Method of particle energy determination based on measurement of waveguide mode frequencies

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A new method of determination of charged particle energy is considered. This method is based on measurement of a waveguide mode frequency.

V.V. Poliektov, A.A. Vetrov, K.A. Trukhanov, V.I. Shvedunov, Instruments and Experimental Techniques 51, p. 191 (2008).

A.V. Tyukhtin, S.P. Antipov, A. Kanareykin, P. Schoessow, PAC'07, Albuquerque, July 2007, p.4156.

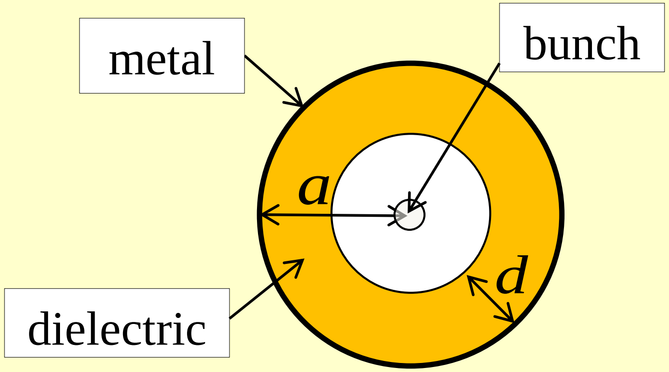
A.V. Tyukhtin, Tech. Phys. Lett. 34 (2008) 884.

For this method, it is important to provide enough strong dependency of mode frequencies on Lorenz-factor of the charged particle. Earlier we developed two variants of this method. One of them is based on use of a thin dielectric layer. Other variant is based on use of a waveguide loading with a system of wires coated with a dielectric material. Here we offer a new version consisting in application of a circular waveguide with a grid wall.

Version 1:
Thin Dielectric Layer

The essential progress can be achieved through the use of simple non-dispersive isotropic dielectric layer. The key factor for this technique consists in optimization of the thickness of the layer d=a-b. Dependence of frequencies on particles energy increases with decreasing the layer thickness.

A.V. Tyukhtin, S.P. Antipov, A. Kanareykin, P. Schoessow, PAC'09, 2009, p.4033.

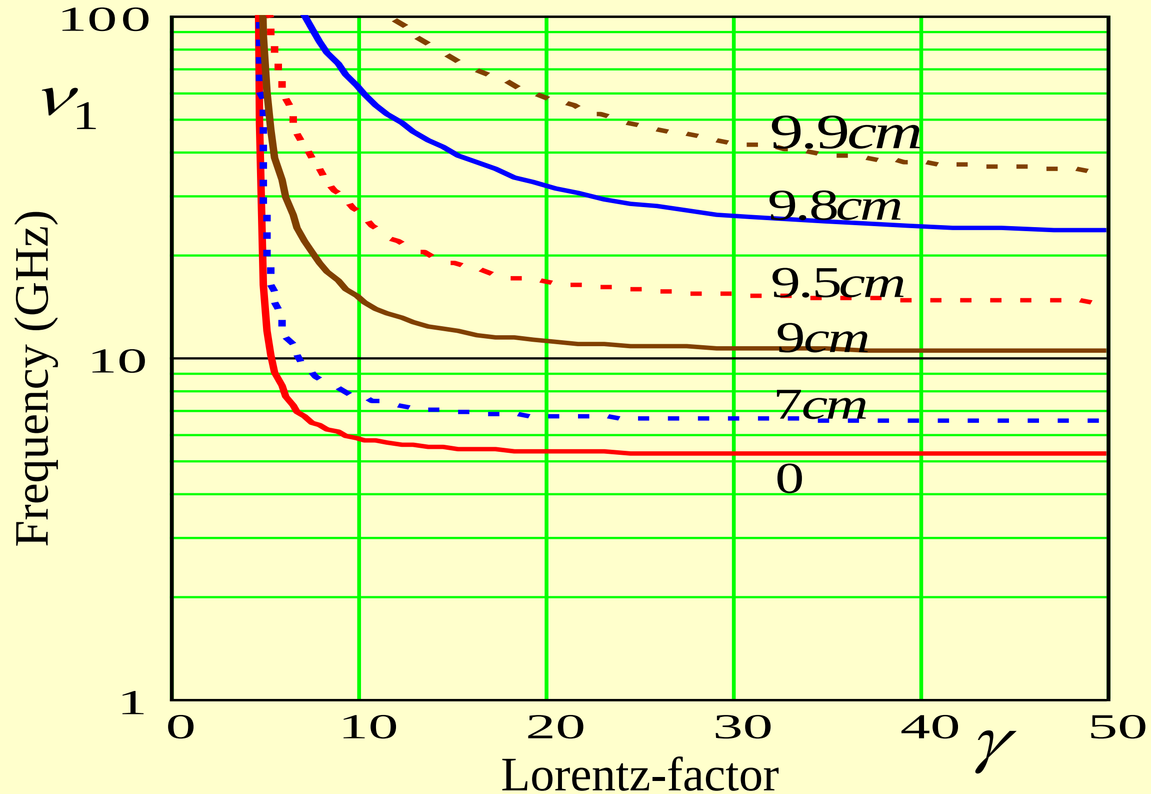


Waveguide radius $a = 10\,\text{cm}$

Channel radius $b = a - d$

is shown near curves.

