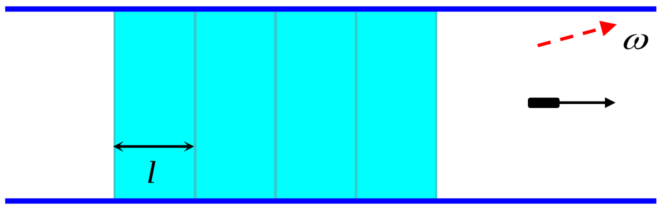
Self-amplified
Cherenkov radiation from a relativistic electron in layered dielectric-filled waveguide



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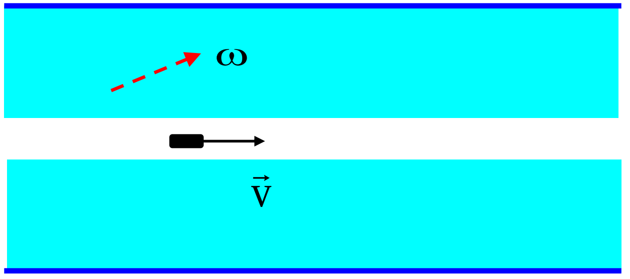
I. The timeliness and novelty of problem

The modern accelerators are promising installations for the generation of THz range radiation. This radiation is generated at the passage of electrons through some structure which force them to radiate.

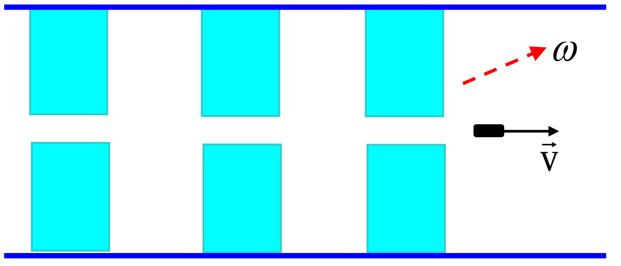
In 2009 J.B. Rosenzweig with co-authors reported the first direct observation of narrow-band terahertz coherent CR driven by a subpicosecond electron bunch traveling along the axis of a hollow cylindrical dielectric-lined waveguide.



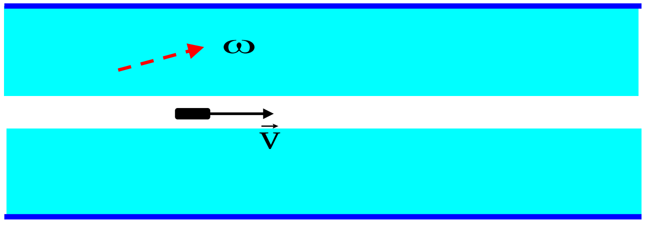
The novelty of our approach

We consider a waveguide filled with a layered (spatially periodic) material

(e.g., a stack of plates)

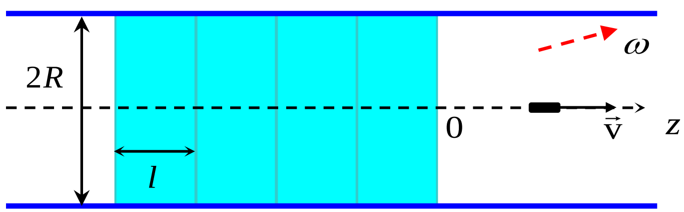


and not a uniform matter.



II. Statement of the problem, the general formula and the goal of the work

Consider a relativistic charged particle uniformly moving along the axis of a cylindrical waveguide.

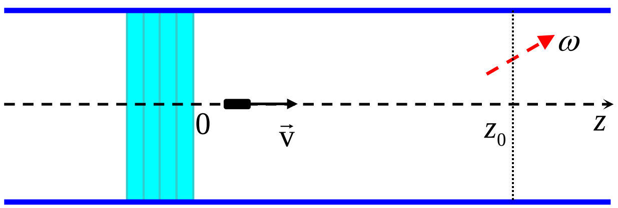


We shall assume that the finite part of the waveguide is filled with a laminated material, the permittivity and permeability of which

$$\varepsilon(z+l)=\varepsilon(z) \qquad \mu(z+l)=\mu(z)$$

are periodic functions of arbitrary shape.

We have studied the energy W of radiation emitted during the whole period of the particle motion and passing through the transverse section of waveguide at large distances from the laminated material:

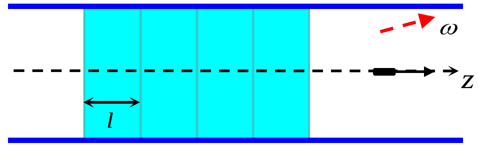


$$W = \sum_{n=1}^{\infty} \int_{\omega_n}^{\infty} I_n(\omega)\, d\omega = \sum_n W_n \qquad (1)$$

W_n is the energy of radiation at the n-th mode of waveguide.

I_n is the spectral distribution of corresponding radiation.

ω_n is the boundary frequency of the n-th waveguide mode.



In a hollow part of waveguide there propagate the plane waves

$$A_n(z) = A_n^q(z) + \frac{i}{\omega} a_n \exp(i\kappa_n z) \qquad (2)$$

The 2-nd summand describes the radiation field.

