Sudakov Shoulder Factorization For Event Shapes

Arindam Bhattacharya

Based on

2205.05702 (PRD) **AB**, Matthew D Schwartz, Xiaoyuan Zhang

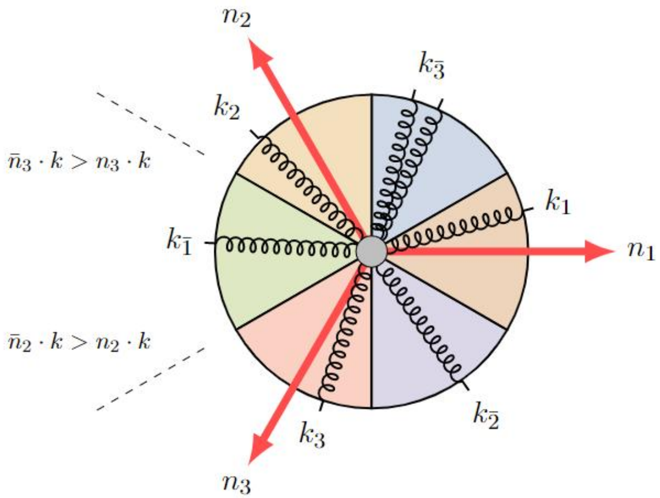
2306.08033 (JHEP) **AB**, Johannes KL Michel, Matthew D Schwartz, Iain Stewart, Xiaoyuan Zhang

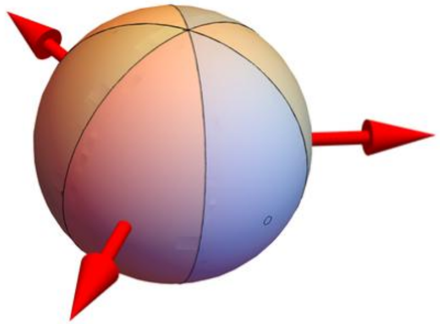
2502.12253 M.A.Benitez, **AB**, A.H.Hoang, V.Mateu, M.D.Schwartz, I.W.Stewart, X. Zhang

$\boldsymbol{\alpha_s}$ 2025 , Aussois

Outline

- Event Shapes Beyond Dijet
- Sudakov Shoulders : Why?
- Sudakov Shoulders : Factorization
- Sudakov Shoulders : Power Corrections in Trijet
- Conclusions





Event Shapes Beyond Dijet

Event Shapes Beyond Dijet

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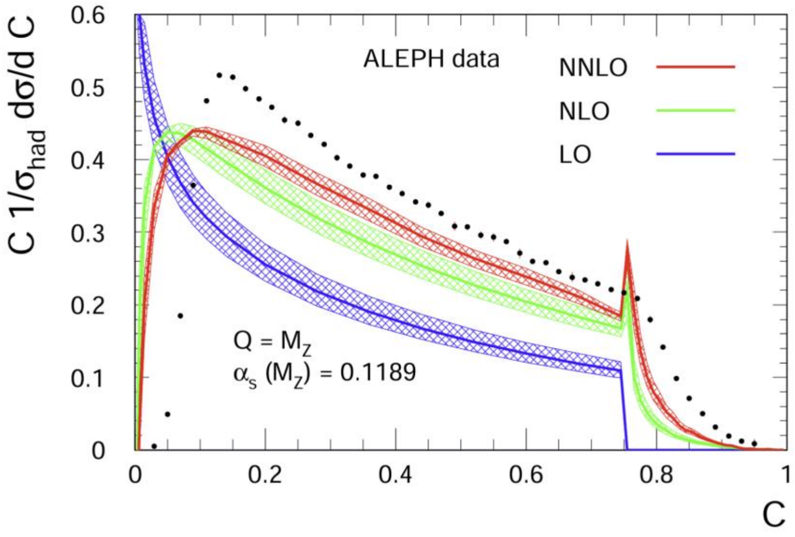
C Parameter



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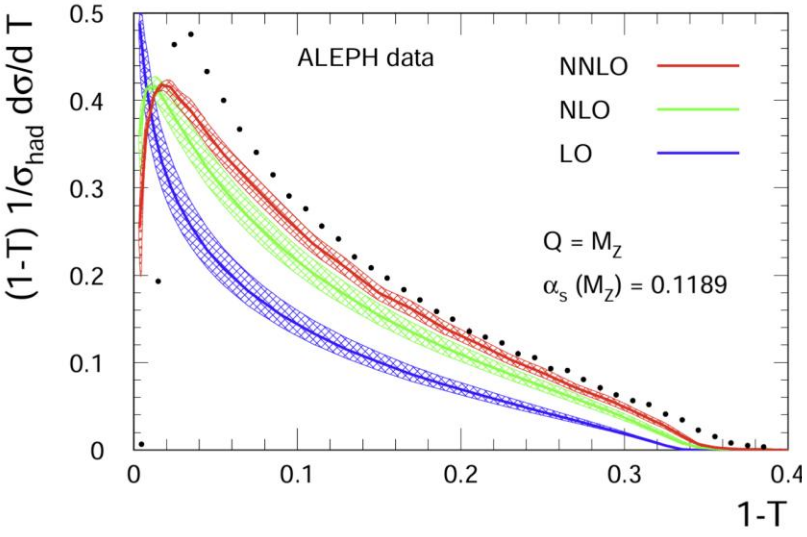
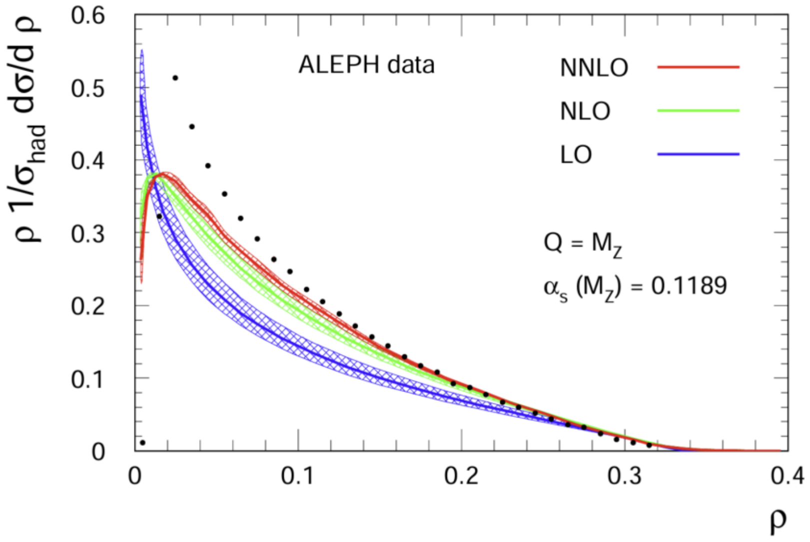


Heavy Jet Mass (HJM)



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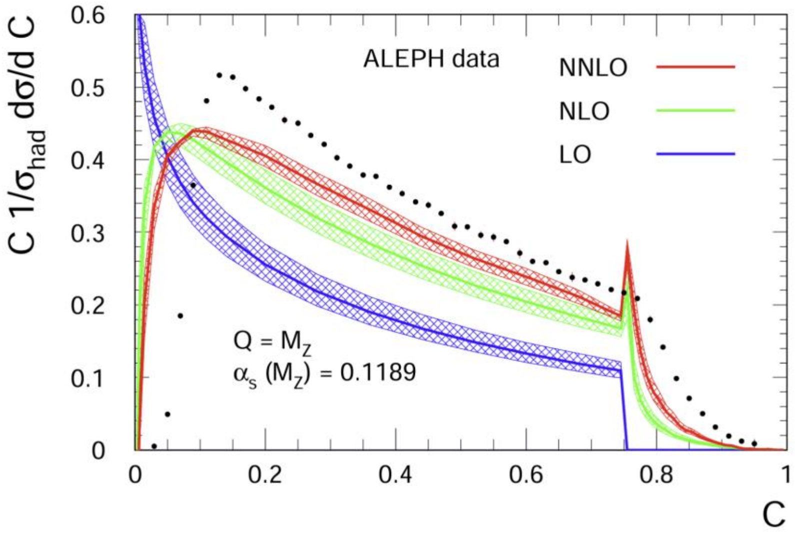

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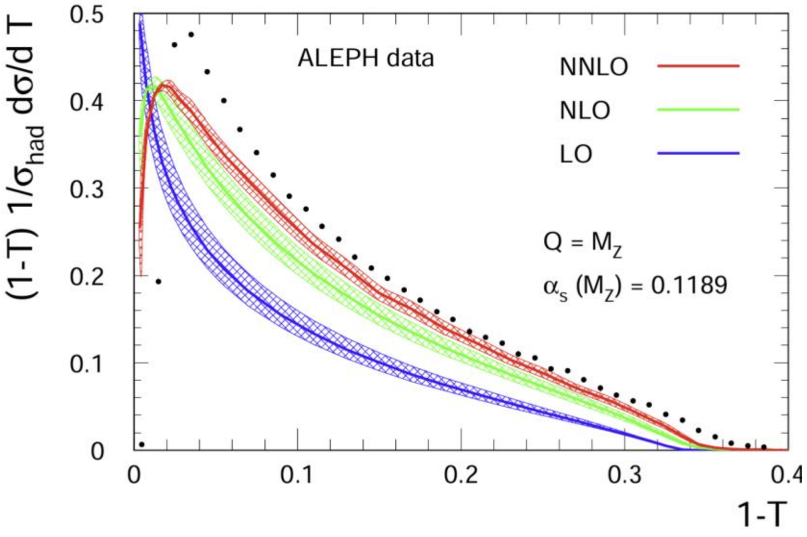

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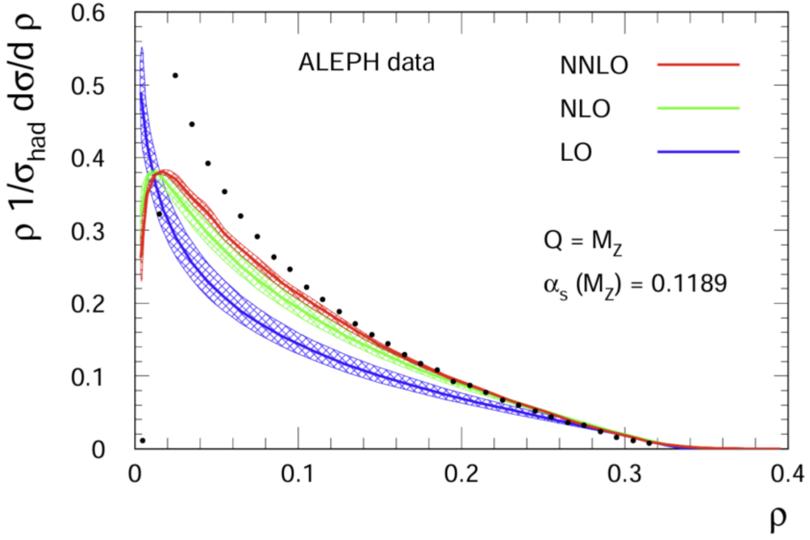

- FO does not suffice everywhere in the spectrum

Event Shapes Beyond Dijet

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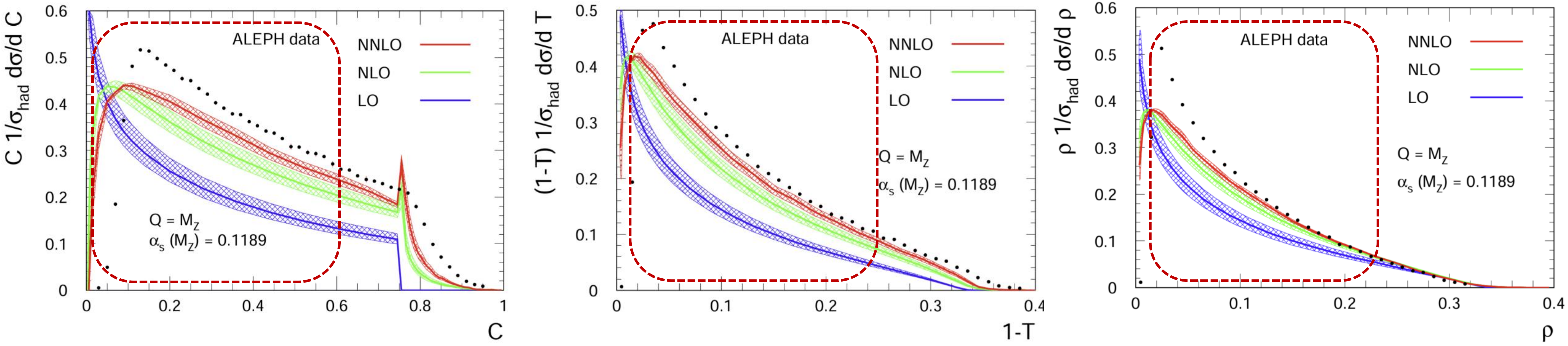
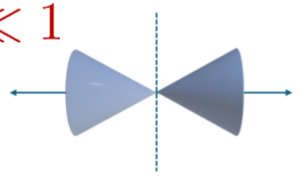
C Parameter — Thrust — Heavy Jet Mass (HJM)



- FO does not suffice everywhere in the spectrum

Large Logs in dijet regions $\sim \frac{1}{e}\left[\alpha_s \ln^2 e\right]^n$

$e \ll 1$



Event Shapes Beyond Dijet

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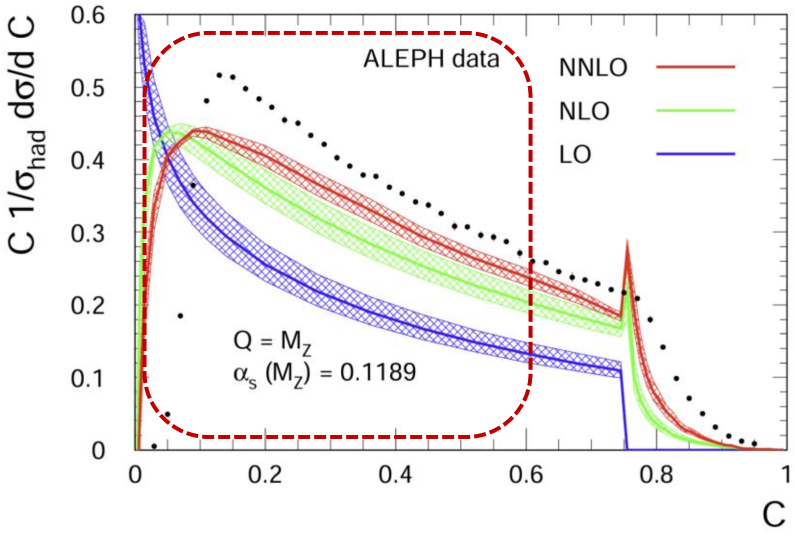

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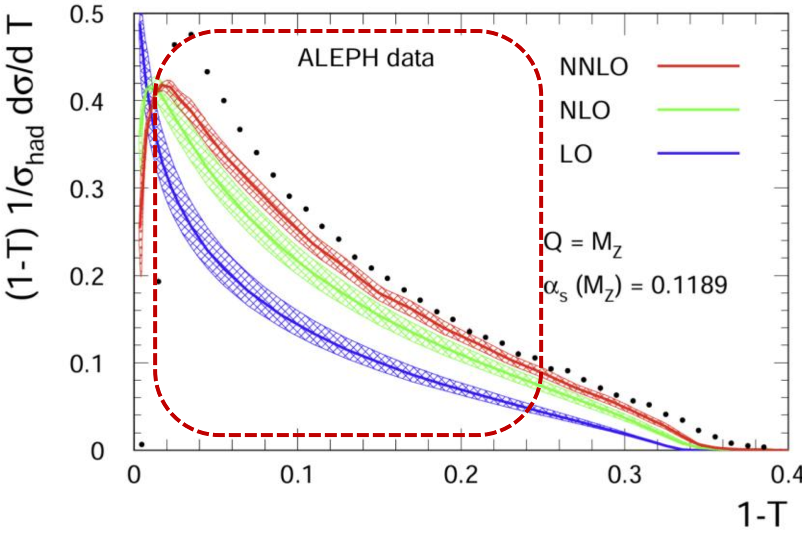

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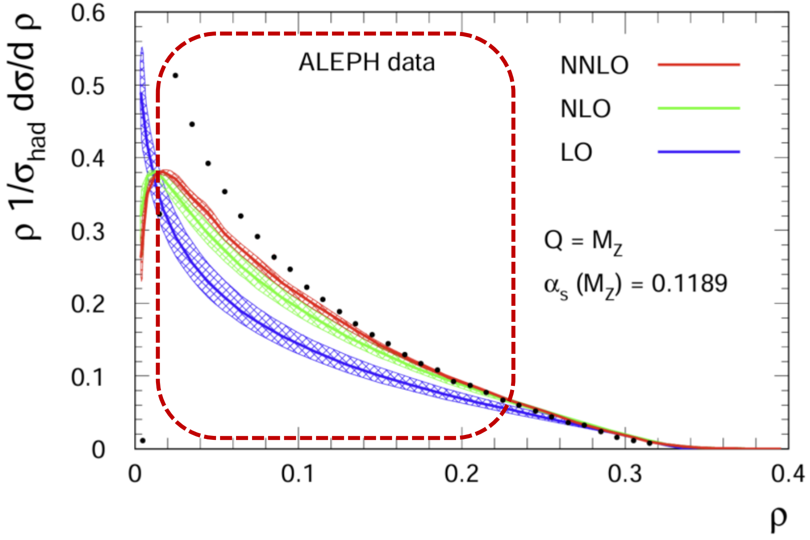

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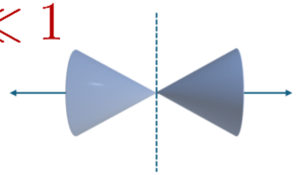