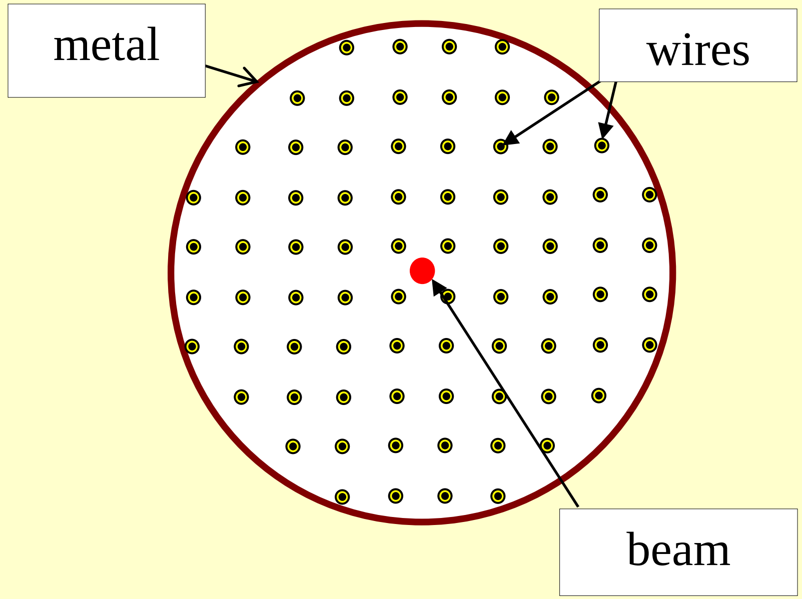
$\varepsilon = 1.05$

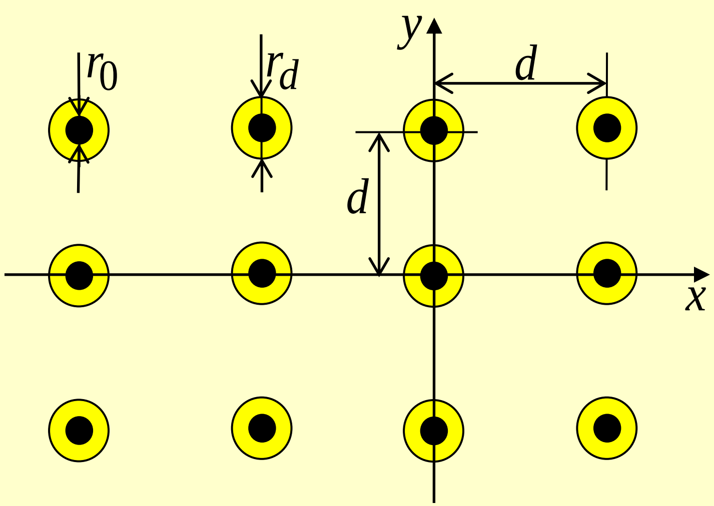


Version 2: Waveguide with Metamaterial

Other variant consists in use of some metamaterial.
A.V. Tyukhtin, Tech. Phys. Lett. 35, p.263 (2009).
A.V. Tyukhtin, P. Schoessow, A. Kanareykin, S. Antipov, AIP Conf. Proceedings 1086 (2009), p.604.
A.V. Tyukhtin, S.P. Antipov, A. Kanareykin, P. Schoessow, PAC'09, 2009, p.4033.

For example, some advantages can be reached with use of a system of parallel wires with dielectric coating.
Tyukhtin A.V., Doil'nitsina E.G., Kanareykin A., IPAC'10, Kyoto, Japan, May 2010, p.1071.