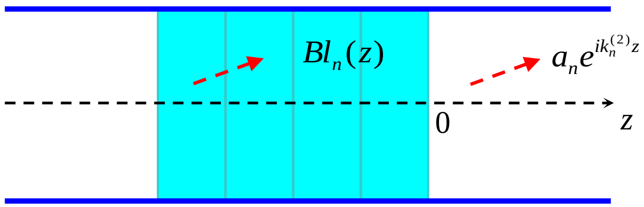
The amplitude a_n determines the spectral distribution of the energy of radiation by means of the following well-known formula

$$I_n(\omega) = \frac{4q^2}{\pi\omega} \frac{\kappa_n |a_n|^2}{\alpha_n^2 J_1^2(\alpha_n)} \qquad (3)$$

For calculation of amplitude it is sufficient to compare (2) with the complete solution of Maxwell equations that is valid for all z . This complete solution we obtained by means of the method of Green functions.

$$\kappa_n = \sqrt{\omega^2 / c^2 - \alpha_n^2 / R^2} \qquad J_0(\alpha_n) = 0$$

General formula

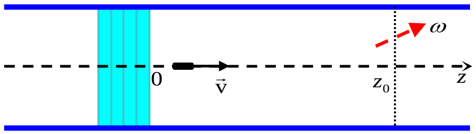


$$Bl(z+l) = Q \cdot Bl(z) \qquad |Q| \le 1$$

The problem was that propagating inside the layered medium are Bloch electromagnetic waves not the plane ones.

We succeed in our calculations of a_n with no limitations on the amplitude and variation profile of the laminated material permittivity.

The formula for determination of a_n is rather cumbersome and I shall not give it here.



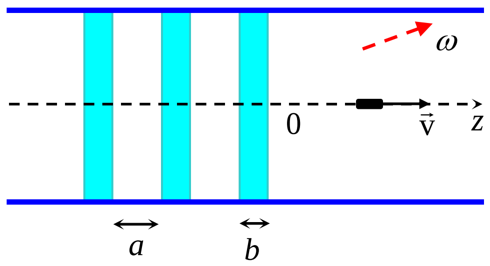
$$W = \sum_{n=1}^{\infty} \int_{\omega_n}^{\infty} I_n(\omega)\, d\omega = \sum_n W_n$$

The aim of the work

is the identification, analysis and clear explanation of cases when the waveguide and the periodical structure of layered material jointly strongly influence the spectral distribution of Cherenkov radiation from a particle:

$$I_n(\omega) = \frac{4q^2}{\pi\omega} \frac{\kappa_n |a_n|^2}{\alpha_n^2 J_1^2(\alpha_n)} \quad (4)$$

Below we shall consider a special, but highly advantageous case of a laminated material consists of *N* plates (each of *b* length) interleaved with vacuum gaps (the length of a gap is a)

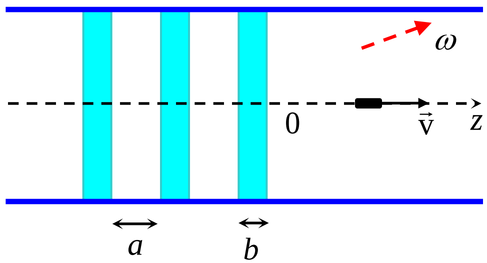


The waveguide filled with a *stack of plates*.

Let us consider a stack of three plates

$$N = 3.$$

Now assume that the permittivity of plate material is



$$\varepsilon_b(\omega) = 1.3 + 0.005i \qquad (5)$$

The values of plate thicknesses and of the gaps in between are given in Tab. 1.

Table 1

	a/R	b/R
A	308.6	13.79

And at last assume that the energy of electron is 1.2 MeV so that

the Cherenkov condition is satisfied

$$v \geq c/\sqrt{\varepsilon_b'} \qquad (6)$$

for the velocity of particle and the material of plates.

IIIa. The radiation at the given waveguide mode.

In Fig.1 the radiation spectrum at the 3rd waveguide mode is shown (curve A).

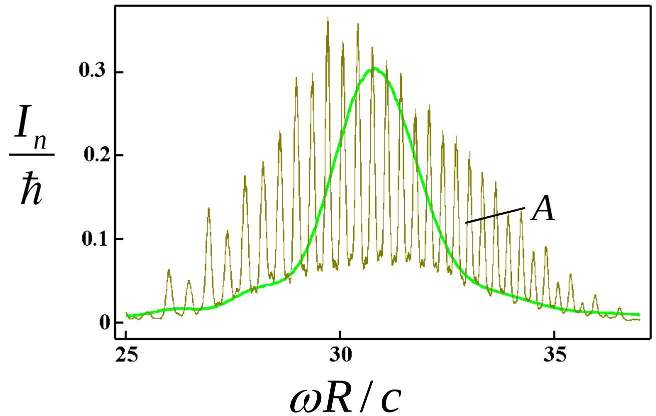
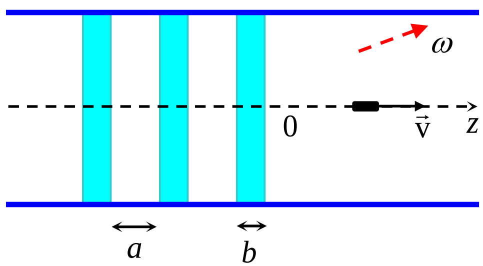


Fig.1a

A:



$a = 308.9R$

$b = 13.79R$

$n = 3$

Here and below we show the results of our numerical calculations based on the exact solutions of Maxwell equations mentioned by me above.

The radiation spectrum is seen here to oscillate.

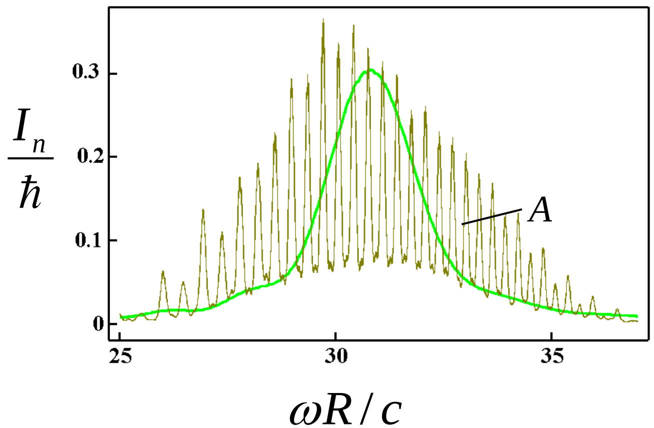
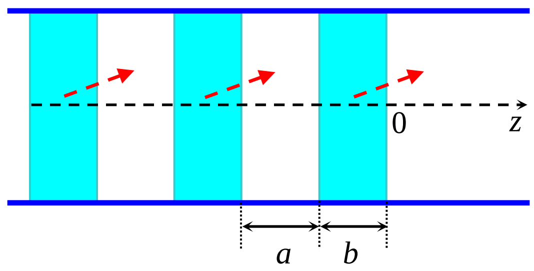


Fig. 1a



It is clear that the pulses of CR generated by the particle in different plates shall superimpose and, hence, the radiation spectrum will be an oscillating curve.

