

Gravitational Noether charges and Immirzi parameter

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Content of the talk

- ① *Gravitational Noether charges*
- ② *MacDowell-Mansouri gravity*
- ③ *Constrained $SO(2,3)$ BF theory*
- ④ *Generalized thermodynamics and Immirzi parameter*



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Black hole thermodynamics

1st law of thermodynamics

$$dE = TdS + dW$$

1st law of black hole dynamics

$$dM = \frac{\kappa}{8\pi G} dA + \Omega dJ$$

We identify

- the surface gravity of a black hole with temperature
- the area of the event horizon with the entropy

Surface gravity κ =acceleration needed to keep an object at horizon.

$$\kappa^2 = -\frac{1}{2}\nabla_\nu\xi^\mu\nabla^\nu\xi_\mu \quad (\text{for example for Schwarzschild } \kappa = \frac{c^4}{4GM}).$$

Beckenstein and Hawking black hole entropy ($l_p = \sqrt{G\hbar/c^3}$)

$$\text{Entropy} = \frac{\text{Area}}{4l_p^2},$$

$$\text{Temperature} = \frac{\kappa}{2\pi}$$



Noether currents and charges

Emmy Noether's theorem

Any differentiable (smooth) symmetry of the action of a physical system has a corresponding conservation law.

$$\delta \mathcal{L}(\varphi, \partial\varphi) = E \cdot \delta\varphi + d\Theta$$

For any diffeomorphism generated by a smooth vector field ξ^μ , one can derive a conserved Noether current 3-form J by

$$J[\xi] = \Theta[\varphi, L_\xi \varphi] - I_\xi \mathcal{L}$$

where L_ξ denotes the Lie derivative in the direction ξ and the contraction operator I_ξ acting on a p-form is $I_\xi \alpha_p = \frac{1}{(p-1)!} \xi^\mu \alpha_{\mu\nu_1\dots\nu^{p-1}} dx^{\nu^1} \wedge \dots \wedge dx^{\nu^{p-1}}$.

On shell the Noether current is closed, and can be written in terms of the Noether charge 2-form Q by the relation $J = dQ$.



R. Wald 1993

Killing vectors as generators of diffeomorphism symmetry.

timelike ∂_t , rotational ∂_φ and generator of the horizon ($\partial_t + \Omega\partial_\varphi$)

Gravitational Noether charges

$$\left\{ \begin{array}{l} Q[\xi_t]_\infty \\ Q[\xi_\varphi]_\infty \\ Q[\xi_t + \Omega\xi_\varphi]_H \end{array} \right. \rightarrow \left\{ \begin{array}{l} \text{Mass} \\ \text{Angular momentum} \\ \text{Temperature} \cdot \text{Entropy} \end{array} \right.$$

However for tetrad formulation...

$$32\pi G S = \int \text{Palatini} + \frac{1}{2\ell^2} \int \text{cosmological}$$

Noether charge for Schwarzschild-AdS black hole

$$\text{Mass} = Q(\partial_t) = \frac{M}{2} + \lim_{r \rightarrow \infty} \frac{r^3}{2G\ell^2}$$



Aros, Contreras, Olea, Zanelli 2000

Euler term as boundary term

$$32\pi G S = \int \text{Palatini} + \frac{1}{2\ell^2} \int \text{cosmological} + \rho \int \text{Euler}_4$$

Noether charge for Schwarzschild-AdS black hole

$$\text{Mass} = Q(\partial_t) = \frac{M}{2} \left(1 + \frac{2}{\ell^2} \rho \right) + \lim_{r \rightarrow \infty} \frac{r^3}{2G\ell^2} \left(1 - \frac{2}{\ell^2} \rho \right)$$

 $\rho = \frac{\ell^2}{2}$ cures the result. Notice that this is MacDowell-Mansouri model!

$$32\pi G S_{MM} = \int \text{Palatini} + \frac{1}{2\ell^2} \int \text{cosmological} + \frac{\ell^2}{2} \int \text{Euler}$$