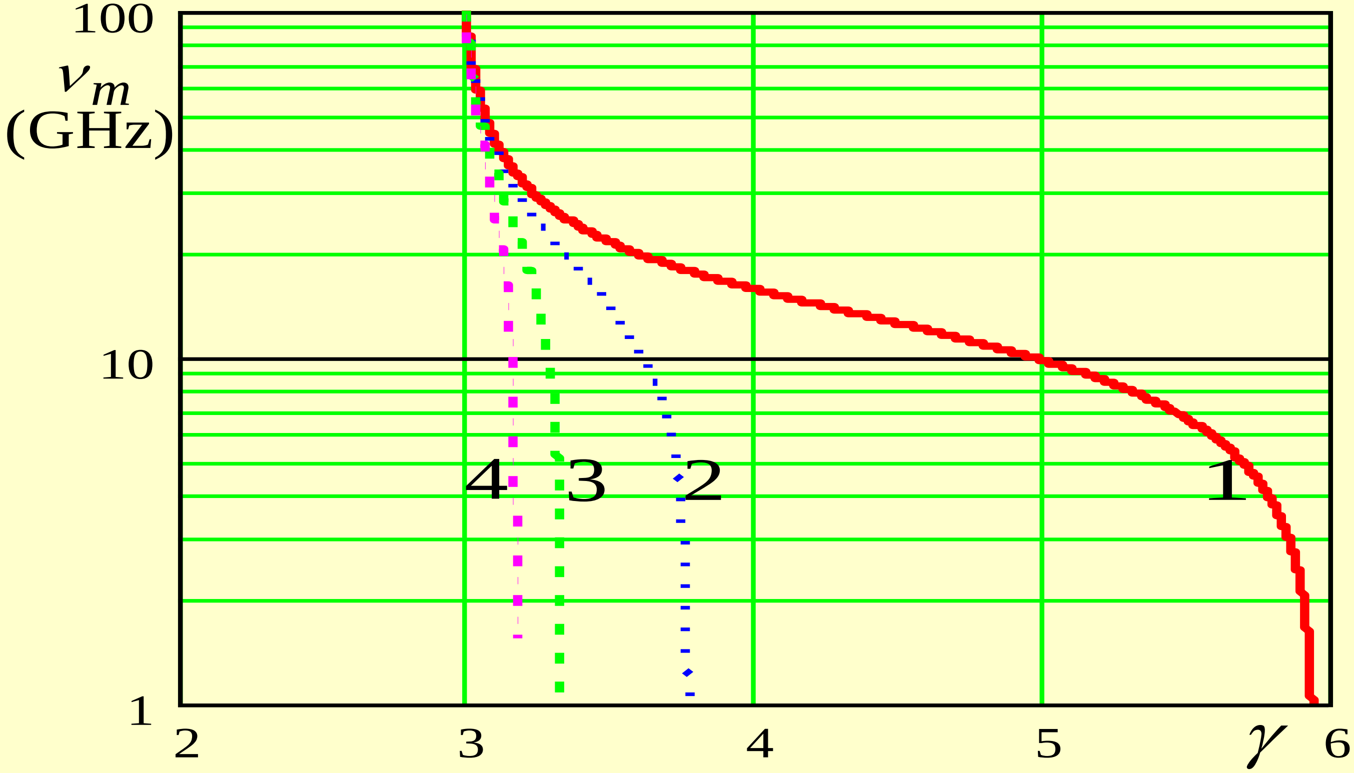




$$r_d \ll d \ll \lambda$$

Theory of this metamaterial:

Tyukhtin A.V., Doil'nitsina E.G., J. Phys. D: Appl. Phys. 44, 265401 (2011).



The mode frequencies depending on Lorentz factor; waveguide radius is 5 cm, coating permittivity = 5;

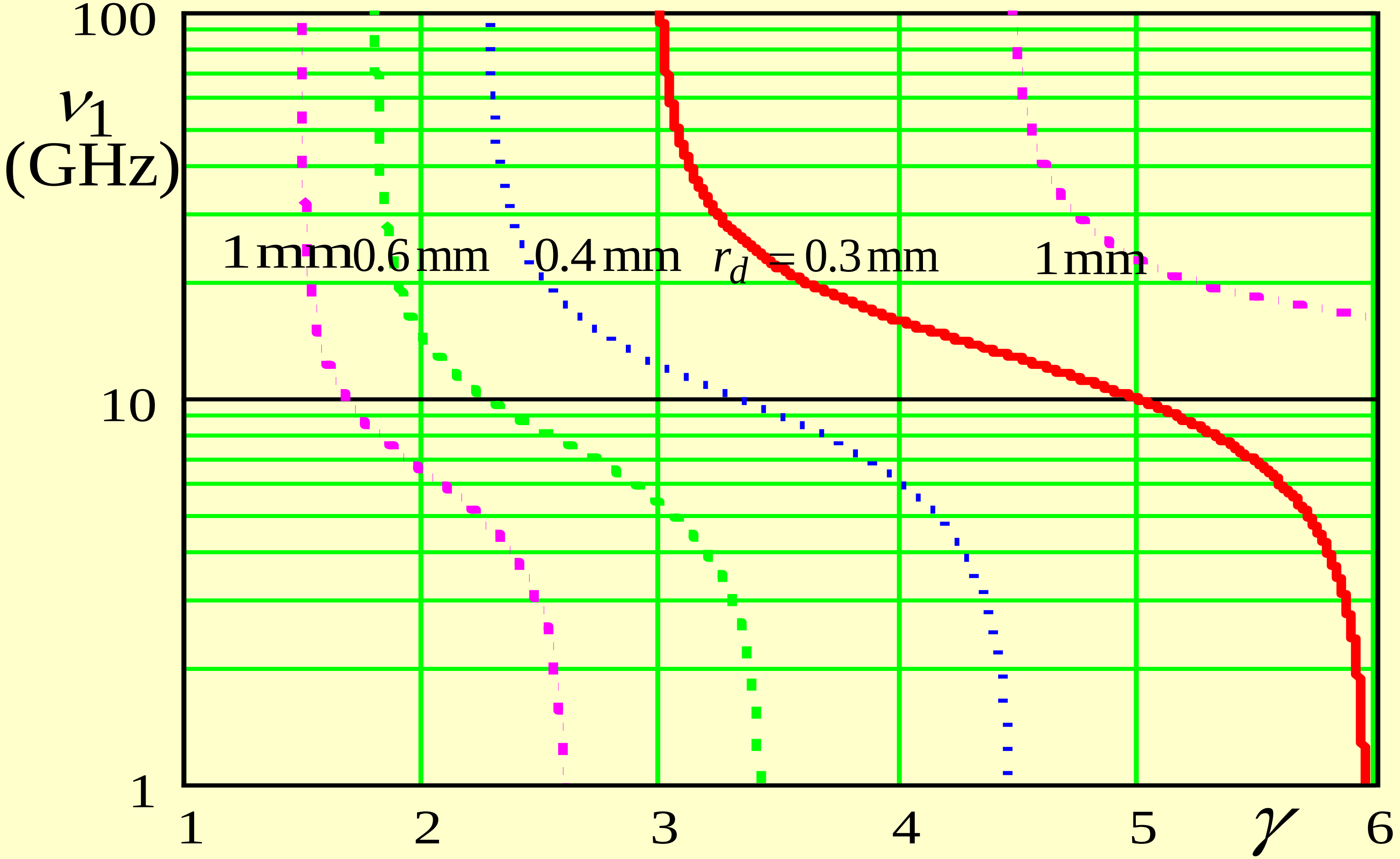
$r_0 = 0.2\,\mathrm{mm}, \quad r_d = 0.3\,\mathrm{mm}, \quad d = 10\,\mathrm{mm},$

mode numbers are indicated near the curves.

