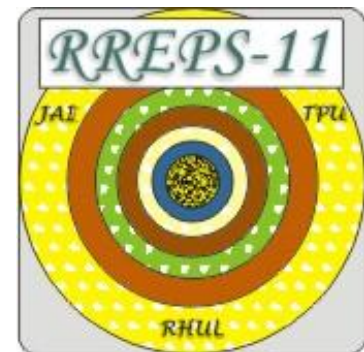


The Combined Effects for PXR at Channeling

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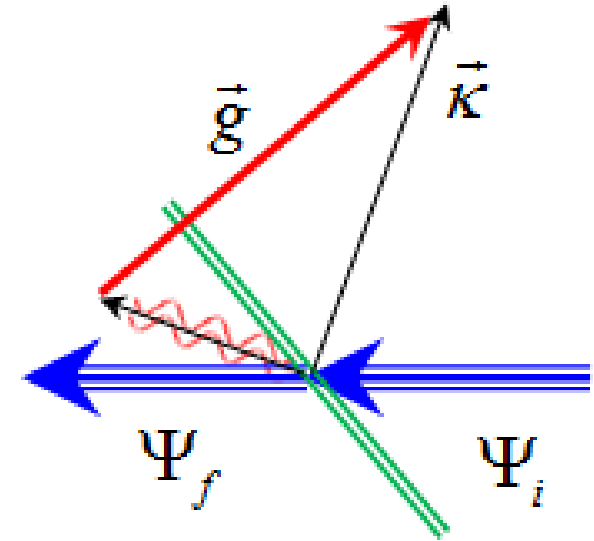
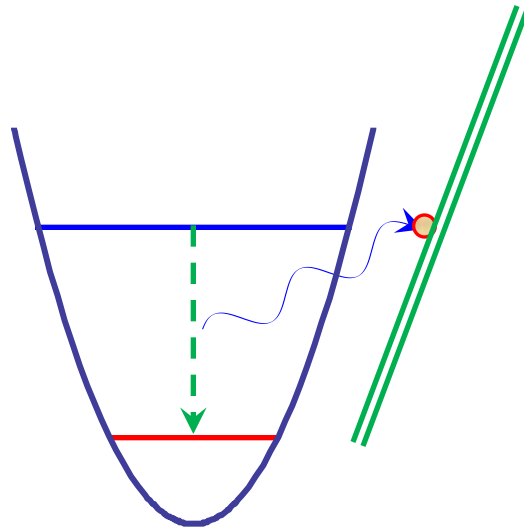
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- **Combined effects for PXR: PXRC and DCR**
- **PXRC:** PXR from channeled electrons (positrons) – arises as very special kind of DCR (diffracted channeling radiation).
- **PXRC:** quantum effect connected with “transverse” form-factor of channeled electron (positron) → modification (quantum correction) of angular distribution of emitted X-ray photons compared to ordinary PXR.
- **PXRC:** orientation dependence on angle of incidence into a crystal (relative to the channeling planes)
- **PXRC:** dependence on initial beam energy (number of quantum sub-barrier channeled states)
- **SAGA-LS:** first experiment devoted to PXRC observation