A_μ^{IJ} connection of the $SO(2, 3)$

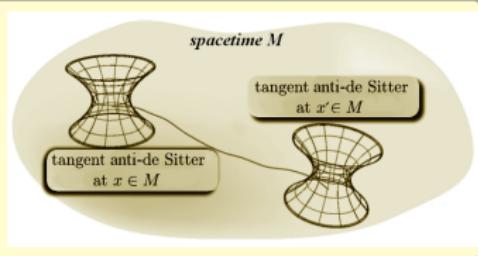
MacDowell and Mansouri gravity

$$A_\mu^{IJ} \rightarrow \begin{cases} A_\mu^{ab} = \omega_\mu^{ab} \\ A_\mu^{a5} = \frac{1}{\ell} e_\mu^a \end{cases} \quad \text{with } \frac{\Lambda}{3} = -\frac{1}{\ell^2}, \quad a, b = (1, 2, 3, 4) \\ I, J = (1, 2, 3, 4, 5)$$

$$\mathbb{A}_\mu = \frac{1}{2} \omega_\mu^{ab} M_{ab} + \frac{1}{\ell} e_\mu^a P_a = \frac{1}{2} A_\mu^{IJ} M_{IJ} \quad M_{a5} = P_a$$

Geometric interpretation of this construction:

$SO(2, 3)$ connection $A = (\omega, e)$
encodes the geometry of the
spacetime \mathcal{M} by "rolling anti-de
Sitter manifold along \mathcal{M} " [Wise]





MacDowell-Mansouri 1977

Connection 1-form A^{IJ}

$$A^{IJ} = \begin{pmatrix} \omega^{ab} & \frac{1}{\ell} e^a \\ -\frac{1}{\ell} e^b & 0 \end{pmatrix}$$

Curvature 2-form $F^{IJ} = dA^{IJ} + A^{IK} \wedge A_K{}^J$

$$F^{IJ} = \begin{pmatrix} R^{ab} + \frac{1}{\ell^2} e^a \wedge e^b & \frac{1}{\ell} T^a \\ -\frac{1}{\ell} T^b & 0 \end{pmatrix}$$

Bianchi identity:

$$D^A F^{IJ} = 0$$



MacDowell-Mansouri 1977

$$F^{IJ} \rightarrow \hat{F}^{IJ} = F^{ab} \quad F^{ab} = R^{ab} + \frac{1}{\ell^2} e^a \wedge e^b$$

General Relativity as gauge symmetry breaking theory

$$S_{MM}(A) = \frac{\ell^2}{64\pi G} \int \text{tr}(\hat{F} \wedge \star \hat{F})$$

$$S_{MM}(A) = \frac{\ell^2}{64\pi G} \int \left(R^{ab} + \frac{1}{\ell^2} e^a \wedge e^b \right) \wedge \left(R^{cd} + \frac{1}{\ell^2} e^c \wedge e^d \right) \epsilon_{abcd}$$

$$32\pi G S_{MM} = \int \text{Palatini} + \frac{1}{2\ell^2} \int \text{cosmological} + \frac{\ell^2}{2} \int \text{Euler}_4$$

Equations of motion : $\left(R^{ab} \wedge e^c + \frac{1}{2\ell^2} e^a \wedge e^b \wedge e^c \right) \epsilon_{abcd} = 0 , \quad T^a = 0$



Boundary conditions

- Boundary term: Euler term
- Boundary condition: AdS asymptotics at the infinity

$$(R^{ab}(\omega) + \frac{1}{\ell^2} e^a \wedge e^b) \Big|_{\infty} = 0$$

This provides differentiability of the action!

$$\delta(Palatini + \Lambda + Euler) = \int_M (f.e.)_a \delta e^a + \int_M (f.e.)_{ab} \delta \omega^{ab} + \int_M d\Theta = 0$$

where

$$\Theta = \epsilon_{abcd} \delta \omega^{ab} \wedge F^{cd}.$$



Perturbed BF theory

Introduce independent $so(2, 3)$ -valued [2-form B] to the action

$$16\pi S(A, B) = \int_{\mathcal{M}} \text{tr} \left(B \wedge F - \frac{G\Lambda}{6} \hat{B} \wedge * \hat{B} \right)$$

$$\delta B_{a5} : F^{a5} = \frac{1}{\ell} T^a = 0, \quad \delta \hat{B}_{ab} : F^{ab} = \frac{G\Lambda}{3} \epsilon^{abcd} B_{cd}$$

$$S_{BF}(A, B) = S_{MM}(A)$$

- In this form Macdowell-Mansouri gravity has the appearance of the deformation of a topological gauge theory.
- The symmetry breaking occurs in the last term with dimensionless coefficient proportional to: $\alpha = G\Lambda/6 \sim 10^{-120}$.



