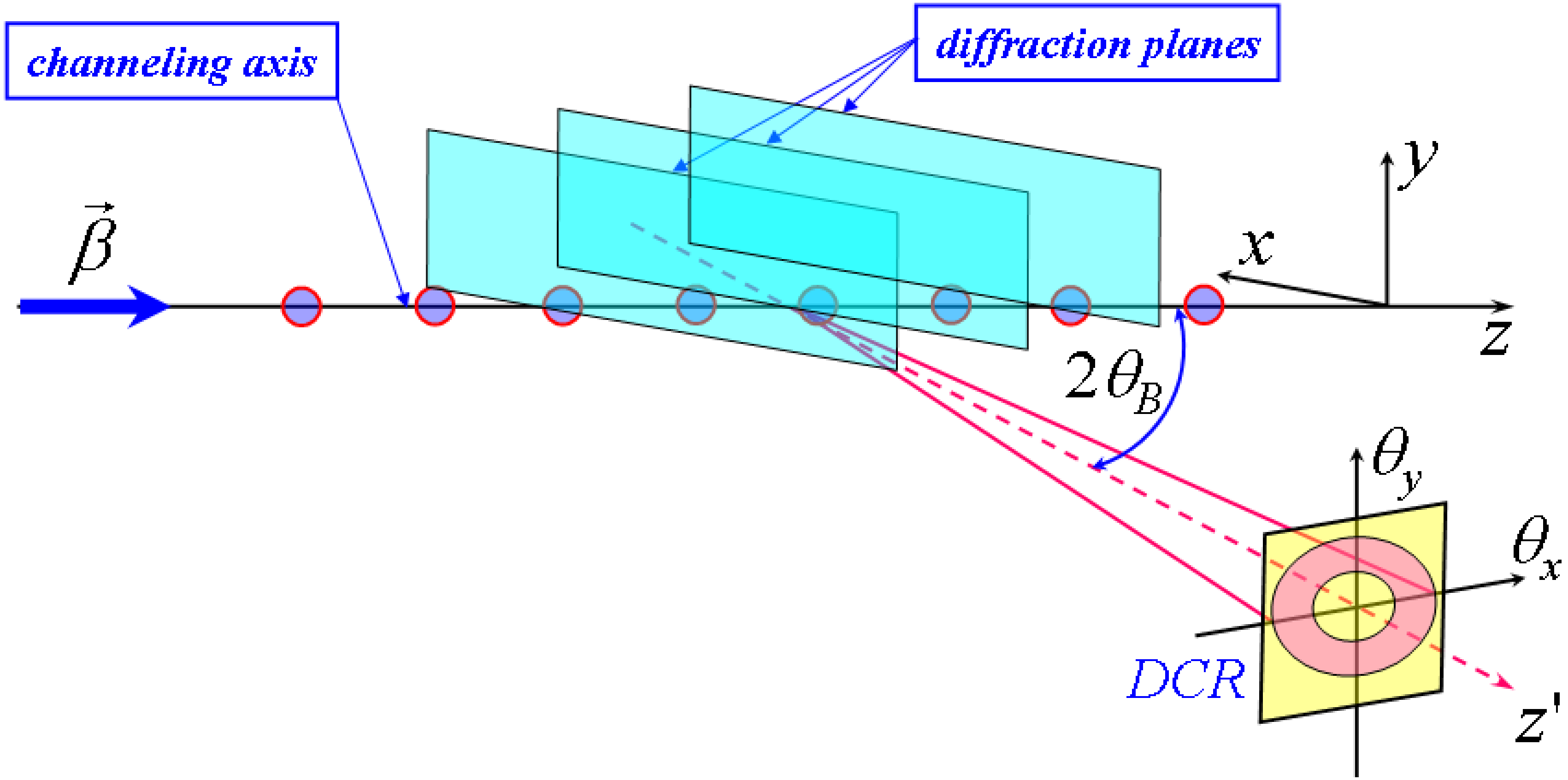
- Emission of virtual CR-photon by channeled electron (positron) due to spontaneous transition $i \rightarrow f$
- Virtual CR-photon Bragg diffraction on the crystallographic planes
- Virtual CR-photon transformation into a real photon





The Feynman diagram for DCR in a first order in α (fine-structure constant) and diagram of diffraction.

