Shape dynamics:
The conformal backbone of general relativity

Sean Gryb

Universiteit Utrecht

Institute for Theoretical Physics, Utrecht University

Grafiti no. 226
ITF, Utrecht University
Oct 10, 2011

Introduction

Forget about Lorentz invariance and spacetime! Can we still get gravity?

Observation

All measurements of lengths are local comparisons

$\therefore$ we expect that experiments are invariant under $g_{ab} \to e^{\phi(x)} g_{ab}$.

This simple idea leads to *shape dynamics* a new...

- approach to GR free of the *local* problem of time.
- symmetry principle of quantum gravity.
- approach to perturbative cosmology.
- view on gauge/gravity duality.

Universiteit Utrecht

Outline

1. Will describe a procedure implementing Mach's principles.
2. Will use this to construct shape dynamics.
3. Will describe recent results / hopes.

Mach's principles

Idea

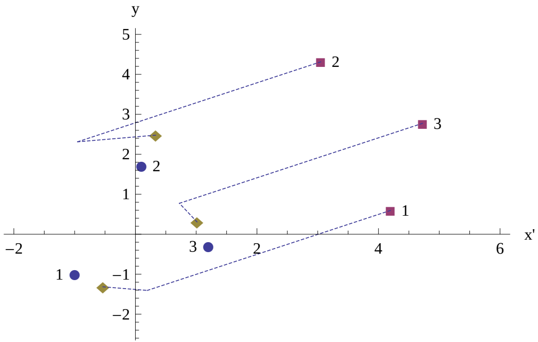
The dynamics of observable quantities should depend only on other observable quantities and no other external structures.

1. "When we say that a body K alters its direction and velocity solely through the influence of another body K', we have inserted a conception that is impossible to come at unless other bodies A, B, C... are present with reference to which the motion of the body K has been estimated."
2. "It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things... "

Best matching: general idea

Goal

Find the “distance” between shapes.



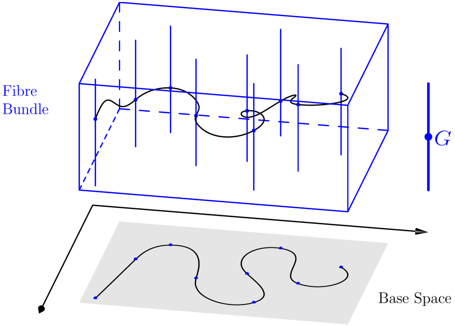
2 steps:

1. Bring to “best–matched” position giving difference in shape (ie, metric).
   $\Rightarrow$ constraints *linear* in momenta.
2. Dynamics: geodesic principle in shape space.

Gauge theory on configuration space (Mach's first principle)

Establish ontology:

- Identify configuration space $\mathcal{A}$.
- Identify symmetry group $\mathcal{G}$.



Best matching connection

- Best–matching procedure: choice of section on fibre bundle.
- Variation wrt best–matching connection $\rightarrow$ linear constraints $pt_\alpha q \approx 0$.

Mach's second principle and geodesics

Dynamics

Geodesic principle on base space.

Eg,

$$S = \int dt \sqrt{G_{ab}(q)\dot{q}^a \dot{q}^b}$$

Geodesics

Specified by point and *direction*.
$\therefore$ length of momenta is irrelevant.

$\Rightarrow$ *quadratic* constraints: $G^{ab} p_a p_b - 1 \approx 0$

Time: *length* of curve (slightly different metric).

Example: standard GR from BM

Fibre Bundle

- Configuration space: $\text{Riem}^3 \equiv$ space of all 3–metrics, g_{ab}.
- Gauge group: Diff^3
- Base space: Superspace $=$ Riem / Diff

Can be made into PFB by removing g's with global isometries [Gomes '11].

Best–matching constraints: $pt_\alpha q \to g_{ab}\mathcal{L}_{N^a}\pi^{ab}$ (after I by P)

Local geodesic: $G^{ab}p_a p_a - 1 = 0 \to \frac{G^{abcd}}{g(R-2\Lambda)}\pi_{ab}\pi_{cd} - 1 = 0.$

$\therefore$ GR $\sim$ gauge theory on Riem with a local geodesic principle on Superspace!

$\boxed{3+1 \text{ has its own beauty!}}$

Universiteit Utrecht

Canonical best matching

Matching procedure ∼ canonical transformation on phase space

Procedure:

- Start with first class Hamiltonian system: $(\Gamma(q,p), H \approx 0, \mathcal{H}_i \approx 0, \chi_j \approx 0)$
- Enlarge phase space $\Gamma(q,p) \to \Gamma_e(q,\phi;p,\pi_\phi)$.
- Introduce constraint $\pi^\phi \approx 0$. (first class)
- Perform canonical transformation T:

$$F(q,\phi;P,\Pi_\phi) = \int dt \left(Pe^\phi q + \phi\Pi_\phi\right)$$

$\Rightarrow q \to e^\phi q$ and $\pi_\phi \to \pi_\phi - ptq$.

- Impose *best–matching constraint* $\pi_\phi \approx 0$.

3 cases

1. $\pi_\phi \approx 0$ first class: standard gauge theory.
2. $\pi_\phi \approx 0$ second class: fix Lagrange multiplier.
3. $\pi_\phi \approx 0$ second class: secondary constraints.