Now compare the curve A with the case (green curve B in Fig.1), when all the three plates are so close to each other that they merge and form one thick plate (one plate filling the waveguide).

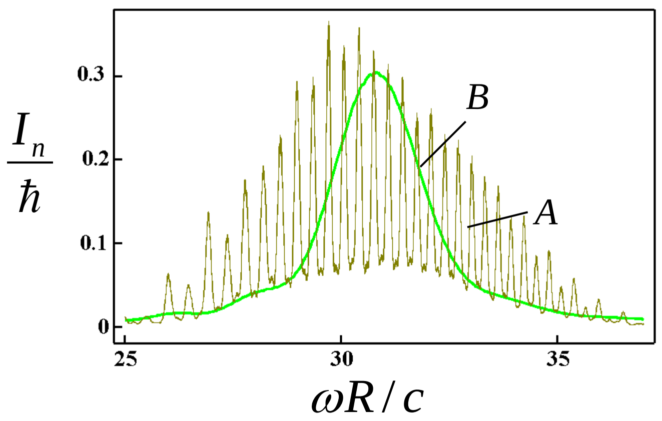
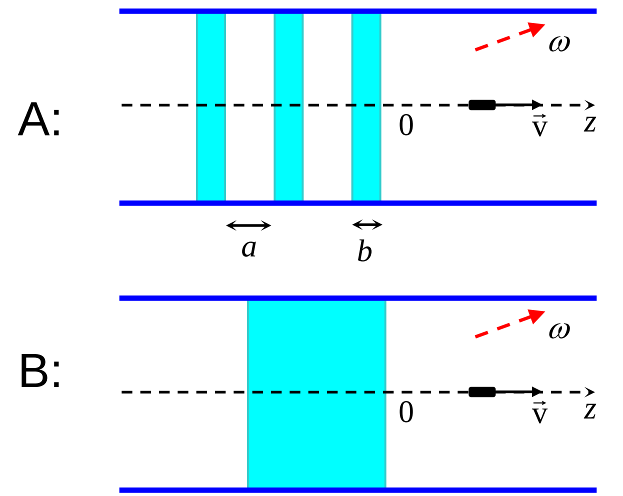


Fig. 1a

Table 2

	a/R	b/R
A	308.6	13.79
B	-	3x13.79



As was to be expected, the oscillations of spectrum vanish.

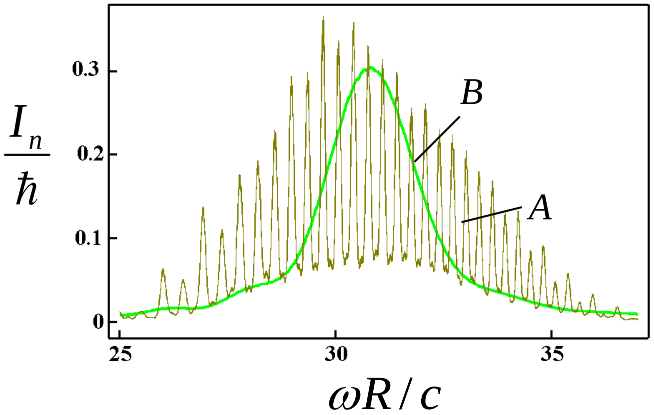
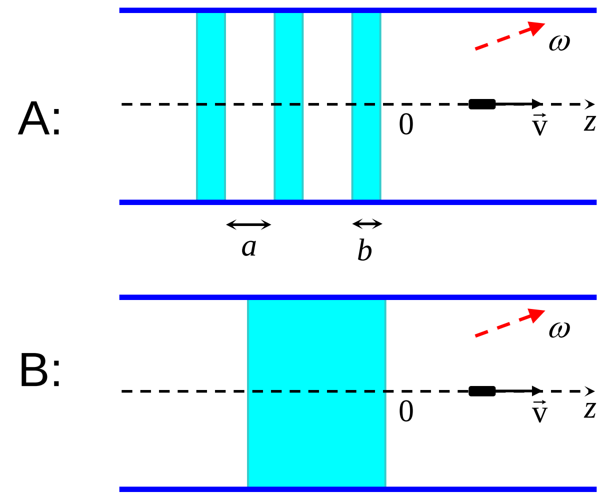


Fig. 1a



And what is clear, the total energy of radiation for the curves A and B is the same

$$W_3^A \approx W_3^B \approx 0.95 c\hbar / R \qquad (7)$$

as, in general, it does not change at interference.

The location of the maximum in CR spectrum is described by the following well-known simple formula:

$$\omega_n^{\max} \approx \frac{\alpha_n \mathrm{v}}{R\sqrt{\varepsilon_b' \mathrm{v}^2 / c^2 - 1}} \qquad J_0(\alpha_n) = 0 \qquad (8)$$

It is quite clear. Nevertheless, another unique situation is possible.

Now return to the case when the waveguide filling is a stack of three plates.

$$N = 3$$

Now select another special value of the thickness of vacuum gaps between the plates as is given in Table 3 (the row C).

