The GB formulation of QFTs and the Unruh Effect

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10. Oktober 2011

Motivation: The Unruh Effect.

In the 60ies, Unruh et.al. discovered that the vacuum state in a QFT is observer-dependent. They found: A uniformly accelerated observer travelling through Minkowski Vacuum sees particles.

Pragmatic definition for this talk

A QFT is in the *vacuum state* for the observer $\mathcal{O}$ if a detector travelling on the worldline of the observer sees no particles.

Consider an initial observer $\mathcal{O}_1$ and a uniformly accelerated observer $\mathcal{O}_2$ in Minkowski space. Say the QFT is in the vacuum state for $\mathcal{O}_1$. The Unruh effect states that $\mathcal{O}_2$ will detect particles.

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Motivation: The Unruh Effect II

- Unruh effect only calculated for very special situations: Scalar Field Theory, 1+1 dimensional spacetime, uniformly accelerated (Rindler-) observer.
- What about observers on arbitrary worldlines?
- What about higher dimensions?
- Idea: Look at a more general reformulation of QFT: The general boundary formulation (GBF). Ask what it can predict about the Unruh Effect!

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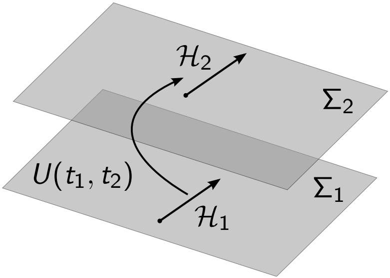
An Example: Quantum Mechanics I

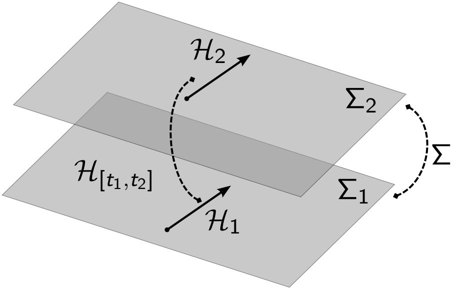
- In standard QM, a Hilbert space $\mathcal{H}_t$ is associated to each time slice Σ_t of a global foliation of spacetime.
- More explicitly, we have Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ associated to the initial time t_1 and final time t_2. Evolution is described by a unitary operator $U(t_1, t_2) : \mathcal{H}_1 \to \mathcal{H}_2$.
- The transition amplitude for an initial state $|\psi_1\rangle \in \mathcal{H}_1$ to evolve into a final state $\langle\psi_2| \in \mathcal{H}_2^*$ is written as $\langle\psi_2|U(t_1, t_2)|\psi_1\rangle$.



An Example: Quantum Mechanics II

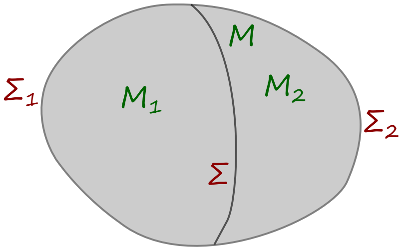
- Idea: Forget about the distinction between initial and final state!
- Consider the generalized state space $\mathcal{H}_{[t_1,t_2]} = \mathcal{H}_1 \otimes \mathcal{H}_2^*$. The transition amplitude is then a map $\rho_{[t_1,t_2]} : \mathcal{H}_{[t_1,t_2]} \to \mathbb{C}$ given by:

$$\rho_{[t_1,t_2]}(\psi_1 \otimes \psi_2) = \langle \psi_2 | U(t_1, t_2) | \psi_1 \rangle$$



An Examle: Quantum Mechanics III

- More generally, consider an arbitrary M with $\Sigma = \partial M$, assign a state space $\mathcal{H}_\Sigma$ to the boundary and encode the dynamics completely in the amplitude $\rho_M : \mathcal{H}_\Sigma \to \mathbb{C}$.
- If $\Sigma = \Sigma_1 \cup \bar{\Sigma}_2$, then require $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}^*_{\Sigma_2}$. Then, any amplitude map $\rho_M : \mathcal{H}_1 \otimes \mathcal{H}^*_2 \to \mathbb{C}$ induces a map $T_M : \mathcal{H}_{\Sigma_1} \to \mathcal{H}_{\Sigma_2}$.
- A composition property is needed: If M is obtained by gluing M_1 and M_2 along Σ, we need $T_M = T_{M_2} \circ T_{M_1}$.



