OPTICAL TRANSITION RADIATION IN PRESENCE OF ACOUSTIC WAVES FOR AN OBLIQUE INCIDENCE

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Content

- Motivation
- Problem statement
- Spectral-angular distributions of the transition radiation
- Special cases
- Numerical examples
- Conclusion

Motivation

- Transition radiation has a number of remarkable properties and has found many important applications
- Optical transition radiation is widely used for the measurement of transverse size, divergence and energy of particle beams
- The intensity of the radiation can be increased considerably by using the interference effects in periodic structures
- From the point of view of controlling the radiation parameters it is of interest to investigate the influence of acoustic waves
- The considerations of *diffraction radiation*, *parametric X-radiation*, *channeling radiation*, *bremsstrahlung*, *electron-positron pair creation* processes, have shown that the acoustic excitations can notably change the spectral-angular-characteristics

Previous research

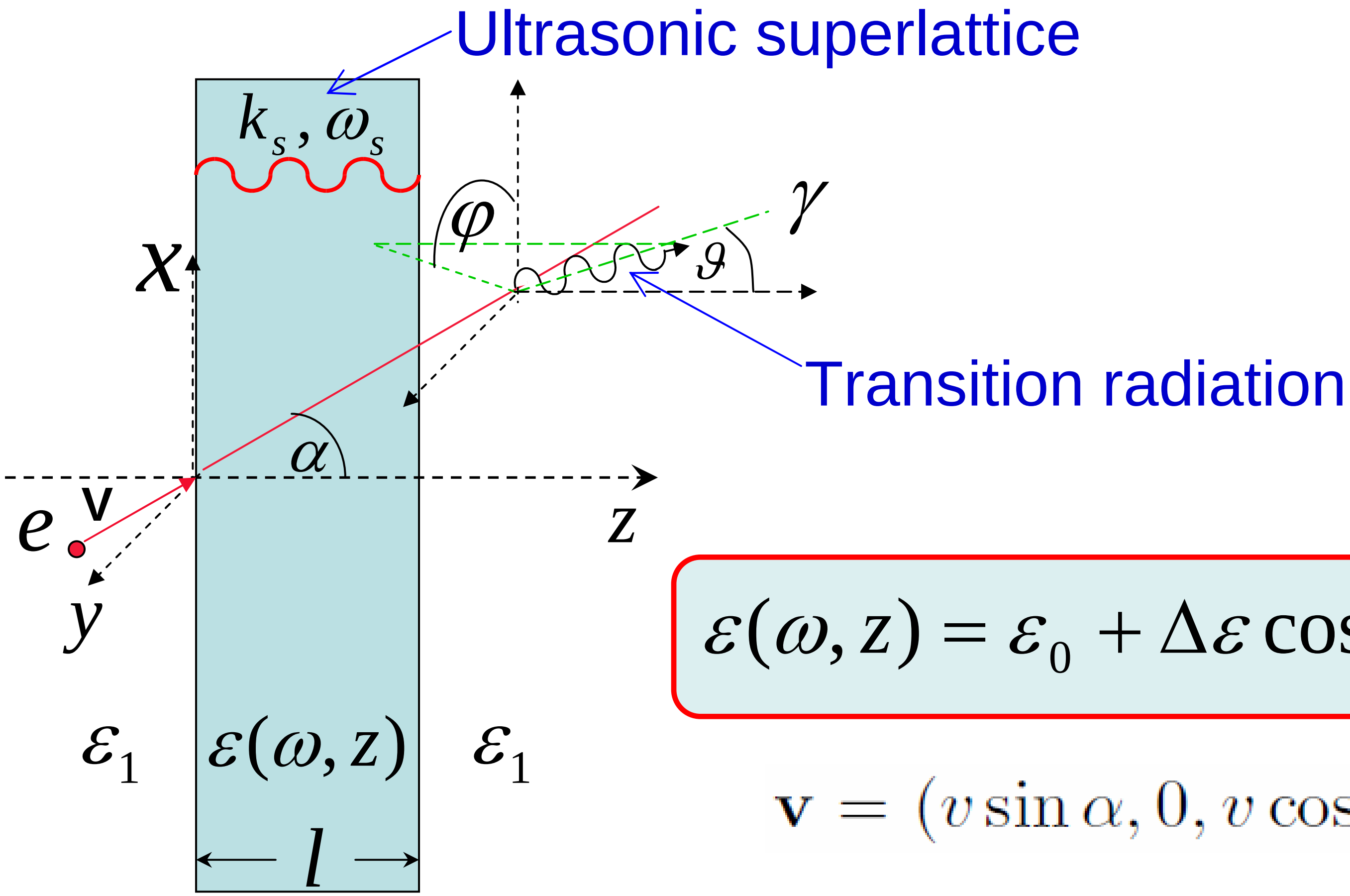
- X-ray transition radiation in an ultrasonic superlattice excited in fused quartz plate has been discussed in