Table 3

	a/R	b/R
A	308.6	13.79
B	-	13.79
C	15.43	13.79

Below is the radiation spectrum at the same 3-rd harmonic. But this time for the specially selected value of parameter $a/R = 15.43$ (curve C)

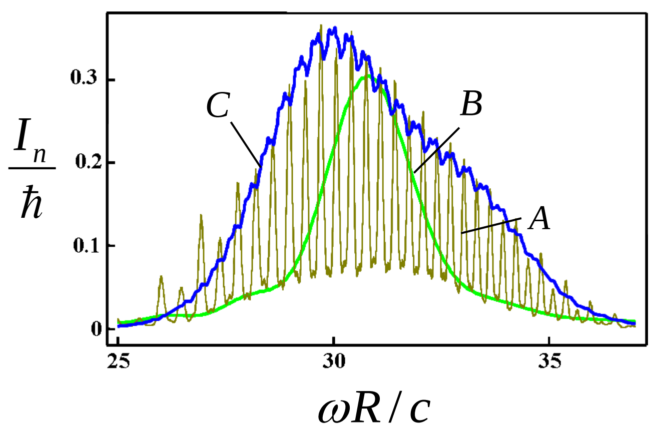
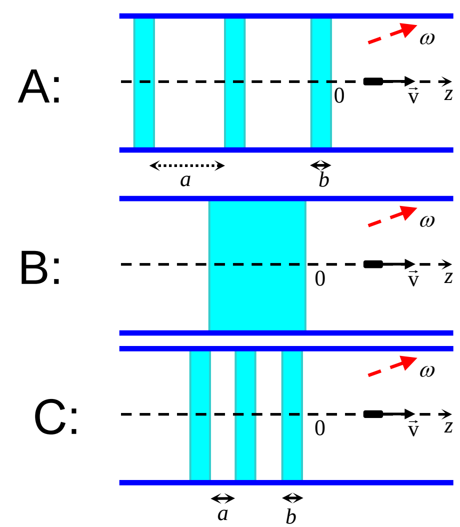


Fig. 1b



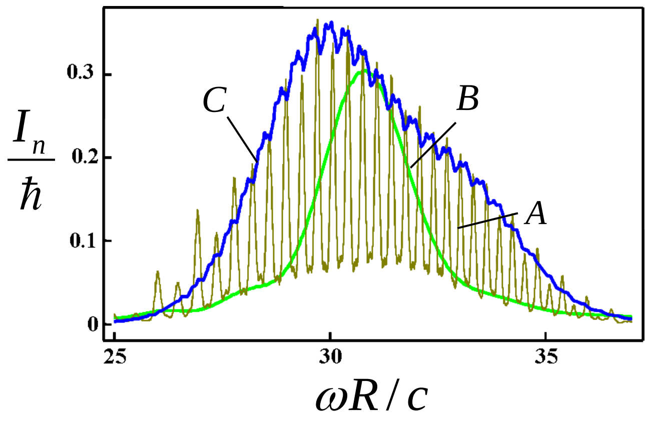


Fig. 1b

It is seen that the oscillations practically vanished.

Moreover, the total energy of radiation (the surface under the curve C) is twice larger than that for the curve A or B.

$$W_3^C = 1.8 c\hbar / R \qquad W_3^A \approx W_3^B \approx 0.95 c\hbar / R \tag{9}$$

But what is the situation at other waveguide modes?