Scheme of the DCR formation at axial channeling



Cross-section of the DCR

DCR cross-section

$$\boxed{d\sigma = \frac{2\pi}{\hbar}\left|M_{if}\right|^2 \delta\left(\hbar\omega - (E_i - E_f)\right) d\rho_f ,}$$

M_{if} is the DCR matrix element,

$$M_{if} = -e\left\langle \Psi_f(\vec{r})\right|\vec{A}^*(\vec{r})\vec{\alpha}\left|\Psi_i(\vec{r})\right\rangle$$

$\vec{A}(\vec{r})$ is the photon wave function (the Bloch function)

$$\vec{A}(\vec{r}) = \sum_{\kappa}\left(\vec{A}_{o\kappa}\exp(\mathbf{i}\vec{\kappa}\vec{r}) + \sum_{g\neq 0}\vec{A}_{g\kappa}\exp\left[\mathbf{i}(\vec{\kappa}+\vec{g})\vec{r}\right]\right),$$

$M_{if}^{(-g)\tau}$ is the DCR matrix element

$$\boxed{M_{if}^{(-g)\tau} = -eC_{if}(C_i + C_f)\vec{\varepsilon}_{g\tau}\vec{I}_{if}/2mc ,}$$

$\vec{\varepsilon}_{g\tau}$ is the polarization vector

$\vec{g}$ is the reciprocal lattice vector

$$\vec{I}_{if} = (2\pi/L)^N A_{g\kappa}^{\tau *}\left\langle \varphi_f(\vec{r}_\perp)\right|\hat{\vec{p}}\left|\varphi_i(\vec{r}_\perp)\right|\mathrm{Exp}(-\mathbf{i}\vec{\kappa}_{-g\perp}\vec{r}_\perp)\rangle_\perp$$
$$\times\,\delta(\Delta\vec{p}_{if\,||}/\hbar - \vec{\kappa}_{-g||}),$$

$$C_{if} = \sqrt{1 + c^2 m/E_f}\,\sqrt{1 + c^2 m/E_i} ,$$

$$C_i = c^2 m(E_i + c^2 m)^{-1}$$

(N = 1 for the case of axial channeling and N = 2 for the case of planar channeling)

DCR matrix element

At the channeling condition $E_{\|} >> U(r_{\perp})$ $E_{\|i} \cong E_{\|f}$.

DCR matrix element

$$\boxed{\begin{aligned}&\sqrt{2\left(1+W_{\tau}^{2}\right)}M_{if}^{(-g)\tau} = \\ &= -C_{if}C_{i}\left(2\pi/L\right)^{N}\frac{e}{mc}A_{0\kappa}^{\tau}\left(\vec{\varepsilon}_{g\tau\|}\vec{p}_{i\|}F_{if} - \mathbf{i}\, m\gamma\,\Omega_{if}\vec{\varepsilon}_{g\tau\perp}\vec{\Delta}_{if}\right) \\ &\times\delta(\Delta\vec{p}_{if\|}/\hbar - \vec{\kappa}_{-g\|}).\end{aligned}}$$

$$\vec{\Delta}_{if} = \left\langle\varphi_{f}(\vec{r}_{\perp})\middle|\vec{r}_{\perp}\exp(-\mathbf{i}\vec{\kappa}_{-g\perp}\vec{r}_{\perp})\middle|\varphi_{i}(\vec{r}_{\perp})\right\rangle_{\perp},$$

$$F_{if} = \left\langle\varphi_{f}(\vec{r}_{\perp})\middle|\exp(-\mathbf{i}\vec{\kappa}_{-g\perp}\vec{r}_{\perp})\middle|\varphi_{i}(\vec{r}_{\perp})\right\rangle_{\perp},$$

$$\Omega_{if} = (E_{i\perp} - E_{f\perp})/\hbar\,.$$

DCR matrix element
spontaneous intraband transition

Matrix element with *i* = *f*

$$\sqrt{2\left(1+W_\tau^2\right)}M_{ii}^{(-g)\tau} = -C_{ii}C_i\left(2\pi/L\right)^N \frac{e}{mc} A_{0\kappa}^{\tau}\vec{\varepsilon}_{g\tau\|}\vec{p}_{i\|}F_{ii}$$
$$\times\,\delta(\Delta p_{ii\|}/\hbar - \vec{\kappa}_{-g\|}).$$