  A. R. Mkrtchyan, L. Sh. Grigoryan, R. G. Gabrielyan, A. H. Mkrtchyan, A. A. Saharian, Preprint AS Arm. SSR (Yerevan, 1987)
  L. Sh. Grigoryan, A. H. Mkrtchyan and A. A. Saharian, *NIMB* 145 (1998) 197

- Optical transition radiation in a finite thickness plate in presence of acoustic waves is considered in

  A. R. Mkrtchyan, V.V.Parazian, A. A. Saharian, *Mod. Phys. Lett. B* 24 (2010) 2693

Problem geometry



$$\varepsilon(\omega, z) = \varepsilon_0 + \Delta\varepsilon \cos(k_s z + \varphi_1)$$

$$\mathbf{v} = (v \sin\alpha, 0, v \cos\alpha)$$

Approach

- Longitudinal ultrasonic vibrations are excited in the plate along the normal to its surface (along the axis *z*)
- Dielectric permittivity

$$\varepsilon(z) = \begin{cases} \varepsilon_0 + \Delta\varepsilon \cos(k_s z + \omega_s t + \phi), & -l \leqslant z \leqslant 0, \\ \varepsilon_1, & z < -l, \ z > 0, \end{cases}$$

- Under the condition $\nu_s l / v \ll 1$ during the transit time of the electron the dielectric permittivity is not notably changed and we will consider it as static: the intensity will be averaged over the corresponding phase
- For relativistic electrons and for the plate thickness $l \lesssim 1 \text{ cm}$ this leads to the constraint $\nu_s \ll 10^{11} \text{ Hz}$
- The presence of the small parameter $k_s c / \omega$ allows one to use the quasi-classical approximation

Total radiation intensity

Total spectral-angular density of the radiation in the region $z > 0$

$$I(\omega,\theta,\varphi) = \frac{e^2 \sin^3\theta\cos^2\alpha}{\pi^2 c\sqrt{\varepsilon_1}} \sum_{m=-\infty}^{+\infty} J_m^2\left(\frac{\omega\Delta\varepsilon/(2ck_s)}{\sqrt{\varepsilon_0 - \varepsilon_1\sin^2\theta}}\right)$$

$$\times \sin^2\left[\frac{\omega l\sqrt{\varepsilon_1}}{2c\cos\alpha}U_m(\theta,\varphi,\alpha)\right]\left(\frac{\mathbf{P}(\theta,\varphi,\alpha)}{V(\theta,\varphi,\alpha)} - \frac{\mathbf{Q}(\theta,\varphi,\alpha)}{U_m(\theta,\varphi,\alpha)}\frac{\cos^{1/2}\theta}{(\varepsilon_0/\varepsilon_1 - \sin^2\theta)^{1/4}}\right)^2$$

$\sin\theta < \sqrt{\varepsilon_0/\varepsilon_1}$ $\quad \mathbf{k} = \frac{\omega}{c}\sqrt{\varepsilon_1}(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ Wave vector

$J_m(a)$ ⇦ Bessel function of the first kind $\quad \theta,\varphi$ Radiation angles

$$U_m(\theta,\varphi,\alpha) = 1/\beta_1 - \sin\theta\cos\varphi\sin\alpha - \cos\alpha\sqrt{\varepsilon_0/\varepsilon_1 - \sin^2\theta} - \frac{mk_s c}{\omega\sqrt{\varepsilon_1}}\cos\alpha,$$

$$V(\theta,\varphi,\alpha) = 1/\beta_1 - \sin\theta\cos\varphi\sin\alpha - \cos\theta\cos\alpha, \qquad \beta_1 = v\sqrt{\varepsilon_1}/c$$

$$\mathbf{P} = (\sin\varphi, \cot\theta\tan\alpha - \cos\varphi, -\sin\varphi\tan\alpha),$$

$$\mathbf{Q} = (\sqrt{\varepsilon_1/\varepsilon_0}\sin\varphi, \cot\theta\tan\alpha - \sqrt{\varepsilon_1/\varepsilon_0}\cos\varphi, -\sin\varphi\tan\alpha)$$