Starodubtsev and Freidel 2005

1-form connection A and independent $so(2, 3)$ -valued 2-form B

$$16\pi \mathcal{S} = \int_{\mathcal{M}} \text{tr} \left(B \wedge F - \frac{\beta}{2} B \wedge B - \frac{\alpha}{4} \hat{B} \wedge * \hat{B} \right)$$

Action proposed by Starodubtsev and Freidel

$$16\pi \mathcal{S} = \int_{\mathcal{M}} \left(B_{IJ} \wedge F^{IJ} - \frac{\beta}{2} B_{IJ} \wedge B^{IJ} - \frac{\alpha}{4} \epsilon_{abcd5} B^{ab} \wedge B^{cd} \right)$$

with constants $(\alpha, \beta \ell) \rightarrow (G, \Lambda, \gamma)$:

$$\gamma = \frac{\beta}{\alpha}, \quad \Lambda = -\frac{3}{\ell^2}, \quad \text{where} \quad \beta = \frac{\gamma G \Lambda}{3(1 + \gamma^2)}, \quad \alpha = \frac{G \Lambda}{3(1 + \gamma^2)}$$



Constrained BF theory

$$\mathcal{S} = \frac{1}{64\pi} \int d^4x \epsilon^{\mu\nu\lambda\rho} \left(B_{\mu\nu IJ} F_{\lambda\rho}^{IJ} - \frac{\beta}{2} B_{\mu\nu IJ} B_{\lambda\rho}^{IJ} - \frac{\alpha}{4} \epsilon_{IJKL5} B_{\mu\nu}^{IJ} B_{\lambda\rho}^{KL} \right)$$

$$\begin{aligned} 32\pi G S = & \int R^{ab} \wedge e^c \wedge e^d \epsilon_{abcd} + \frac{1}{2\ell^2} \int e^a \wedge e^b \wedge e^c \wedge e^d \epsilon_{abcd} \\ & + \frac{\ell^2}{2} \int R^{ab} \wedge R^{cd} \epsilon_{abcd} + \frac{2}{\gamma} \int R^{ab} \wedge e_a \wedge e_b \\ & - \ell^2 \gamma \int R^{ab} \wedge R_{ab} + 2 \frac{\gamma^2 + 1}{\gamma} \int (T^a \wedge T_a - R^{ab} \wedge e_a \wedge e_b) \end{aligned}$$

Structure standing behind the constrained *BF* model

$$\begin{aligned} 32\pi G S = & \int \boxed{\text{Palatini}} + \frac{1}{2\ell^2} \int \boxed{\text{cosmological}} + \frac{\ell^2}{2} \int \boxed{\text{Euler}} \\ & + \frac{2}{\gamma} \int \boxed{\text{Holst}} - \ell^2 \gamma \int \boxed{\text{Pontryagin}} + 2 \frac{\gamma^2 + 1}{\gamma} \int \boxed{\text{Nieh/Yan}} \end{aligned}$$



Generalized Noether charge

Action

$$32\pi G S = \int \text{Palatini} + \frac{1}{2\ell^2} \int \text{cosmological} + \frac{\ell^2}{2} \int \text{Euler} \\ + \frac{2}{\gamma} \int \text{Holst} - \ell^2 \gamma \int \text{Pontryagin} + 2 \frac{\gamma^2 + 1}{\gamma} \int \text{Nieh/Yan}$$

with interesting scheme

$$\boxed{(\text{Palatini} + \Lambda) + \frac{\ell^2}{2} \text{Euler}} - \gamma \boxed{(\text{Holst} + \text{Nieh/Yan}) + \frac{\ell^2}{2} \text{Pontryagin}}$$

More useful form of the action

$$S_{BF}(\omega, e) = \frac{1}{16\pi} \int_M \left(\frac{1}{4} M^{abcd} F_{ab} \wedge F_{cd} - \frac{1}{\beta\ell^2} T^a \wedge T_a \right)$$

with use of

$$M^{ab}_{cd} = \frac{\alpha}{(\alpha^2 + \beta^2)} (\gamma \delta^{ab}_{cd} - \epsilon^{ab}_{cd})$$