Angular distribution of this part of DCR

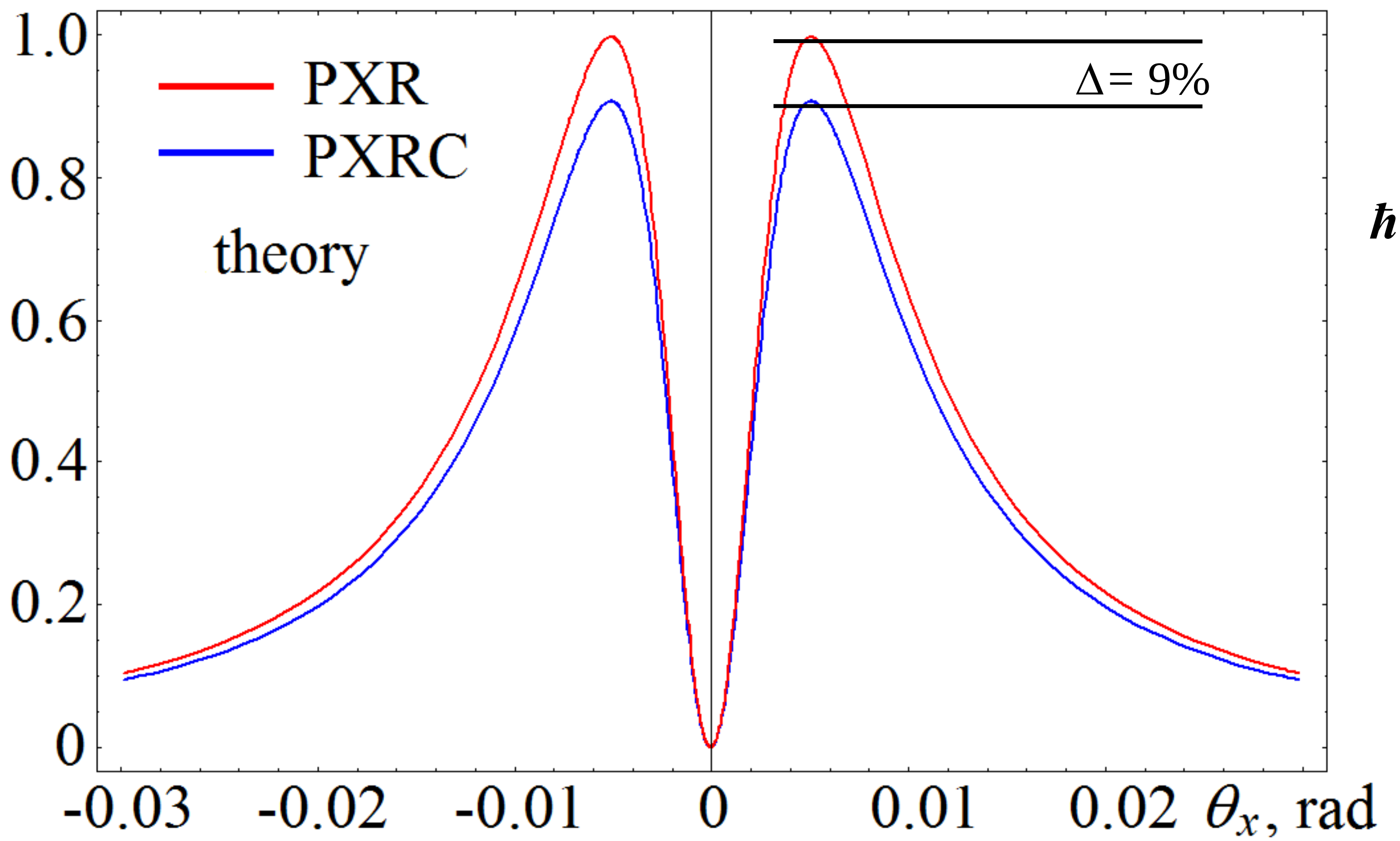
$$\boxed{\frac{d^3N}{d\theta_x d\theta_y dz} = \frac{\alpha\omega_B}{4\pi c\sin^2\theta_B}F_{ii}^2\left[\frac{\theta_x^2}{4(1+W_\pi^2)}+\frac{\theta_y^2}{4(1+W_\sigma^2)}\right]}$$