Perpendicular polarization

- In the problem under consideration we have two different *polarizations*: Perpendicular and Parallel
- For the perpendicular polarization the electric field is perpendicular to the radiation plane (plane (k,v))
- Radiation intensity

$$I_{\perp}(\omega,\theta,\varphi) = \frac{e^2}{4\pi^2 c}\frac{\left(1-\sqrt{\varepsilon_1/\varepsilon_0}\right)^2}{\sqrt{\varepsilon_0-\varepsilon_1\sin^2\theta}}\sin^3\theta\cos\theta\sin^2\varphi\sin^2(2\alpha)$$
$$\times\sum_{m=-\infty}^{+\infty}J_m^2\left(\frac{\omega\Delta\varepsilon/(2ck_s)}{\sqrt{\varepsilon_0-\varepsilon_1\sin^2\theta}}\right)U_m^{-2}(\theta,\varphi)\sin^2\left[\frac{\omega l\sqrt{\varepsilon_1}}{2c\cos\alpha}U_m(\theta,\varphi)\right]$$

- For parallel polarization

$$I_{\parallel}(\omega,\theta,\varphi) = I(\omega,\theta,\varphi) - I_{\perp}(\omega,\theta,\varphi)$$

Peaks in the radiation intensity

- Strong peaks are present in the radiation intensity (zeros $U_m(\theta, \varphi, \alpha)$)
- Angular location of the peaks is determined from the equation

$$\sin\theta\cos\varphi\sin\alpha + \cos\alpha\sqrt{\varepsilon_0/\varepsilon_1 - \sin^2\theta} = 1/\beta_1 - \frac{mk_s c}{\omega\sqrt{\varepsilon_1}}\cos\alpha$$

- Acoustic waves lead to the generation of new peaks
- Location of the peaks does not depend on the amplitude of the acoustic oscillations
- Amplitude determines the heights of peaks
- Location of the peaks can be controlled by tuning the incidence angle and the frequency of acoustic oscillations

Peaks: Features

- Peaks correspond to the Cherenkov radiation emitted inside the plate and refracted from the boundary
- Wave vector for photon inside the plate

$$\mathbf{k}_0 = \frac{\omega}{c}\sqrt{\varepsilon_0}\left(\sin\theta_0\cos\varphi, \sin\theta_0\sin\varphi, \cos\theta_0\right) \quad \sin\theta_0 = \sqrt{\varepsilon_1/\varepsilon_0}\sin\theta$$

- Condition for the peaks is written as

$$\mathbf{k}_{0m}\mathbf{v} = \omega,\ \mathbf{k}_{0m} \equiv \mathbf{k}_0 + m\mathbf{k}_s \qquad \mathbf{k}_s = (0, 0, k_s)$$

- We can have a situation where the Cherenkov radiation emitted inside the plate is completely reflected from the boundary and in the exterior region there are no peaks
- There are cases when the Cherenkov radiation is confined inside the plate in the absence of acoustic excitations and the peaks appear as a result of the influence of the acoustic waves

Radiation intensity at the peaks

- Total radiation intensity at the peak

$$I\left(\omega,\theta^{(m)},\varphi\right) = \frac{e^2\varepsilon_1 l^2\omega^2\sin^3\theta^{(m)}\cos\theta^{(m)}}{4\pi^2c^3(\varepsilon_0-\varepsilon_1\sin^2\theta^{(m)})^{1/2}} \times\mathbf{Q}^2(\theta^{(m)},\varphi,\alpha)J_m^2\left(\frac{\omega\Delta\varepsilon/(2ck_s)}{\sqrt{\varepsilon_0-\varepsilon_1\sin^2\theta^{(m)}}}\right)$$

