Pair Creation by Polarized Photons in Constant and Homogeneous Electromagnetic Fields

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September 7, 2011

Abstract

The process of pair creation by a photon in a constant and homogeneous electromagnetic field of an arbitrary configuration is investigating. At high energy the correction to the standard quasiclassical approximation (SQA) has been calculated. In the region of intermediate photon energies where SQA is inapplicable the new approximation, developed recently by authors, is used. The influence of weak electric field on the process in a magnetic field is considered. In particular, in the presence of electric field the root divergence in the probability of pair creation on the Landau energy levels is vanished. For smaller photon energies the low energy approximation is used. The found probability describes the absorption of soft photon by particles created by field. At low photon energy the electric field action dominates and the influence of magnetic field on the process is connected with the interaction of it and the magnetic moment of creating particles.

1 Introduction

The pair photoproduction in an electromagnetic field is the basic QED reaction which can play the significant role in many processes.

This process was considered first in a magnetic field. The investigation was started in 1952 independently by Klepikov and Toll [1, 2]. In Klepikov's paper [3], which was based on the solution of the Dirac equation, the probability of photoproduction had been obtained on the mass shell ($k^2 = 0, k$ is the 4-momentum of photon. We use the system of units with $\hbar = c = 1$ and the metric $ab = a^\mu b_\mu = a^0 b^0 - \boldsymbol{ab}$). In 1971 Adler [4] had calculated the photon polarization operator in a magnetic field using the proper-time technique developed by Schwinger [5] and Batalin and Shabad [6] had calculated this operator in an electromagnetic field using the Green function found by Schwinger [5]. In 1975 the contribution of charged-particles loop in an electromagnetic field with n external photon lines had been calculated by Baier, Katkov and Strakhovenko [7]. For $n = 2$ the explicit expressions for the contribution of scalar and spinor particles to the polarization operator of photon were given in this work. Making use of the imaginary part of this operator for spinor particles the pair photoproduction probability was analyzed by Baier and Katkov in the pure magnetic field [8] and the pure electric field [9].

The probability of pair photoproduction in a constant and homogeneous electric field in the quasiclassical approximation had been found in 1968 by Narozhny [10] using the solution of the Dirac equation in the Sauter potential [11]. Nikishov [12] had obtained in 1970 the differential distribution of this process also using the solution of the Dirac equation in the indicated field.

In the present paper we consider the integral probability of pair creation in a constant and homogeneous electromagnetic field of an arbitrary configuration basing on the polarization operator [7]. In Sec.2 the exact expression for this probability has been received for the general case $k^2 \neq 0$.In Sec.3 the standard quasiclassical approximation (SQA) is outlined for the high-energy photon $\omega \gg m$ (m is the electron mass). The corrections to SQA, determined also the applicability region of SQA, have been calculated. The found expressions, given in the Lorentz invariant form, contain two invariant parameters. In Sec.4 the new approach has been developed for the relatively low energies where SQA is not applicable. This approach is based on the method proposed in [8]. The obtained probability is valid in the wide interval of photon energy, which is overlapped with SQA. In Sec.5 the case of the "nonrelativistic" photon $\omega \ll m$ is analyzed. In particular, in the energy region $\omega \lesssim eE/m$ where the previous approach is inapplicable, the low energy approximation has been developed basing on the analysis in [9]. In tern the found results have an overlapping region of applicability with the previous approach. So just as in [9] we have three overlapping approximations which include all photon energies. At the photon energy $\omega \ll eE/m$ the probability has been found for arbitrary values of fields E and H.