In the well known formula for the angular distribution of PXR [1] “transverse” formfactor F_{ii} is absence

PXRC is special case of DCR
and its amplitude is always smaller than the amplitude of PXR ($F_{ii} < 1$)

1. Yabuki R., Nitta H., Ikeda T. and Ohtsuki Y.H. // Physical Review B 63, 174112 (2001).

Angular distribution of the PXRC and PXR electron beam with energy 255 MeV at (220) Si channeling



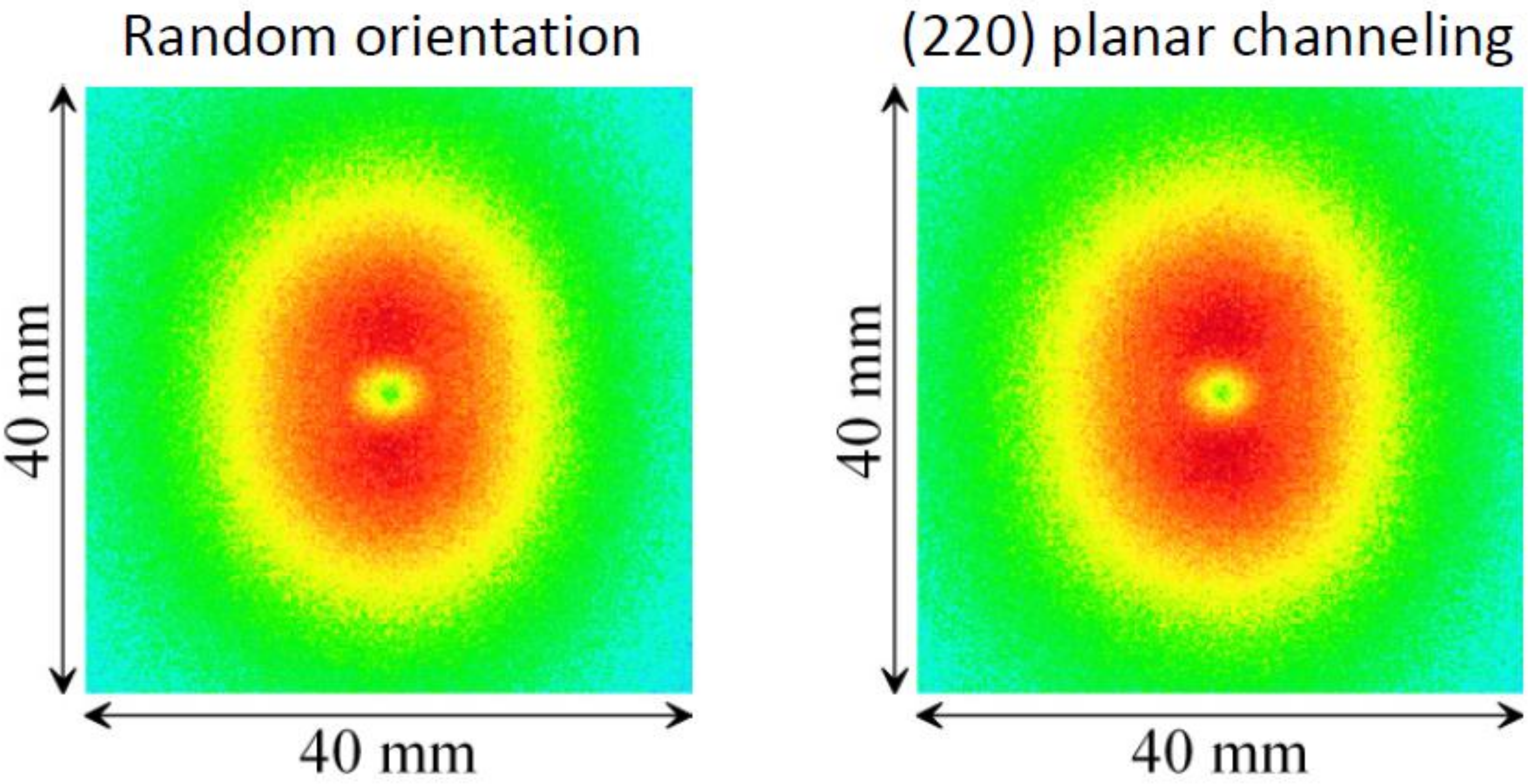
$\hbar\omega_B = 7.133$ keV

$\theta_B = 16.1^o$

Beam angular divergence = 0 !

Preliminary experimental results on PXRC

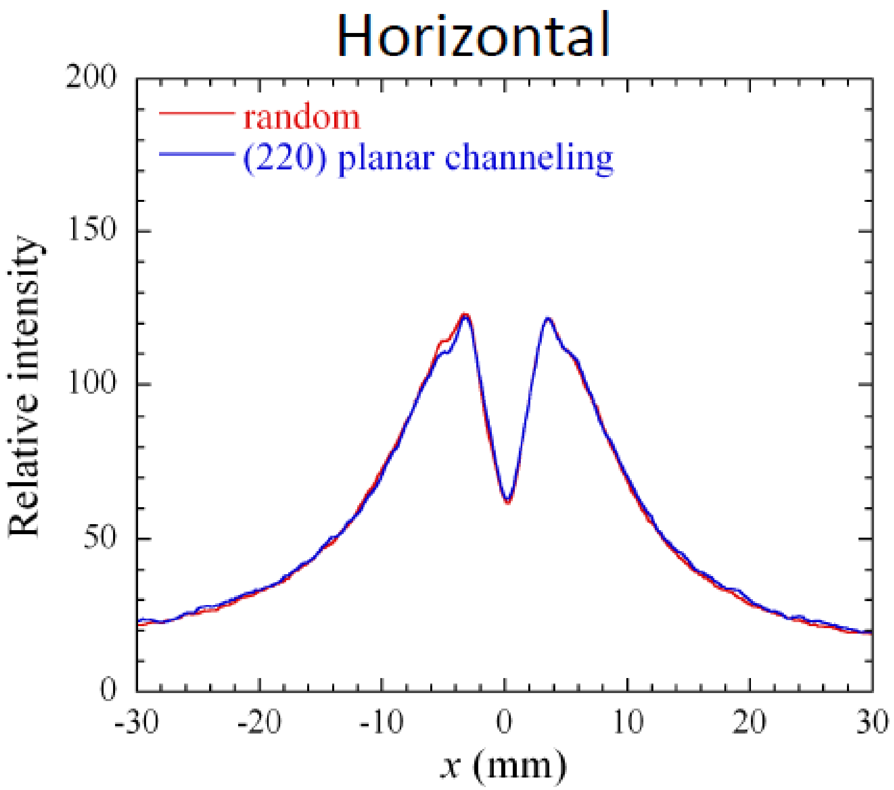
255 MeV $e^- \rightarrow$ 20-μm-thick Si crystal