- What about beyond it? Lets zoom in

Event Shapes Beyond Dijet

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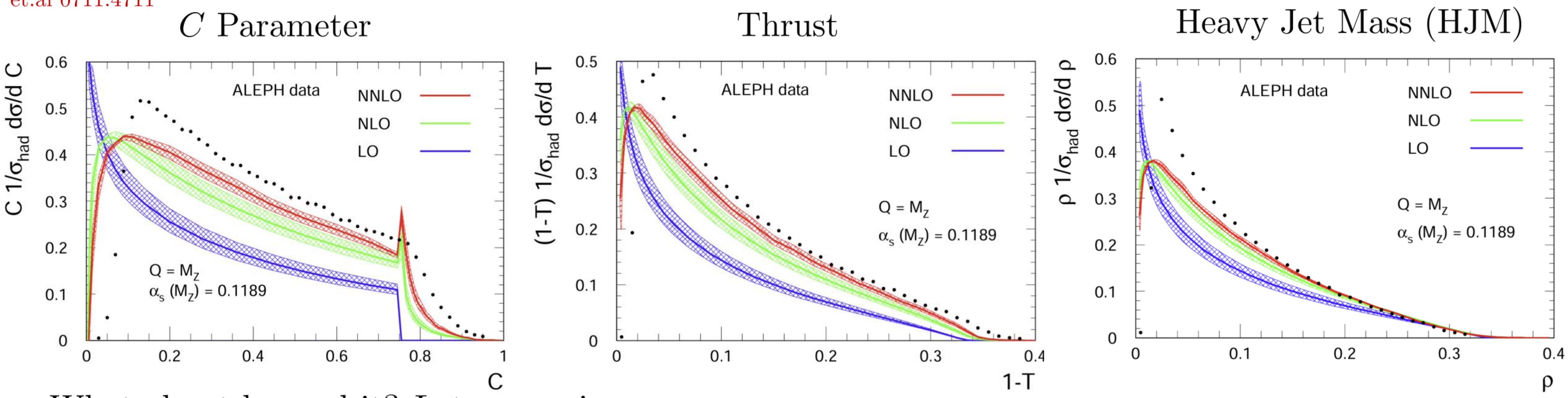

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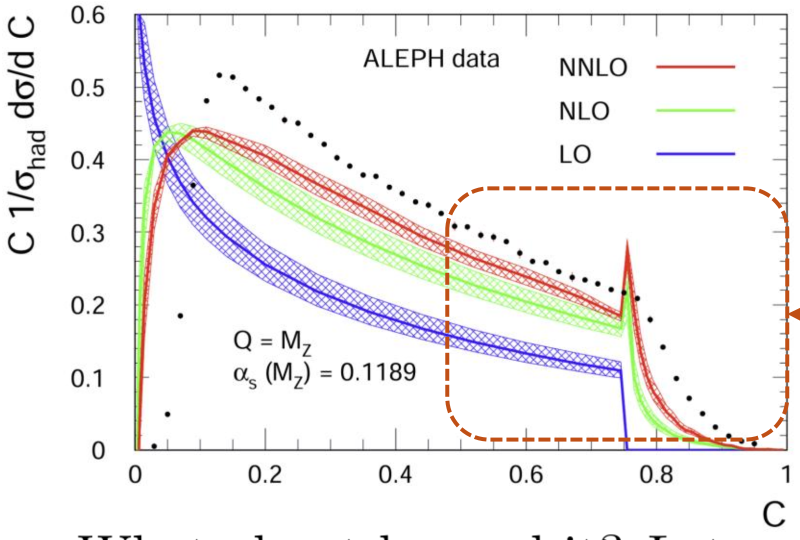

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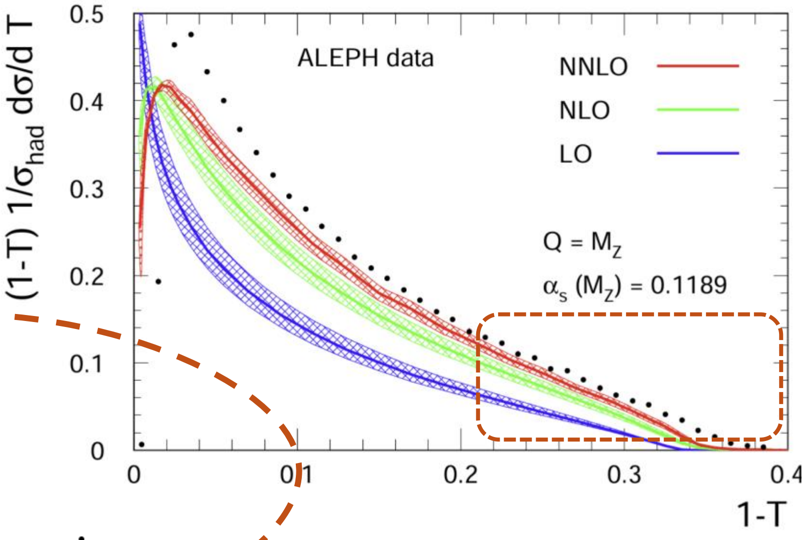

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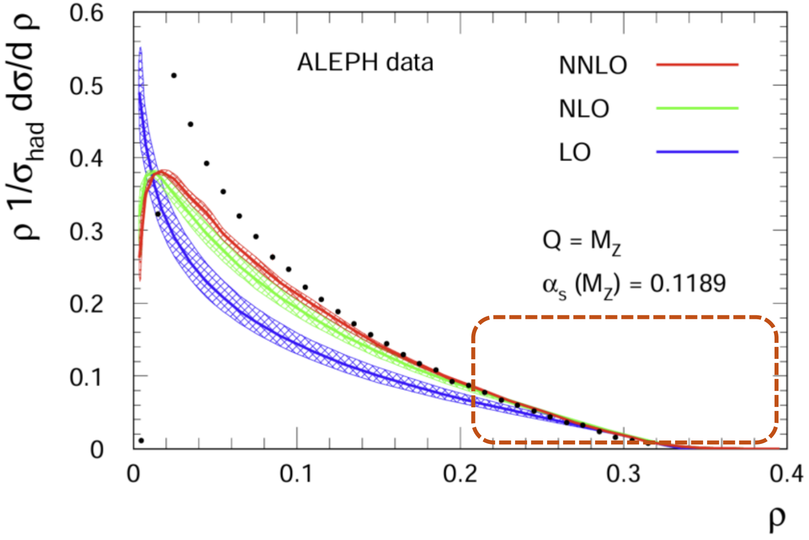

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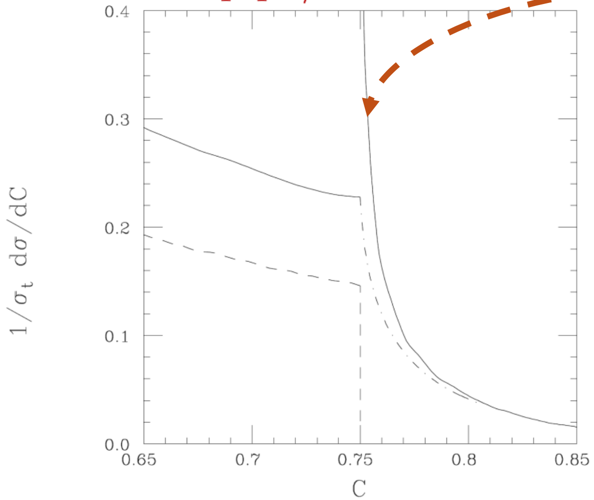

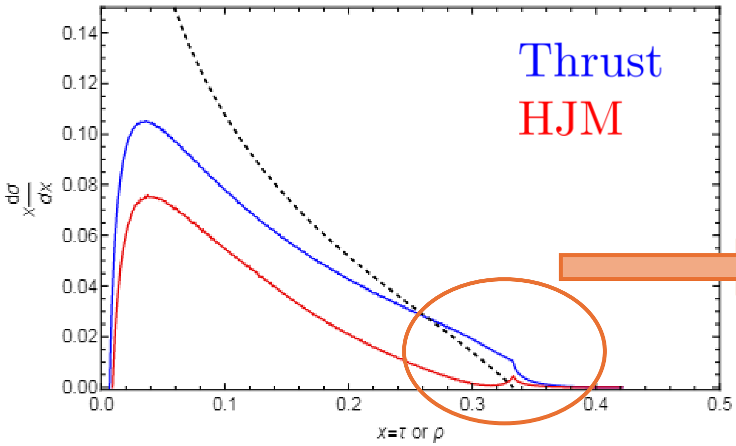

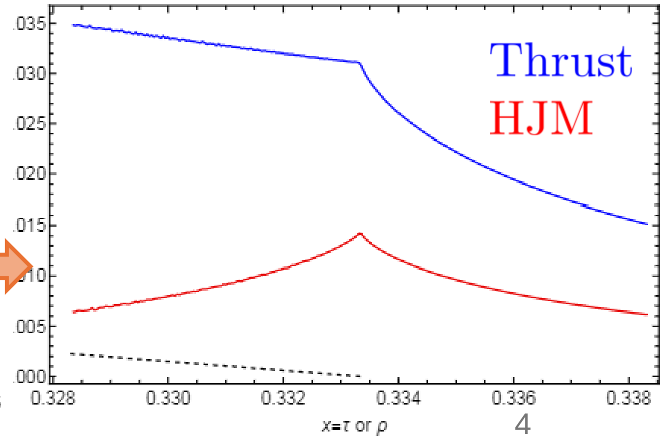

Event Shapes Beyond Dijet

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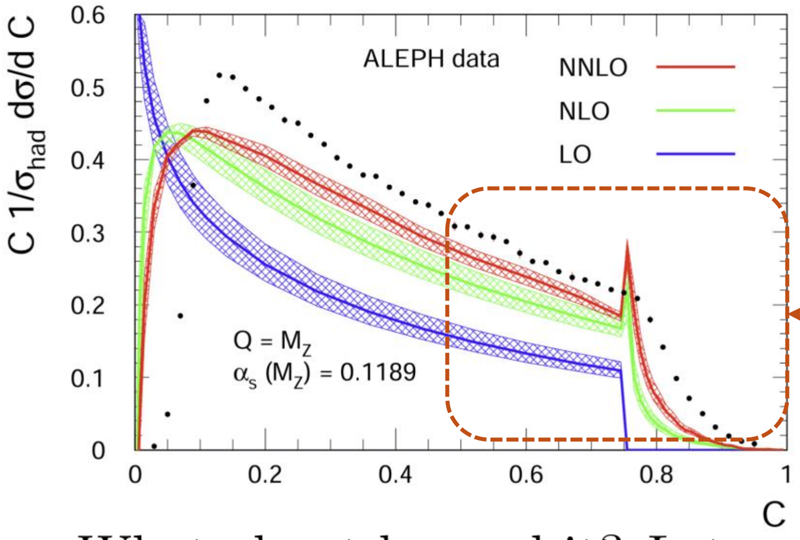

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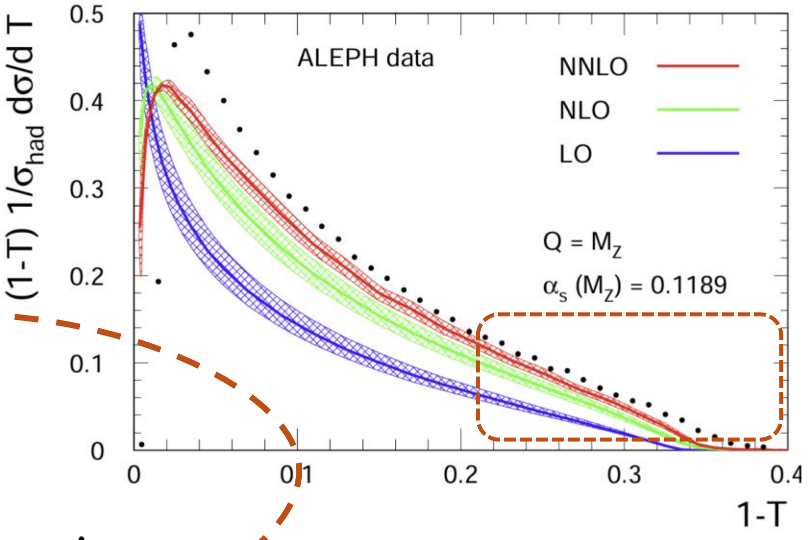

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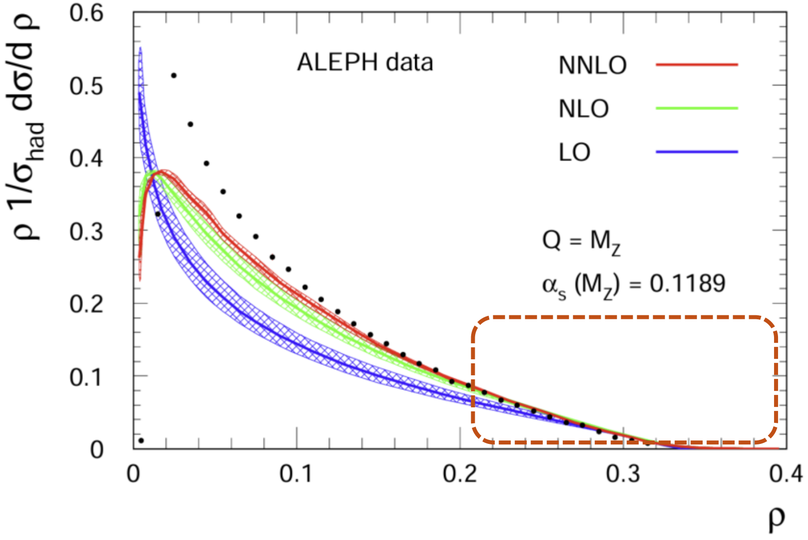

- What about beyond it? Lets zoom in

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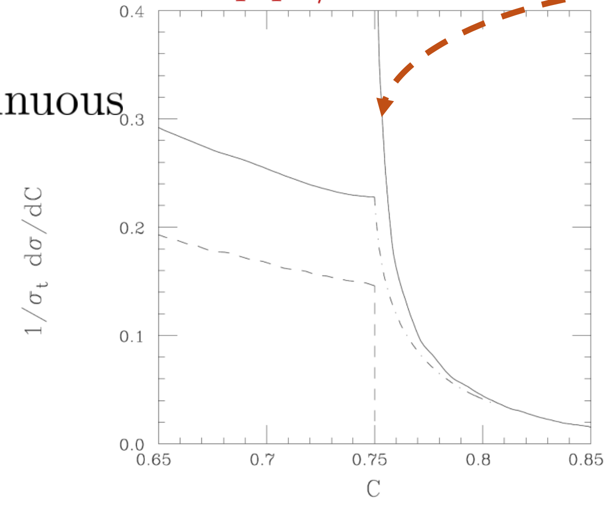
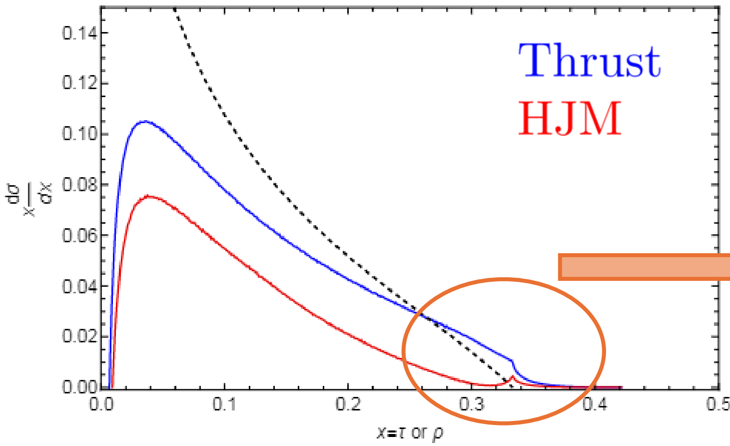
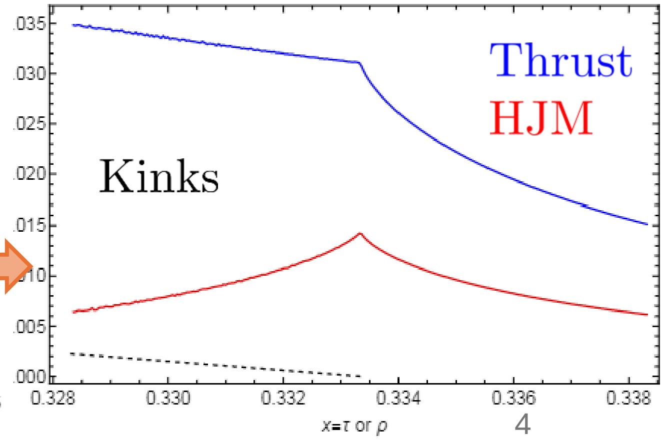
Discontinuous



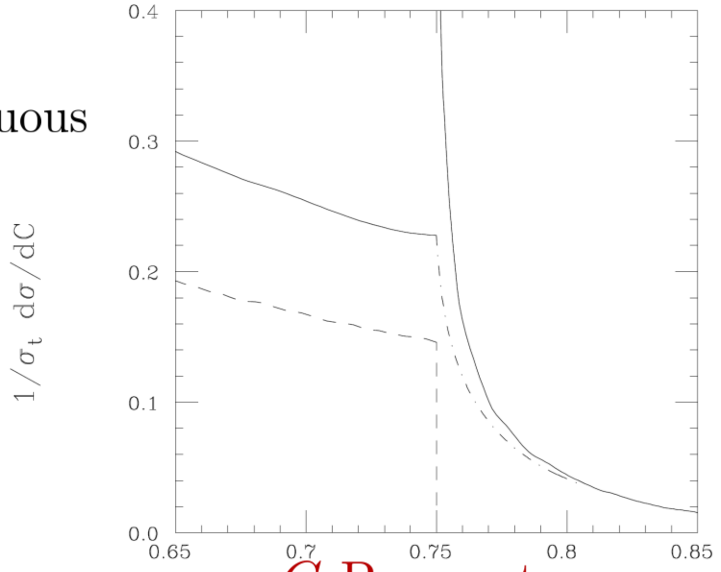




Sudakov Shoulders

- What about beyond it? Lets zoom in

Discontinuous



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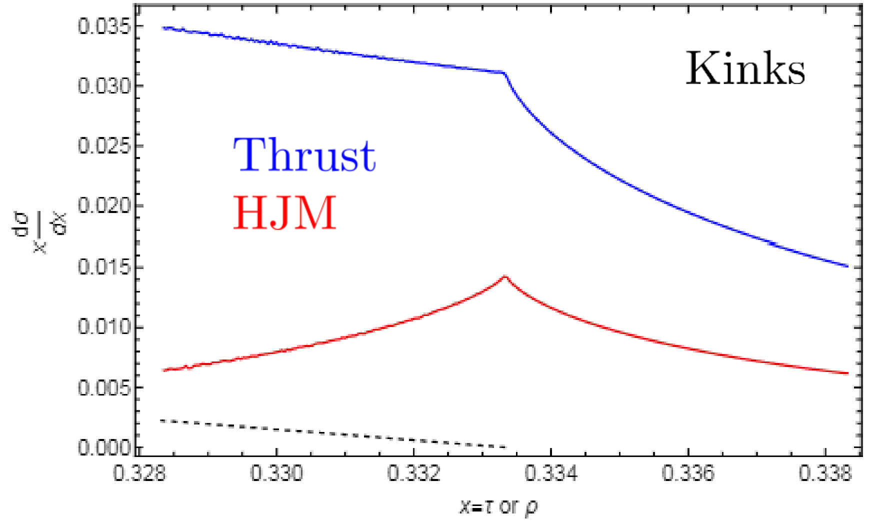