2 General expressions for the probability of process

Our analysis is based on the general expression for the contribution of spinor particles to the polarization operator obtained in a diagonal form in [7] (see Eqs. (3.19), (3.33)). The imaginary part of the eigenvalue κ_i of this operator on the mass shell ($k^2 = 0$) determines the probability per unit length W_i of e^-e^+ pair creation by the real photon with the polarization e_i directed along the corresponding eigenvector:

$$W_i = -\frac{\mathrm{Im}\kappa_i}{\omega}; \quad e_i^\mu = \frac{b_i^\mu}{\sqrt{-b_i^2}}, \quad b_2^\mu = (Bk)^\mu + \frac{2\Omega_4}{\Omega}(Ck)^\mu, \tag{1}$$

$$b_3^\mu = (Ck)^\mu - \frac{2\Omega_4}{\Omega}\,(Bk)^\mu\,;$$

$$-\mathrm{Im}\kappa_2 = r\left(\Omega_2 - \frac{2\Omega_4^2}{\Omega}\right), \quad -\mathrm{Im}\ \kappa_3 = r\left(\Omega_3 + \frac{2\Omega_4^2}{\Omega}\right), \tag{2}$$

$$\Omega = \Omega_3 - \Omega_2 + \sqrt{(\Omega_3 - \Omega_2)^2 + 4\Omega_4^2}, \quad r = \frac{\omega^2 - k_3^2}{4m^2}.$$

The consideration realizes in the frame where electric **E** and magnetic **H** fields are parallel and directed along the axis 3. In this frame the tensor of electromagnetic field $F_{\mu\nu}$ and tensors $F^*_{\mu\nu}$, $B_{\mu\nu}$ and $C_{\mu\nu}$ have a form

$$\begin{aligned} F_{\mu\nu} &= C_{\mu\nu}E + B_{\mu\nu}H, \quad F^*_{\mu\nu} = C_{\mu\nu}H - B_{\mu\nu}E, \\ C_{\mu\nu} &= g^0_\mu g^3_\nu - g^3_\mu g^0_\nu, \\ B_{\mu\nu} &= g^2_\mu g^1_\nu - g^1_\mu g^2_\nu; \quad eE/m^2 = E/E_0 \equiv \nu, \end{aligned} \tag{3}$$

$$eH/m^2 = H/H_0 \equiv \mu;$$

$$\Omega_i = \frac{\alpha m^2}{2\pi \mathrm{i}}\mu\nu \int\limits_{-1}^{1} dv \int\limits_{-\infty - \mathrm{i}0}^{\infty - \mathrm{i}0} f_i(v,x)\exp(\mathrm{i}\psi(v,x))x dx. \tag{4}$$

Here

$$
\begin{aligned}
f_1 &= \frac{\cos(\mu x v)\cosh(\nu x v)}{\sin(\mu x)\sinh(\nu x)} \\
&- \frac{\cos(\mu x)\cosh(\nu x)\sin(\mu x v)\sinh(\nu x v)}{\sin^2(\mu x)\sinh^2(\nu x)}, \\
f_2 &= 2\frac{\cosh(\nu x)(\cos(\mu x) - \cos(\mu x v))}{\sinh(\nu x)\sin^3(\mu x)} + f_1, \\
f_3 &= 2\frac{\cos(\mu x)(\cosh(\nu x) - \cosh(\nu x v))}{\sin(\mu x)\sinh^3(\nu x)} - f_1, \\
f_4 &= \frac{\cos(\mu x)\cos(\mu x v) - 1}{\sin^2(\mu x)}\frac{\cosh(\nu x)\cosh(\nu x v) - 1}{\sinh^2(\nu x)} \\
&+ \frac{\sin(\mu x v)\sinh(\nu x v)}{\sin(\mu x)\sinh(\nu x)}; \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
\psi(v,x) &= 2r\left(\frac{\cosh(\nu x) - \cosh(\nu x v)}{\nu\sinh(\nu x)} + \frac{\cos(\mu x) - \cos(\mu x v)}{\mu\sin(\mu x)}\right) \qquad (6) \\
&- x.
\end{aligned}
$$

Let us note that the integration contour in Eq.(4) is passing slightly below the real axis.