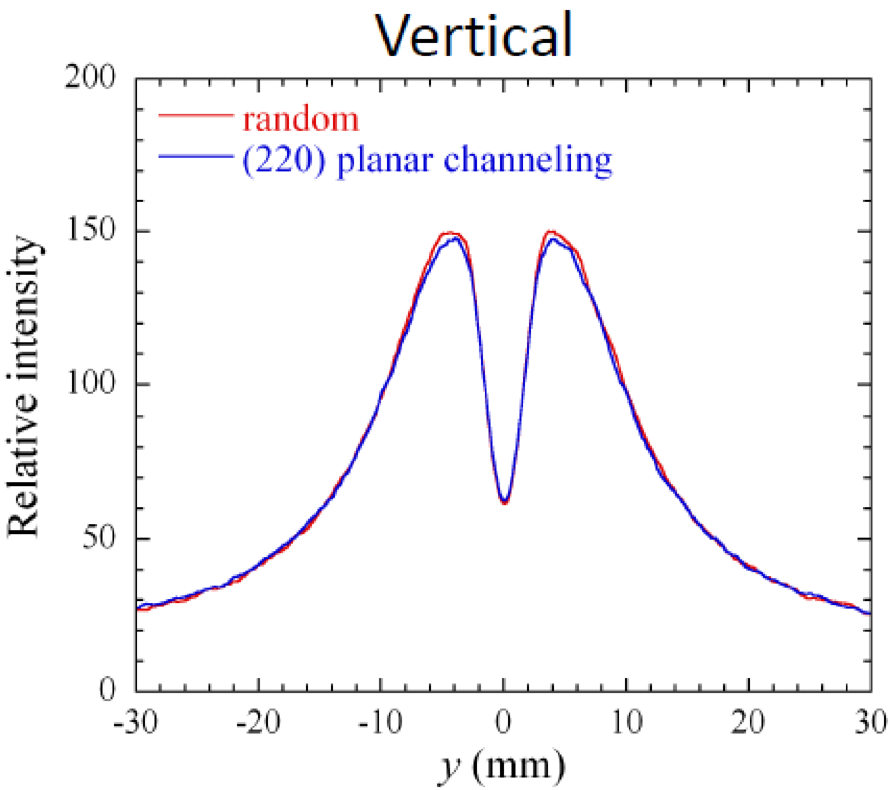


The experimental data obtained at SAGA Light Source (Japan)

Preliminary experimental results on PXRC

255 MeV $e^{-} \rightarrow$ 20-μm-thick Si crystal





Angular divergence of the electron beam

$$w(k_o) = Exp[-k_o^2 / 2\sigma^2],$$

w is the probability that angle between channeling plane and the electron momentum equals θ_o = $k_o\theta_C$ (θ_C is critical channeling angle) σ is the dispersion in θ_C units

Angular distribution of PXRC for electrons

$$\left.\frac{d^3 N_{PXRC}}{d\theta_x d\theta_y dz}\right|_{beam} = \left\langle w(\theta_o) P(i,\theta_o) W_{PXRC}(i,\theta_o)\right\rangle_{\theta_o},$$

$W_{PXRC}(i, \theta_o)$ angular distribution of PXRC due to one electron,

$P(i, \theta_o)$ is the initial population of the i^{th} level for the electron incident at the angle θ_o to the channeling plane