Generalized Noether charge

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Gravity as a constrained BF theory: Noether charges and Immirzi parameter

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Noether charge from Wald's approach generalized to the case of first order gravity with Immirzi parameter:

$$Q[\xi] = \int_{\partial\Sigma} (\xi^\mu A_\mu^{IJ}) \frac{\delta \mathcal{L}}{\delta F^{IJ}} = \frac{1}{32\pi} \int_{\partial\Sigma} (\xi^\mu \omega_\mu^{ab}) M_{abcd} F^{cd}$$

Generalized Noether charge

$$Q[\xi] = \frac{\ell^2}{32\pi G} \int_{\partial\Sigma} (\xi^\mu \omega_\mu{}_{ab}) (\epsilon^{ab}{}_{cd} F^{cd}_{jk} - 2\gamma F^{ab}_{jk}) dx^j \wedge dx^k.$$



Schwarzschild–AdS spacetime

$$ds^2 = -f(r)^2 dt^2 + f(r)^{-2} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$f(r)^2 = \left(1 - \frac{2GM}{r} + \frac{r^2}{\ell^2}\right)$$

There is no Immirzi parameter in the black hole thermodynamics

- $Q[\xi_t]_\infty = M$
- $Q[\xi_\varphi]_\infty = 0$
- $Q[\xi_t + \Omega \xi_\varphi]_H = \frac{\kappa}{2\pi} \cdot \frac{4\pi(r_H^2 + \ell^2)}{4G}$
- Mass
- Angular momentum
- Temperature · Entropy

Entropy shifted by a constant

$$\text{Entropy} = \frac{\text{Area}}{4G} + \frac{4\pi\ell^2}{4G}$$



Details of Schwarzschild–AdS

$$ds^2 = -f(r)^2 dt^2 + f(r)^{-2} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$f(r)^2 = \left(1 - \frac{2GM}{r} + \frac{r^2}{\ell^2}\right)$$

$$Q[\partial_t] = -\frac{4\ell^2}{32\pi G} \int \omega_t^{01} (\gamma F_{jk01} - \epsilon_{0134} F_{jk}^{34}) dx^j \wedge dx^k$$

$$Q[\partial_t] = \frac{4\ell^2 \epsilon_{0134}}{32\pi G} \int_{\partial\Sigma} \left(\frac{1}{2} \frac{\partial f(r)^2}{\partial r} \right) \left(\textcolor{red}{1 - f(r)^2} + \frac{r^2}{\ell^2} \right) \sin \theta d\theta \wedge d\varphi$$

□ The charge calculated at the Schwarzschild–AdS black hole horizon:

$$Q[\xi]_H = \frac{\ell^2}{8\pi G} \kappa \left(1 + \frac{r_H^2}{\ell^2} \right) \int_{\partial\Sigma} \sin \theta d\theta \wedge d\varphi = \frac{\kappa}{2\pi} \frac{4\pi(r_H^2 + \ell^2)}{4G}$$

□ The value of charge calculated at the boundary at infinity:

$$Q[\xi]_\infty = \lim_{r \rightarrow \infty} \frac{1}{4\pi} \int_{\partial\Sigma} \left(M + \frac{\ell^2 GM^2}{r^3} \right) \sin \theta d\theta \wedge d\varphi = M$$



Immirzi parameter

Entropy calculated in LQG framework

$$S_{LQG} = \frac{\ln 2}{\gamma \pi \sqrt{3}} \frac{Area}{4G}$$

Immirzi parameter might be present in black hole thermodynamics

$$\int_{\partial\Sigma} \partial_\theta g_{t\varphi} \neq 0$$

Under investigation: AdS–Taub–NUT metric with NUT charge

$$ds^2 = -f(r)^2(dt + 4n \sin^2 \frac{\theta}{2} d\varphi)^2 + \frac{dr^2}{f(r)^2} + (n^2 + r^2)(r^2 d\theta^2 + r^2 \sin \theta d\varphi^2)$$

$$\text{where } f(r)^2 = \frac{r^2 - 2GMr - n^2 + (r^4 - 3n^4 + 6n^2r^2)\ell^{-2}}{n^2 + r^2}$$



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