$\theta^{(m)} \Leftarrow$ Polar angle of a peak for given values of α and φ

- Relative contribution of the perpendicular polarization

$$\frac{I_\perp\left(\omega,\theta^{(m)},\varphi\right)}{I\left(\omega,\theta^{(m)},\varphi\right)} = \left(1-\sqrt{\varepsilon_1/\varepsilon_0}\right)^2\frac{\sin^2\varphi\sin^2\alpha}{\mathbf{Q}^2(\theta^{(m)},\varphi,\alpha)}$$

- Peak in the absence of untrasonic waves is reduced by the factor

$$J_0^2\left(\omega\Delta\varepsilon/(2ck_s\sqrt{\varepsilon_0-\varepsilon_1\sin^2\theta^{(0)}})\right)$$

- For a given radiation frequency, the frequency or the amplitude of the ultrasound can be tuned to eliminate this peak

Numerical examples

- We consider the transition radiation in fused quartz immersed in vacuum ($\varepsilon_1 = 1$)
- Dispersion for the melted quartz is described by the Sellmeier formula

$$\epsilon_0 = 1 + \sum_{i=1}^{3} \frac{a_i \lambda^2}{\lambda^2 - l_i^2} \quad 0.5\times 10^{14}\ \text{Hz} \le \omega \le 2\times 10^{16}\ \text{Hz}$$

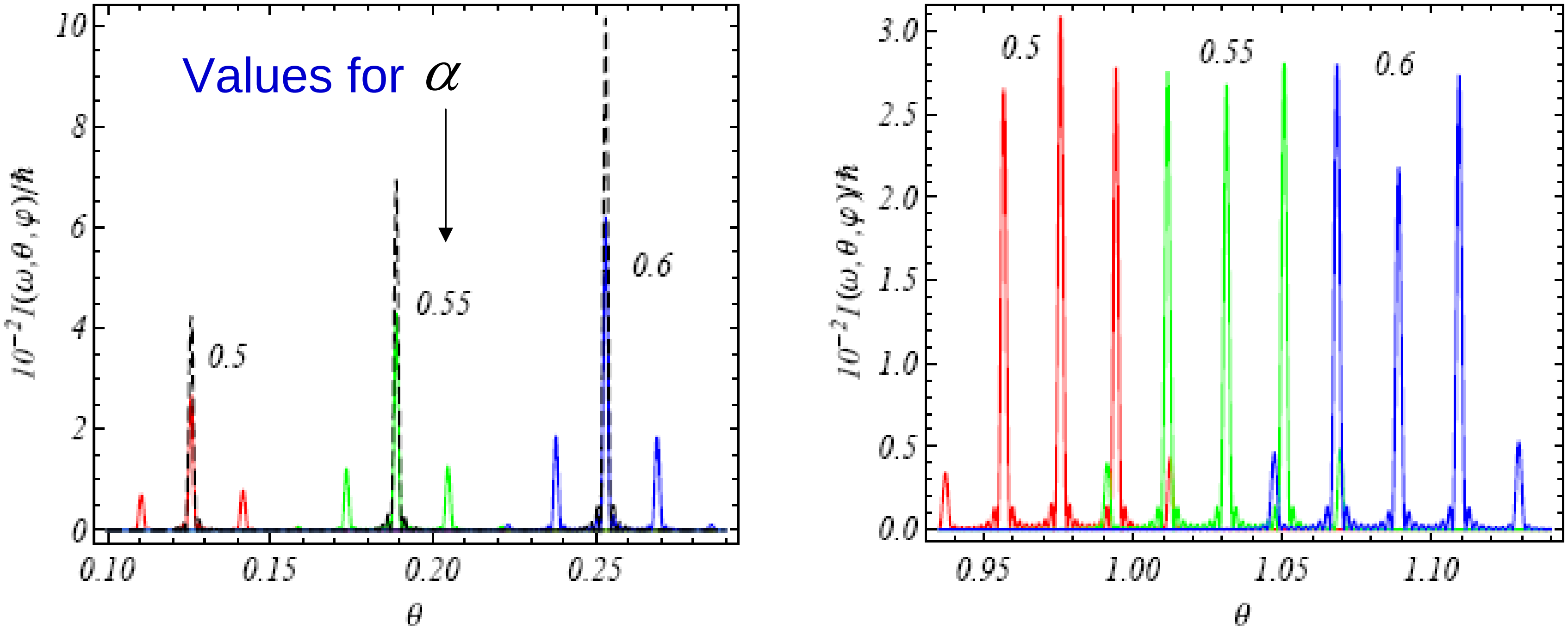
- Velocity of acoustic waves $\mathrm{v}_s = 5.6\times 10^5\ \text{cm/sec}$
- Oscillation amplitude we have taken the value $\Delta n/n_0 = 0.05$, where n_0 is the number of electrons per unit volume for fused quartz.
- Plate thickness $l = 1\ \text{cm}$