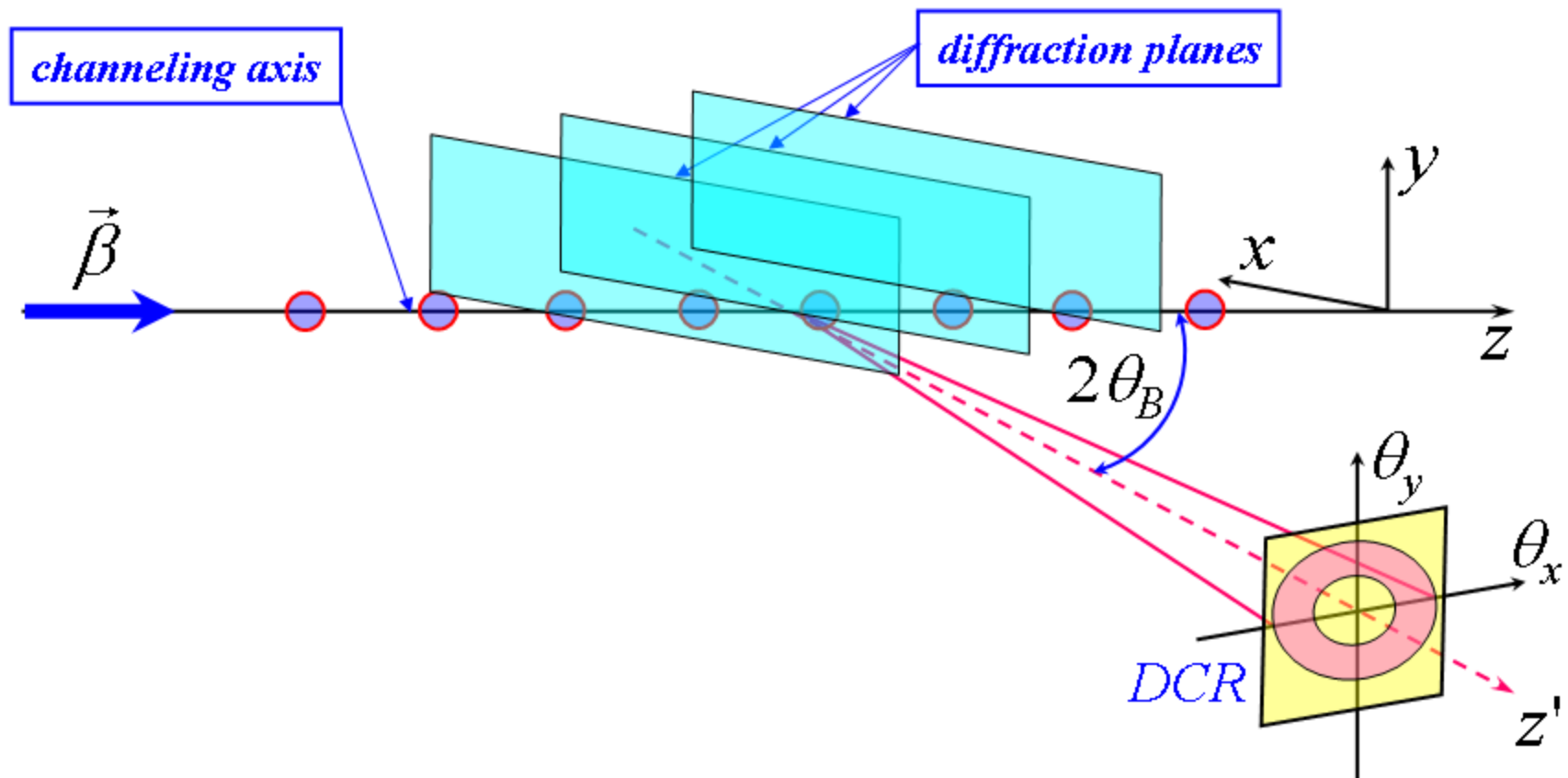
The formation of DCR

- Emission of virtual CR-photon by channeled electron (positron) due to spontaneous transition $i \rightarrow f$
- Virtual CR-photon Bragg diffraction on the crystallographic planes
- Virtual CR-photon transformation into a real photon



The Feynman diagram for DCR
in a first order in α
(fine-structure constant)
and diagram of diffraction.

Scheme of the DCR formation at axial channeling



Cross-section of the DCR

DCR cross-section

$$d\sigma = \frac{2\pi}{\hbar} |M_{if}|^2 \delta(\hbar\omega - (E_i - E_f)) d\rho_f,$$

M_{if} is the DCR matrix element,

$\vec{A}(\vec{r})$ is the photon wave function
(the Bloch function)

$$M_{if} = -e \langle \Psi_f(\vec{r}) | \vec{A}^*(\vec{r}) \vec{\alpha} | \Psi_i(\vec{r}) \rangle$$

$$\vec{A}(\vec{r}) = \sum_{\kappa} \left(\vec{A}_{\kappa} \exp(\mathbf{i}\vec{\kappa}\vec{r}) + \sum_{g \neq 0} \vec{A}_{g\kappa} \exp[\mathbf{i}(\vec{\kappa} + \vec{g})\vec{r}] \right),$$