The 1st mode frequency depending on Lorentz factor; waveguide radius is 5 cm, coating permittivity = 5;

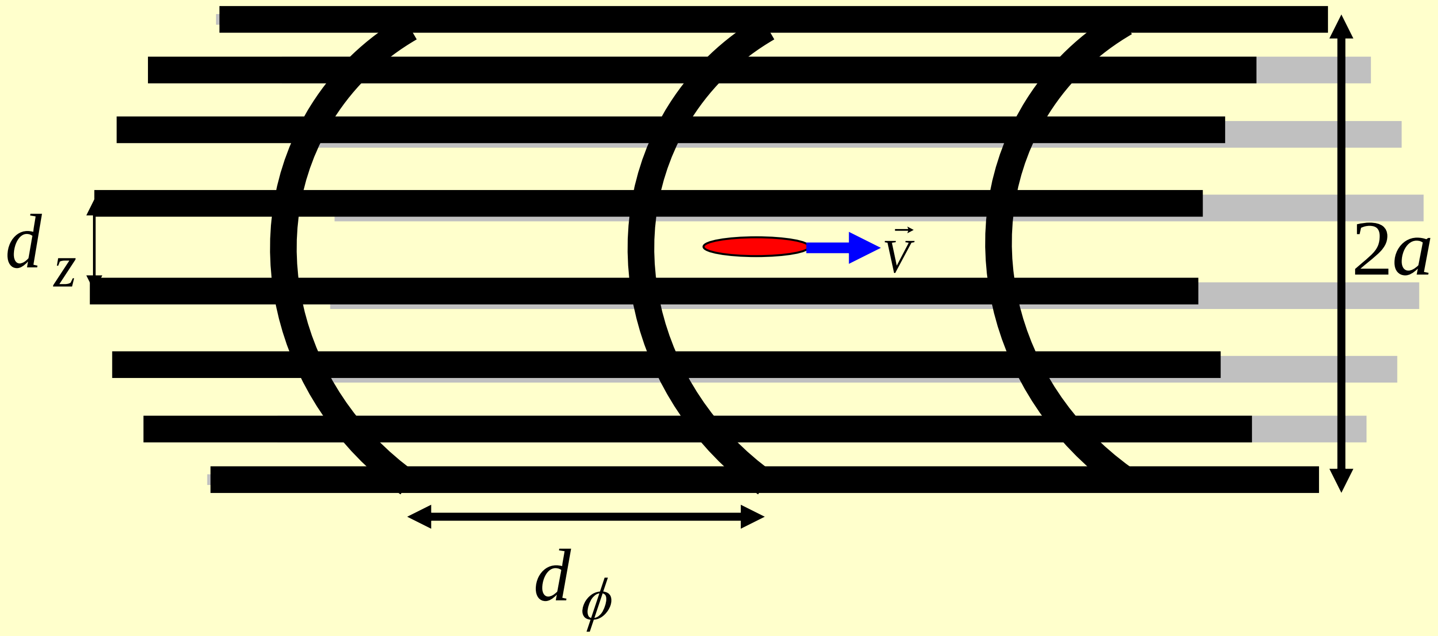
$r_0 = 0.2\,\mathrm{mm}, \quad d = 10\,\mathrm{mm},$

Coating radius are indicated near the curves.



Version 3 (new): Grid Waveguide

Waveguide radius: a
Wire radius: r_0
Period for z-wires: d_z
Period for ϕ**-wires:** d_ϕ



$$r_0 << d_{z,\phi} << a, c/\omega$$

Averaged Boundary Conditions

M.I. Kontorovich etc., Electrodynamics of grid structures. Moscow, 1987 (in Russian).

$$E_{\omega z}\Big|_{r=\pm a} = -\frac{i\omega d_z}{2\pi c}\ln\left(\frac{d_z}{2\pi r_0}\right)\left(1+\frac{c^2}{\omega^2\delta}\frac{\partial^2}{\partial z^2}\right)\left(H_{\omega\phi}\Big|_{r=a+0} - H_{\omega\phi}\Big|_{r=a-0}\right),$$

$$\delta = \frac{1+d_\phi/d_z+\kappa}{d_\phi/d_z+\kappa}$$

For perfect contact in intersections of wires: $\kappa = 0, \quad \delta = \dfrac{d_z + d_\phi}{d_\phi}$