Numerical examples

Spectral-angular density of the radiation

Electron energy = 2 MeV

$$10^{-2} I(\omega,\theta,\varphi)/\hbar$$



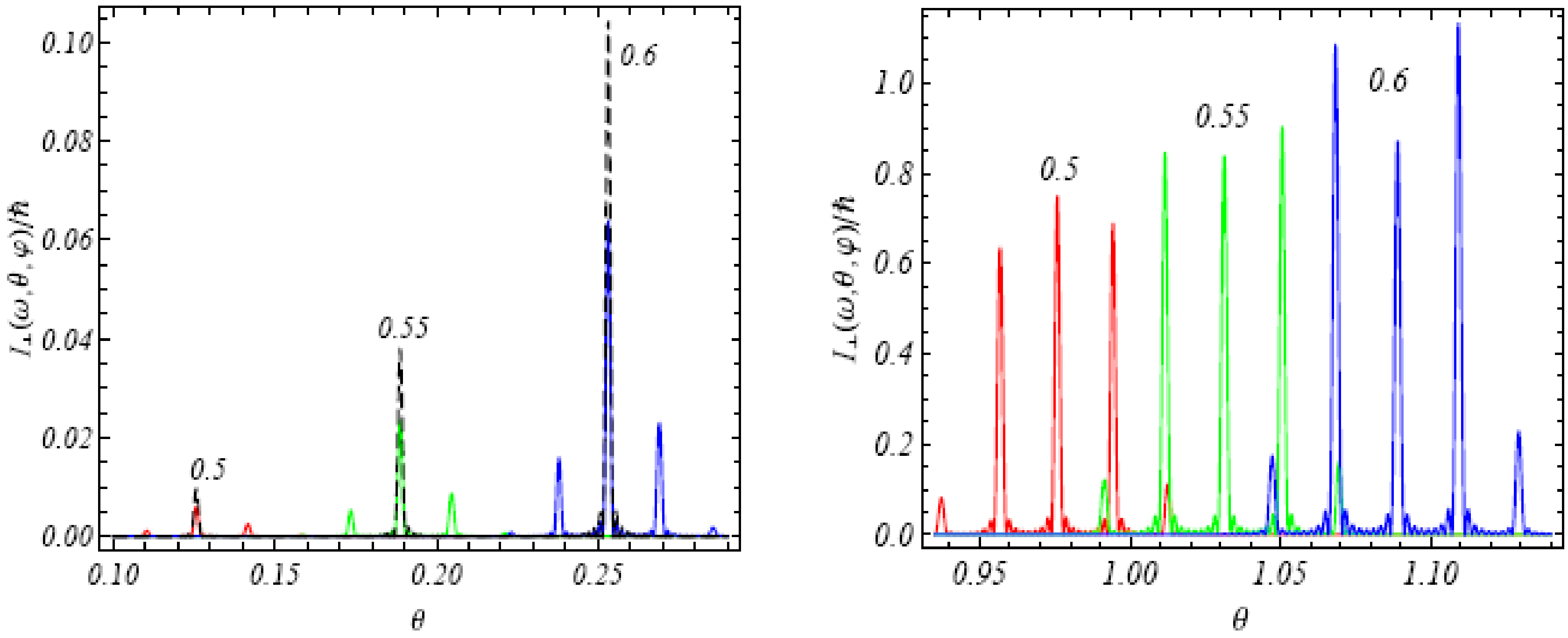
Angular distributions of the radiation intensity for two separate sets of peaks