$M_{if}^{(-g)\tau}$ is the DCR matrix element

$$M_{if}^{(-g)\tau} = -e C_{if} (C_i + C_f) \vec{\varepsilon}_{g\tau} \vec{I}_{if} / 2mc,$$

$\vec{\varepsilon}_{g\tau}$ is the polarization vector

\vec{g} is the reciprocal lattice vector

$$\vec{I}_{if} = (2\pi/L)^N A_{g\kappa}^{\tau*} \langle \varphi_f(\vec{r}_{\perp}) | \hat{p} | \varphi_i(\vec{r}_{\perp}) \rangle \text{Exp}(-\mathbf{i}\vec{\kappa}_{-g\perp}\vec{r}_{\perp}) \Big|_{\perp} \\ \times \delta(\Delta\vec{p}_{if\parallel} / \hbar - \vec{\kappa}_{-g\parallel}),$$

$$C_{if} = \sqrt{1 + c^2 m / E_f} \sqrt{1 + c^2 m / E_i},$$

$$C_i = c^2 m (E_i + c^2 m)^{-1}$$

($N = 1$ for the case of axial channeling and $N = 2$ for the case of planar channeling)

DCR matrix element

At the channeling condition $E_{\parallel} \gg U(r_{\perp})$ $E_{\parallel i} \approx E_{\parallel f}$.