IIIb. Radiation at the neighboring waveguide modes

Table 4

	a/R	b/R
B	-	3x13.79
C	15.43	13.79
D	14.90	13.79

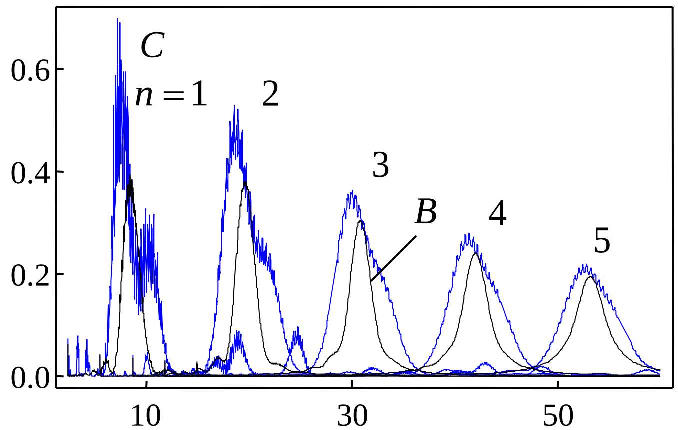


Fig. 2a

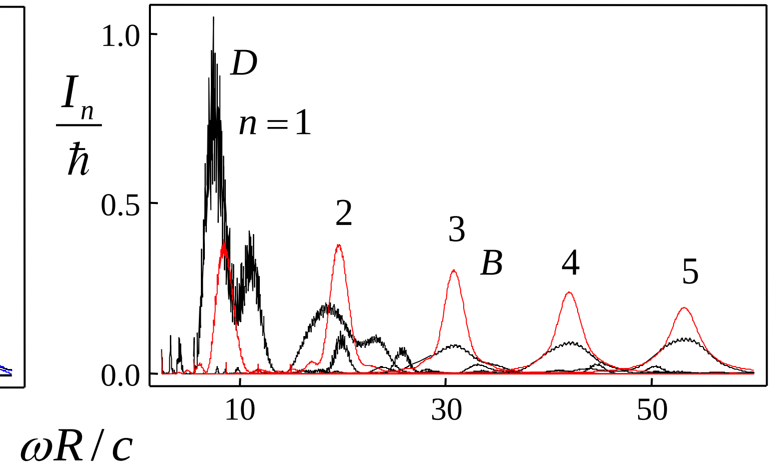
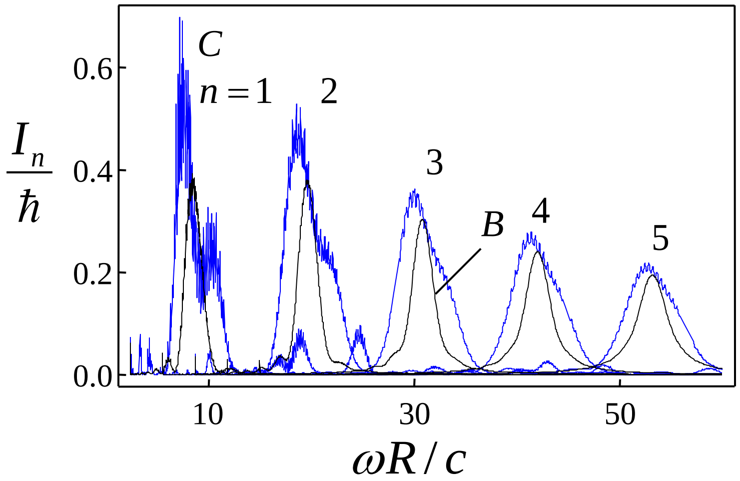
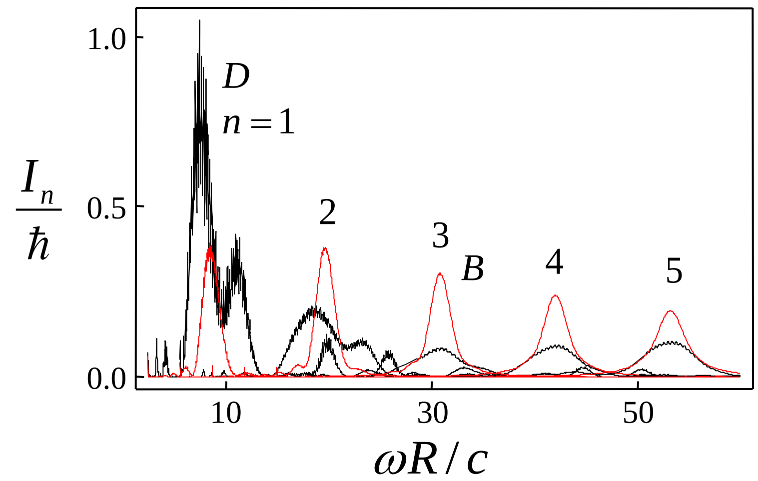


Fig. 2b

In Figs.3a,3b the radiation spectra at the first five modes of waveguide are shown. The number of mode *n* is seen by the curves. The curves C and D differ by a small difference in the thickness of vacuum gap between plates (a few percent).

The curve B (waveguide filled with one thicker dielectric) in Figs.3a,3b (black and red respectively) is given for comparison.





In Fig. 2a at the transition from the black curve B to the blue curve C, the total energy of CR increases at all five modes.

In Fig.2b at the transition from the red B curve to the black curve D, the total energy of CR increases in 3.3 times at the 1st mode and does not increase at the subsequent modes (2,3,4.5).

Hence, at an insignificant (a few percent) variation in the thickness of vacuum gap between the plates the waveguide modes are changed at which the energy of CR increases.

What is the matter?

What is the reason of radiation enhancement?

IIIc. Visual explanation

Now I shall submit to you a simple model that reproduces the basic futures of numerical results

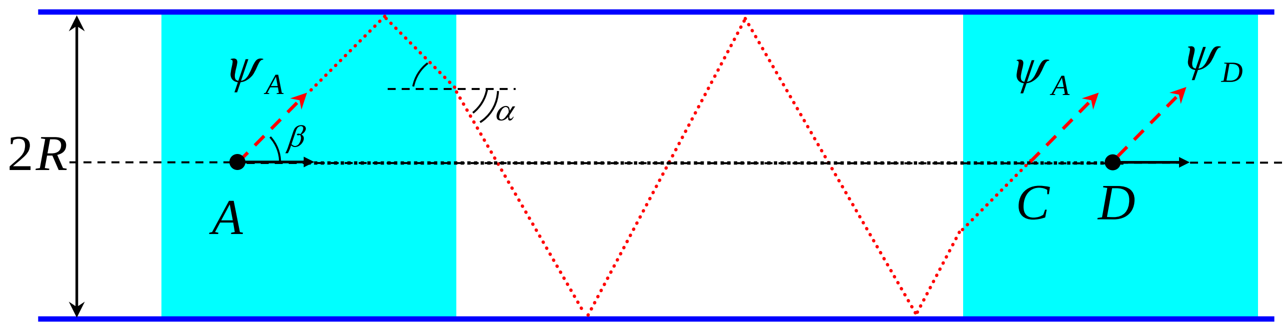
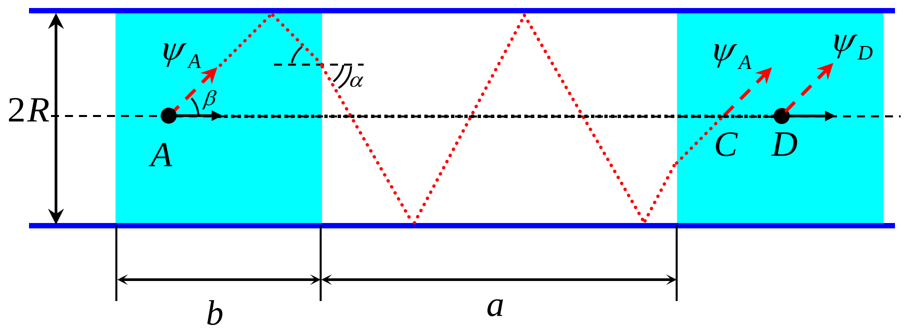


Fig.3

Let us consider (Fig.3) the instant when the charged particle is in the left plate and observe Ψ*A pulse of CR*, generated by a relativistic particle in the vicinity of point A (red dashed line). At propagation this pulse is found in the right plate and at some instant of time crosses the particle trajectory in a certain point C.