After all calculations have been fulfilled we can return to a covariant form of the process description using the following expressions

$$E^2, H^2 = \left(\mathcal{F}^2 + \mathcal{G}^2\right)^{1/2} \pm \mathcal{F}, \quad \mathcal{F} = \left(\mathbf{E}^2 - \mathbf{H}^2\right) \diagup 2,$$
$$\mathcal{G}{=}\mathbf{EH}, \quad \left(C^2\right)_{\mu\nu} - \left(B^2\right)_{\mu\nu} = g_{\mu\nu}.$$
$$\left(C^2\right)_{\mu\nu} = \left(F^2_{\mu\nu} + H^2 g_{\mu\nu}\right) \diagup \left(E^2 + H^2\right), \tag{7}$$

3 Quasiclassical approximation

The standard quasiclassical approximation (SQA) was developed first for a magnetic field by Klepikov [3], Baier and Katkov [13], Tsai and Erber [14]. The SQA is valid for ultrarelativistic created particles ($r \gg 1$) and can be derived from Eqs.(4)-(6) by expanding the functions $f_i(v, x)$, $\psi(v, x)$ over x powers. To get the correction to the probability in SQA we shall keep leading to leading powers of x. We have

$$b_2^\mu = \ (Bk)^\mu + \frac{\nu}{\mu}\,(Ck)^\mu \propto F^{\mu\nu} k_\nu, \tag{8}$$
$$b_3^\mu = (Ck)^\mu - \frac{\nu}{\mu}\,(Bk)^\mu \propto F^{*\mu\nu} k_\nu;$$

$$
\begin{aligned}
-\mathrm{Im}\kappa_i &= \mathrm{i}\frac{\alpha m^2}{12\pi} r(\mu^2+\nu^2) \int\limits_{-1}^{1} dv \left(1-v^2\right) \\
&\times \int\limits_{-\infty}^{\infty} h_i(v,x) \left[-\mathrm{i}\gamma\left(v,x\right)\right] x dx; \\
\gamma\left(v,x\right) &= x + \frac{x^3}{12} r \left(1-v^2\right)^2 \left(\nu^2+\mu^2\right), \qquad (9) \\
h_2(v,x) &= \frac{3+v^2}{2} + \frac{1}{30}\left(15-6v^2-v^4\right)\left(\mu^2-\nu^2\right)x^2 \\
&- \frac{\mathrm{i}}{720} r(\mu^2+\nu^2)\left(1-v^2\right)^2\left(9-v^2\right)\left(\mu^2-\nu^2\right)x^5, \\
h_3(v,x) &= 3-v^2 + \frac{1}{60}\left(15-2v^2+3v^4\right)\left(\mu^2-\nu^2\right)x^2 \\
&- \frac{\mathrm{i}}{360} r(\mu^2+\nu^2)\left(1-v^2\right)^2\left(3-v^2\right)^2\left(\mu^2-\nu^2\right)x^5. \qquad (10)
\end{aligned}
$$

We are using the known integrals:

$$\int_{-\infty}^{\infty} \cos\left(x + \frac{ax^3}{3}\right) dx = \frac{2}{\sqrt{3a}} K_{1/3}\left(\frac{2}{3\sqrt{a}}\right),$$

$$\int_{-\infty}^{\infty} x \sin\left(x + \frac{ax^3}{3}\right) dx = \frac{2}{\sqrt{3a}} K_{2/3}\left(\frac{2}{3\sqrt{a}}\right). \qquad (11)$$