DCR matrix element

$$\begin{aligned} \sqrt{2(1+W_{\tau}^2)} M_{if}^{(-g)\tau} = \\ = -C_{if} C_i (2\pi/L)^N \frac{e}{mc} A_{0\kappa}^{\tau} (\vec{\varepsilon}_{g\tau\parallel} \vec{p}_{i\parallel} F_{if} - \mathbf{i} m \gamma \Omega_{if} \vec{\varepsilon}_{g\tau\perp} \vec{\Delta}_{if}) \\ \times \delta(\Delta \vec{p}_{if\parallel} / \hbar - \vec{\kappa}_{-g\parallel}). \end{aligned}$$

$$\begin{aligned} \vec{\Delta}_{if} &= \langle \varphi_f(\vec{r}_{\perp}) | \vec{r}_{\perp} \exp(-\mathbf{i} \vec{\kappa}_{-g\perp} \vec{r}_{\perp}) | \varphi_i(\vec{r}_{\perp}) \rangle_{\perp}, \\ F_{if} &= \langle \varphi_f(\vec{r}_{\perp}) | \exp(-\mathbf{i} \vec{\kappa}_{-g\perp} \vec{r}_{\perp}) | \varphi_i(\vec{r}_{\perp}) \rangle_{\perp}, \\ \Omega_{if} &= (E_{i\perp} - E_{f\perp}) / \hbar. \end{aligned}$$

DCR matrix element spontaneous intraband transition

Matrix element with $i = f$

$$\sqrt{2(1+W_\tau^2)}M_{ii}^{(-g)\tau} = -C_{ii}C_i(2\pi/L)^N \frac{e}{mc} A_{0\kappa}^\tau \vec{\epsilon}_{g\tau||} \vec{p}_{i||} F_{ii} \\ \times \delta(\Delta p_{i||}/\hbar - \vec{\kappa}_{-g||}).$$

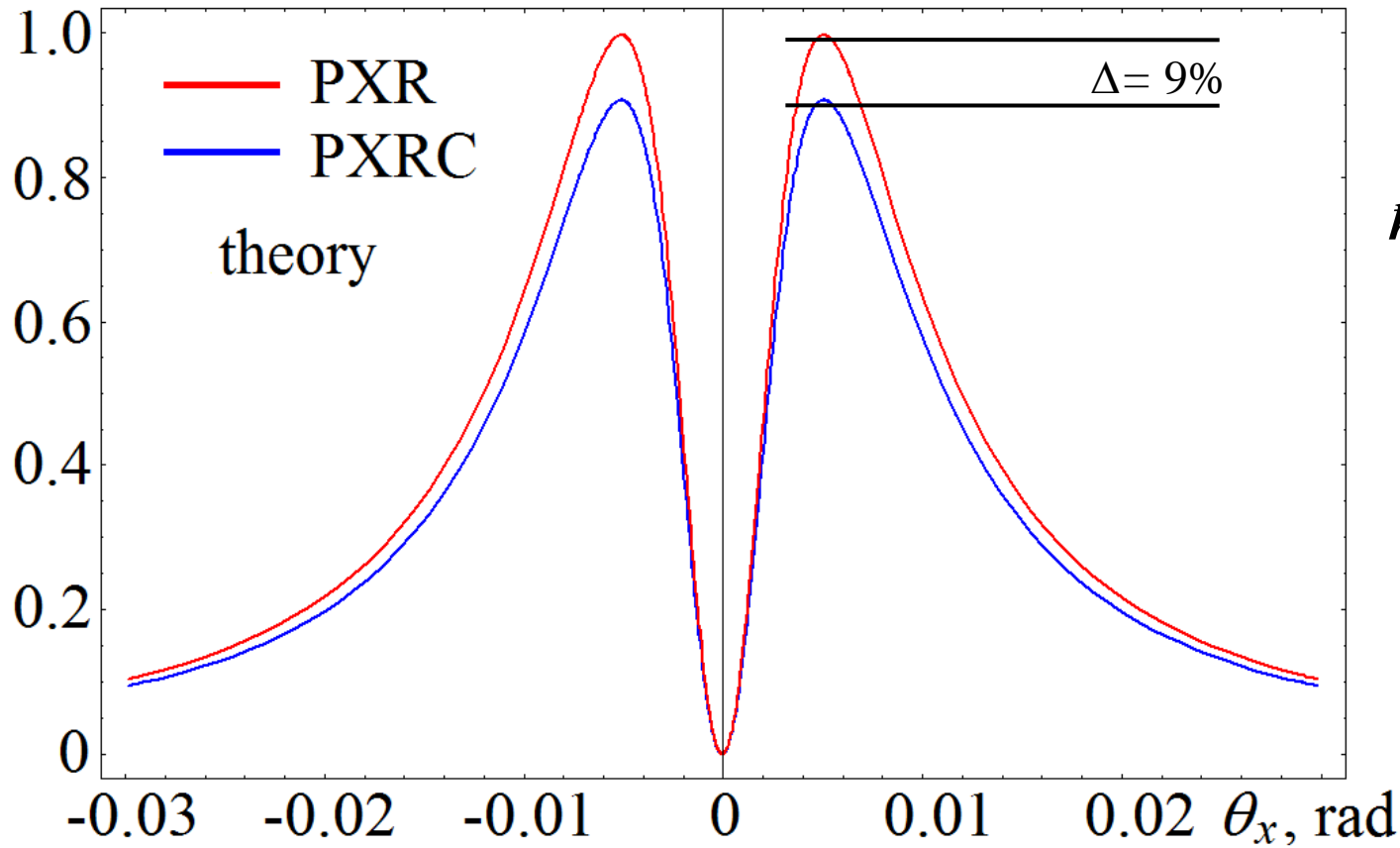
Angular distribution of this
part of DCR