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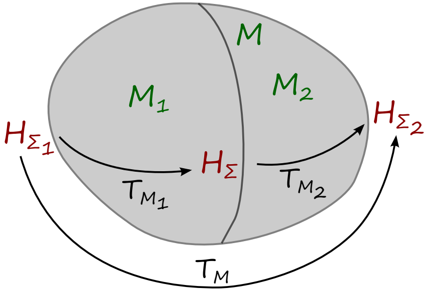

Axiomatic Setup for the GBF

We want to generalize this to a general boundary formulation for Quantum Field Theories. The following axioms have to hold [Oeckl 03'-10']:

- For every hypersurface Σ there is a complex separable Hilbert space $\mathcal{H}_\Sigma$ with inner product $\langle\cdot,\cdot\rangle_\Sigma$, the *state space*.
- If $\bar{\Sigma}$ is Σ with reversed orientation, then $\mathcal{H}_{\bar{\Sigma}} = \mathcal{H}_\Sigma^*$.
- If $\Sigma = \Sigma_1 \cup \ldots \cup \Sigma_m$, then $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1} \otimes \ldots \otimes \mathcal{H}_{\Sigma_m}$.
- Associated with each region M there is a linear map $\rho_M : \mathcal{H}^\circ_{\partial M} \to \mathbb{C}$ where $\mathcal{H}^\circ_{\partial M}$ is dense in $\mathcal{H}_{\partial M}$, the *amplitude map*.
- Gluing property: If M is obtained by gluing M_1 and M_2 along Σ and T_M, T_{M_1} and T_{M_2} are the induced maps from ρ_M, ρ_{M_1} and ρ_{M_2}, then $T_M = T_{M_2} \circ T_{M_1}$.

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Construction of $\mathcal{H}_\Sigma$

We wish to construct the state space $\mathcal{H}_\Sigma$. A Quantization prescription is needed! KG theory review:

- Consider $\Sigma = \Sigma_1 \cup \bar{\Sigma}_2$ with $\Sigma_{1/2}$ Cauchy. Then $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}^*_{\Sigma_2}$. We wish to construct $\mathcal{H}_{\Sigma_{1/2}}$.
- First consider the *space of Classical Solutions* L_{Σ_1} to the KG-equation on Σ_1. That is, all real linear combinations of plane waves $e^{\pm i(Et-kx)}$.
- Next, find the positive frequency plane waves! Those are $e^{i(Et-kx)}$. Write

$$\phi(x,t) = \int dk \left(\phi(k) e^{-i(Et-kx)} + \overline{\phi(k)} e^{i(Et-kx)} \right)$$

- Replace $\phi(k) \to a_k$ and $\overline{\phi(k)} = a_k^\dagger$ to obtain $\hat{\phi}$ as an operator on $\mathcal{H}_\Sigma \approx L^2(L_\Sigma)$.

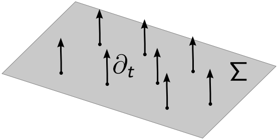
Positive Frequency, what's that?

How do you find the positive frequency plane waves?