Conserving the first (independent on x) terms in Eq.(10) we obtain the probabilities in SQA

$$W_i^{(SQA)} = -\frac{\mathrm{Im}\kappa_i}{\omega} = \frac{\alpha m^2}{3\sqrt{3}\pi\omega} \int_{-1}^{1} \frac{s_i}{1-v^2} K_{2/3}(z)\, dv,$$

$$z = \frac{8}{3(1-v^2)\kappa}, \;\; s_2 = 2(3-v^2), \quad s_3 = 3+v^2, \qquad (12)$$

$$\kappa^2 = 4r(\mu^2+\nu^2) = -\frac{e^2}{m^6}\left(F^{\mu\nu}k_\nu\right)^2.$$

The correction to SQA has a form

$$W_i^{(1)} = -\frac{\alpha m^2 \widetilde{\mathcal{F}}}{15\sqrt{3}\pi\omega\kappa} \int_{-1}^{1} \frac{dv}{1-v^2} G_i(v,z), \;\; \widetilde{\mathcal{F}} = \frac{e^2\mathcal{F}}{m^4} = \frac{\nu^2-\mu^2}{2}, \qquad (13)$$

where

$$G_2(v,z) = \left(36 + 4v^2 - 18z^2\right) K_{1/3}(z) + \left(3v^2 - 57\right) z K_{2/3}(z),$$
$$G_3(v,z) = -\left(34 + 2v^2 + 36z^2\right) K_{1/3}(z) + \left(78 - 6v^2\right) z K_{2/3}(z). \tag{14}$$

The mathematical transformations of integrals can be found in Appendix C [8]. It is seen that in this order of decomposition the correction does not depend on the invariant parameter $\mathcal{G}$, because $\mathcal{G}$ is the pseudoscalar. The asimptotic of the integrals incoming in the correction terms have been given in the mentioned Appendix C. The asymptotic at $\kappa \ll 1$ will become necessary further

$$W_2^{(1)} = \frac{4\alpha m^2 \widetilde{\mathcal{F}}}{5\omega\kappa^2}\sqrt{\frac{2}{3}}\exp\left(-\frac{8}{3\kappa}\right), \quad W_3^{(1)} = 2W_2^{(1)}, \tag{15}$$

$$\frac{W_i^{(1)}}{W_i^{(SQA)}} = \frac{64\widetilde{\mathcal{F}}}{15\kappa^3}.$$

4 Region of intermediate photon energies

In the field which is weak comparing with the critical field $E/E_0 = \nu \ll 1$ ($E_0 = 1.32 \cdot 10^{16}\,\mathrm{V\,/\,cm}$), $H/H_0 = \mu \ll 1$ ($H_0 = 4.41 \cdot 10^{13}\,\mathrm{G}$) and at the relatively low photon energies $r \lesssim \nu^{-2/3}$ the standard quasiclassical approximation Eq.(12) is non-applicable. This follows from the last equality in Eq.(15). For these energies, if the condition $r \gg \nu^2$ is fulfilled, the method of stationary phase can be applied at integration over x in Eq.(4). In this case the small values of v contribute to the integral over v. So one can expand the phase $\psi(v,x)$ over v and extend the integration limit to the infinity. We get

$$\Omega_i = \frac{\alpha m^2}{2\pi \mathrm{i}}\mu\nu \int\limits_{-\infty}^{\infty} dv \int\limits_{-\infty}^{\infty} f_i(0,x) \exp\left\{-\mathrm{i}\left[\varphi\left(x\right) + v^2\chi\left(x\right)\right]\right\} x dx, \tag{16}$$

where

$$\varphi(x) = 2r\left(\frac{1}{\mu}\tan\frac{\mu x}{2} - \frac{1}{\nu}\tanh\frac{\nu x}{2}\right) + x,$$
$$\chi(x) = rx^2\left(\frac{\nu}{\sinh(\nu x)} - \frac{\mu}{\sin(\mu x)}\right). \tag{17}$$

From the equation $\varphi'(x_0) = 0$ we find the saddle point x_0

$$\tan^2\frac{\nu s}{2} + \tanh^2\frac{\mu s}{2} = \frac{1}{r}, \quad x_0 = -\mathrm{i}s. \tag{18}$$