Dashed curves ⇒ radiation in the absence of acoustic waves

Parameters ⇒ $\omega = 2.73\times 10^{14}$ Hz $\varphi = 0.369$ $\nu_s = 5$ MHz

Perpendicular polarization

$$I_{\perp}(\omega,\theta,\varphi)/\hbar$$

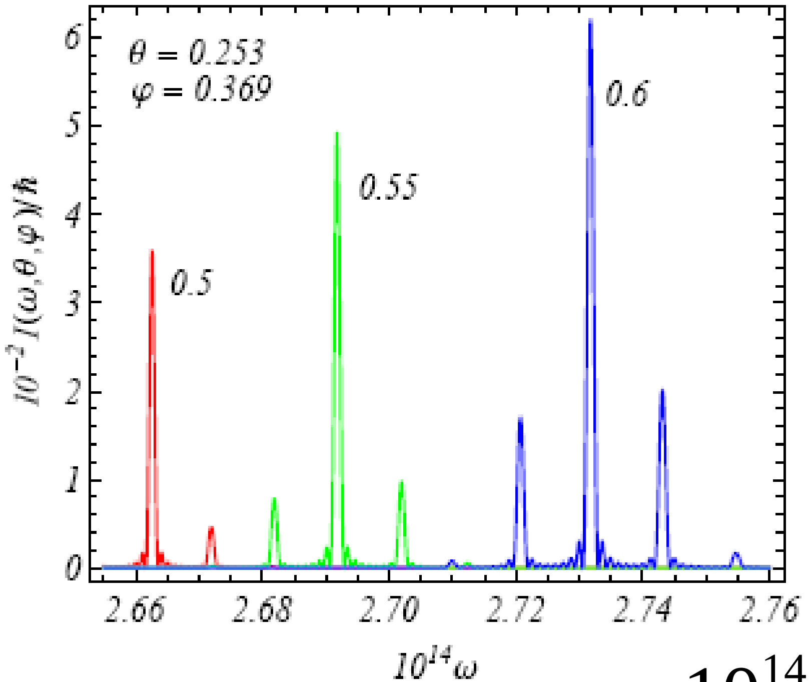


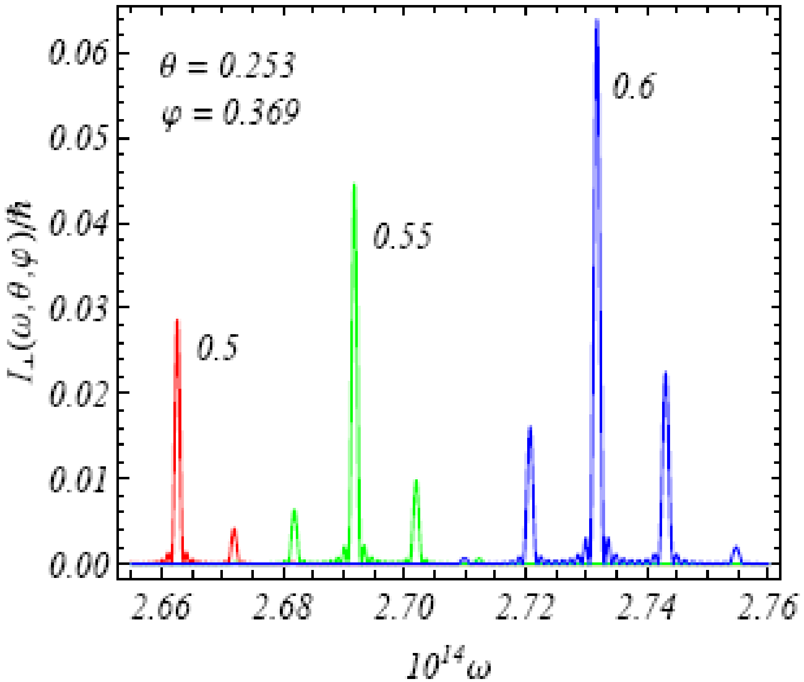
The same as before for perpendicular polarization