$$\frac{d^3N}{d\theta_x d\theta_y dz} = \frac{\alpha\omega_B}{4\pi c \sin^2 \theta_B} F_{ii}^2 \left[\frac{\theta_x^2}{4(1+W_\pi^2)} + \frac{\theta_y^2}{4(1+W_\sigma^2)} \right]$$

In the well known formula for the angular distribution of PXR [1] “transverse” formfactor F_{ii} is absence

PXRC is special case of DCR
and its amplitude is always smaller than the amplitude of PXR ($F_{ii} < 1$)

Angular distribution of the PXRC and PXR
electron beam with energy 255 MeV at (220) Si channeling



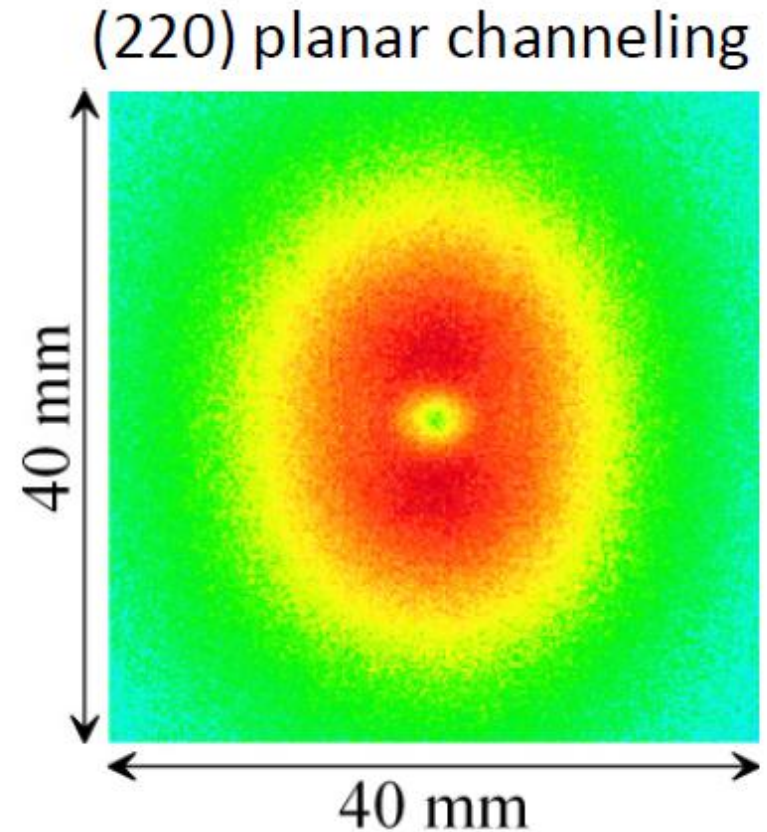
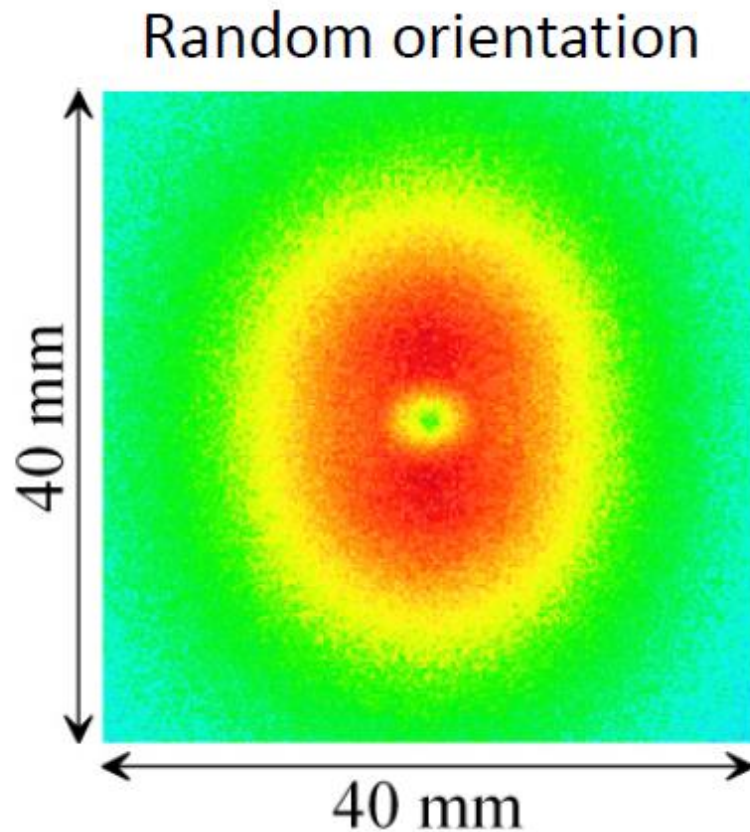
$$\hbar\omega_B = 7.133 \text{ keV}$$

$$\theta_B = 16.1^\circ$$

Beam angular divergence = 0 !