Substituting this value of x_0 in the expressions determined the integrals in Eq.(16) we have

$$\mathrm{i}\varphi(x_0) = 2r\left(\frac{1}{\mu}\tanh\frac{\mu s}{2} - \frac{1}{\nu}\tan\frac{\nu s}{2}\right) + s \equiv b(s), \tag{19}$$

$$\mathrm{i}\chi(x_0) = rs^2\left(\frac{\nu}{\sin(\nu s)} - \frac{\mu}{\sinh(\mu s)}\right) \equiv \frac{1}{2}rs^2 A(s), \tag{20}$$

$$\mathrm{i}\varphi''(x_0) = r\left[\nu\sin\frac{\nu s}{2}\Big/\cos^3\frac{\nu s}{2} + \mu\sinh\frac{\mu s}{2}\Big/\cosh^3\frac{\mu s}{2}\right] \tag{21}$$

$$\equiv rD(s),$$

$$f_2(0,x_0) = \frac{1}{\sinh(\mu s)\sin(\nu s)}\left[\cos(\nu s)\big/\cosh^2\frac{\mu s}{2} - 1\right] \equiv -a_2(s),$$
$$f_3(0,x_0) = \frac{1}{\sinh(\mu s)\sin(\nu s)}\left[1 - \cosh\mu s\big/\cos^2\frac{\nu s}{2}\right] \equiv -a_3(s),$$
$$f_4(0,x_0) = -\left(4\cos^2\frac{\nu s}{2}\cosh^2\frac{\mu s}{2}\right)^{-1} \equiv -a_4(s). \qquad (22)$$

Performing the standard procedure of the stationary phase method and using Eqs.(1)-(2) one obtains the following expressions

$$\Omega_i = a_i\frac{\alpha m^2\mu\nu}{r\sqrt{AB}}\exp\left(-b\right), \quad W_i = \lambda_i\frac{\alpha m^2\mu\nu}{\omega\sqrt{AB}}\exp\left(-b\right); \qquad (23)$$

$$\lambda_2 = a_2 - \frac{2a_4^2}{a}, \quad \lambda_3 = a_3 + \frac{2a_4^2}{a},$$
$$a = a_3 - a_2 + \sqrt{\left(a_3 - a_2\right)^2 + 4a_4^2};$$
$$b_2^\mu = (Bk)^\mu + \frac{2a_4}{a}\left(Ck\right)^\mu, \quad b_3^\mu = (Ck)^\mu - \frac{2a_4}{a}\left(Bk\right)^\mu \qquad (24)$$

These equations is valid at $r \gg 1$ if the condition $b \gg 1$ is fulfilled. The first two terms of the decomposition of the functions $s\left(r\right)$ Eq.(18)) and $b\left(s\left(r\right)\right)$ Eq.(19) over $1/r$ are

$$s(r) \simeq \frac{4}{\kappa}\left(1-\frac{8\widetilde{\mathcal{F}}}{3\kappa^2}\right),\ \ b(r) \simeq \frac{8}{3\kappa}-\frac{64\widetilde{\mathcal{F}}}{15\kappa^3},\ \ \kappa^2=4(\mu^2+\nu^2)r. \tag{25}$$

It is follows from this formula that the applicability of Eq.(23) is limited by the condition $\kappa \ll 1$. The main values of the rest terms in Eqs.(23),(24) have a form

$$\begin{gathered}A=\frac{1}{3}\left(\mu^2+\nu^2\right)s,\ D=\frac{3}{2}A;\ a_2=\frac{\mu^2+2\nu^2}{4\mu\nu},\ a_3=\frac{2\mu^2+\nu^2}{4\mu\nu}, \\ a_4=\frac{1}{4},\quad a=\frac{\mu}{2\nu},\quad \lambda_2=\frac{\mu^2+\nu^2}{4\mu\nu},\quad \lambda_3=2\lambda_2, \end{gathered} \tag{26}$$

and the vectors of polarization are given by Eq.(8). Substituting this values into equation for W_i we have

$$W_2=\frac{\alpha m^2\kappa}{8\omega}\sqrt{\frac{3}{2}}\exp\left(-\frac{8}{3\kappa}+\frac{64\widetilde{\mathcal{F}}}{15\kappa^3}\right),\quad W_3=2W_2. \tag{27}$$

In the region of the SQA applicability and for $\kappa \ll 1$ this probability coincides with the results of the previous section and so the overlapping region of both approximations exists.