Shape dynamics: construction

Construct *linking theory*

1. Start with local geodesic principle on ADM phase space: $(\Gamma(g_{ab}, \pi^{ab}), S(N) \approx 0)$
2. BM diffeos $g_{ab} \to g_{ab} + \mathcal{L}_\xi g_{ab}$.
3. Diffeo constraint $H(\xi) \approx 0$ is first class. $\therefore$ *case 1*.
4. BM *vpcts*: $g_{ab} \to e^{4\hat{\phi}} g_{ab}$.
5. Vpct constraints $D(\rho)$ are second class wrt $S(N)$. $\therefore$ *case 2*.

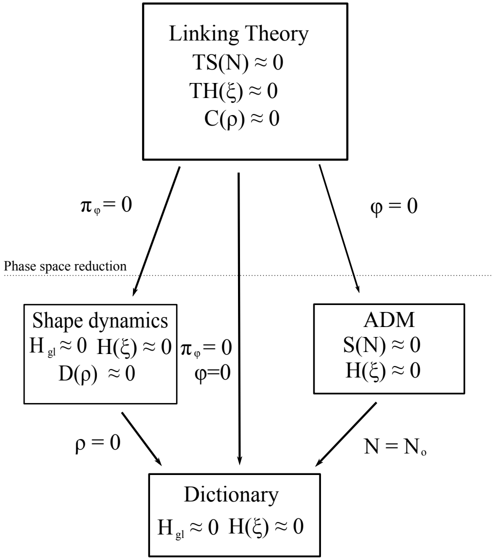
Partial gauge fixing of $S(N)$

Note: Vol preserving condition $\to$ global restriction on $D(\rho)$.

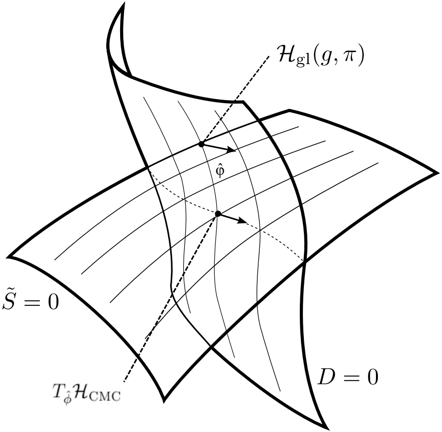
$\therefore$ Decompose $S = S_{\text{CMC}} + \tilde{S}$

$\tilde{S} \equiv$ part of S that is second class wrt to D!

Shape dynamics: *linking* theory



Shape dynamics basics: ADM phase space



Definition of $\mathcal{H}_{\rm gl}$

$$\mathcal{H}_{\rm gl}(g,\pi) = \mathcal{S}_{\rm CMC}(T_\phi g,\, T_\phi \pi)$$

Notes:

- $\tilde{S}$ = part of S gauge fixed by $D=0$.
- Foliation invariance is *traded* for vpct.
- Intersection: CMC (soap bubbles).

Large volume expansion

Problem

The equation

$$\mathcal{H}_{\text{gl}} = S(T_\phi g, T_\phi \pi)$$

is a non–linear elliptic PDE (because $T_\phi R = e^{-4\hat{\phi}} \left(R - 8 \frac{\nabla^2(e^{\hat{\phi}})}{e^{\hat{\phi}}} \right)$).

$\therefore \mathcal{H}_{\text{gl}}$ is non–local!

Perturbative expansion

$$\mathcal{H}_{\text{gl}} = \sum_n V^{-n} \mathcal{H}_{\text{gl}}^{(n)}$$

Solve the PDE order by order in $1/V$:

$$\mathcal{H}_{\text{gl}} = \left(2\Lambda - \frac{1}{6} \langle \pi \rangle^2 \right) + \dots$$

dS/CFT correspondence

$$\boxed{\text{Large } V\text{: } \mathcal{H}_{\text{gl}} + D(x) \approx 0 \rightarrow g_{ab}\pi^{ab} \approx \pm\sqrt{12\Lambda}!}$$

$\therefore$ full (inhomogeneous) conformal constraints!

dS/CFT correspondence

Semi–classical: $S_{\text{HJ}} \sim Z_{\text{CFT}}$ $\therefore$ HJ eq'n $\sim$ Conformal Ward identity

- We can repeat standard Holographic RG cals $\Rightarrow$ easier in SD.
- Potential construction principle for shape dynamics!?

Universiteit Utrecht

Current projects / discussions

- Perturbative SD: perts about a background and cosmo pert theory.
- Matter coupling.
- Ashtekar variables + LQSD
- Connection with AdS/CFT.
- New variables: conformal Cartan connections (slide).

Conformal Cartan connection

Motivation

Ashtekar connection $\to$ ugly under vpcts!!

Find variables that transform trivially under vpcts.

Idea: Find frame fields, e^I_a that give the metric only up to a conformal factor

$$g_{ab} = e^{4\phi} e^I_a e^I_b.$$

Thus:

- $e^I_a \in$ fund rep of conformal group $SO(4,1)$.
- Connection $A^{IJ}_a \in SO(4,1)$ valued 1–form.

Advantages:

- Ham of linking theory is naturally written in these variables and $F(A)$.
- Natural *physical* Hilbert space: conformal spinnets (conformal nets??).
- $SO(4,1)$ isometry group of (Euclidean) AdS $\to$ AdS/CFT correspondence?!

Universiteit Utrecht