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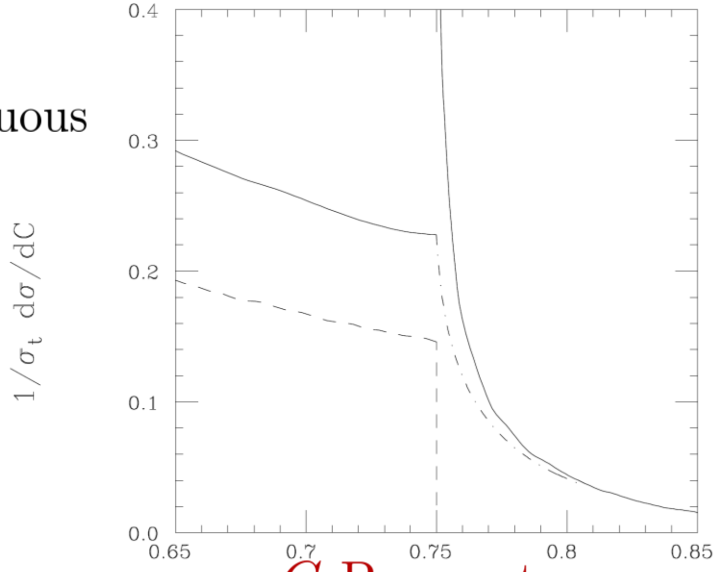

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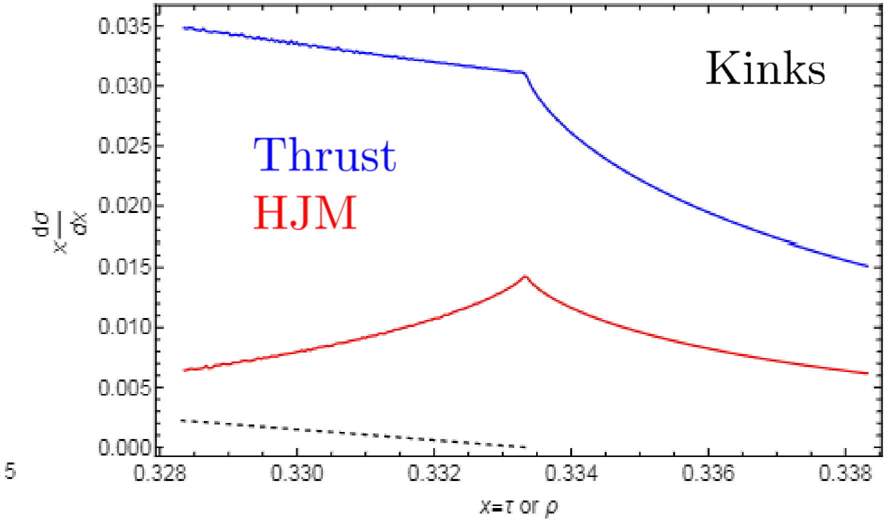

- Non analytic behavior $\Longrightarrow$ Large Logs

Sudakov Shoulders

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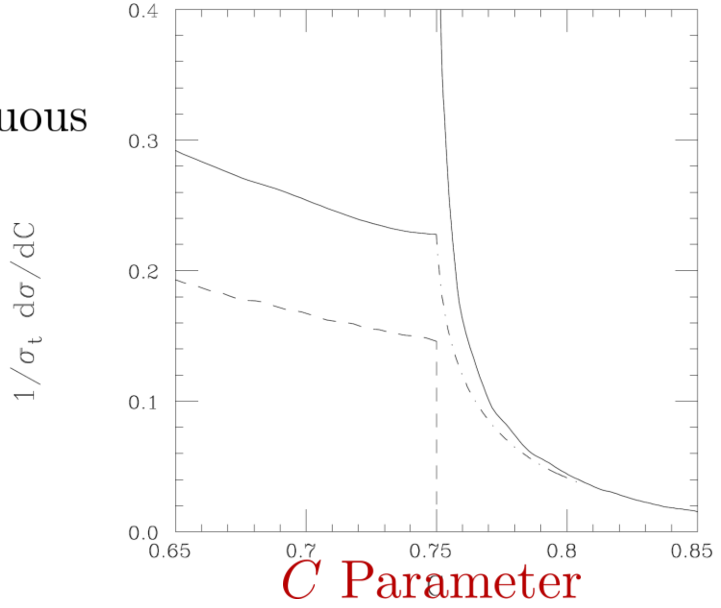
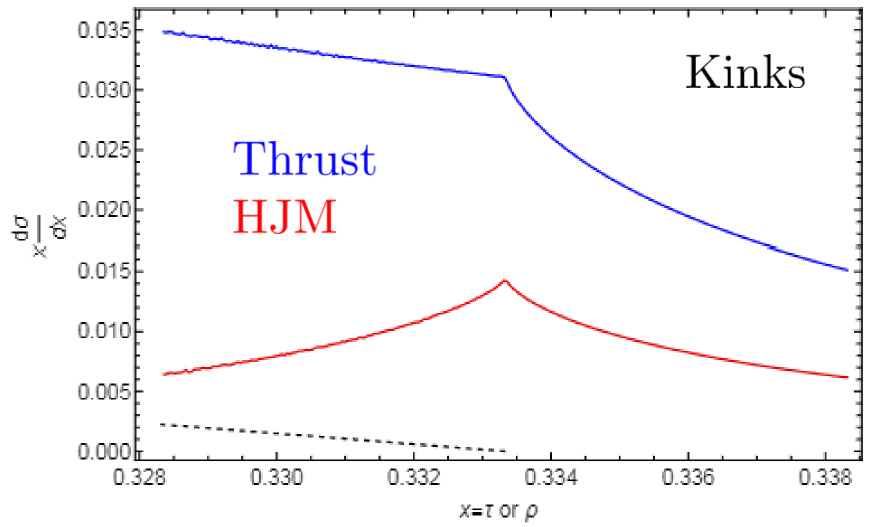
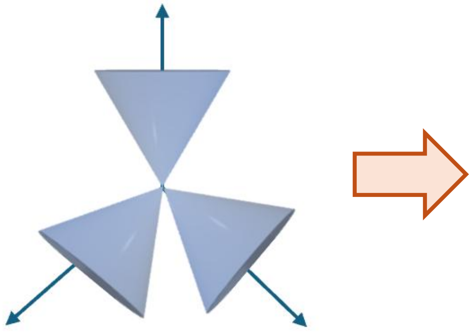
Discontinuous







- Non analytic behavior $\Longrightarrow$ Large Logs
- Radiation off symmetric trijet events

Sudakov Shoulders

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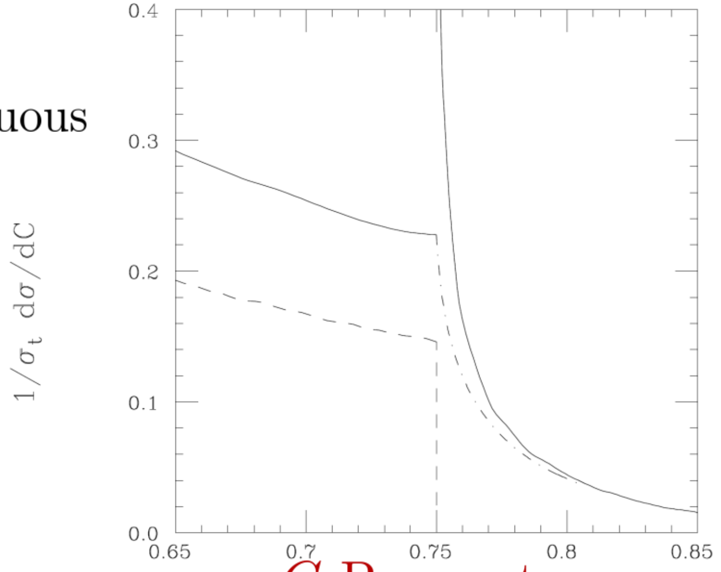

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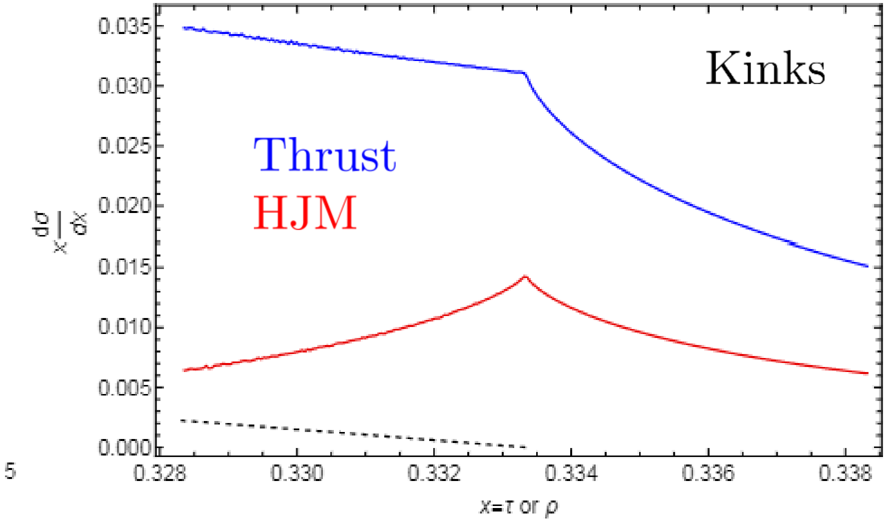

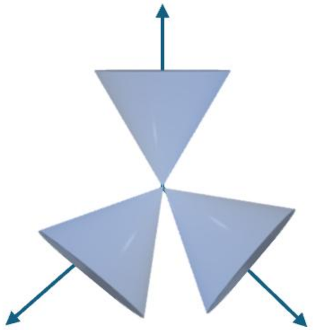

- Non analytic behavior $\implies$ Large Logs
- Radiation off symmetric trijet events
- Christened **Sudakov Shoulders**

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Sudakov Shoulders

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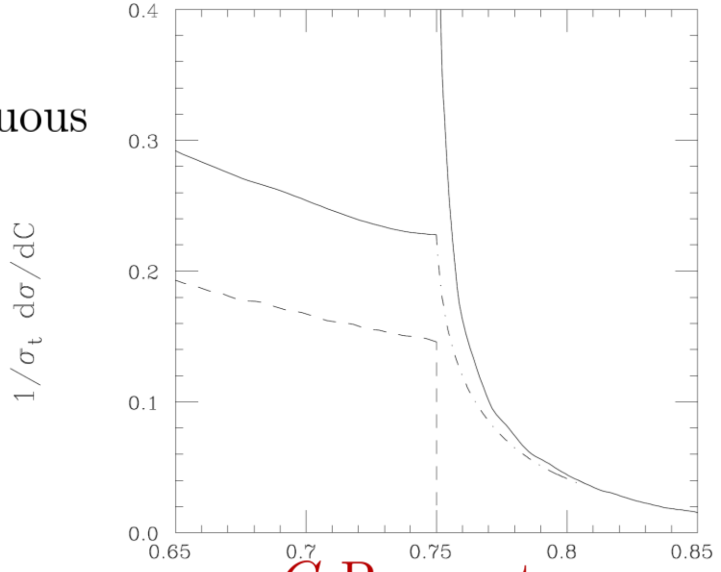
Discontinuous



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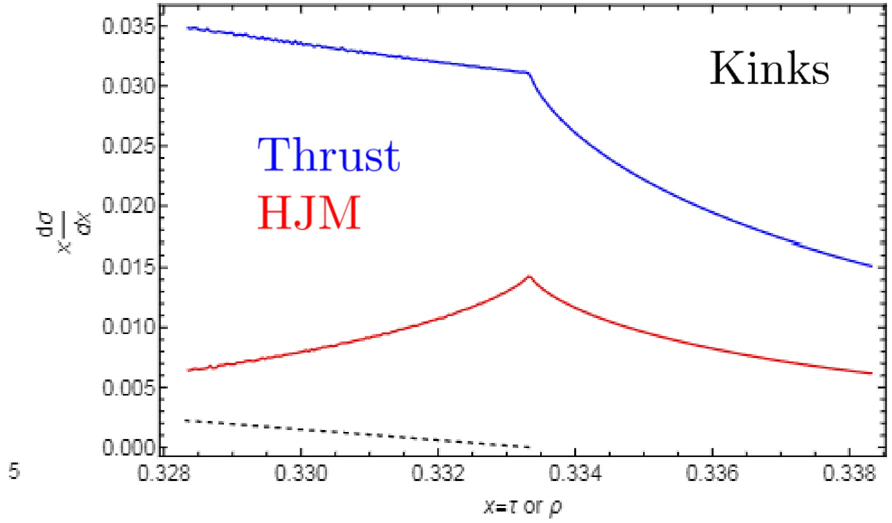

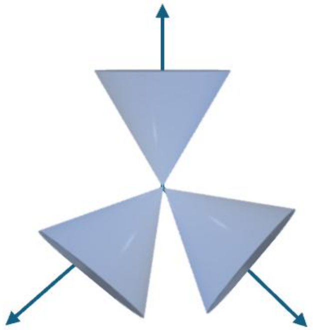

- Non analytic behavior $\implies$ Large Logs
- Radiation off symmetric trijet events
- Christened **Sudakov Shoulders**

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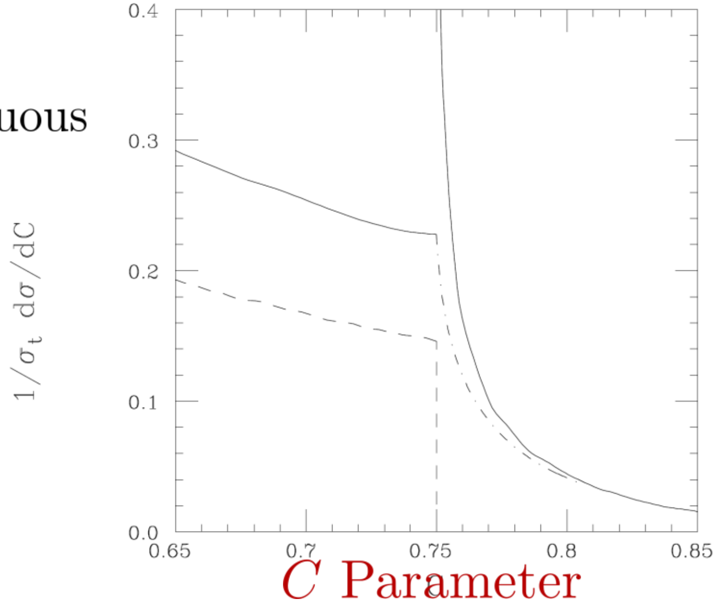
C Parameter $\quad \alpha_s \ln^2(C - \frac{3}{4})$

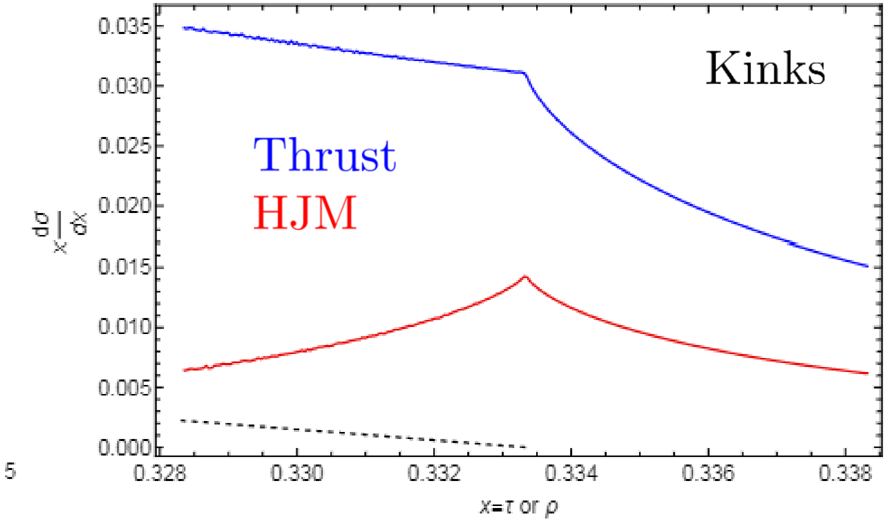
Thrust $\quad \alpha_s \left(\tau - \frac{1}{3}\right) \ln^2(\tau - \frac{1}{3})$

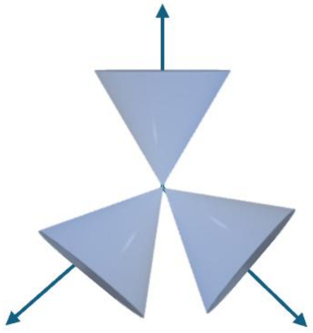
Sudakov Shoulders

- What about beyond it? Lets zoom in

Discontinuous







- Non analytic behavior $\Longrightarrow$ Large Logs
- Radiation off symmetric trijet events
- Christened **Sudakov Shoulders**

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C Parameter	$\alpha_s \ln^2(C - \frac{3}{4})$
Thrust	$\alpha_s\, (\tau - \frac{1}{3}) \ln^2(\tau - \frac{1}{3})$
HJM	$\alpha_s \left\lvert \rho - \frac{1}{3} \right\rvert \ln^2 \left\lvert \rho - \frac{1}{3} \right\rvert$

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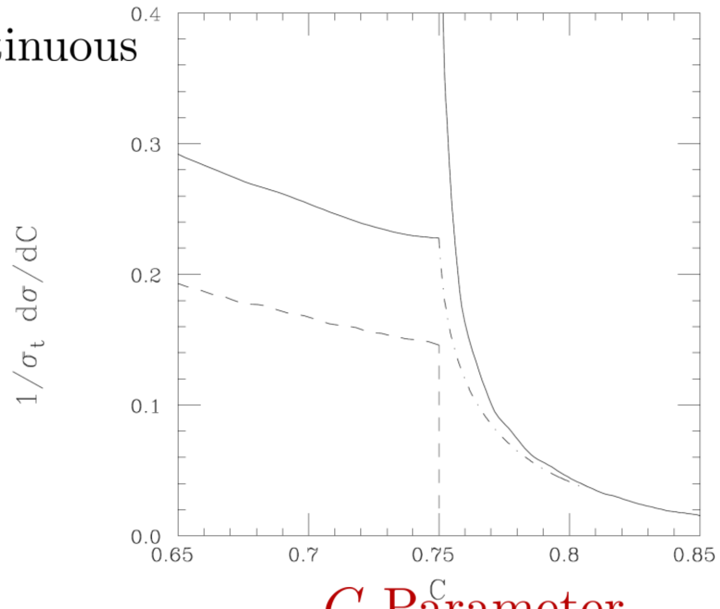
Sudakov Shoulders