At low photon energy $r \ll 1$ ($\nu^2 \ll r \ll \nu^{2/3}$) we have

$$\nu s \simeq \pi - 2\sqrt{r} + r^{3/2}\left(\frac{2}{3} - \tanh^2\frac{\pi\eta}{2}\right),$$

$$b \simeq \frac{1}{\nu}\left(\pi - 4\sqrt{r} + \frac{2r}{\eta}\tanh\frac{\pi\eta}{2}\right); \tag{28}$$

$$a_2 = \frac{1}{\sqrt{r}\sinh(\pi\eta)}\left(1 - \frac{1}{2}\tanh^2\frac{\eta\pi}{2} + \frac{\mu}{4r}\coth\pi\eta\right),$$

$$a_3 = \frac{\coth(\pi\eta)}{2r^{3/2}}\left(1 + \frac{4\eta\sqrt{r}}{\sinh(2\pi\eta)}\right) \simeq a, \quad a_4 = \left(4r\cosh^2\frac{\eta\pi}{2}\right)^{-1}, \tag{29}$$

$$\lambda_2 = \frac{1}{\sqrt{r}\sinh(\pi\eta)} \times \left[1 - \left(\frac{1}{2} + \frac{1}{\cosh(\pi\eta)}\right)\tanh^2\frac{\eta\pi}{2} + \frac{\mu}{4r}\coth(\pi\eta)\right],$$

$$\lambda_3 \simeq a_3, \quad A = \frac{\nu}{\sqrt{r}}\left(1 - \frac{2\eta\sqrt{r}}{\sinh(\pi\eta)}\right), \quad D = \frac{\nu}{r^{3/2}}, \quad \eta = \frac{\mu}{\nu}. \tag{30}$$

Here we have retained the leading and the leading to leading terms of decomposition. The term $\propto \mu$ in a_2 has appeared as the contribution of the second term in f_1 ($\propto v^2$) in Eq.(5). Substituting these values into Eq.(23) one obtains the following expression for the probability of the process

$$W_3 = \frac{\alpha m^2 \mu}{2\omega\sqrt{r}} \coth\left(\pi\eta\right) \left(1 + \frac{\eta\sqrt{r}}{\sinh(\pi\eta)} + \frac{4\eta\sqrt{r}}{\sinh(2\pi\eta)}\right) \exp\left(-b\right),$$

$$W_2 = \frac{\alpha m^2 \mu\sqrt{r}}{\omega \sinh(\pi\eta)} \qquad (31)$$

$$\times \left[1 - \frac{2+\cosh(\pi\eta)}{2\cosh(\pi\eta)} \tanh^2 \frac{\eta\pi}{2} + \frac{\mu}{4r} \coth(\pi\eta)\right] \exp\left(-b\right),$$

where b is given by Eq.(28). One can see out of this equation that $W_2 \ll W_3$.At $\eta \gg 1$ the probability W_3 has been increased by the factor $\eta\pi \exp\left(\pi r/\nu\right)$ in comparison with the case of the absence of magnetic field. The probability W_2 has been reduced by the additional factor $(\exp\left(-\pi\mu/\nu\right))$ and becomes non-applicable at $\mu \gtrsim \sqrt{r} \gg \nu$. In that case for the probability W_2 one can use Eq.(35) which will be get below.