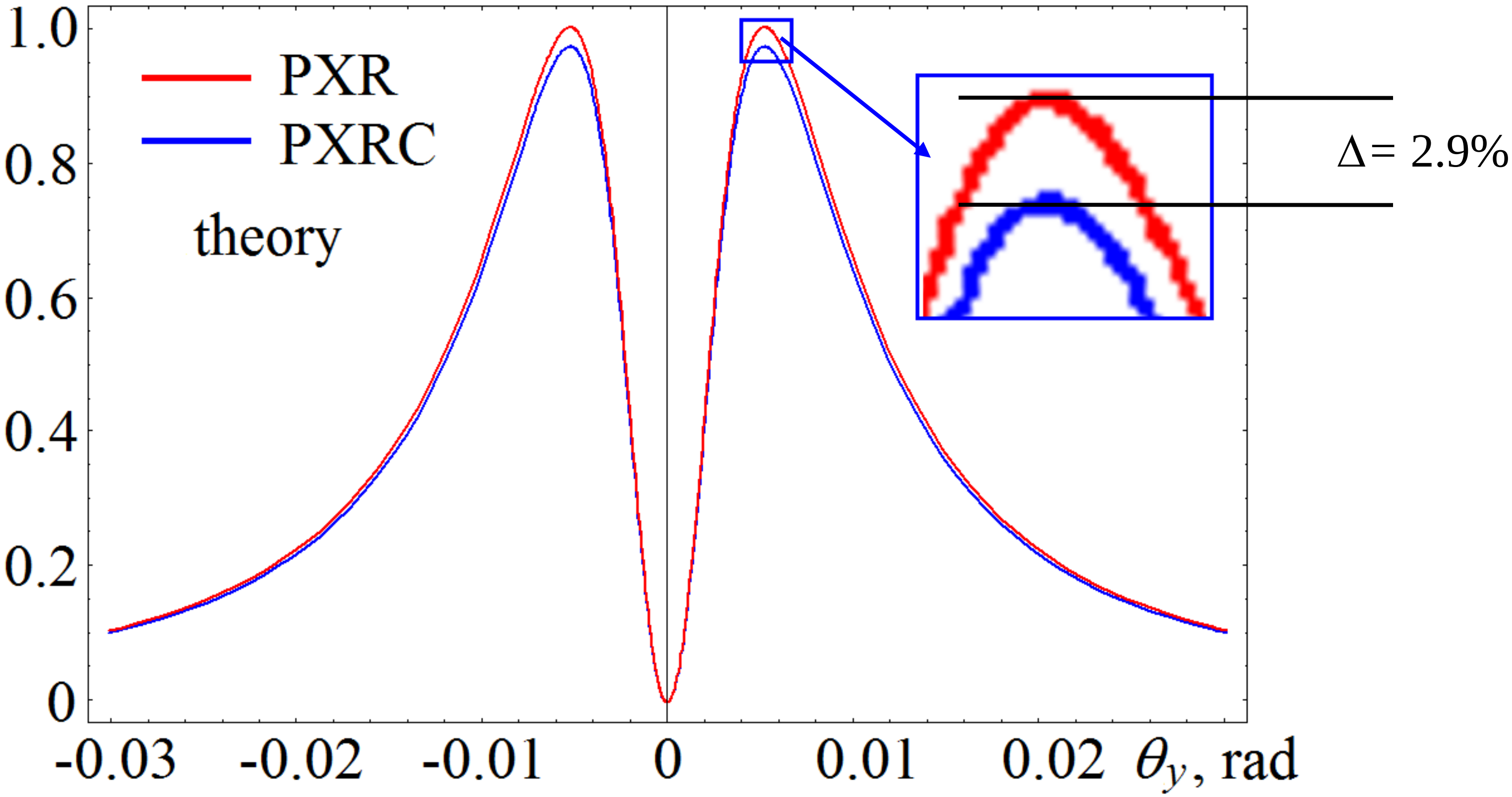
$\langle\ldots\rangle_{\theta_o}$ – averaging over the angle θ_o

Only the part of the electrons are captured to the channeling regime (that is to the sub-barrier levels) and its involved to the generation PXRC the other electrons generate the general PXR

$$\left.\frac{d^3 N}{d\theta_x d\theta_y dz}\right|_{beam} = \left.\frac{d^3 N_{PXRC}}{d\theta_x d\theta_y dz}\right|_{beam} + \left\langle w(\theta_o) W_{PXR} (1 - P(i,\theta_o)\right\rangle_{\theta_o},$$