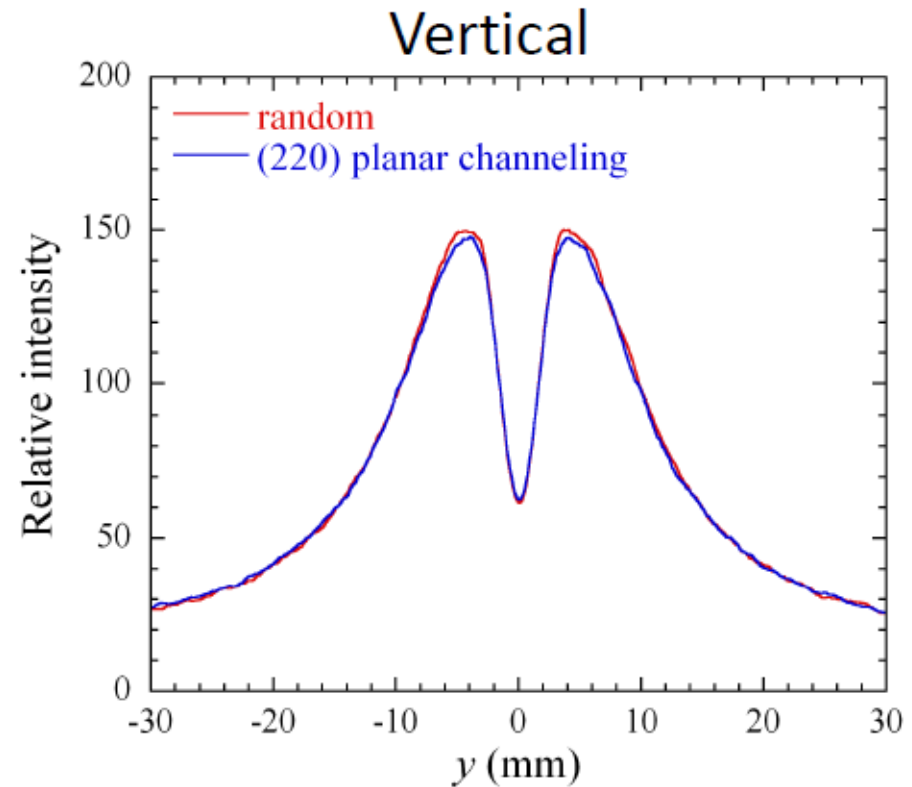
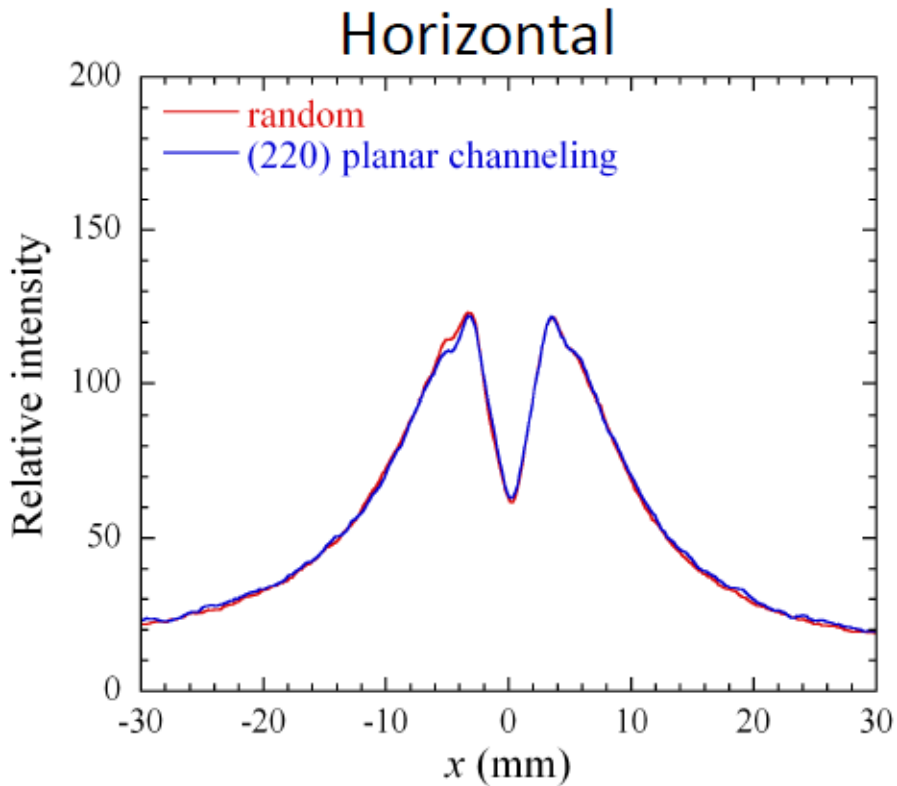
Preliminary experimental results on PXRC

255 MeV $e^- \rightarrow$ 20- μm -thick Si crystal



The experimental data obtained at SAGA Light Source (Japan)

Preliminary experimental results on PXRC

255 MeV $e^- \rightarrow$ 20- μm -thick Si crystal

Angular divergence of the electron beam

$$w(k_o) = \text{Exp}[-k_o^2 / 2\sigma^2],$$

w is the probability that angle between channeling plane and the electron momentum equals $\theta_o = k_o\theta_C$ (θ_C is critical channeling angle) σ is the dispersion in θ_C units

Angular distribution of PXRC for electrons

$$\left. \frac{d^3 N_{PXRC}}{d\theta_x d\theta_y dz} \right|_{beam} = \langle w(\theta_o) P(i, \theta_o) W_{PXRC}(i, \theta_o) \rangle_{\theta_o},$$