5 Approximation at low photon energy

At $r \sim \nu^2$ the above approximation becomes non-applicable and another approach has to be. We close the integration over x contour in Eq.(4) in the lower half-plane and represent this equation in the following form

$$\Omega_i = \frac{\alpha m^2}{2\pi \mathrm{i}} \mu\nu \int\limits_{-1}^{1} dv \sum_{n=1}^{\infty} \oint f_i(v,x) \exp(\mathrm{i}\psi(v,x)) x dx, \tag{32}$$

where the path of integration is any simple closed contour around the point $-\mathrm{i}\pi n/\nu$. Let us choose the contour near this point in the following way $\nu x = -\mathrm{i}\pi n + \xi_n, \quad |\xi_n| \sim \sqrt{r} \sim \nu$ and expand the function entering in over the variables ξ_n. In the case $\nu \ll 1$, because of appearance of the factor $\exp\left(-\mathrm{i}\pi n/\nu\right)$, the main contribution to the sum gives the term $n = 1$. Near the point $-\mathrm{i}\pi/\nu$ the main terms of expansion such as $(\xi \equiv \xi_1)$

$$
\begin{aligned}
f_3 &= \frac{4\mathrm{i}}{\xi^3}\coth(\pi\eta)\cos^2\frac{\pi v}{2}, \\
f_2 &= -\frac{1}{\xi^2}\frac{\coth(\pi\eta)}{\sinh(\pi\eta)}\sinh(v\pi\eta)\ \sin(v\pi), \\
f_4 &= \frac{2}{\xi^2}\frac{\cosh(\pi\eta)-\cosh(v\pi\eta)}{\sinh^2(\pi\eta)}\cos^2\frac{\pi v}{2}, \qquad (33) \\
\psi &= \frac{4r}{\xi\nu}\cos^2\frac{\pi v}{2} - \frac{\xi}{\nu} + \frac{\mathrm{i}\pi}{\nu}.
\end{aligned}
$$

We find

$$
W_3 = 2\frac{\alpha m^2}{\omega}\eta\pi\coth(\pi\eta)\ \exp\left(-\frac{\pi}{\nu}\right) I_1^2(z), \qquad (34)
$$

$$
z = \frac{2\sqrt{r}}{\nu},
$$

$$
\begin{aligned}
W_2 &= \frac{\alpha m^2}{\omega}\mu\coth(\pi\eta)\exp\left(-\frac{\pi}{\nu}\right) \qquad (35) \\
&\times\left[\frac{\pi\eta}{\sinh(\pi\eta)}\int_0^1\cosh(v\pi\eta)I_0\left(2z\cos\frac{\pi v}{2}\right)dv - 1\right],
\end{aligned}
$$

where $\mathrm{I}_n(z)$ is the Bessel function of imaginary argument. At calculation W_2 the integration by parts over v has been performed. For $\eta \ll 1$ one obtains

$$W_2 = \frac{\alpha m^2}{\omega} \frac{\nu}{\pi} \exp\left(-\frac{\pi}{\nu}\right) \left(I_0^2(z) - 1\right). \qquad (36)$$

The found probability is applicable for $r \ll \nu$. Here we have kept the main terms in W_i only.

For $r \gg \nu^2$ the asymptotic representation $\mathrm{I}_n(z) \simeq \exp(z) \big/ \sqrt{2\pi z}$ can be used. As a result one obtains the probability Eq.(31) where the leading terms have to be retained. At very low photon energy $r \ll \nu^2$, using the expansion of the Bessel functions for the small value of argument, we have

$$W_3 = 2\frac{\alpha m^2 r}{\omega \nu^2} \eta\pi \coth(\pi\eta) \exp\left(-\frac{\pi}{\nu}\right), \qquad (37)$$
$$W_2 = \frac{\nu}{\pi\left(1+\eta^2\right)} W_3.$$