If there are only wires parallel to z-axis: $\delta = 1$

If cells are square $(d_z = d_\phi)$ **:** $\delta = 2$

Field of Point Charge

$$E_r = \frac{q\sqrt{1-\beta^2}}{\pi c^2 \beta^2} \int\limits_{-\infty}^{\infty} |\omega| \begin{Bmatrix} K_1(kr) - R\, I_1(kr) \ \text{ for } \ r < a \\ T\, K_1(kr) \ \text{ for } \ r > a \end{Bmatrix} \exp\left(\frac{i\omega\zeta}{V}\right) d\omega,$$

$$E_z = -\frac{i\, q(1-\beta^2)}{\pi c^2 \beta^2} \int\limits_{-\infty}^{\infty} \omega \begin{Bmatrix} K_0(kr) + R\, I_0(kr) \ \text{ for } \ r < a \\ T\, K_0(kr) \ \text{ for } \ r > a \end{Bmatrix} \exp\left(\frac{i\omega\zeta}{V}\right) d\omega,$$

$$B_\phi = \beta E_r .$$

$$R = -\frac{K_0^2(ka)}{I_0(ka) K_0(ka) - \chi} \qquad T = -\frac{\chi}{I_0(ka) K_0(ka) - \chi}$$

$$\chi = \frac{\delta \beta^2 - 1}{\delta(1-\beta^2)} \frac{d_z}{2\pi a} \ln\left(\frac{d_z}{2\pi r_0}\right) \qquad k = \frac{|\omega|}{c\beta}\sqrt{1-\beta^2}$$

$$\beta = V/c \qquad \zeta = z - Vt$$

Dispersion Equation

$$I_0(ka)K_0(ka) = \chi$$

This equation can have only a single real root

$$k = k_0 = \frac{\omega_0}{c\beta}\sqrt{1-\beta^2}$$

This root is presented only in the case when $\chi > 0$ **,**
that is $\delta\beta^2 > 1$ **.**
Thus, radiation can be generated only in the case of grid possessing both z-wires and φ-wires.

Wakefield (= wave field = radiation field) of Thin Gaussian Bunch Moving along the Axis

Charge density of bunch: $\rho = \dfrac{q}{\sqrt{2\pi}\sigma}\delta(x)\delta(y)\exp\left(-\dfrac{\zeta^2}{2\sigma^2}\right)$

$$E_r^W = \frac{4q\gamma}{a^2}\frac{k_0 a}{W(k_0 a)}\exp\left(-\frac{\omega_0^2\sigma^2}{2V^2}\right)\sin\left(\frac{\omega_0\zeta}{V}\right)\begin{Bmatrix} K_0^2(k_0 a)I_1(k_0 r) \text{ for } r<a \\ -K_0(k_0 a)I_0(k_0 a)K_1(k_0 r) \text{ for } r>a \end{Bmatrix},$$

$$E_z^W = \frac{4q}{a^2}\frac{k_0 a}{W(k_0 a)}\exp\left(-\frac{\omega_0^2\sigma^2}{2V^2}\right)\cos\left(\frac{\omega_0\zeta}{V}\right)\begin{Bmatrix} K_0^2(k_0 a)I_0(k_0 r) \text{ for } r<a \\ K_0(k_0 a)I_0(k_0 a)K_0(k_0 r) \text{ for } r>a \end{Bmatrix},$$

$$B_\phi^W = \beta E_r^W ,$$

$$W(x) = I_1(x)K_0(x) - I_0(x)K_1(x),$$

$$k_0 = \omega_0 V^{-1}\gamma^{-1}, \qquad \gamma = \left(1-\beta^2\right)^{-1/2}, \qquad \zeta = z - Vt,$$

The case $k_0 a >> 1, \quad \chi << 1$ **(substantially nonrelativistic velocity):**

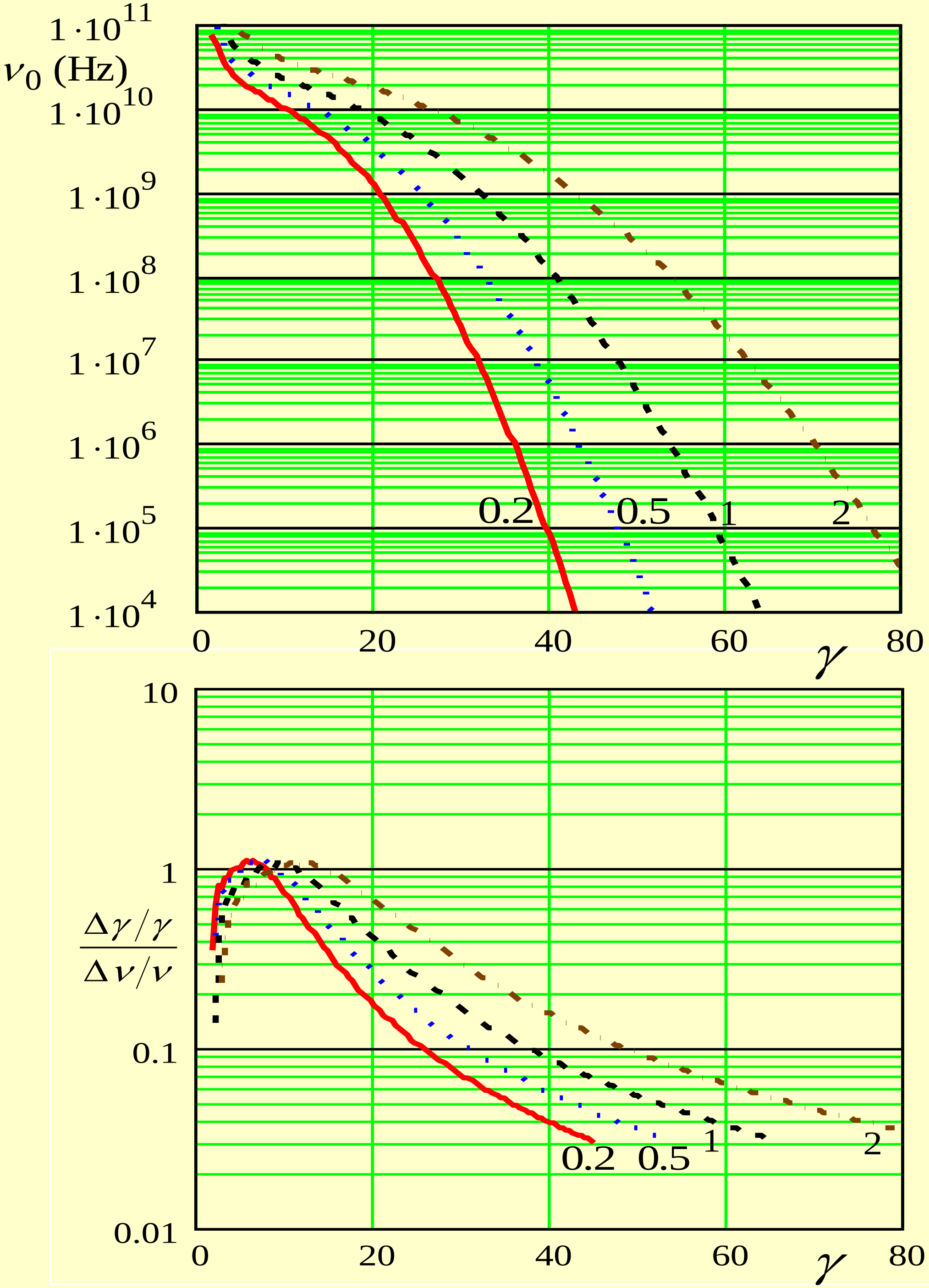
$$k_0 a \approx 1/(2\chi) \qquad \omega_0 \approx \frac{c}{2a}\frac{\delta\beta\sqrt{1-\beta^2}}{\delta\beta^2 - 1}\frac{2\pi a}{d_z}\frac{1}{\ln\left(\dfrac{d_z}{2\pi r_0}\right)}$$