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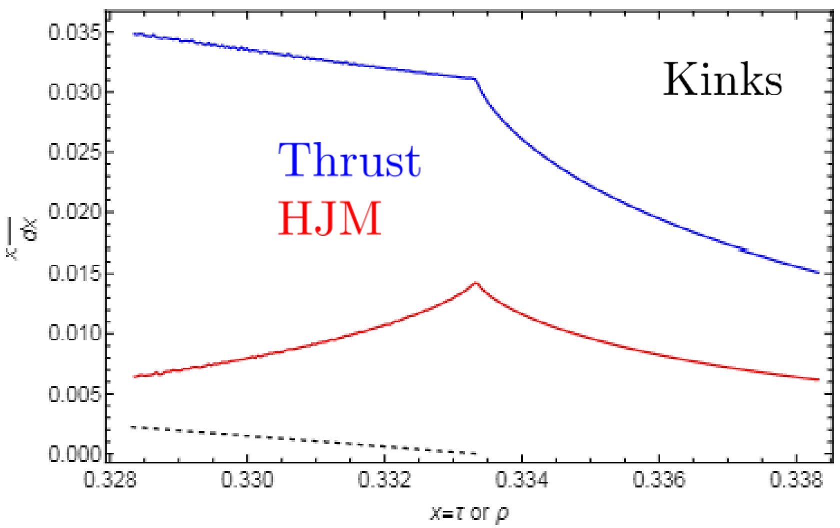

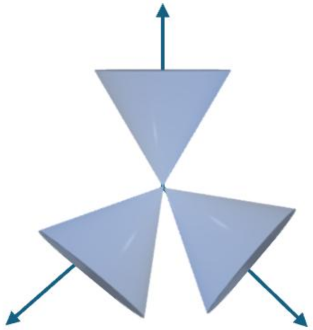

- Non analytic behavior $\Longrightarrow$ Large Logs
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C Parameter	$\alpha_s \ln^2(C - \frac{3}{4})$
Thrust	$\alpha_s\, (\tau - \frac{1}{3}) \ln^2(\tau - \frac{1}{3})$
HJM	$\alpha_s \left|\rho - \frac{1}{3}\right| \ln^2 \left|\rho - \frac{1}{3}\right|$

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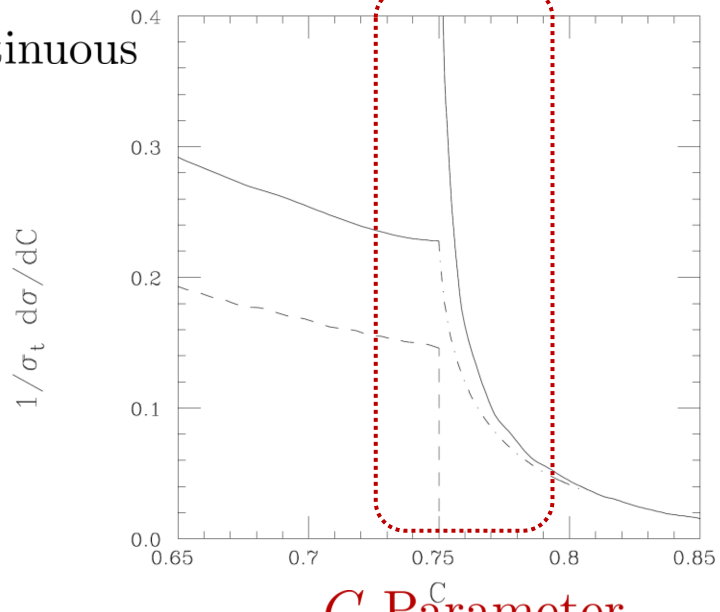
Sudakov Shoulders

Discontinuous



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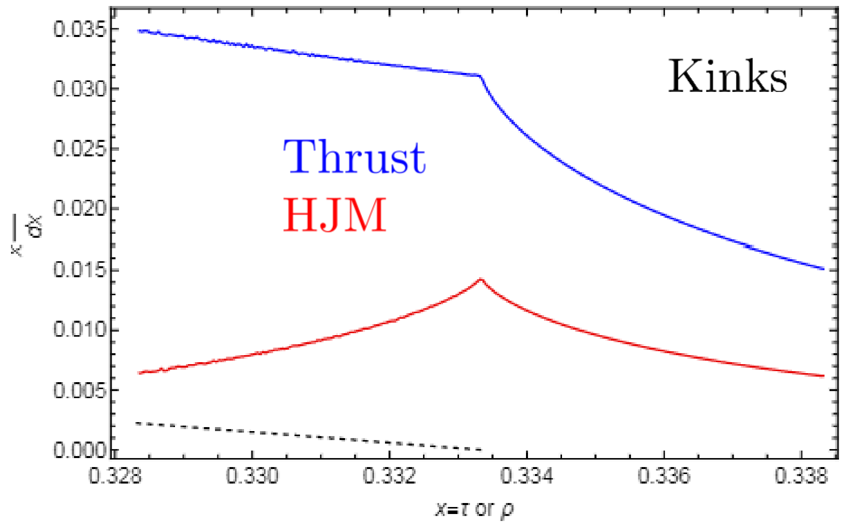

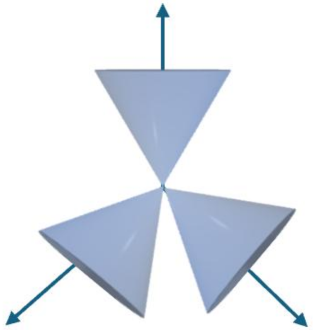

- Non analytic behavior $\implies$ Large Logs
- Radiation off symmetric trijet events

C Parameter	$\alpha_s \ln^2(C - \frac{3}{4})$	Numerically most severe
Thrust	$\alpha_s \left(\tau - \frac{1}{3}\right) \ln^2(\tau - \frac{1}{3})$	
HJM	$\alpha_s \left\|\rho - \frac{1}{3}\right\| \ln^2 \left\|\rho - \frac{1}{3}\right\|$	

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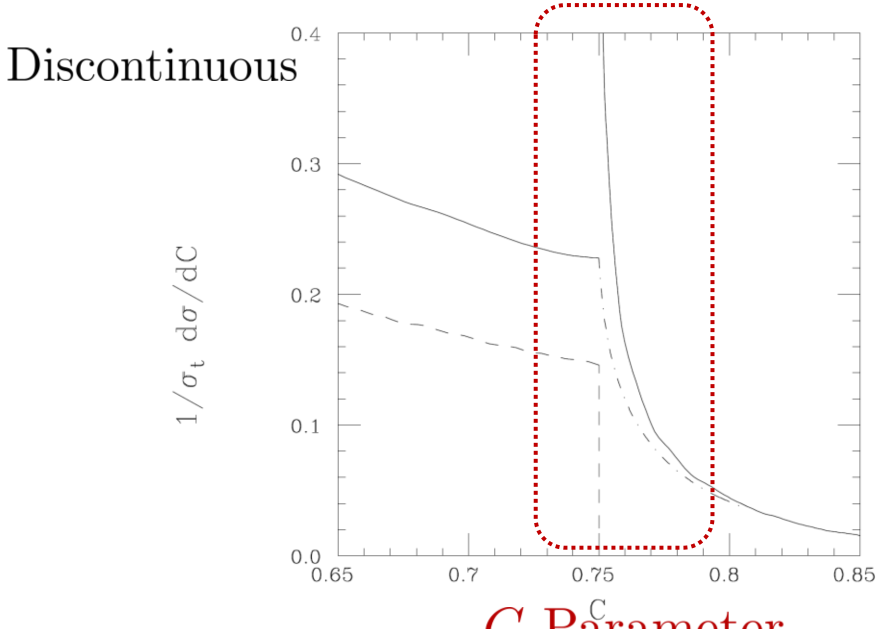
Sudakov Shoulders



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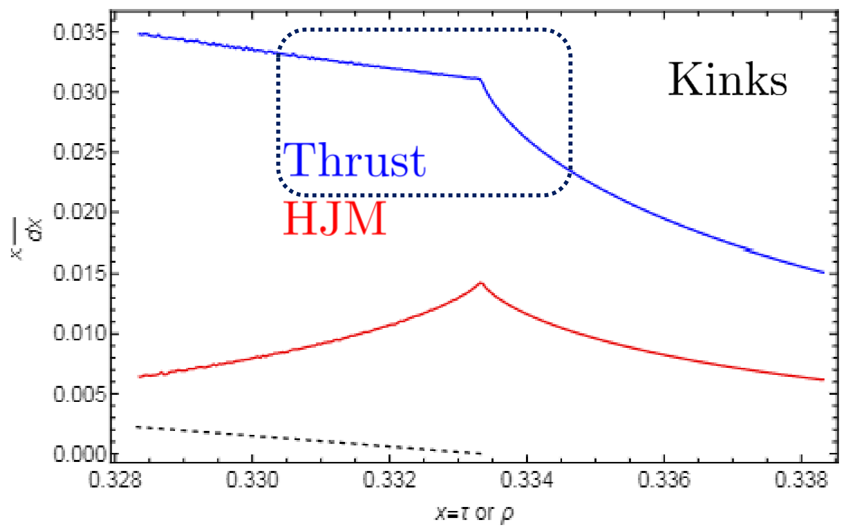

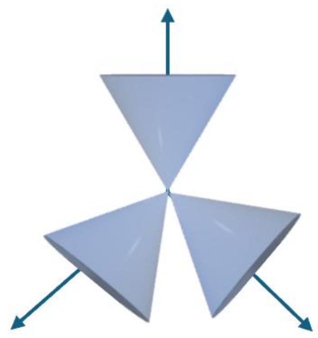

- Non analytic behavior $\Longrightarrow$ Large Logs
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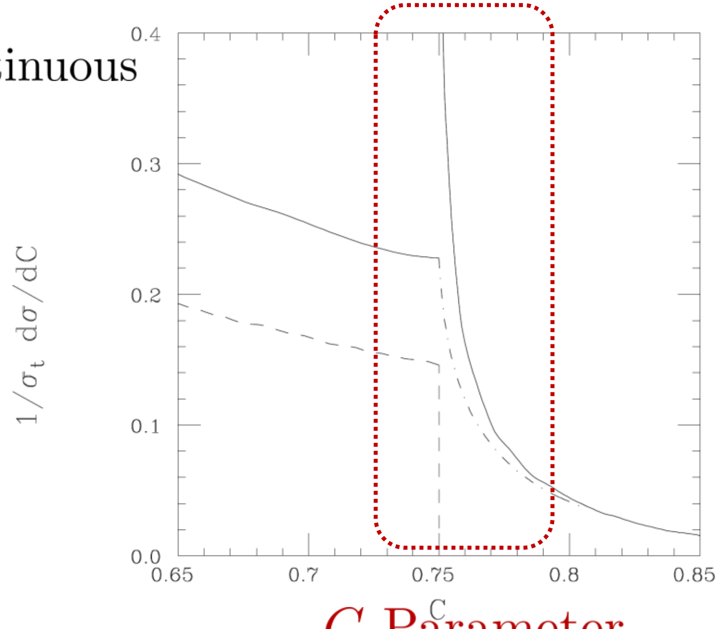
C Parameter	$\alpha_s \ln^2(C - \frac{3}{4})$	Numerically most severe
Thrust	$\alpha_s\,(\tau - \frac{1}{3})\ln^2(\tau - \frac{1}{3})$	Right shoulder only
HJM	$\alpha_s\,\lvert\rho - \frac{1}{3}\rvert \ln^2 \lvert\rho - \frac{1}{3}\rvert$	

Sudakov Shoulders

Discontinuous



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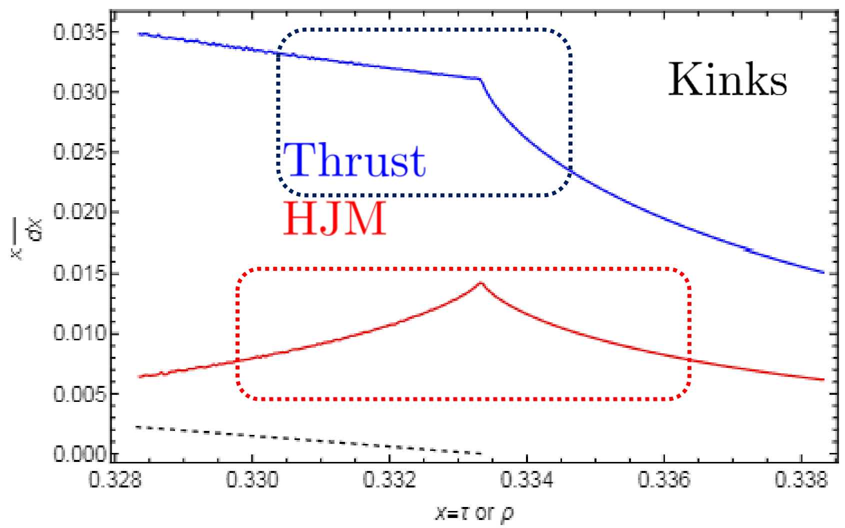

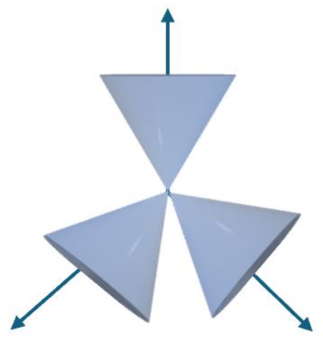

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Thrust	$\alpha_s \left(\tau - \frac{1}{3}\right) \ln^2(\tau - \frac{1}{3})$	Right shoulder only
HJM	$\alpha_s \left\lvert\rho - \frac{1}{3}\right\rvert \ln^2 \left\lvert\rho - \frac{1}{3}\right\rvert$	

Left and right shoulder! Creeps into regions where one performs fits

Sudakov Shoulders : Why?

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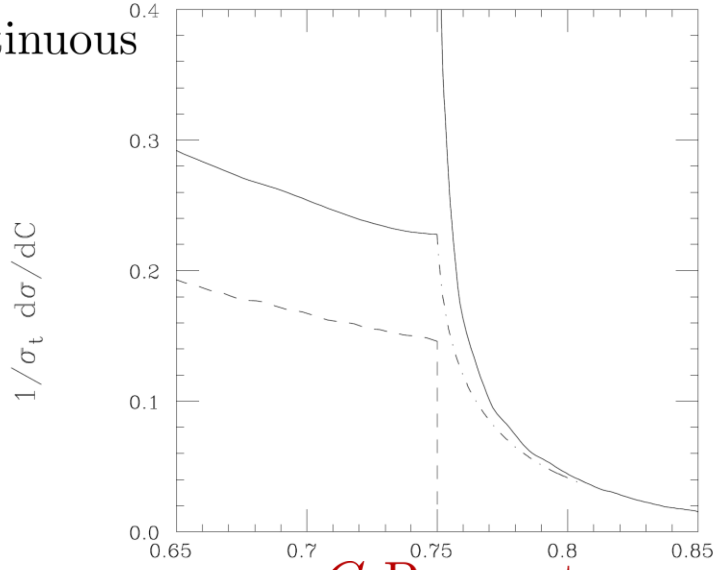
Discontinuous



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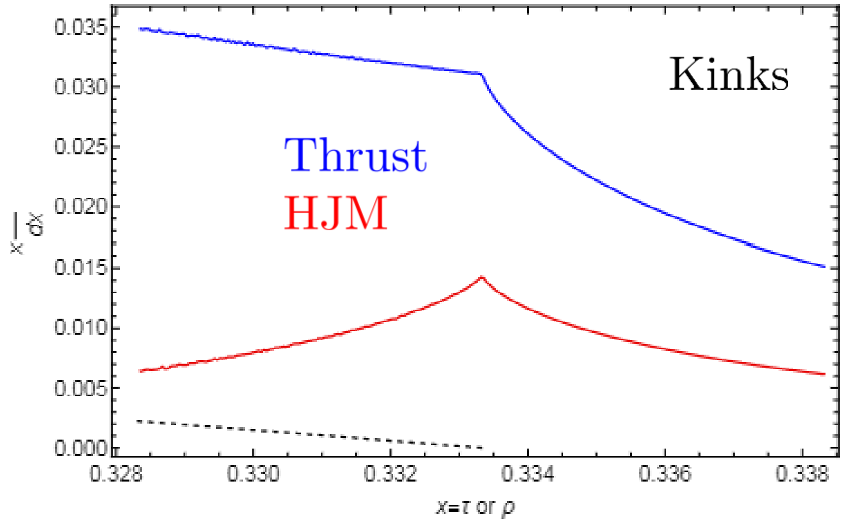
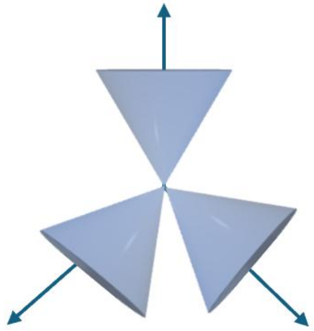


Radiation off symmetric trijet events



Sudakov Shoulders : Why?