Spectral distribution

$10^{-2} I(\omega,\theta\varphi)/\hbar$

$I_{\perp}(\omega,\theta,\varphi)/\hbar$





$10^{14}\,\omega$ →

Total intensity

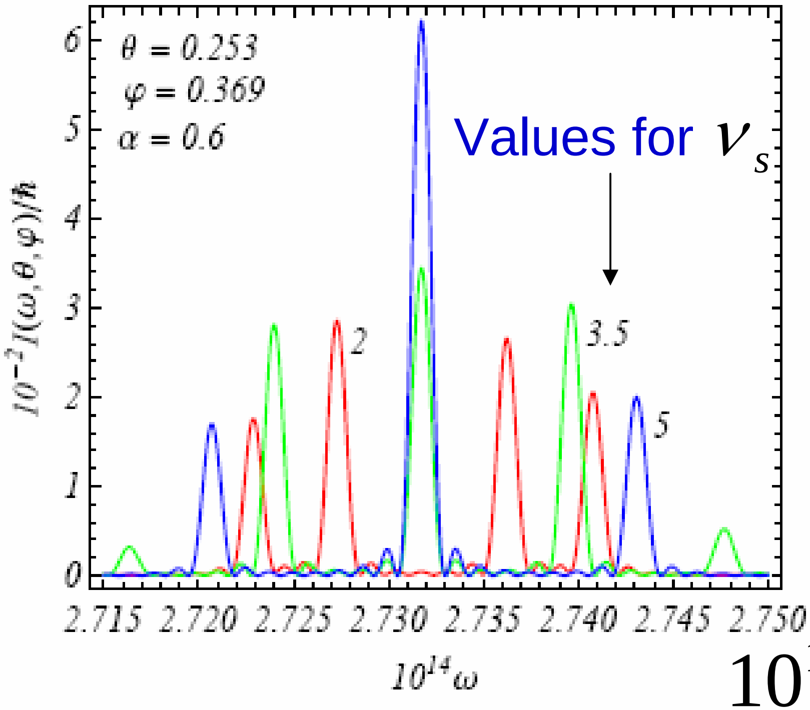
Perpendicular polarization

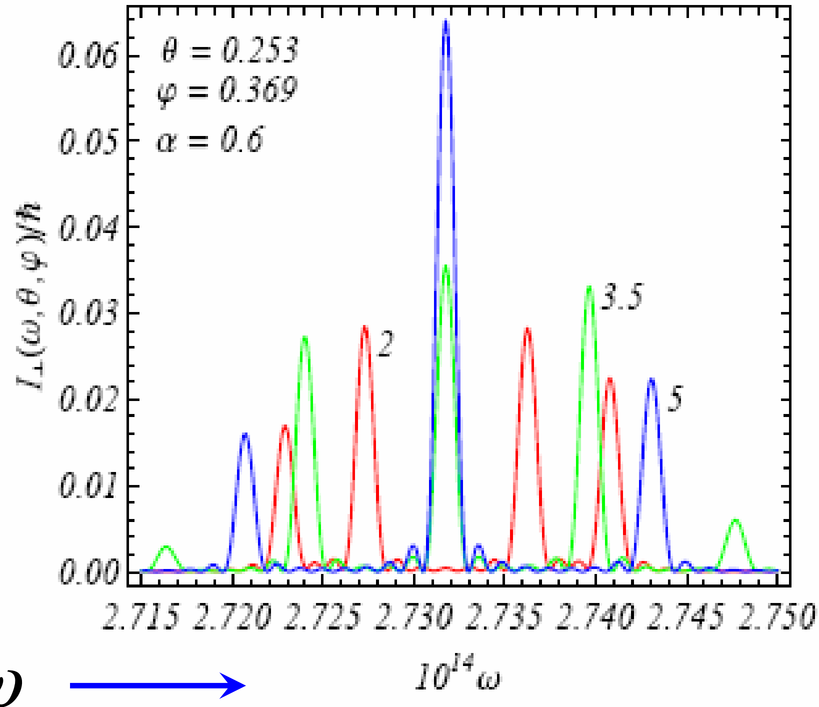
⇨ $\theta = 0.253$ $\varphi = 0.369$ $\nu_s = 5\,\mathrm{MHz}$