The case $k_0 a << 1, \quad \chi > 1$ **(substantially relativistic velocity):**

$$k_0 a \approx 2\exp(-C-\chi)$$

$$\omega_0 \approx \frac{2c}{a}\frac{\beta}{\sqrt{1-\beta^2}}\exp(-C-\chi) \approx$$

$$\approx \frac{2c}{a}e^{-C}\sqrt{\gamma^2 - 1}\exp\left[-\left(\gamma^2 - 1 - \frac{\gamma^2}{\delta}\right)\frac{d_z}{2\pi a}\ln\left(\frac{d_z}{2\pi r_0}\right)\right]$$



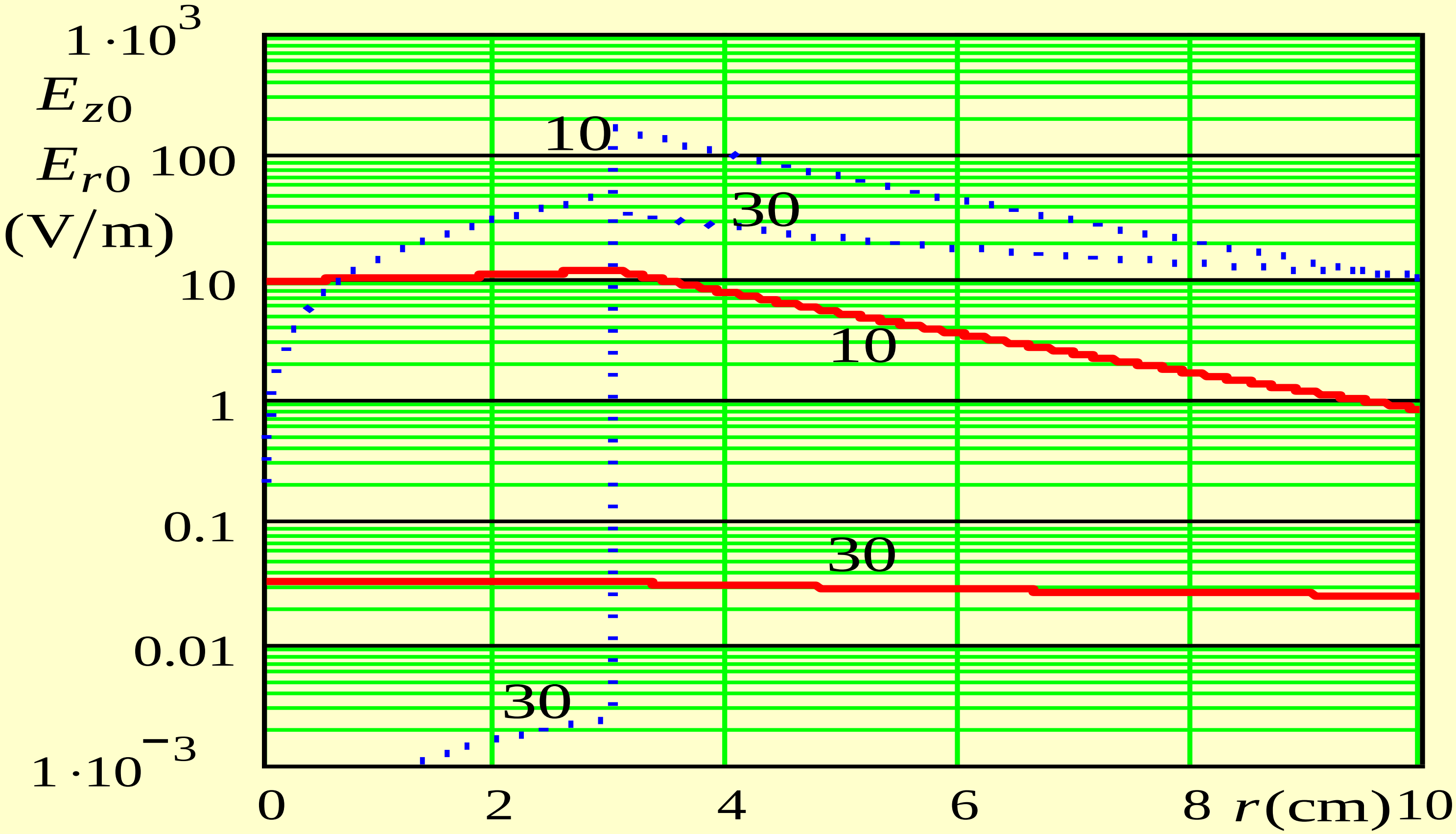
The mode frequency (top) and the relative accuracy of determination of γ (bottom).