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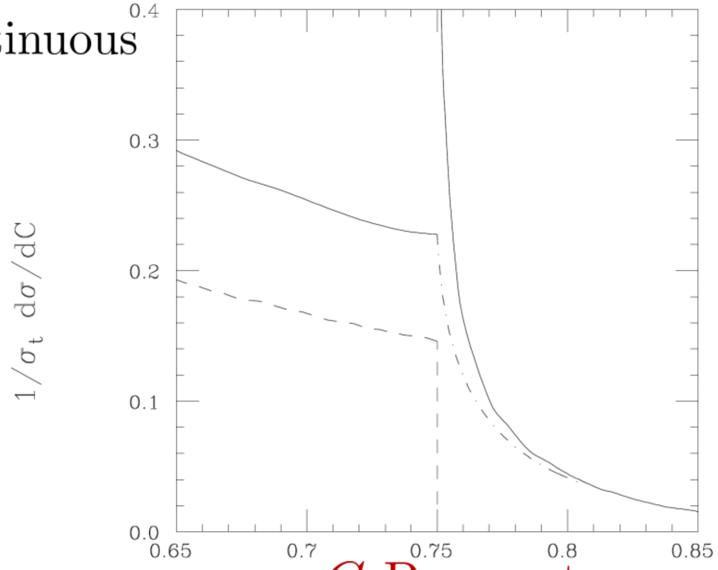

C Parameter

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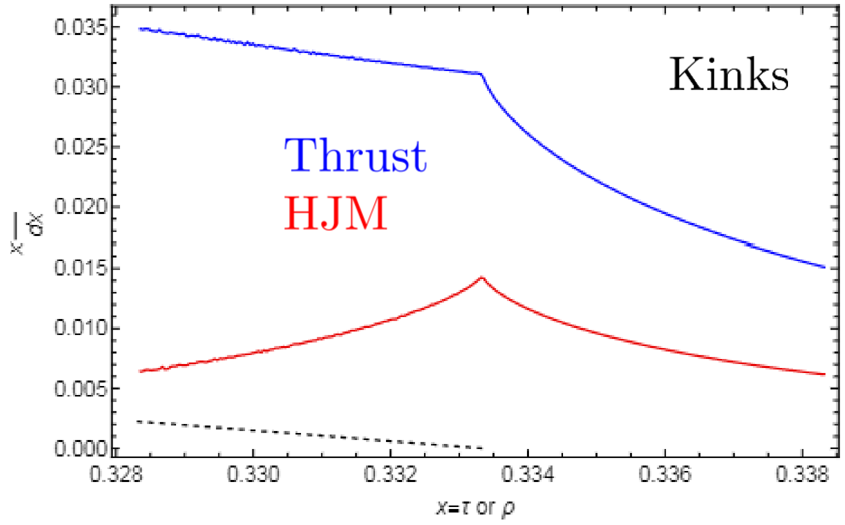
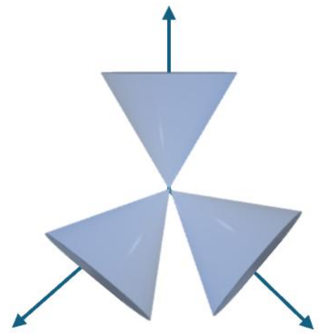


Radiation off symmetric trijet events

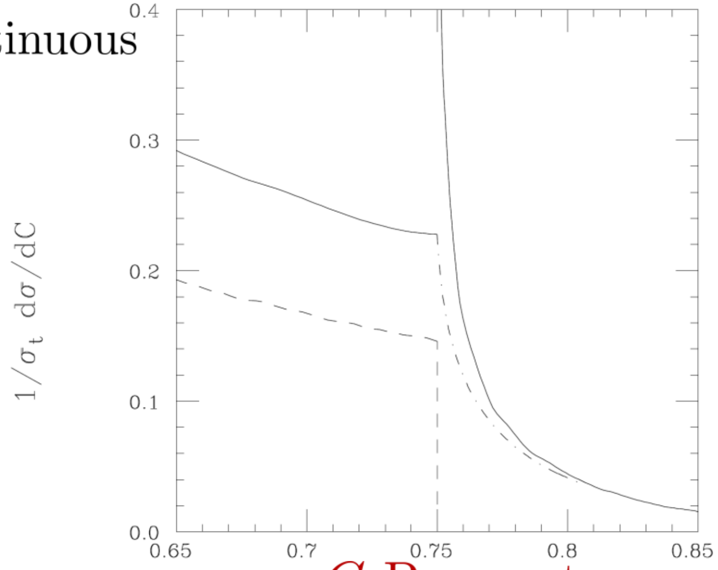


- Event shape allowed range is bounded from above in phase space.

Sudakov Shoulders : Why?

Catani and Webber hep-ph/9710333

Discontinuous



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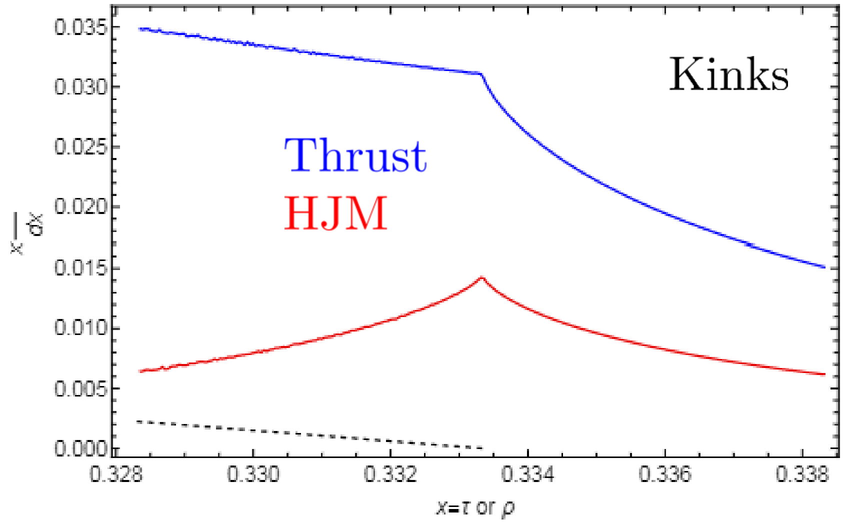
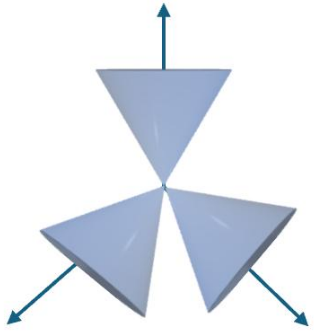


Radiation off symmetric trijet events



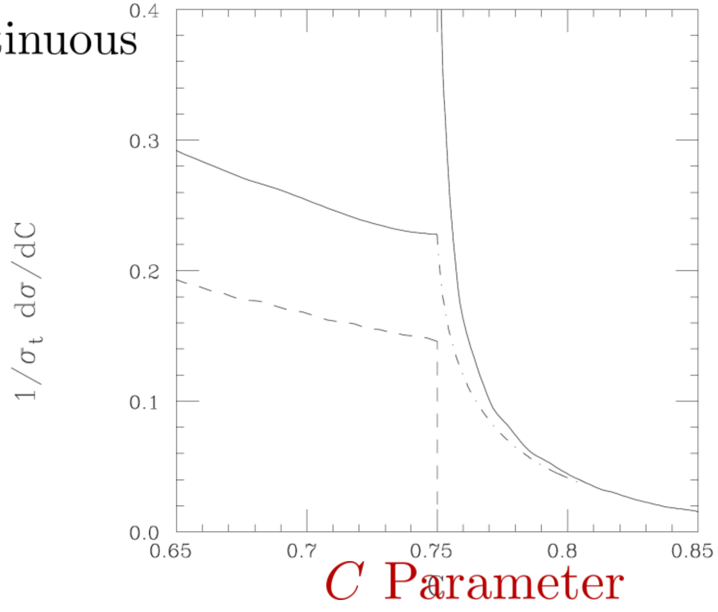
- Event shape allowed range is bounded from above in phase space.

3 Massless final state partons $C \leq \frac{3}{4}$ $\tau,\ \rho \leq \frac{1}{3}$

Sudakov Shoulders : Why?

Catani and Webber hep-ph/9710333

Discontinuous



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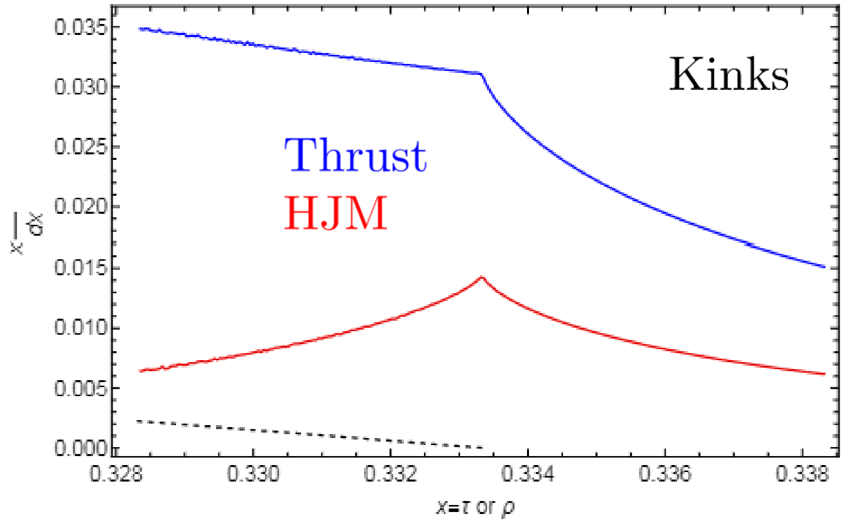
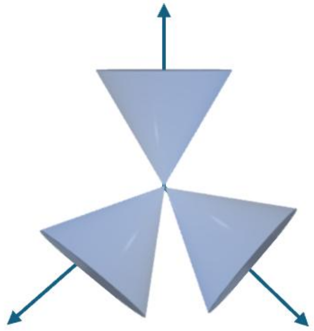


Radiation off symmetric trijet events



- Event shape allowed range is bounded from above in phase space.

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Sudakov Shoulders : Why?

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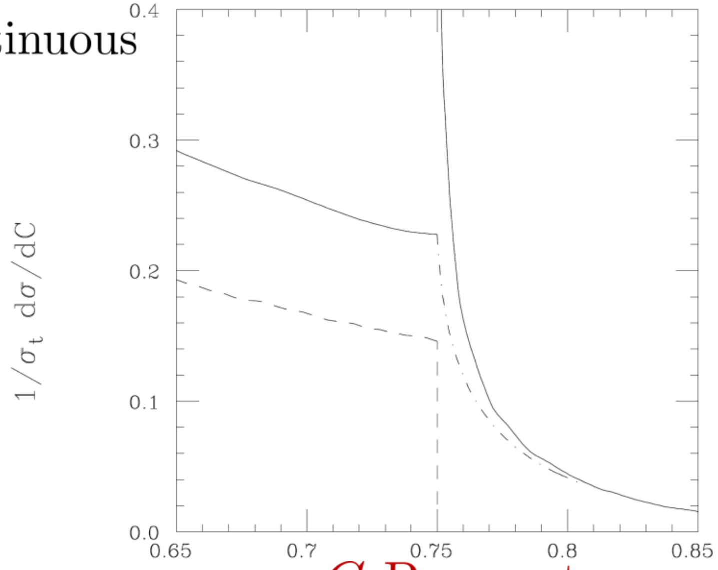

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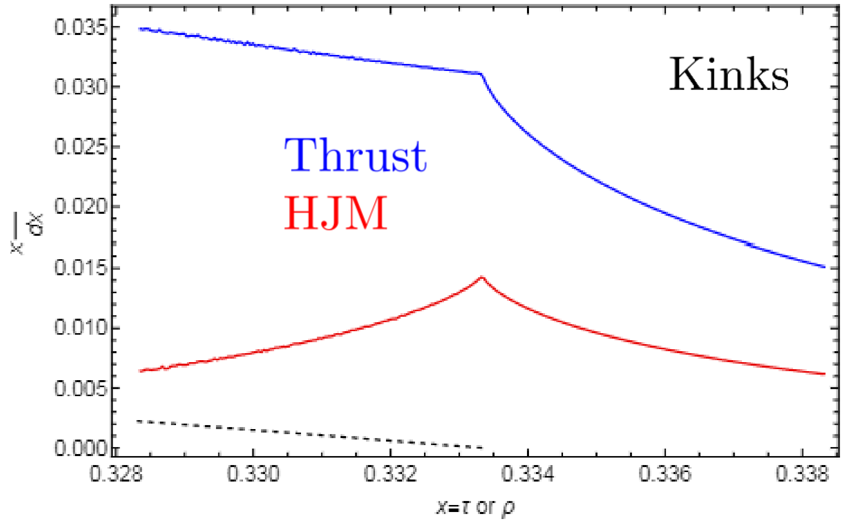

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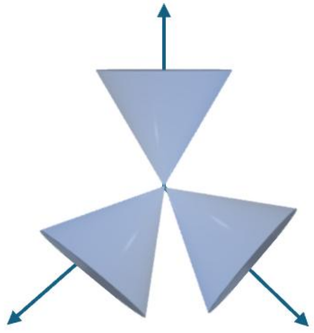

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3 Massless final state partons $C \leq \frac{3}{4}$ $\tau,\ \rho \leq \frac{1}{3}$

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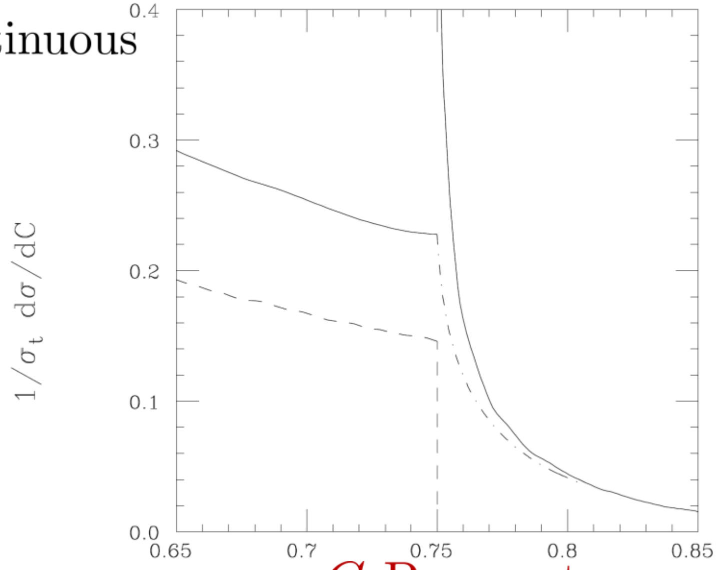
$$\int d\Pi \sim (e - e_{\mathrm{bd}})$$

Phase space closing off near kinematic boundary

Sudakov Shoulders : Why?

Catani and Webber hep-ph/9710333

Discontinuous



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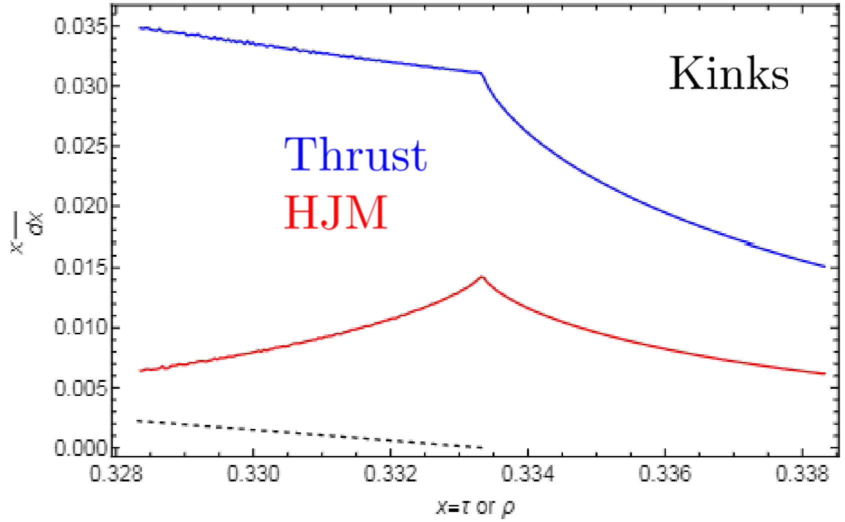

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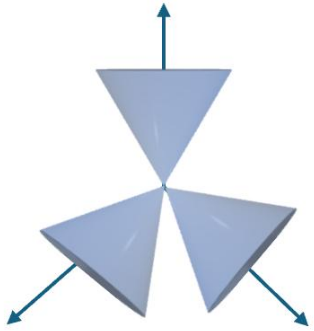

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Soft and Collinear emissions →

Phase space closing off near kinematic boundary

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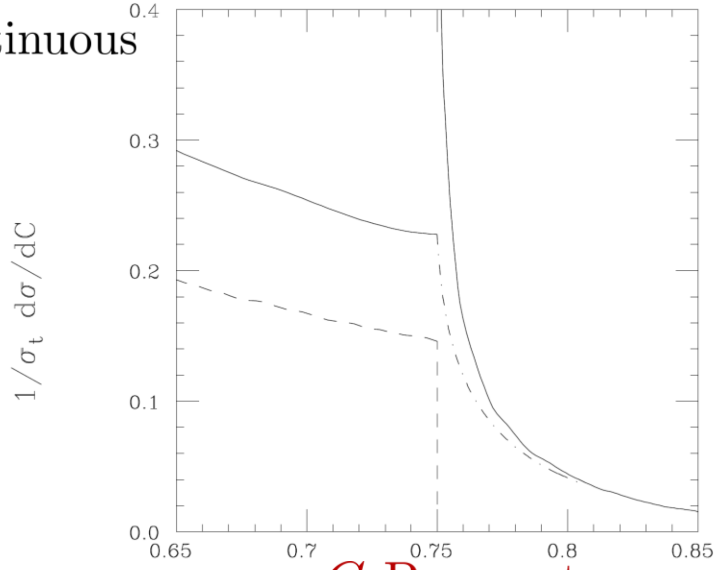

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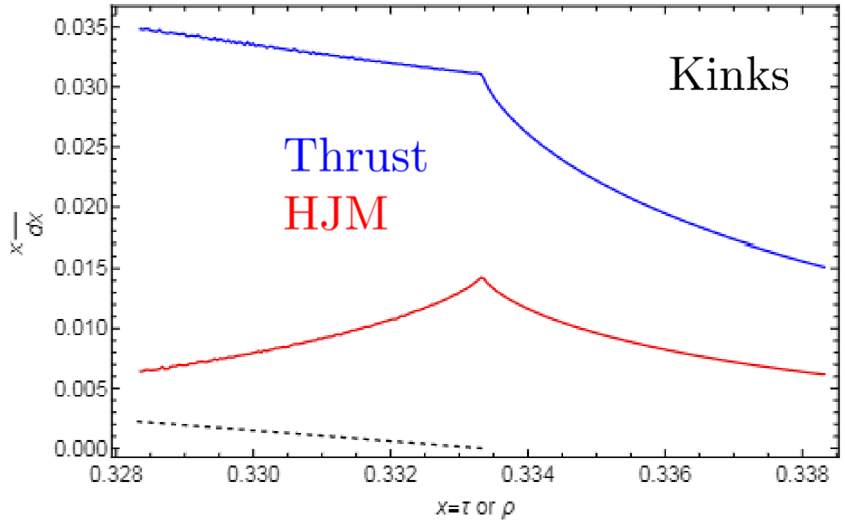
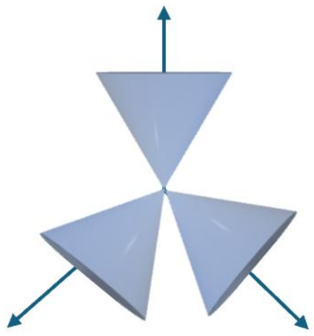


Radiation off symmetric trijet events



- Event shape allowed range is bounded from above in phase space.