The probability under consideration is of interest of theoretics for arbitrary values μ and ν. For $r \ll \nu^2 \big/ \left(1+\nu^2\right)$ one can conserve in the phase $\psi(v,x)$ the term $-x$ only. After integrating over v we get the following equation for the probability averaged over the photon polarizations

$$
\begin{aligned}
W &= \frac{W_2 + W_3}{2} \\
&= \frac{\alpha m^2 r}{\mathrm{i}\pi\omega} \sum_{n=1}^{\infty} \oint F(y_n) \exp\left(-\mathrm{i}\frac{y_n}{\nu}\right) dy_n, \\
F(y) &= \frac{\cosh(y)\left(\eta y \cos\left(\eta y\right) - \sin\left(\eta y\right)\right)}{\sinh y \sin^3 \eta y} \qquad (38) \\
&+ \frac{\eta \cos(\eta y)\left(y \cosh y - \sinh y\right)}{\sinh^3 y \sin(\eta y)}.
\end{aligned}
$$

Summing the residues in the points $y_n = -in\pi$ one obtains

$$
W = \frac{\alpha m^2 r}{\omega} \sum_{n=1}^{\infty} \exp\left(-\frac{\pi n}{\nu}\right) \Phi\left(z_n\right), \quad z_n = \eta\pi n, \qquad (39)
$$

$$
\begin{aligned}
\Phi\left(z_n\right) &= \frac{z_n}{\nu^2} \coth z_n \qquad (40) \\
&+ \frac{2}{\sinh^2 z_n}\left[\frac{\eta z_n}{\nu} + \left(1 + \eta^2\right) z_n \coth z_n - 1\right].
\end{aligned}
$$

In the absence of magnetic field ($\eta \to 0$, $z_n \to 0$) we have

$$
\begin{aligned}
\Phi &= \frac{1}{\nu^2} + \frac{2}{\nu \pi n} + \frac{2}{\pi^2 n^2} + \frac{2}{3}, \\
W &= \frac{\alpha m^2 r}{\omega} [\left(\frac{1}{\nu^2} + \frac{2}{3} \right) \frac{1}{e^{\pi/\nu} - 1} \\
&\quad - \frac{2}{\pi \nu} \ln \left(1 - e^{-\pi/\nu} \right) + \frac{2}{\pi^2} \mathrm{Li}_2 \left(e^{-\pi/\nu} \right)],
\end{aligned}
\tag{41}
$$

where $\mathrm{Li}_2(z)$ is the Euler dilogarithm. In the opposite case $\eta \gg 1$ one obtains

$$
\Phi = \frac{\pi \eta n}{\nu^2}, \quad W = \frac{\alpha m^2 r}{\omega \nu^2} \frac{\pi \eta}{4} \sinh^{-2} \frac{\pi}{2\nu}. \tag{42}
$$

6 Conclusion

We have considered the process of pair creation in constant and homogeneous electromagnetic fields with a real photon taking part in. The probability of the process has been calculated using four different overlapping approximation. In the region of SQA applicability the created by a photon particles have ultrarelativistic energies. The role of fields in this case is to transfer the required transverse momentum and the electric and magnetic field actions are equivalent. But even in this case it is necessary to note a special significance of a weak electric field

$E = \xi H$ ($\xi \ll 1$) in the removal of the root divergence of the probability when the particles of pair are created on the Landau levels with the electron and positron momentum $p_3 = 0$. The frame is used where $k_3 = 0$.

In the region $\omega \lesssim 2m$ ($r \lesssim 1$) the energy transfer from electric field to the created particles becomes appreciable and for $\omega \ll m$ it determines the probability of the process mainly. At $\omega \ll eE/m$ the photon assistance in the pair creation comes to the end and the probability under consideration defines the probability of photon absorption by the particles created by electromagnetic fields. The influence of a magnetic field on the process is connected with the interaction of the magnetic moment of the created particles and magnetic field. This interaction, in particular, has appeared in the distinction of the pair creation probability by field for scalar and spinor particles.

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