Angular distribution of the PXRC

electron beam with energy 255 MeV at (220) Si channeling

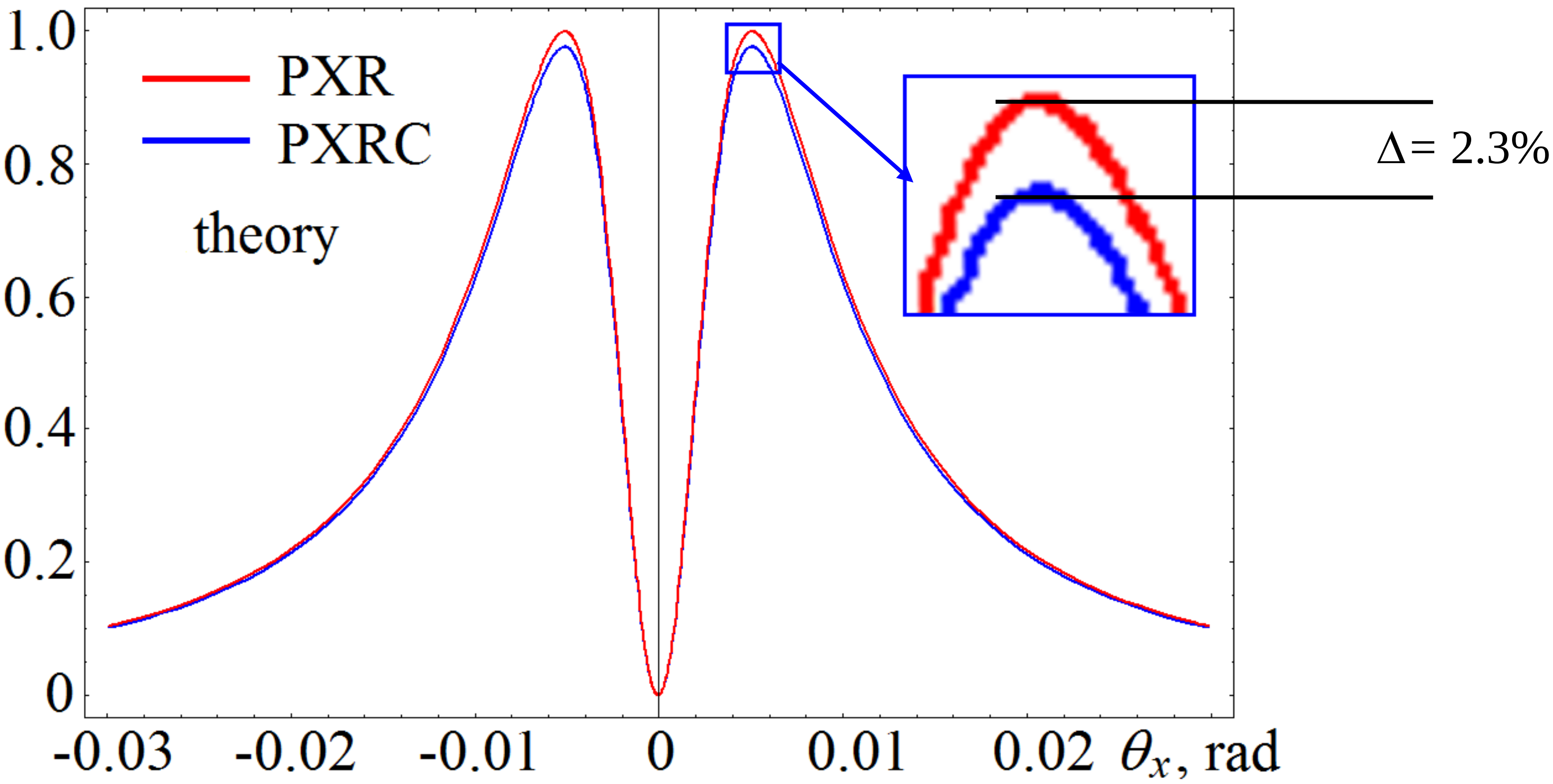


$$\sigma_{theor} = 0.7362\ \theta_C = 0.3 \text{ mrad}$$

$$\theta_C = 0.4075 \text{ mrad}$$

Angular distribution of the PXRC

electron beam with energy 255 MeV at (220) Si channeling



$$\sigma_{theor} = 0.49\ \theta_C = 0.2 \text{ mrad}$$

$$\theta_C = 0.4075 \text{ mrad}$$

Conclusions

- Preliminary experimental results (SAGA-LS Linac) on angular distributions of PXR from channeling electrons with energy 255 MeV show small deviations from ordinary PXR angular distribution

- Probably, the deviations are explained by manifestation of the new Combined effect for PXR at channeling, i.e. by PXRC: quantum effect connected with “transverse” form-factor of channeled electron (positron) which leads to modification (quantum correction) of angular distribution of emitted X-ray photons compared to ordinary PXR.

- Further experiments at SAGA-LS are planned using thinner crystal and changing electron beam energy

Thank you for attention

Two-dimensional X-ray detector

Imaging plate
[$BaFX:Eu^{2+}$(X=Cl, Br, I)]

- Reusable
- Digitally readable
- Size: 20 × 20 cm
- Nominal position resolution: 50 μm

Imaging plate reader
[FUJIFILM BAS-2500]

Imaging plate eraser