  3 Massless final state partons $\quad C \leq \frac{3}{4} \quad \tau,\ \rho \leq \frac{1}{3}$

- Incomplete cancellation of real emissions and virtual right at kinematical boundary

$$\int d\Pi \sim (e - e_{\rm bd}) \xrightarrow{\text{Soft and Collinear emissions}} \sigma \sim (e - e_{\rm bd}) \ \left[\alpha_s \ln^2(e - e_{\rm bd})\right]^n$$

Phase space closing off near kinematic boundary

Sudakov Shoulders : Factorization

2205.05702 **AB**, Matthew D Schwartz, Xiaoyuan Zhang

Sudakov Shoulders : Factorization

- Factorization using SCET

2205.05702 **AB**, Matthew D Schwartz, Xiaoyuan Zhang

$$\frac{1}{\sigma_{\text{LO}}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\; dk_L\; dk_H\; J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\; S^e(k_L, k_H) \times \mathscr{M}_e$$

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- Factorization using SCET

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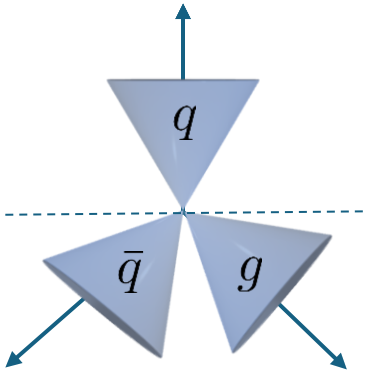
$$\frac{1}{\sigma_{\mathrm{LO}}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\, dk_L\, dk_H\, J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\, S^e(k_L,k_H)\times \mathscr{M}_e$$



Note : The full shoulder cross section is given by

$$\frac{1}{\sigma_{\mathrm{LO}}}\frac{d\sigma^{\mathrm{sh}}}{de} = \frac{1}{\sigma_{\mathrm{LO}}}\frac{d\sigma_g}{de} + 2\times\frac{1}{\sigma_{\mathrm{LO}}}\frac{d\sigma_q}{de}$$

Sudakov Shoulders : Factorization

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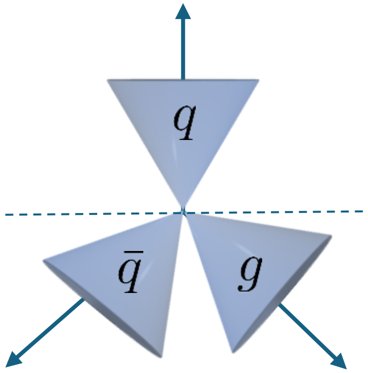
$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{de} = H(Q^2)\int \left(\prod_{i=1}^{3} dm_i^2\right)\; dk_L\; dk_H\; J_q(m_1^2)J_{\bar q}(m_2^2)J_g(m_3^2)\; S^e(k_L,k_H)\times \mathscr{M}_e$$



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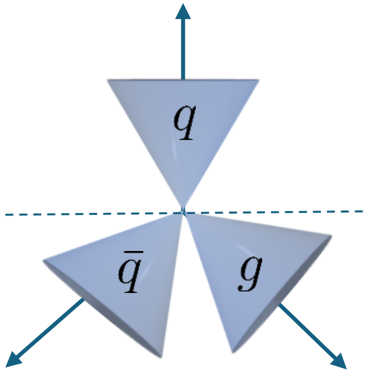
Normalization set by

$$\sigma_{\rm LO} = 48\,\frac{\alpha_s}{4\pi}\;C_F\;\sigma_{\rm Born}(e^+e^-\to q\bar q)$$

Sudakov Shoulders : Factorization

- Factorization using SCET

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{de} = H(Q^2)\int \left(\prod_{i=1}^{3} dm_i^2\right)\, dk_L\, dk_H\, J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\, S^e(k_L,k_H)\times \mathscr{M}_e$$



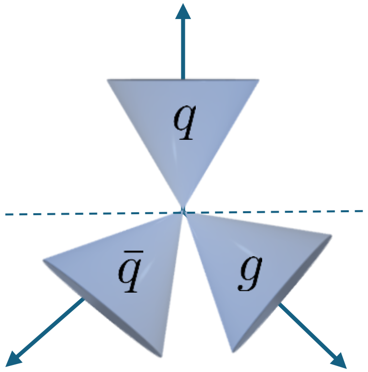
Obtained from three parton amplitudes in QCD Ellis, Ross, Terrano 1981
Garland et.al hep-ph/0206067

3-jet Hard Function
Same as direct photon Becher Schwartz 0911.0681
3-Jettiness Jouttenus et.al. 1102.4344

Sudakov Shoulders : Factorization

- Factorization using SCET

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{de} = H(Q^2)\int \left(\prod_{i=1}^{3} dm_i^2\right)\, dk_L\, dk_H\, J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\, S^e(k_L,k_H)\times \mathscr{M}_e$$



Inclusive jet function in $\mathrm{SCET_I}$

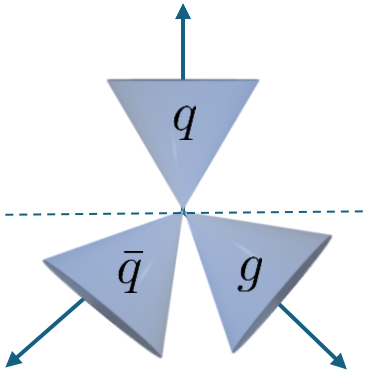
Known to 3 loop order

Brüser,Liu, Stahlhofen 1804.09722

Sudakov Shoulders : Factorization

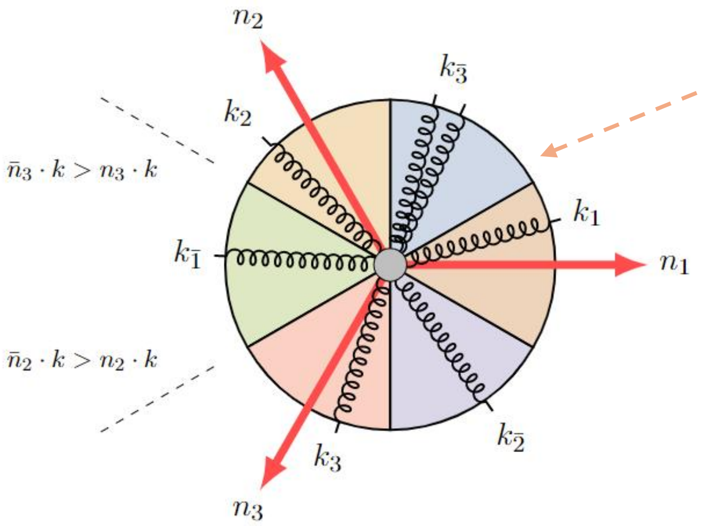
- Factorization using SCET

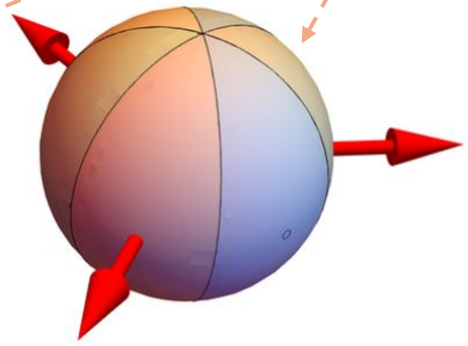
$$\frac{1}{\sigma_{\mathrm{LO}}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\, dk_L\, dk_H\, J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\, S^e(k_L,k_H)\times \mathscr{M}_e$$



New Ingredient : Trijet Hemisphere Soft Function

Projection of soft radiation into 'orange like wedges' dictated by measurement function $\mathscr{M}_e$





Sudakov Shoulders : Factorization

- Factorization using SCET

$$\frac{1}{\sigma_{\mathrm{LO}}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\ dk_L\ dk_H\ J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\ S^e(k_L,k_H)\times \mathscr{M}_e$$

Sudakov Shoulders : Factorization

- Factorization using SCET

$$\frac{1}{\sigma_{\mathrm{LO}}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\, dk_L\, dk_H\, J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\, S^e(k_L,k_H)\times \mathscr{M}_e$$

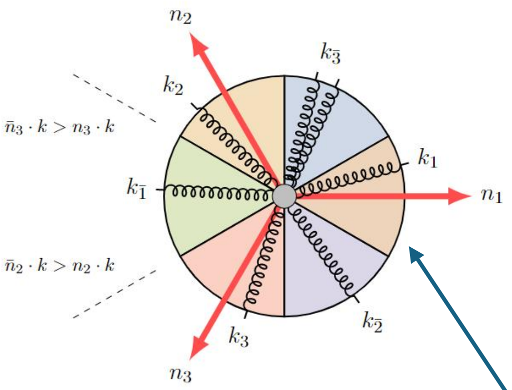
$$S_g(k_\ell,k_h) = \frac{1}{NC_F}\sum_{X_s}\left|\langle X_s|\mathbf{Tr}(Y_{n_2}^\dagger Y_{n_1} T^a Y_{n_1}^\dagger Y_{n_3})|0\rangle\right|^2 \hat{\mathscr{M}}(k_\ell,k_h)\ .$$

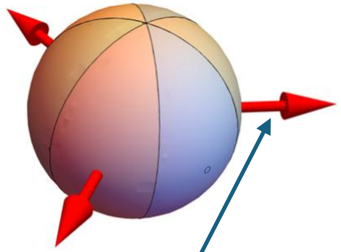
Sudakov Shoulders : Factorization

- Factorization using SCET

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\; dk_L\; dk_H\; J_q(m_1^2)J_{\bar q}(m_2^2)J_g(m_3^2)\; S^e(k_L,k_H) \times \mathscr{M}_e$$

$$S_g(k_\ell,k_h) = \frac{1}{NC_F}\sum_{X_s}\left|\langle X_s|\mathbf{Tr}(Y^\dagger_{n_2}Y_{n_1}T^aY^\dagger_{n_1}Y_{n_3})|0\rangle\right|^2\;\hat{\mathscr{M}}(k_\ell,k_h)\;.$$





Projection defined via operators

$$\hat{\mathscr{M}}(k_\ell,k_h) = \delta\left(k_\ell - \sum_{k_i\in|X_s\rangle}\left[\theta_{\mathbf{1}}(k_i)\left(\frac{2}{3}n_1\cdot k_i\right)+\theta_{\bar{\mathbf{2}}}(k_i)\left(\frac{2}{3}N_2\cdot k_i\right)+\theta_{\bar{\mathbf{3}}}(k_i)\left(\frac{2}{3}N_3\cdot k_i\right)\right]\right) \times\delta\left(k_h - \sum_{k_m\in|X_s\rangle}\left[\theta_{\bar{\mathbf{1}}}(k_m)\left(\frac{2}{3}\bar n_1\cdot k_m\right)+\theta_{\mathbf{2}}(k_m)\left(\frac{2}{3}n_2\cdot k_m\right)+\theta_{\mathbf{3}}(k_m)\left(\frac{2}{3}n_3\cdot k_m\right)\right]\right).$$

Projection onto 'slice'

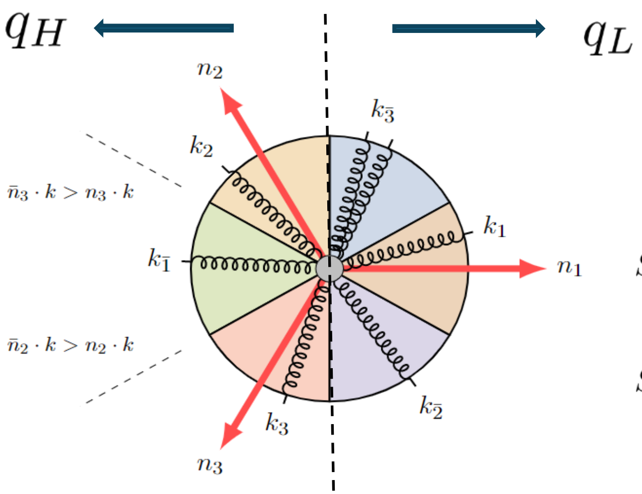
Slight difference of projections for HJM vs thrust

$$N_2^\mu = \bar n_3^\mu + \frac{1}{2}(n_1^\mu - \bar n_1^\mu),$$

$$N_3^\mu = \bar n_2^\mu + \frac{1}{2}(n_1^\mu - \bar n_1^\mu)\,.$$

$$\theta_{\mathbf{1}}(k) = \theta(n_2\cdot k - \bar n_2\cdot k)\theta(n_3\cdot k - \bar n_3\cdot k)$$

Sudakov Shoulders : Factorization



$$S_g^{\text{one-loop , hemi}}(q_L, q_H, \mu) = \delta(q_L)\delta(q_H) + \frac{\alpha_s(\mu)}{4\pi}\delta(q_L)\left[\frac{-4C_F\Gamma_0 \ln\frac{k_H}{\mu} + \gamma_{sqq}}{k_H}\right]_\star + \frac{\alpha_s(\mu)}{4\pi}\delta(q_H)\left[\frac{-2C_A\Gamma_0 \ln\frac{k_L}{\mu} + \gamma_{sg}}{k_L}\right]_\star + \cdots$$

$$S_q^{\text{one-loop , hemi}}(q_L, q_H, \mu) = \delta(q_L)\delta(q_H) + \frac{\alpha_s(\mu)}{4\pi}\delta(q_L)\left[\frac{-2(C_F+C_A)\Gamma_0 \ln\frac{k_H}{\mu} + \gamma_{sqg}}{k_H}\right]_\star + \frac{\alpha_s(\mu)}{4\pi}\delta(q_H)\left[\frac{-2C_F\Gamma_0 \ln\frac{k_L}{\mu} + \gamma_{sq}}{k_L}\right]_\star$$

$\gamma_{sqq} = -4C_F \ln 6,$ $\gamma_{sg} = -2C_A \ln 3 + 4C_F \ln 2$

$\gamma_{sqg} = -2(C_A + C_F)\ln 6,$ $\gamma_{sq} = -2C_F \ln\frac{3}{2} + 2C_A \ln 2$

RGE Consistency check ✓

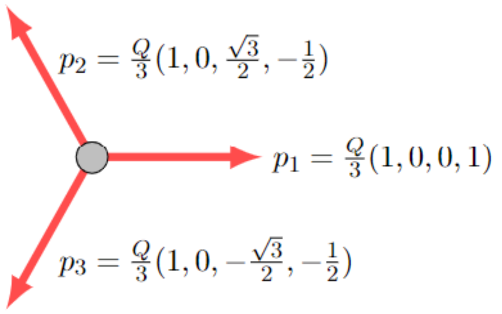
$$\gamma_h = \gamma_{jg} + 2\gamma_{jq} + \gamma_{sqq} + \gamma_{sg} = \gamma_{jg} + 2\gamma_{jq} + \gamma_{sqg} + \gamma_{sq}$$

Previously computed in the literature

Sudakov Shoulders : Factorization

- Factorization using SCET

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\, dk_L\, dk_H\, J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\, S^e(k_L,k_H)\times \mathscr{M}_e$$



Measurement function set by thrust axis

Ex) Single collinear emission

$$T_1 \approx \frac{2}{3} + r - 2m_1^2 \qquad T_2 = \frac{1}{3} - r + s_{12} - m_1^2 \qquad T_3 \approx \frac{2}{3} - (s_{12} - \frac{1}{3})$$

$$r \equiv \frac{1}{3} - \rho\,, \qquad t \equiv \tau - \frac{1}{3}\,, \qquad T_1 > T_2\,,\ T_1 > T_3\,.$$

$\Longrightarrow$ $m_1^2 < r$ $\quad$ $t < m_1^2$

$\mathscr{M}_\rho$ $\quad$ $\mathscr{M}_\tau$

Sudakov Shoulders : Factorization

- Factorization using SCET

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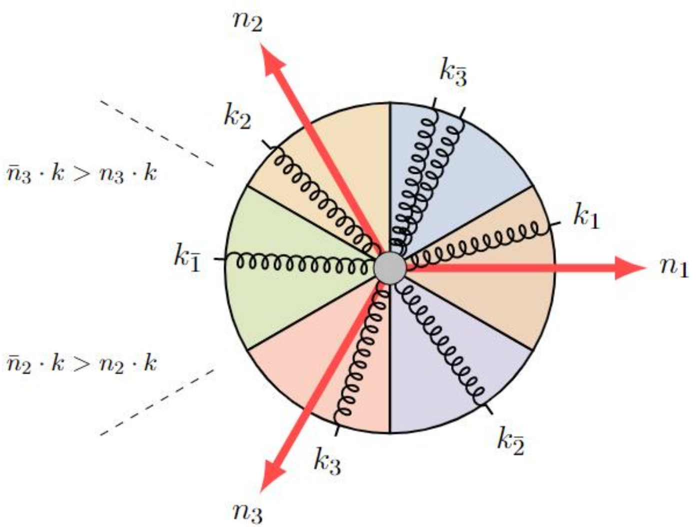
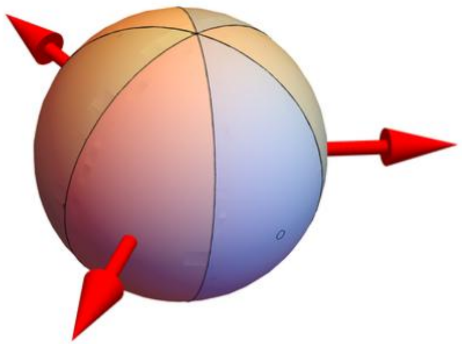
Measurement function set by thrust axis

$\mathscr{M}_\rho$	$W\theta(W)$	$W = r - m_1^2 + m_2^2 + m_3^2 + \frac{2Q}{3}(k_H - k_L)$
$\mathscr{M}_\tau$	$T\theta(T)$	$T = m_1^2 + m_2^2 + m_3^2 + \frac{2Q}{3}(k_L + k_H) - t$





Sudakov Shoulders : Factorization

- Factorization using SCET

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Measurement function set by thrust axis

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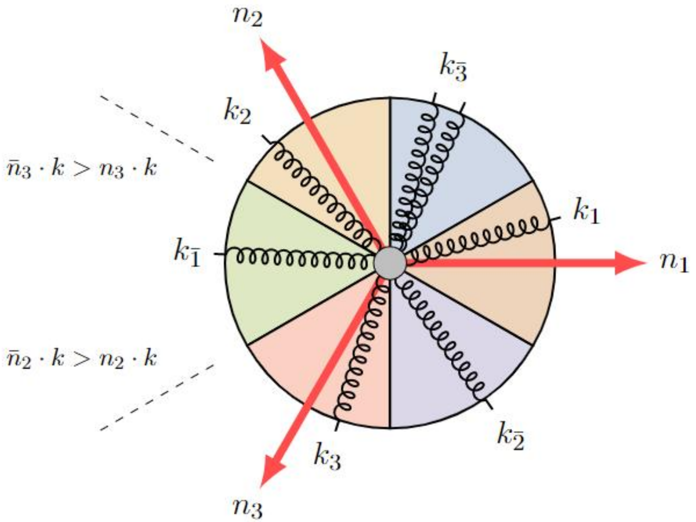