At this moment the relativistic particle emits ΨD pulse of CR in the vicinity of some other point D.



However, if the following two equations

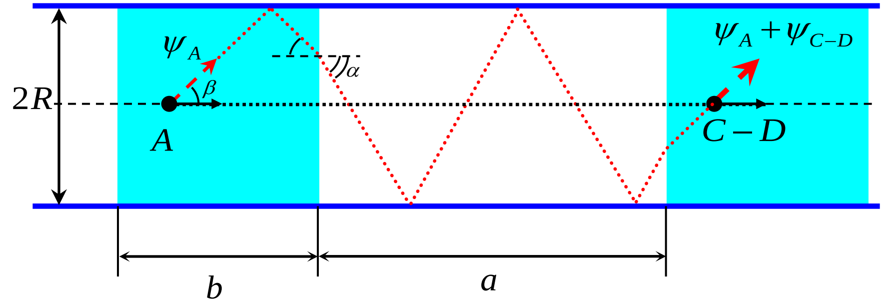
$$CD = 0 \quad \text{and} \quad AC = a + b \quad \Leftrightarrow \quad \frac{a\,tg\,\alpha + b\,tg\,\beta}{2R} = N \tag{11}$$

are satisfied, then an exceptional situation arises when:

1. *the point D coincides with the point C*

and

2. *this "joined" point C-D in the right-side plate is located in the same place where the point A is located in the left-side plate*.



$$CD = 0 \quad \text{and} \quad AC = a + b \quad \Leftrightarrow \quad \frac{a\,tg\,\alpha + b\,tg\,\beta}{2R} = N \qquad (11)$$

For this reason ΨC-D pulse generated in the vicinity of point C-D will "superimpose" on Ψ*A* pulse emitted earlier in the 1st plate.

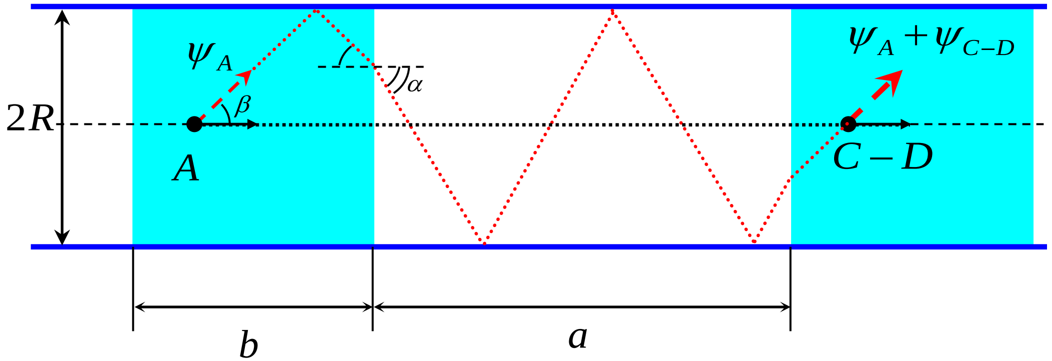
The amplification takes place if the following condition

$$N = 4s \qquad s = 1,2,3... \qquad (12)$$

is satisfied. Here the factor 4 ensures the constructive superposition of pulses.

From (11) and (12) there follow two equations

$$\boxed{CD = 0} \qquad \boxed{\frac{a\,tg\,\alpha + b\,tg\,\beta}{2R} = 4s} \qquad (13)$$



Hence, one may conclude that if two equations

$$CD = 0 \qquad \frac{a\,tg\,\alpha + b\,tg\,\beta}{2R} = 4s \qquad (13)$$

are met, then

the Cherenkov waves are generated inside the plate and (*at the same time*) constructively interfere with those emitted by the particle in the preceding plate.

In other words, the *in-phase superposition of CR pulses takes place in the zone of radiation formation*.

Fig. 2a

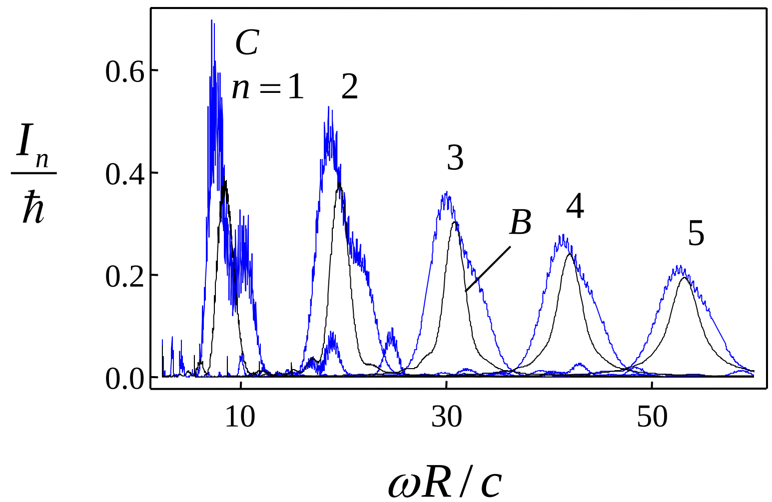


Table 4

	a/R	b/R
B	-	3x13.79
C	15.43	13.79
D	14.90	13.79

The thicknesses of vacuum gaps and plates for the curve C have been determined from

$$CD = 0 \qquad \frac{atg\alpha + btg\beta}{2R} = 4s \tag{13}$$

This fact explains the reason of CR amplification (more precisely, self-amplification) at different waveguide modes.

Equations

$$CD = 0 \qquad \frac{a\,tg\,\alpha + b\,tg\,\beta}{2R} = 4s \tag{13}$$

are valid within the framework of geometrical optics. At the first waveguide mode the laws of geometrical optics need a specification.