$a = 3\,\mathrm{cm},$

$d_z = 2\pi a/38 \approx 5\,\mathrm{mm},$

$r_0 = 0.5\,\mathrm{mm}.$

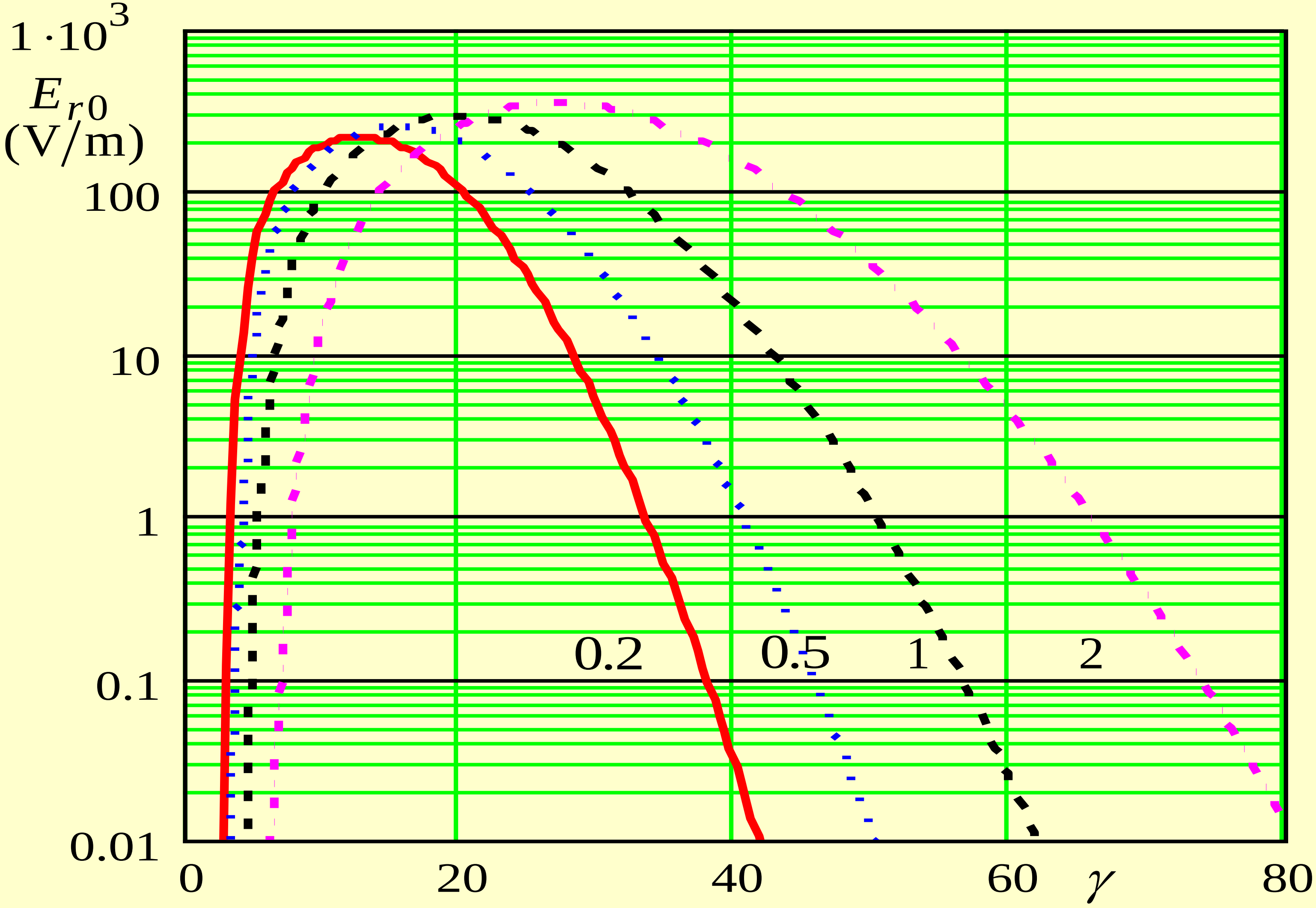
Magnitudes of d_ϕ (cm) **are given close to the curves.**