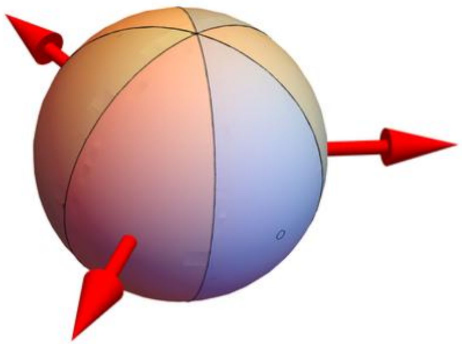

Essentially measurement functions

$$m_1^2 < r + m_2^2 + m_3^2$$

$$t < m_1^2 + m_2^2 + m_3^2$$

Sudakov Shoulders : Factorization

- Factorization using SCET

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\; dk_L\; dk_H\; J_q(m_1^2)J_{\bar{q}}(m_2^2)J_g(m_3^2)\; S^e(k_L,k_H)\times \mathscr{M}_e$$

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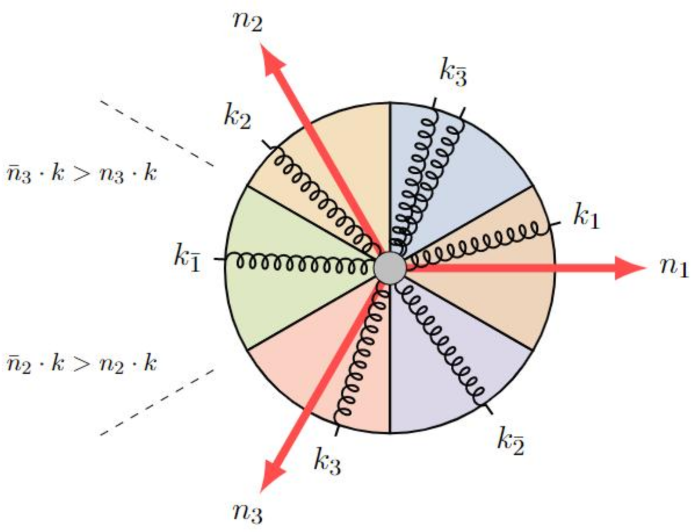
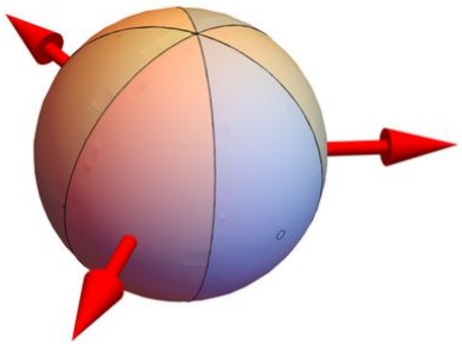




Essentially measurement functions

$$m_1^2 < r + m_2^2 + m_3^2$$

Allows for $r > 0$ and $r < 0$
Left and right shoulders in HJM

$$t < m_1^2 + m_2^2 + m_3^2$$

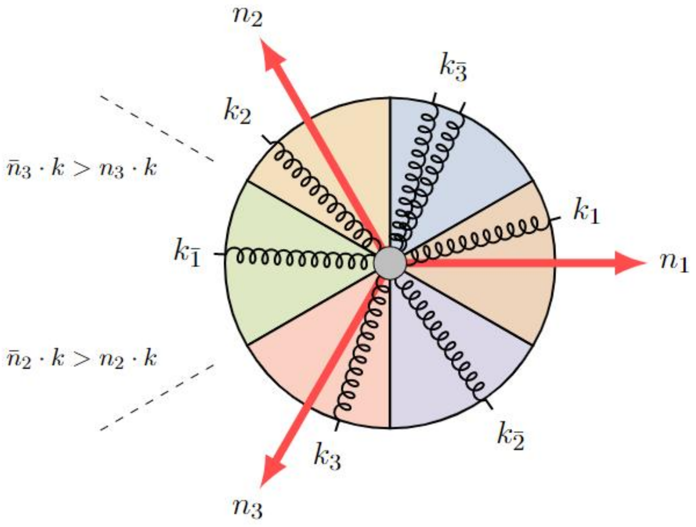
Sudakov Shoulders : Factorization

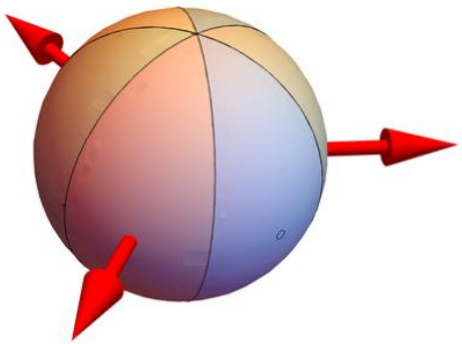
- Factorization using SCET

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{de} = H(Q^2)\int\left(\prod_{i=1}^{3} dm_i^2\right)\; dk_L\; dk_H\; J_q(m_1^2)J_{\bar q}(m_2^2)J_g(m_3^2)\; S^e(k_L,k_H)\times \mathscr{M}_e$$

Measurement function set by thrust axis

$\mathscr{M}_\rho$	$W\theta(W)$	$W = r - m_1^2 + m_2^2 + m_3^2 + \frac{2Q}{3}(k_H - k_L)$
$\mathscr{M}_\tau$	$T\theta(T)$	$T = m_1^2 + m_2^2 + m_3^2 + \frac{2Q}{3}(k_L + k_H) - t$





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Left and right shoulders in HJM

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$t < 0$ does not generate logs

Only right shoulder in thrust

Sudakov Shoulders HJM

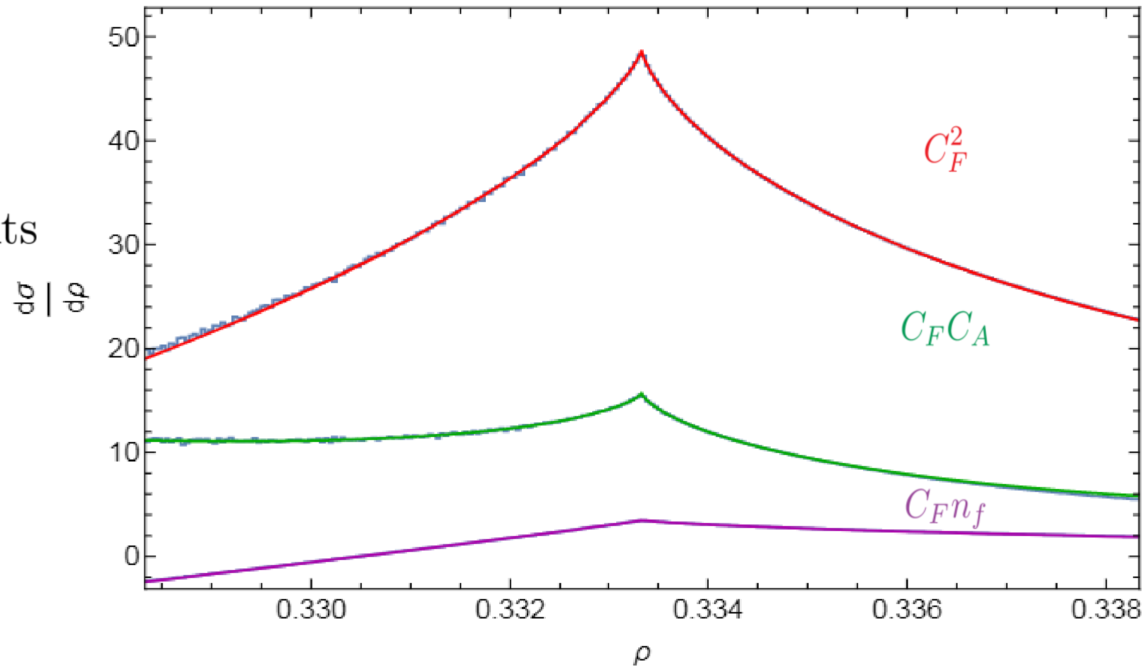
$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{dr} = -96C_F\left(\frac{\alpha_s}{4\pi}\right)^2(2C_F+C_A)\,r\ln^2 r + 48C_F\left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{4}{3}n_fT_F + \frac{C_A}{3}\left(1+3\ln\frac{256}{81}\right) + C_F\left(2+2\ln\frac{256}{81}\right)\right\} r\ln r$$

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma}{ds} = -192C_F\left(\frac{\alpha_s}{4\pi}\right)^2(2C_F+C_A)s\ln^2 s + 48C_F\left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{8}{3}n_fT_F + \frac{2}{3}C_F(6-24\ln 6) + \frac{2}{3}C_A(1-12\ln 6)\right\} s\ln s$$

$$r \equiv \frac{1}{3} - \rho \qquad\qquad s \equiv \rho - \frac{1}{3}$$

- Expansion of the factorization thm in FO gives NLO logs
- Compared to EVENT2, IR cutoff at 10^{-12}, 12 trillion events

Log Fits on EVENT2



Sudakov Shoulders : Resummation

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dt} = \Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h})\, t \left(\frac{tQ}{\mu_s}\right)^{\eta_\ell+\eta_h} \frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)}$$

$$\frac{1}{\sigma_1}\frac{d\sigma_g}{dr} = \Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h})\, r \left(\frac{rQ}{\mu_s}\right)^{\eta_\ell+\eta_h} \frac{e^{-\gamma_E(\eta_\ell+\eta_h)}}{\Gamma(2+\eta_\ell+\eta_h)} \frac{\sin(\pi\eta_\ell)}{\sin(\pi(\eta_\ell+\eta_h))}$$

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$$\Pi_g(\partial_{\eta_\ell}, \partial_{\eta_h}) = \exp\left[4C_F S(\mu_h, \mu_{jh}) + 4C_F S(\mu_{s\ell}, \mu_{jh}) + 2C_A S(\mu_h, \mu_{j\ell}) + 2C_A S(\mu_{s\ell}, \mu_{j\ell})\right]$$
$$\times \exp\left[2A_{\gamma sg}(\mu_{s\ell}, \mu_h) + 2A_{\gamma sqq}(\mu_{sh}, \mu_h) + 2A_{\gamma jg}(\mu_{j\ell}, \mu_h) + 4A_{\gamma jq}(\mu_{jh}, \mu_h)\right]$$
$$\times H(Q, \mu_h)\tilde{j}_q\left(\partial_{\eta_h} + \ln\frac{Q\mu_{sh}}{\mu_{jh}^2}\right)\tilde{j}_{\bar{q}}\left(\partial_{\eta_h} + \ln\frac{Q\mu_{sh}}{\mu_{jh}^2}\right)\tilde{j}_g\left(\partial_{\eta_\ell} + \ln\frac{Q\mu_{s\ell}}{\mu_{j\ell}^2}\right)\tilde{s}_{qq}(\partial_{\eta_h})\tilde{s}_g(\partial_{\eta_\ell})$$

$$\eta_\ell = 2C_A\ A_\Gamma(\mu_j, \mu_s) \sim -2C_A\left(\frac{\alpha_s}{4\pi}\right)\ln r$$

$$\eta_h = 4C_F\ A_\Gamma(\mu_j, \mu_s) \sim -4C_F\left(\frac{\alpha_s}{4\pi}\right)\ln r$$

- RG evolve hard, jet and soft functions from scales $Q, Q\sqrt{r}, Qr$ to resum
- Run into spurious Sudakov Landau singularity! $\eta_\ell + \eta_h \in \mathbb{Z}$ Poles even if $\beta_0 = 0$!
- Expansion in α_s recovers logs in FO. Singularity reminiscent of q_T resummation poles

Frixione, Nason and Ridolfi hep-ph/9808367
Becher, Neubert 1007.4005
Monni, Re, Torielli 1604.02191
Ebert, Tackmann 1611.08610

Sudakov Shoulders : Resummation

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- Why the singularity? Space in which scales are set sometime matters.

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$$
\begin{aligned}
\widetilde{\sigma}_g(z) &= \int_{-\infty}^{\infty} dr\; e^{izr}\, \frac{1}{\sigma_{\rm LO}} \frac{d^3\sigma_g}{d\rho^3} \\
&= e^{-2\hat{\alpha} C_F \ln^2 \frac{\mu_h}{\mu_{jh}} - 2\hat{\alpha} C_F \ln^2 \frac{\mu_{sh}}{\mu_{jh}} - \hat{\alpha} C_A \ln^2 \frac{\mu_h}{\mu_{j\ell}} - \hat{\alpha} C_A \ln^2 \frac{\mu_{s\ell}}{\mu_{j\ell}}} \left(-iz \frac{\mu_{s\ell} e^{\gamma_E}}{Q}\right)^{-a} \left(iz \frac{\mu_{sh} e^{\gamma_E}}{Q}\right)^{-b} .
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$$\mu_{s,\ell/h} = |r|Q, \quad \mu^2_{j,\ell/h} = Q\mu_{s,\ell/h}$$

$$\mu_{s,\ell/h} = \pm i\frac{Qe^{-\gamma_E}}{z} \qquad \mu_{s,\ell/h} = \frac{Qe^{-\gamma_E}}{|z|}$$

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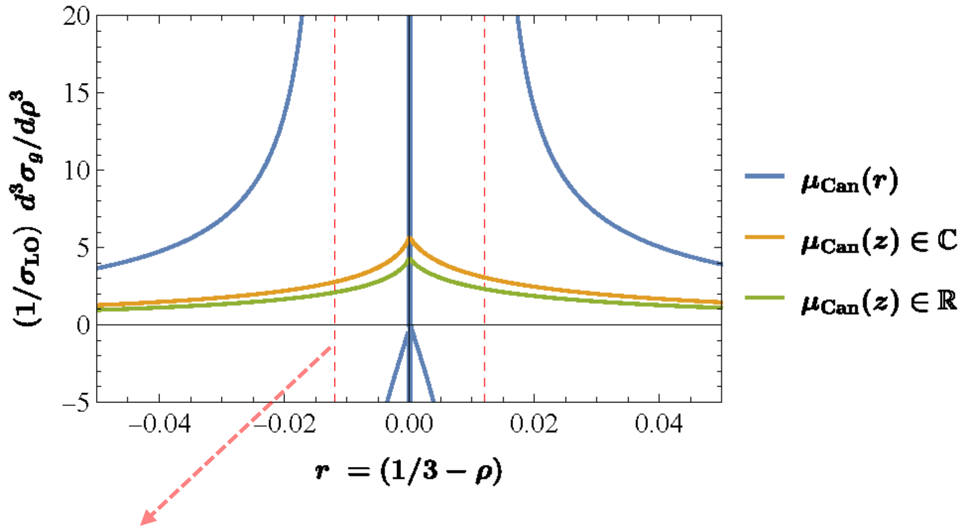
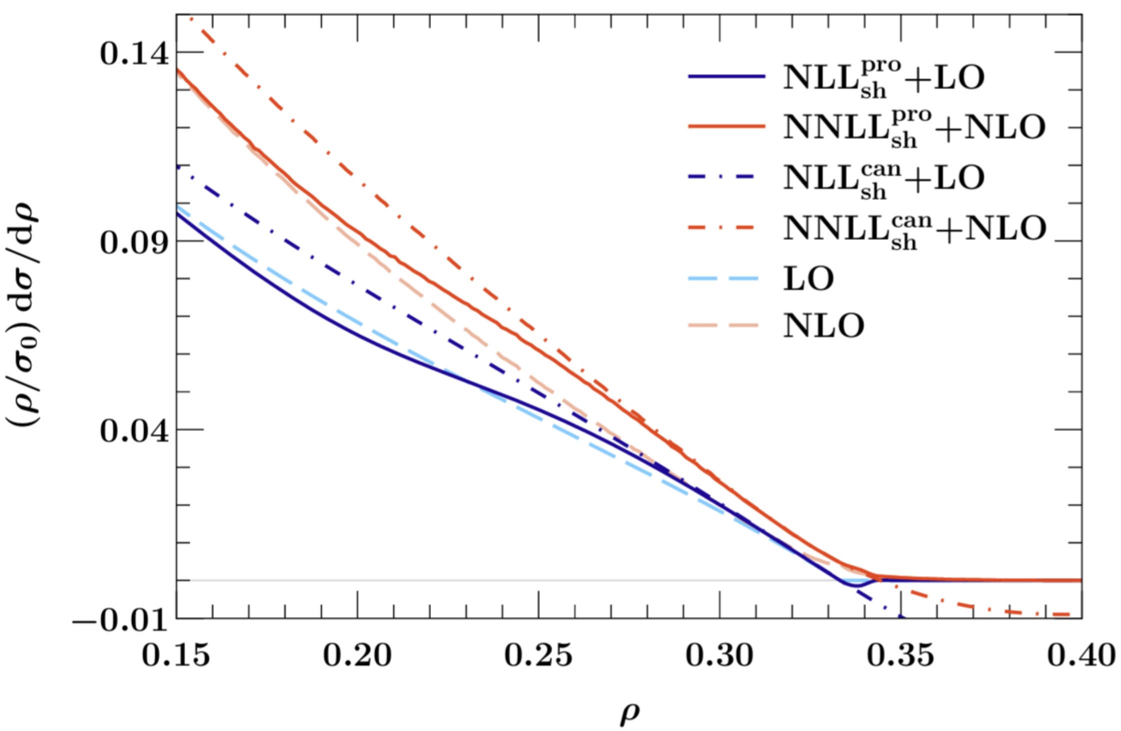
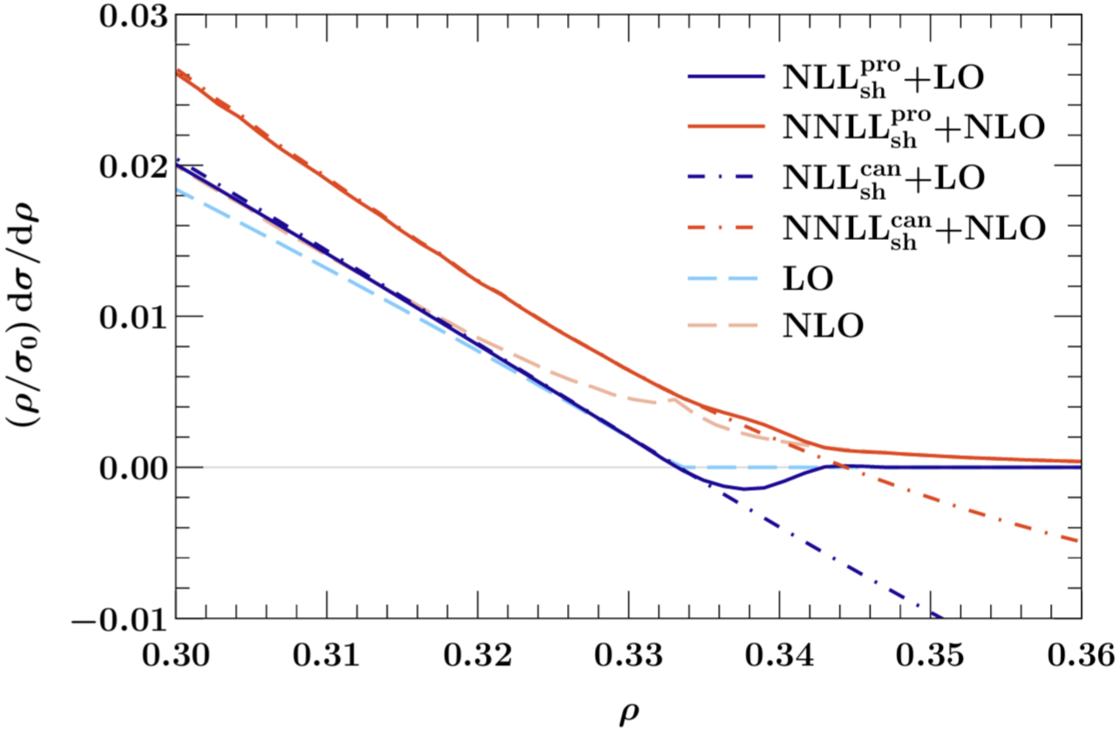