Fig. 2b

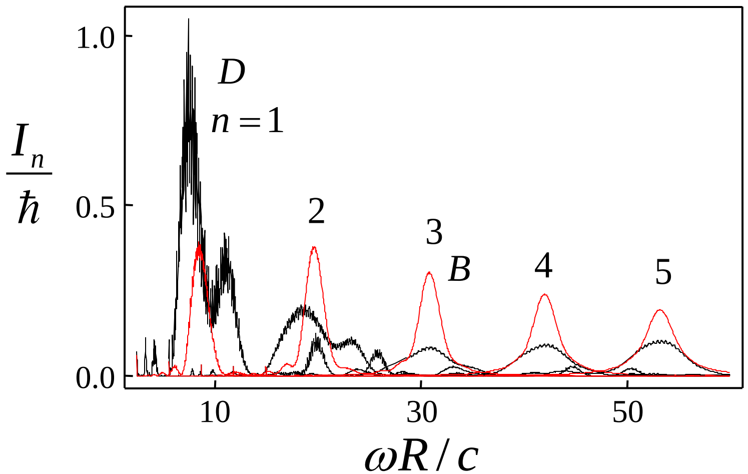
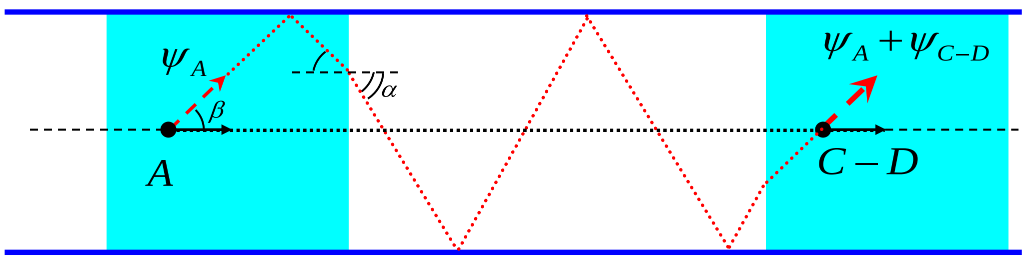


Table 4

	a/R	b/R
B	-	3x13.79
C	15.43	13.79
D	14.90	13.79

So, it is natural that the amplification of CR at the 1st waveguide mode takes place for *a/R* slightly different from the value obtained from the laws of geometrical optics.



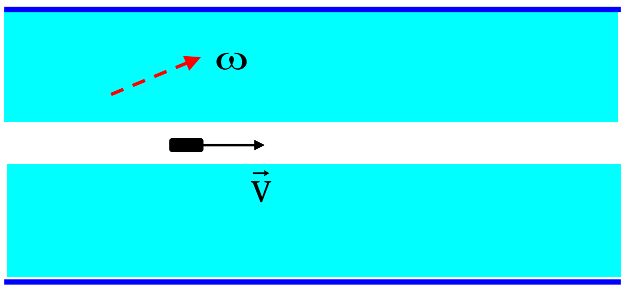
The in-phase superposition of electromagnetic oscillations at point C-D entails the total field increase in the zone of radiation formation. The force that brakes the motion of bunch will also increase and

the extra work of external force compelling the particle to move uniformly along the waveguide axis will be spent for generation of more powerful CR.

This is the reason of amplification (to be more exact, self-amplification) of CR from the electron moving inside a layered dielectric filled-waveguide.

IV. Discussion

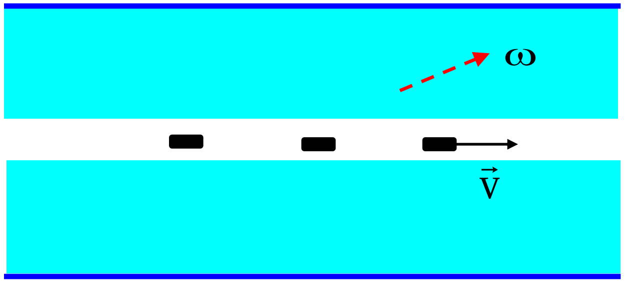
Recall that in 2009 J.B. Rosenzweig with co-authors reported [1] the fist direct observation of narrow-band terahertz coherent CR driven by a subpicosecond electron bunch traveling along the axis of a hollow cylindrical dielectric-lined waveguide.



The measurements indicate a peak power of 150 kW.

[1] A.M. Cook, R. Tikhoplav, S.Y. Tochitsky, G. Travish, O.B. Williams, J.B. Rosenzweig, Phys. Rev. Lett. **103** (2009) 095003.

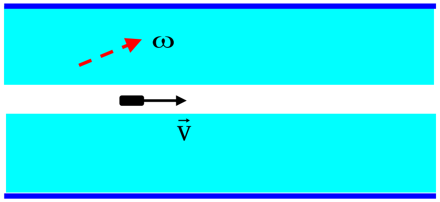
In 2011 J.B. Rosenzweig with co-authors [2] implemented structure variations that gave improved result.



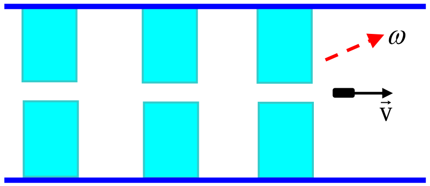
$$d = 2\pi v/\omega_n$$

[2] G. Andonian…,, J. B. Rosenzweig .. , Appl.Phys.Lett. **98** (2011) 202901

We propose the following modernization of the experimental setup:



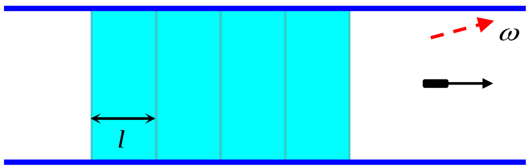
The hollow cylindrical dielectric inside the waveguide shall be cut in *N* identical parts (tubes) and then a periodic structure consisting of these tubes alternated with *vacuum gaps* shall be combined.