- If M is Minkowski and Σ a Cauchy hypersurface, there is a nowhere vanishing timelike KVF ∂_t (the normal direction to Σ).
- The object $J = \frac{\partial_t}{\sqrt{-\partial_t^2}}$ acts on plane waves as

$$Je^{i(Et-kx)} = ie^{i(Et-kx)}, \quad Je^{-i(Et-kx)} = -ie^{-i(Et-kx)}.$$

  Thus positive frequency plane waves = eigenvectors of J with eigenvalue $+i$.
- J is what is called a *complex structure*: $J : L_\Sigma \to L_\Sigma$ is linear and $J^2 = -id_\Sigma$.



Geometric Quantization

- Given a hypersurface Σ and an action functional $S(\phi)$, consider the space L_Σ of classical solutions near that hypersurface.
- L_Σ comes equipped with a natural symplectic structure $\omega_\Sigma = ddS_\Sigma$.
- Choose a *complex structure* $J : L_\Sigma \to L_\Sigma$ compatible with ω_Σ. That is, J linear, $J^2 = -id_\Sigma$ and $g(\cdot,\cdot) = \omega(\cdot, J(\cdot))$ is a positive definite symmetric bilinear form.
- Then the complex inner product

$$\{\phi, \eta\}_\Sigma := g_\Sigma(\phi, \eta) + 2i\omega_\Sigma(\phi, \eta)$$

  turns L_Σ into a complex Hilbert space, the *one-particle H-space*.
- There is a unique Gaussian translation invariant measure μ_Σ allowing us to define $\mathcal{H}_\Sigma = L^2_{Hol}(L_\Sigma, \mu_\Sigma)$ with inner product

$$\langle \psi, \psi' \rangle_\Sigma = \int_{L_\Sigma} \psi(\phi)\overline{\psi'(\phi)} \exp\left(-\frac{1}{2} g_\Sigma(\phi,\phi)\right) d\mu(\phi)$$

Vacuum and Coherent States

The coherent state $K_\xi \in \mathcal{H}_\Sigma$ corresponding to $\xi \in L_\Sigma$ is given by

$$K_\xi(\phi) := \exp\left(\frac{1}{2}\{\xi, \phi\}_\Sigma\right) \quad \forall \phi \in L_\Sigma.$$

For each hypersurface Σ, we require the existence of a vacuum state $\psi_{\Sigma,0} \in \mathcal{H}_\Sigma$ satisfying a number of axioms, most notably

$$\rho_M(\psi_{\partial M,0}) = 1 \quad \text{for any region } M.$$

It turns out that a preferred (but not the only) choice for a vacuum state is $\psi_{\Sigma,0} = K_0 = 1_{L_\Sigma}$ for $0 \in L_\Sigma$.

Discussion

- completely covariant
- problem of time disappears
- technically involved
- Everything dependent on the choice of complex structure J
- Where does J come from? Existence and Uniqueness is the main question in geometric quantization!

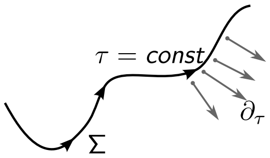
The Ashtekar-Magnon Condition

Condition by A.Ashtekar and A. Magnon on the choice of J for spacelike Σ [80ies]:

AM-condition

There is a unique complex structure J_Σ on Σ such that $\langle\phi|H|\phi\rangle = \int_\Sigma d\Sigma_a T_{ab} t^a n^b$ for all $\phi \in (L_\Sigma, \{\cdot,\cdot\})$.

In 1+1 dimensions, it can be shown that there always is a global orthogonal coordinate system (x, τ) such that $\tau = const$ on Σ and $J_\Sigma = \frac{\partial_\tau}{\sqrt{-\partial_\tau^2}}$.



GBF and the Unruh effect?

- The Unruh effect tells us that the vacuum in a QFT is *observer-dependent*.
- The GB formulation of QFT tells us how QFT effects can be observed on different *hypersurfaces*.
- In 1+1 dimensions,

$$\text{worldlines of observers} = \text{hypersurfaces!}$$

- Central Question: Does GBF allow to model observer-dependent phenomena? At least in 1+1 dimensions?
- Arena: Massless scalar field theory in 1+1 Minkowski spacetime. Conformal Invariance!