Spectral distribution

$10^{-2} I(\omega,\theta)/\hbar$

$I_{\perp}(\omega,\theta)/\hbar$





$10^{14}\,\omega$ →

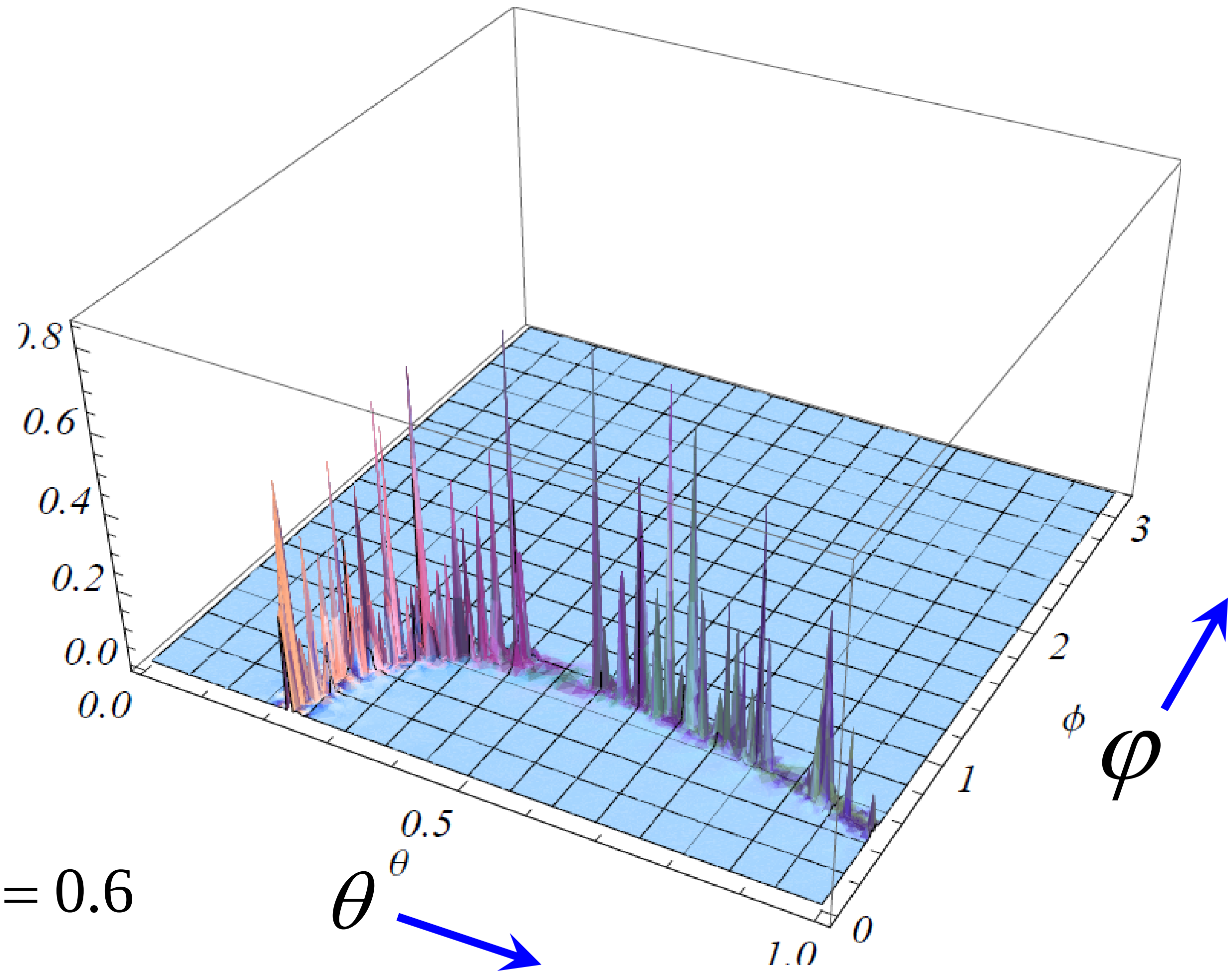
ν_s ⇨ acoustic wave frequency in MHz

⇨ $\theta = 0.253$ $\varphi = 0.369$ $\alpha = 0.6$

Angular distribution

Electron energy = 2 MeV

$$10^{-3} I(\omega,\theta)/\hbar$$



acoustic wave
frequency = 5 MHz

Incidence angle $\alpha = 0.6$

Conclusion

- ✓ We have investigated the transition radiation in an optical range in presence of acoustic waves and in the case of oblique incidence
- ✓ By the choice of parameters of an acoustic wave it is possible to control the both of the spectral and angular distributions of the radiation intensity