If the length of each *vacuum gap* between the tubes *is selected correctly*, then nearly a *N-fold increase in the peak power of coherent CR* from electron bunch is expected [3].

[3] **L. Sh. Grigoryan**, H.F. Khachatryan, S.R. Arzumanyan, Proc. Intern. Conf. IRPhE 2010, Ashtarak-Aghveran, Armenia, Sept. 23-25, 2010, pp.32-37.

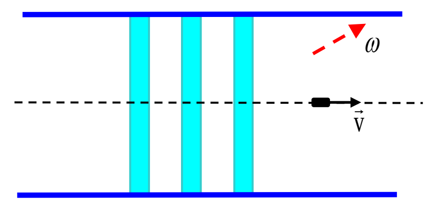
V. Conclusions



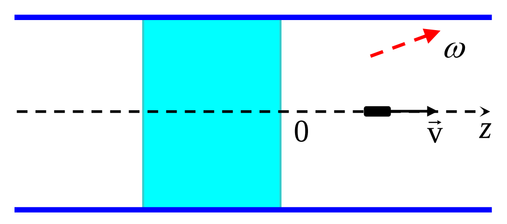
In our report the Cherenkov radiation from a relativistic electron moving along the cylindrical waveguide axis is investigated under the assumption that the waveguide is filled with a laminated material of finite length, the permittivity and permeability of which are periodic functions of arbitrary shape.

1. Expressions for calculation of the spectral distribution of total energy of radiation passing throw the transverse section of waveguide at large distances from the boundary of layered medium are derived with no limitations on the amplitude and variation profile of the layered material permittivity.

Numerical results for Cherenkov radiation from a charged particle uniformly moving along the axis of cylindrical waveguide filled with a stack of dielectric plates alternated with vacuum gaps are given.



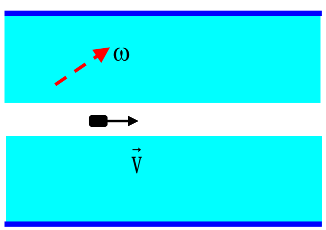
2. It is shown that due to the presence of vacuum gaps between the plates the power of CR from an electron may be many times greater than that produced, when the plates in the stack are so close that one thick plate is formed

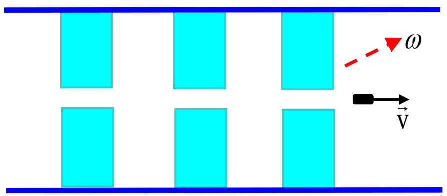


(waveguide filled with a solid dielectric without vacuum gaps).

3. The visual explanation of this effect of self-amplification effect of CR is given.

4. It is proposed to use the above effect for amplification of coherent CR observed by J.B. Rosenzweig with co-authors in 2009 [1] and 2011 [2].

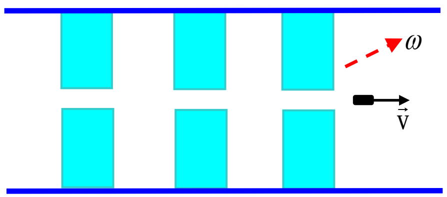




Appendix 1

<u>Comparison with the Traveling-Wave Tube</u> <u>(TWT)</u>

The self-amplification of particle CR inside the waveguide filled with a stack of plates



is similar to the amplification of microwaves in the TWT.

In both the cases:

1. There is an unbound, fast electron
2. There is a system that retards the propagation of electromagnetic waves
3. The radiation is amplified on the account of coherent (in-phase) superposition of waves.

However, there are essential differences. In our case:

1. The radiation is not form from the fast electron, but from the bound electrons inside the material of plates excited by the fast electron.
2. The radiation is amplified at rectilinear uniform motion of fast electron, whereas in TWT the amplification in this case is impossible.

Appendix 2

Some historical background

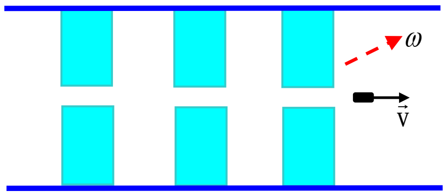
1957

I should like to draw attention to the review article by Y.B. Fainberg and N.A. Khizhniak. Here the radiation of a particle inside the waveguide filled with layered (periodical in space) material have been studed.

However:

1. The radiation that was investigated and termed by authors as the “parametric CR” was not the Cherenkov radiation. The authors openly point out that “the parametric effect… IS IRREDUCIBLE to the ordinary Cherenkov effect…”.

2. The authors considered the case of a waveguide completely filled with periodical medium while in our case the filling is partial.



Appendix 2

Some historical background

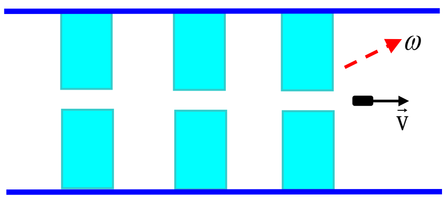
1974

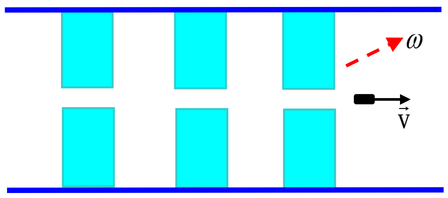
At last, take note also of the article «Radiation from a charged particle in flight through a stack of dielectric plates in the waveguide» by E.A. Begloyan, E.D. Gazazyan and E.M. Laziev.

The authors discussed the case of a waveguide filled with a stack of plates. However:

1. The authors did not *investigate the Cherenkov radiation from the particle in detail*. The authors only *assert* that "…the energy flux from the stack of plates turns out to be N^2 times as large as that from one plate" (not large values of N are implied).

2. No physical explanations of the obtained conclusions are given in the work.





Thank you indeed for your attention

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