A word about detector models

We model a particle detector as a two level system: $|0\rangle$ ground state, $|1\rangle$ excited state, energies 0 and w. The detector is coupled to the scalar field via

$$H_{int} = c\chi(\tau)\mu(\tau)\phi(x(\tau))$$

where $\chi(\tau)$ is a switching function, $\mu(\tau)$ is the monopole moment and $x(\tau)$ the position of the detector at proper time τ.

If the field ϕ is in the state $|A\rangle$, then in first order perturbation theory the detection probability is

$$P(w) = c^2|\langle 0|\mu(0)|1\rangle|^2 F(w)$$

with the response function

$$F(w) = \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau'' e^{-iw(\tau'-\tau'')}\chi(\tau')\chi(\tau'')\langle A|\phi(x(\tau'))\phi(x(\tau''))|A\rangle.$$

Vacua

Consider a detector $\mathcal{O}$ with worldline Σ in 1+1 Minkowski space where a massless scalar matter field is present. Two different notions of vacuum:

- The QFT is in the vacuum state for $\mathcal{O}$ if the detection rate is zero ('no-click condition')
- Pick a complex structure J_Σ and use geometric quantization to construct $\mathcal{H}_\Sigma$. Then there is a unique vacuum state $K_0(J_\Sigma) \in \mathcal{H}_\Sigma$.

Question: For which choice of J_Σ are these notions compatible?

Conjecture

There is (exactly) one complex structure J_Σ such that the corresponding vacuum state $K_0(J_\Sigma)$ satisfies the no-click condition, namely the one satisfying the Ashtekar-Magnon condition!

Remark: Σ is timelike. However, massless scalar field theory in 1+1 is symmetric under $(t, x) \leftrightarrow (x, t)$.

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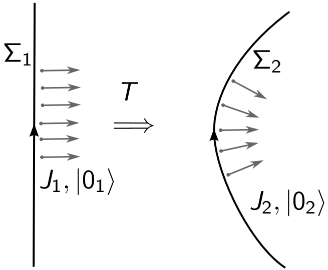
Hypersurfaces, hypersurfaces

- consider a region M bounded by two detector worldlines Σ_1 and Σ_2.
- inertial detector on worldline Σ_1, accelerated detector on worldline Σ_2. We can choose J_1 on Σ_1 and J_2 on Σ_2 according to the AM condition. This will give AM vacua $|0_1\rangle \in \mathcal{H}_{\Sigma_1}$ and $|0_2\rangle \in \mathcal{H}_{\Sigma_2}$.
- But these are not dynamically consistent! Consider the evolution map $T : \mathcal{H}_{\Sigma_1} \to \mathcal{H}_{\Sigma_2}$. In general, $|0_2\rangle \neq T|0_1\rangle$, and $TJ_{\Sigma_1}T^{-1} \neq J_{\Sigma_2}$. One detector must violate 'no-click'!



Conclusion

- The General Boundary Formulation provides a framework for QFTs with data prepared on arbitrary hypersurfaces.
- Quantization is ambiguous. One has to choose a compatible complex structure J_Σ on at least one hypersurface Σ, which can then be propagated with the time evolution map.
- We propose to fix this ambiguity by imposing the (rotated) AM-condition on one hypersurface Σ. This gives rise to a unique vacuum state K_0 on Σ.
- We conjecture that in 1+1 dimensions, a detector travelling along Σ will not click if and only if the QFT is in the state K_0.
- The Unruh effect arises in the sense that for an initial and an accelerated worldline, the AM-condition cannot be realised simultaneously in a dynamically consistent way.

Outlook

- We hope to prove our conjecture!
- Dynamical inconsistency argument is conceptually clear, but a fully explicit calculation is still missing.
- What about higher dimensions? Additional complications arise.