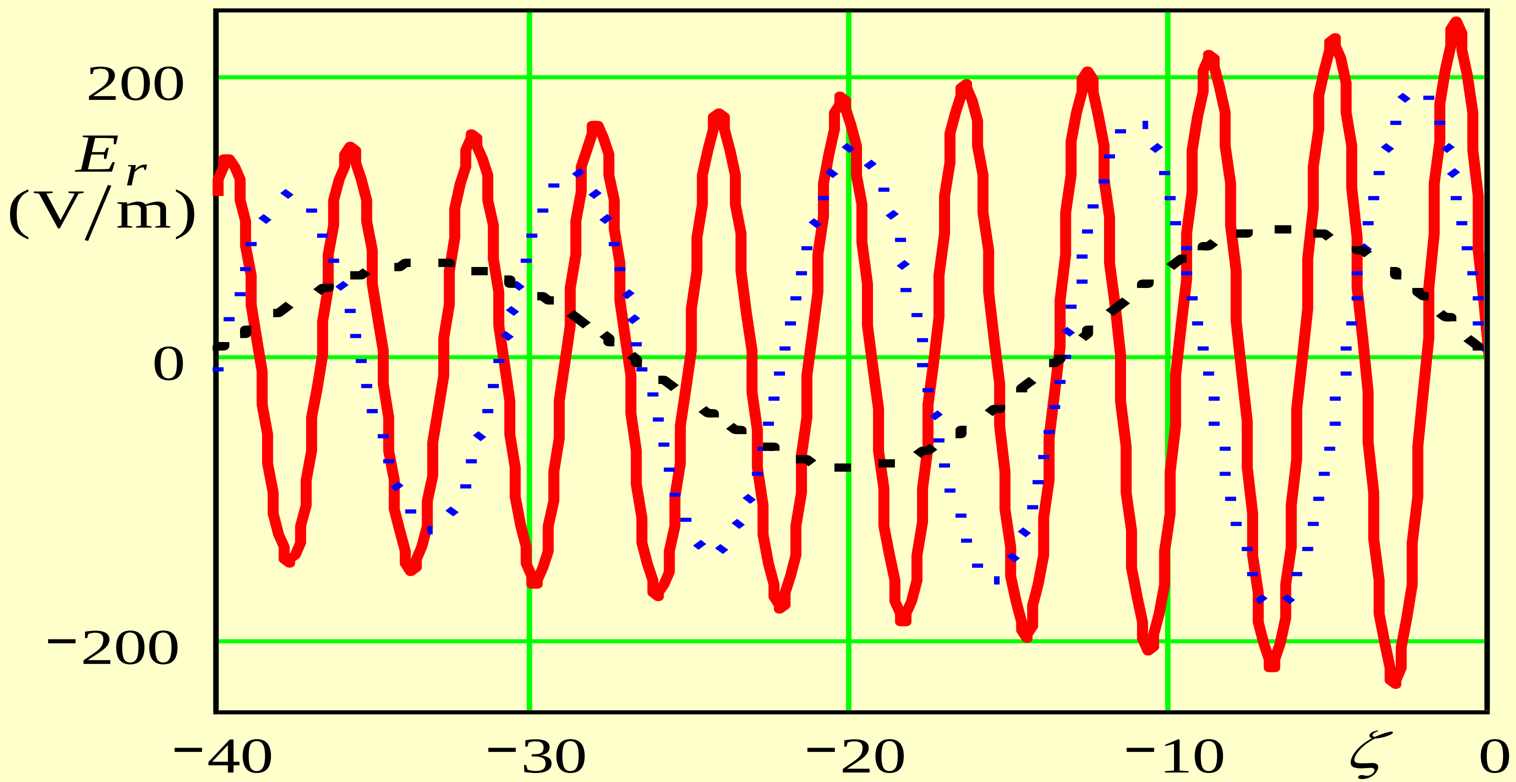
Dependency of amplitude of component E_z^W (solid red) and E_r^W (dotted blue) on distance from waveguide axis;

$\sigma = 3\text{mm},\ q = 1\text{pC},\ a = 3\,\text{cm},\ r_0 = 0.5\,\text{mm},\ d_z = d_\phi \approx 5\,\text{mm},$

γ are indicated near the curves.



Amplitude of the component E_r^W **on the outward surface of waveguide depending on γ; magnitudes of** d_ϕ (cm) **are indicated near the curves.**



Typical wakefield.
Component E_r^W on the outward surface of waveguide depending on the distance $\zeta = z - Vt$ for γ=15 (solid red), γ=20 (dotted blue), γ=25 (dashed black).
Conductivity of wires is $5 \cdot 10^7 (\text{Om m})^{-1}$
Other parameters are the same as early.

Conclusion

We consider a new method of determination of charged particle energy. This method is based on measurement of a waveguide mode frequency. For this method, it is important to provide an enough strong dependency of mode frequencies on Lorenz-factor of the charged particle.

Earlier we developed two variants of this method.
1. Use of a thin dielectric layer.
2. Use of a waveguide loading with a system of wires coated with a dielectric material.

3. New version is waveguide with a grid wall with rectangular small cells. In this case a single propagating mode can be generated. Its frequency depends on the Lorentz factor enough strongly in wide range.

As well, this structure can be used for generation of a monochromatic radiation with tunable frequency depending on the bunch velocity.

Thank you for attention!