$W_{PXRC}(i, \theta_o)$ angular distribution of PXRC due to one electron,

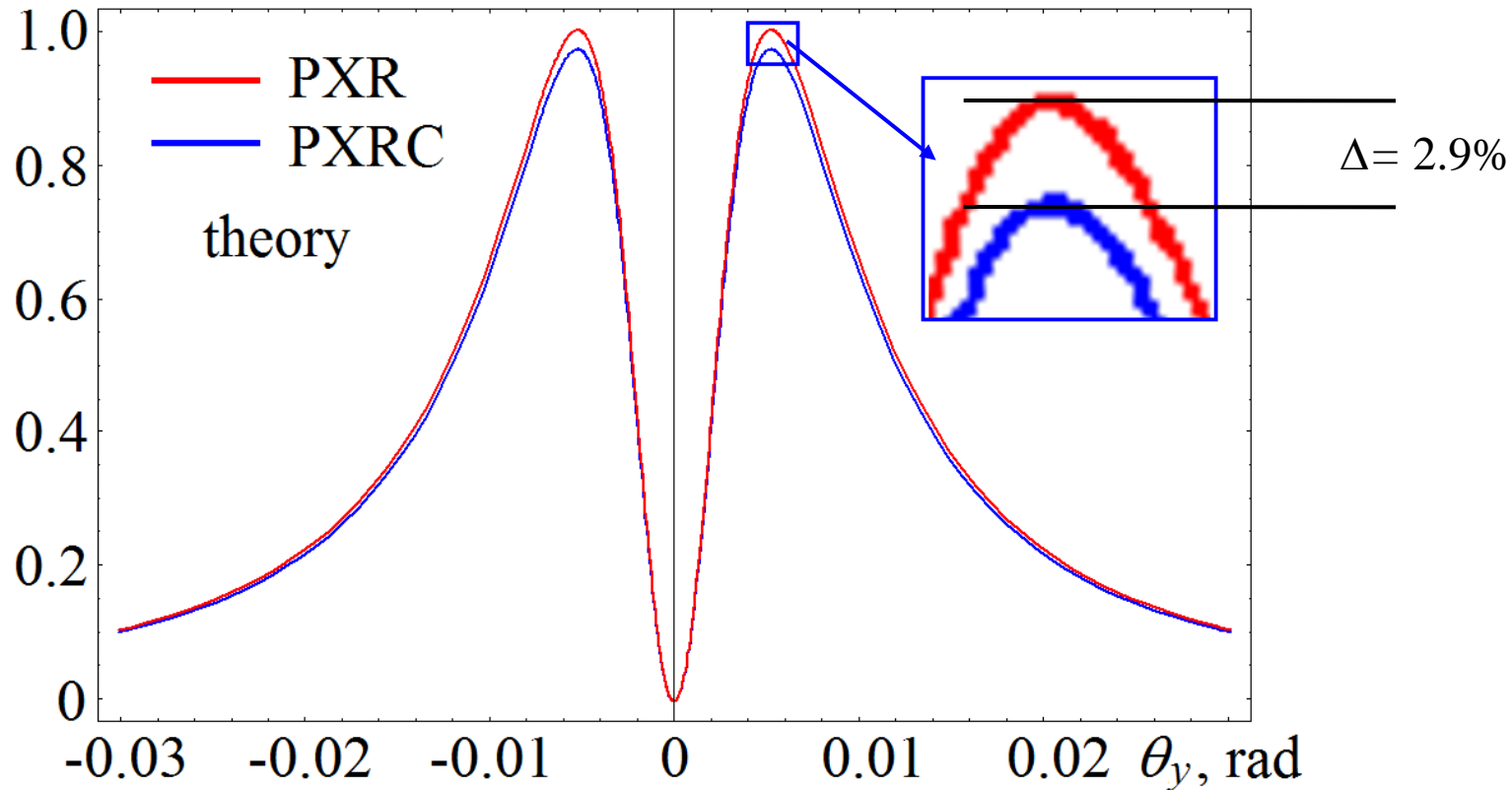
$P(i, \theta_o)$ is the initial population of the i^{th} level for the electron incident at the angle θ_o to the channeling plane

$\langle \dots \rangle_{\theta_o}$ – averaging over the angle θ_o

Only the part of the electrons are captured to the channeling regime (that is to the sub-barrier levels) and its involved to the generation PXRC the other electrons generate the general PXR

$$\left. \frac{d^3 N}{d\theta_x d\theta_y dz} \right|_{beam} = \left. \frac{d^3 N_{PXRC}}{d\theta_x d\theta_y dz} \right|_{beam} + \langle w(\theta_o) W_{PXR}(1 - P(i, \theta_o)) \rangle_{\theta_o},$$

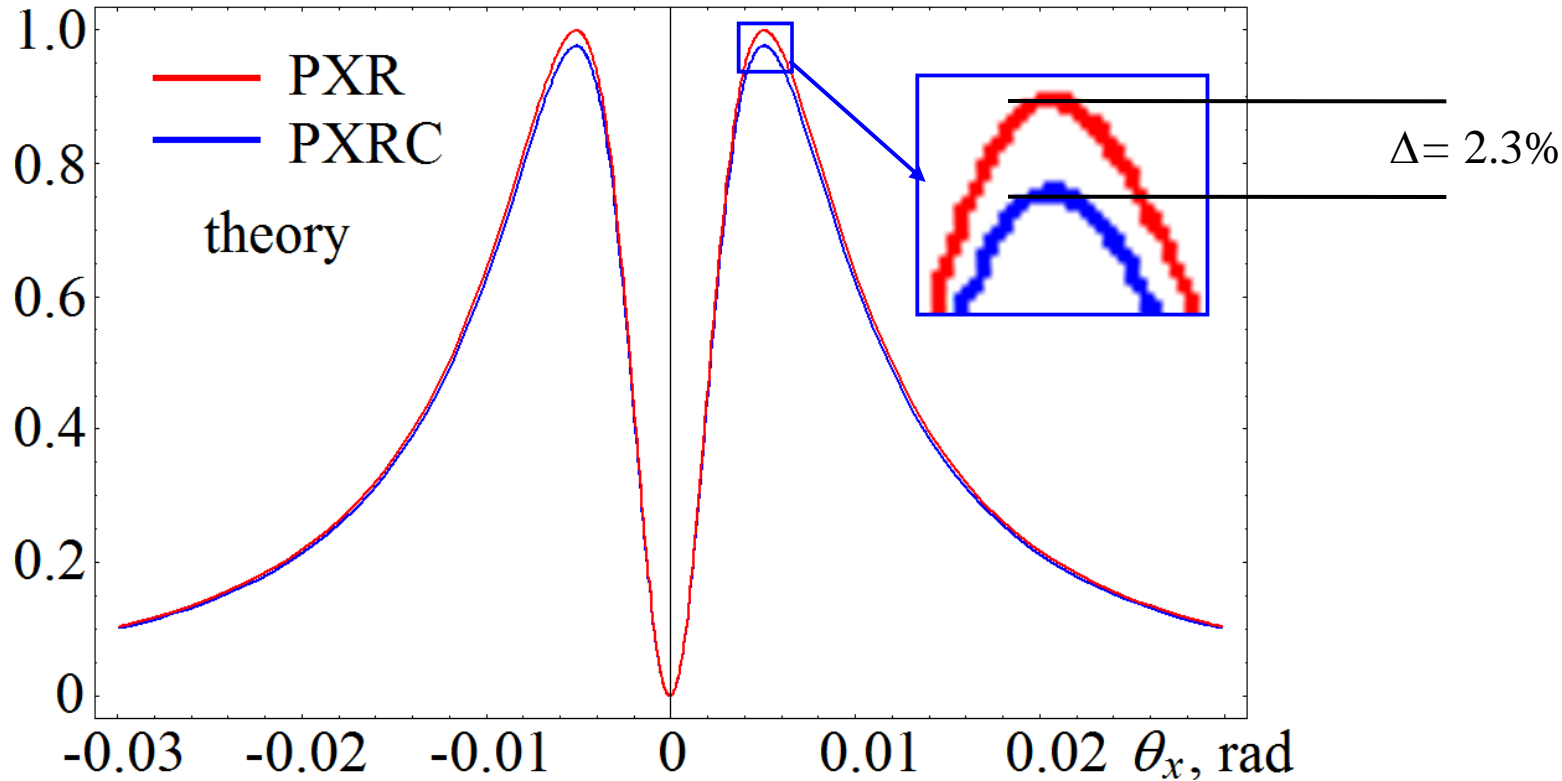
Angular distribution of the PXRC electron beam with energy 255 MeV at (220) Si channeling



$$\sigma_{theor} = 0.7362 \quad \theta_C = 0.3 \text{ mrad}$$

$$\theta_C = 0.4075 \text{ mrad}$$

Angular distribution of the PXRC electron beam with energy 255 MeV at (220) Si channeling



$$\sigma_{theor} = 0.49 \quad \theta_C = 0.2 \text{ mrad}$$

$$\theta_C = 0.4075 \text{ mrad}$$

Conclusions

- Preliminary experimental results (SAGA-LS Linac) on angular distributions of PXR from channeling electrons with energy 255 MeV show small deviations from ordinary PXR angular distribution
- Probably, the deviations are explained by manifestation of the new Combined effect for PXR at channeling, i.e. by PXRC: quantum effect connected with “transverse” form-factor of channeled electron (positron) which leads to modification (quantum correction) of angular distribution of emitted X-ray photons compared to ordinary PXR.
- Further experiments at SAGA-LS are planned using thinner crystal and changing electron beam energy



Thank you for attention

Two-dimensional X-ray detector

Imaging plate

[BaFX:Eu²⁺ (X=Cl, Br, I)]



- Reusable
- Digitally readable
- Size: 20 × 20 cm
- Nominal position resolution:
50 μm

Imaging plate reader
[FUJIFILM BAS-2500]



Imaging plate eraser