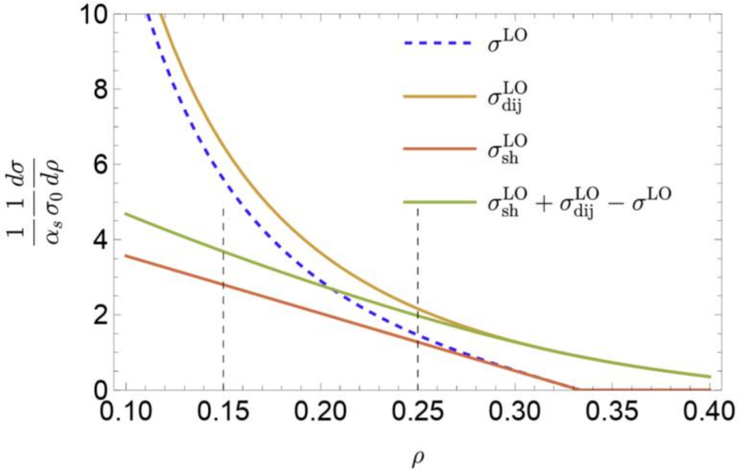
Sudakov Landau Pole is spurious!

2306.08033 **AB**, Johannes KL Michel, Matthew D Schwartz, Iain Stewart, Xiaoyuan Zhang

Sudakov Shoulders : Resummation







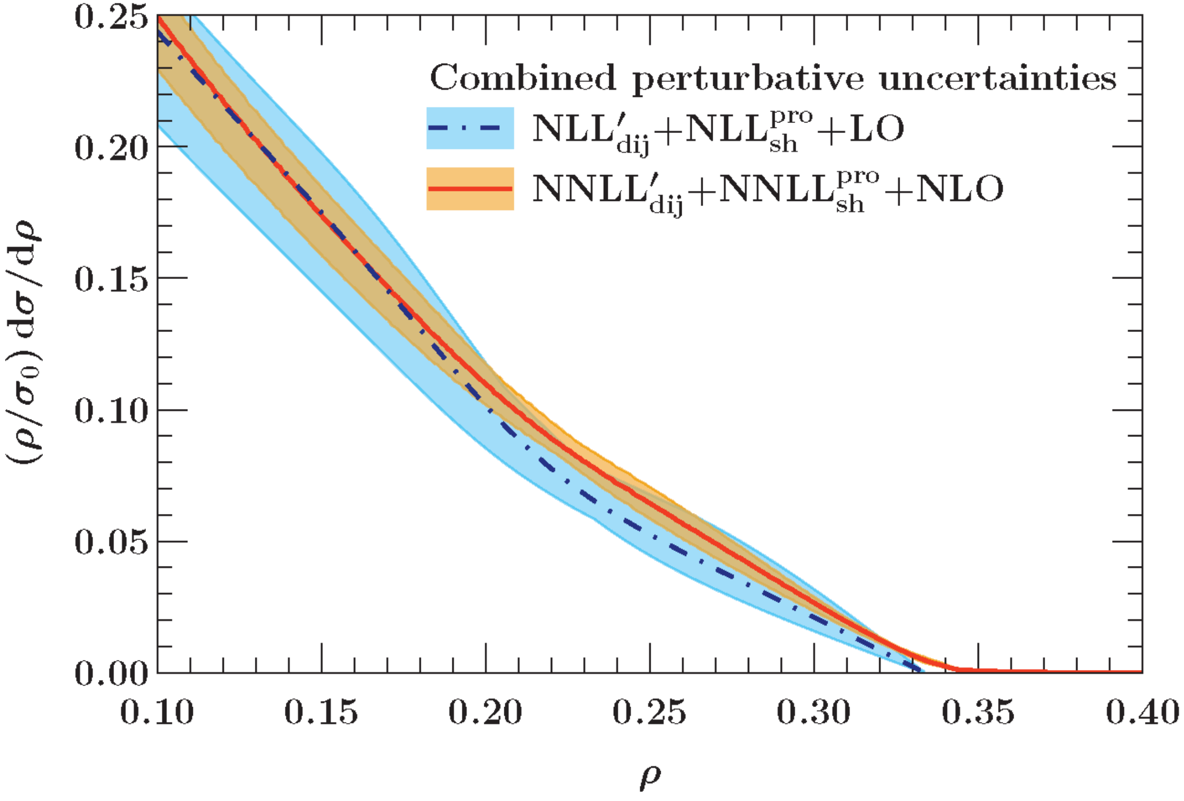
- Effect of shoulder resummation of HJM is $\sim 20\%$ near $\rho \geq 0.25$
- Profile scales to transition to dijet log regions

Sudakov Shoulders : Resummation

- Last piece : Match and ensure that distribution transitions to dijet logs as well

$$\frac{d\sigma^{\rm match}}{d\rho} = \frac{d\sigma^{\rm dij}(\mu_{\rm dij}^{\rm pro})}{d\rho} + \frac{d\sigma^{\rm sh}(\mu_{\rm sh}^{\rm pro})}{d\rho} + \left[\frac{d\sigma^{\rm FO}(\mu_{\rm FO})}{d\rho} - \frac{d\sigma^{\rm dij}(\mu_{\rm FO})}{d\rho} - \frac{d\sigma^{\rm sh}(\mu_{\rm FO})}{d\rho}\right]$$

Remove logs from FO to avoid double counting



Sudakov Shoulders : Three Jet Power Corrections

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- 2-jet power corrections validity is questionable near 3-jet configurations

2012.00622 Luisoni, Monni, Salam
2108.08897, 2204.02247 Caola et.al.

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Cumulative σ

$$\Sigma_{\mathrm{B+NP}}(v) - \Sigma_B(v) = \Sigma_B\left(v - \delta v\right) - \Sigma_B(v) = -\frac{\mathrm{d}\sigma_B}{\mathrm{d}v}\delta v, \qquad \delta v = H_{\mathrm{NP}}\zeta(v)$$

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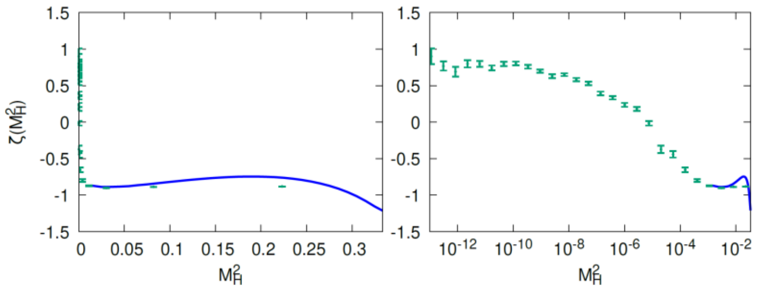

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Shift profile

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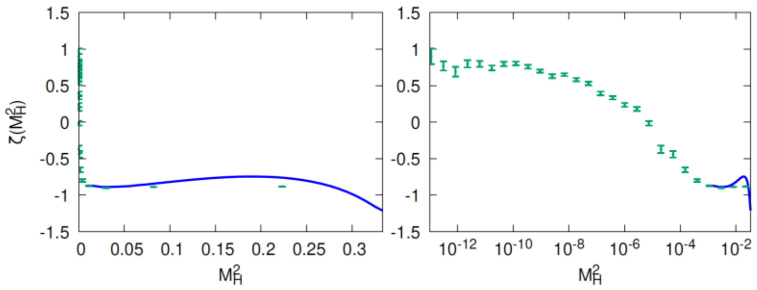

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Shift profile

- Mostly constant for thrust. Interestingly negative throughout for heavy jet mass
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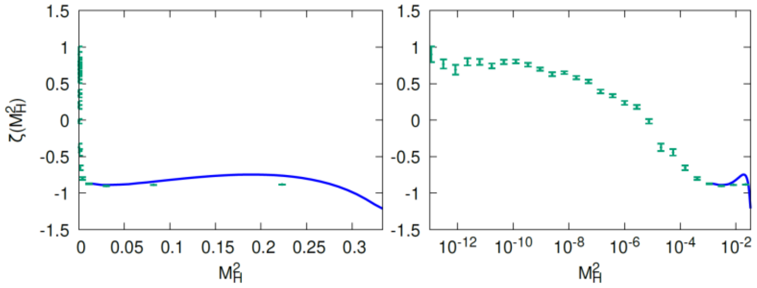

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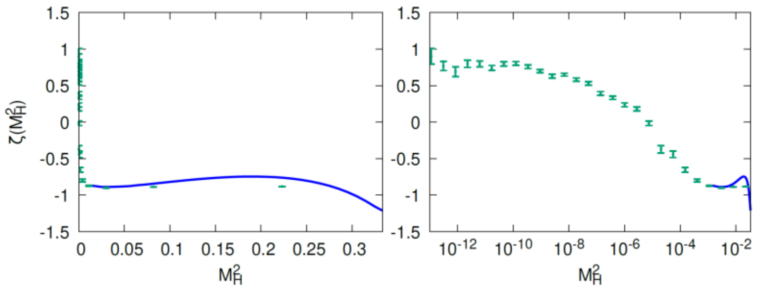

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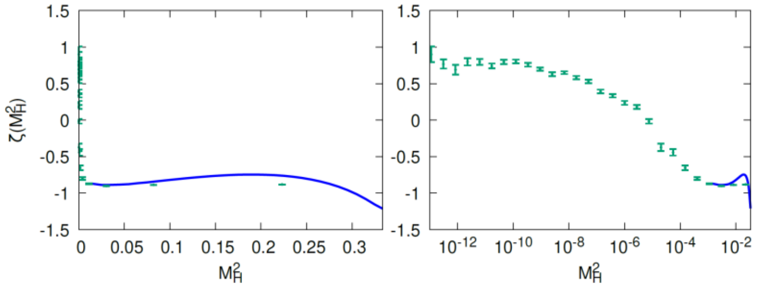

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Renormalon indication from trijet hemisphere soft function. Admits OPE

$$\ln S^{\rm NP}(z,\mu) = \ln S^{\rm P}(z,\mu) - \frac{iz}{Q}\Theta_1 + \cdots$$

AB, M.D.Schwartz, X. Zhang, Work in progress

$$\Longrightarrow \frac{{\rm d}^3\sigma^{\rm NP}}{{\rm d}\rho^3}\bigg|_r \sim \frac{{\rm d}^3\sigma^{\rm P}}{{\rm d}\rho^3}\bigg|_{(r-\frac{\Theta_1}{Q})}$$

3-jet NP parameter

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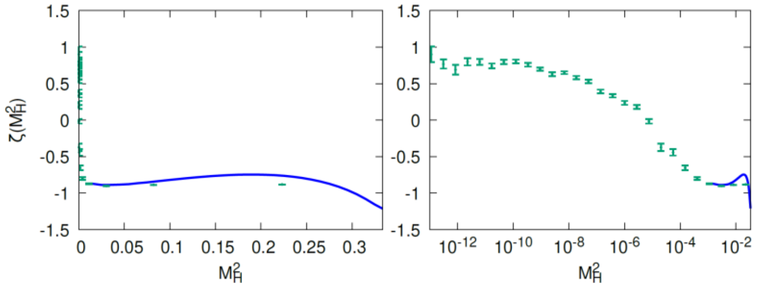

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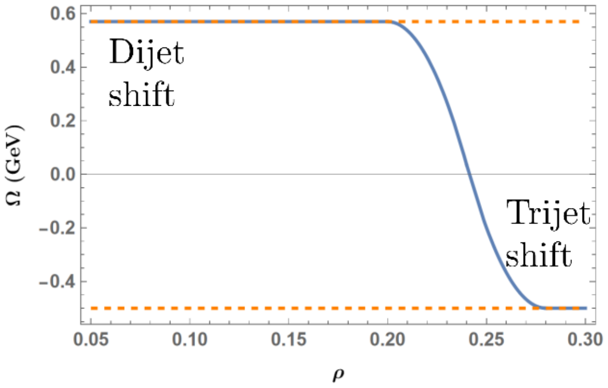

2502.12253 Benitez et.al

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AB, M.D.Schwartz, X. Zhang, Work in progress

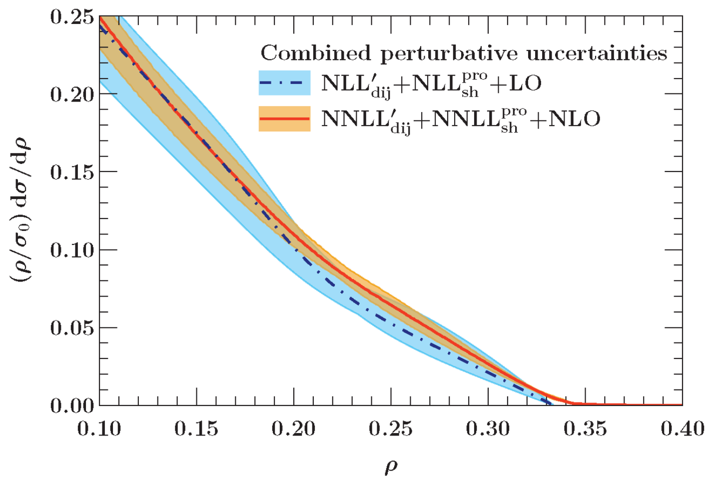
$$\Longrightarrow \left.\frac{\mathrm{d}^3\sigma^{\mathrm{NP}}}{\mathrm{d}\rho^3}\right|_r \sim \left.\frac{\mathrm{d}^3\sigma^{\mathrm{P}}}{\mathrm{d}\rho^3}\right|_{\left(r - \frac{\Theta_1}{Q}\right)}$$

3-jet NP parameter

Conclusions

- Important ingredient for global fit of α_s from HJM

More in Xiaoyuan's talk



- Sudakov Shoulders can be used to describe 3 jet regions in event shapes

Capture as much as possible of the partonic distribution
Hadronic Power Corrections using EFT Operators

Future

- Ongoing work : Renormalon analysis of trijet soft function to rigorously define $\boldsymbol{\Theta_1}$ **AB**, M.D.Schwartz, X. Zhang
- NNLL resummation of C parameter shoulder **AB**, X. Zhang Work in Progress
- Interesting NGL structure worth exploring!
- Shoulders are generic anytime event shape range is bounded by number of partons. Helps analyze multijet regions!

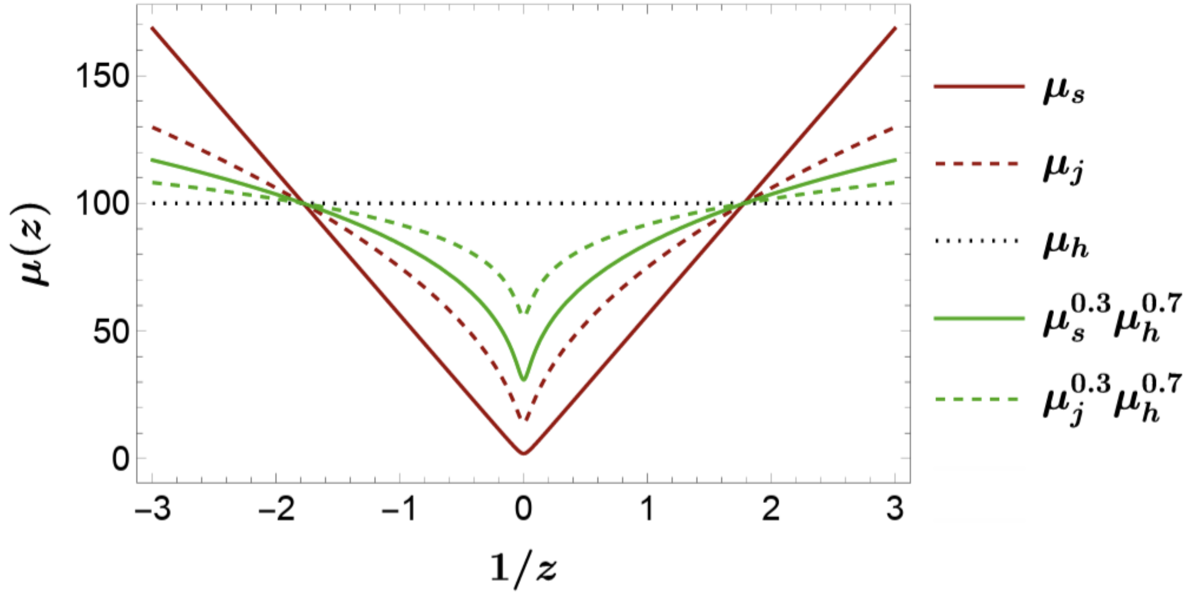
Thanks for listening! Questions?

Backup

Profile Functions

$$\mu_S^{\rm sh}(\rho,z) = [\mu_H]^{1-g(\rho)} \left[\sqrt{\left(2^{v_s}\frac{\mu_H e^{-\gamma_E}}{|z|}\right)^2 + \left(\mu_S^{\min}\right)^2}\,\right]^{g(\rho)}, \qquad \mu_J^{\rm sh} = [\mu_S^{\rm sh}]^{v_j}[\mu_H]^{1-v_j}.$$

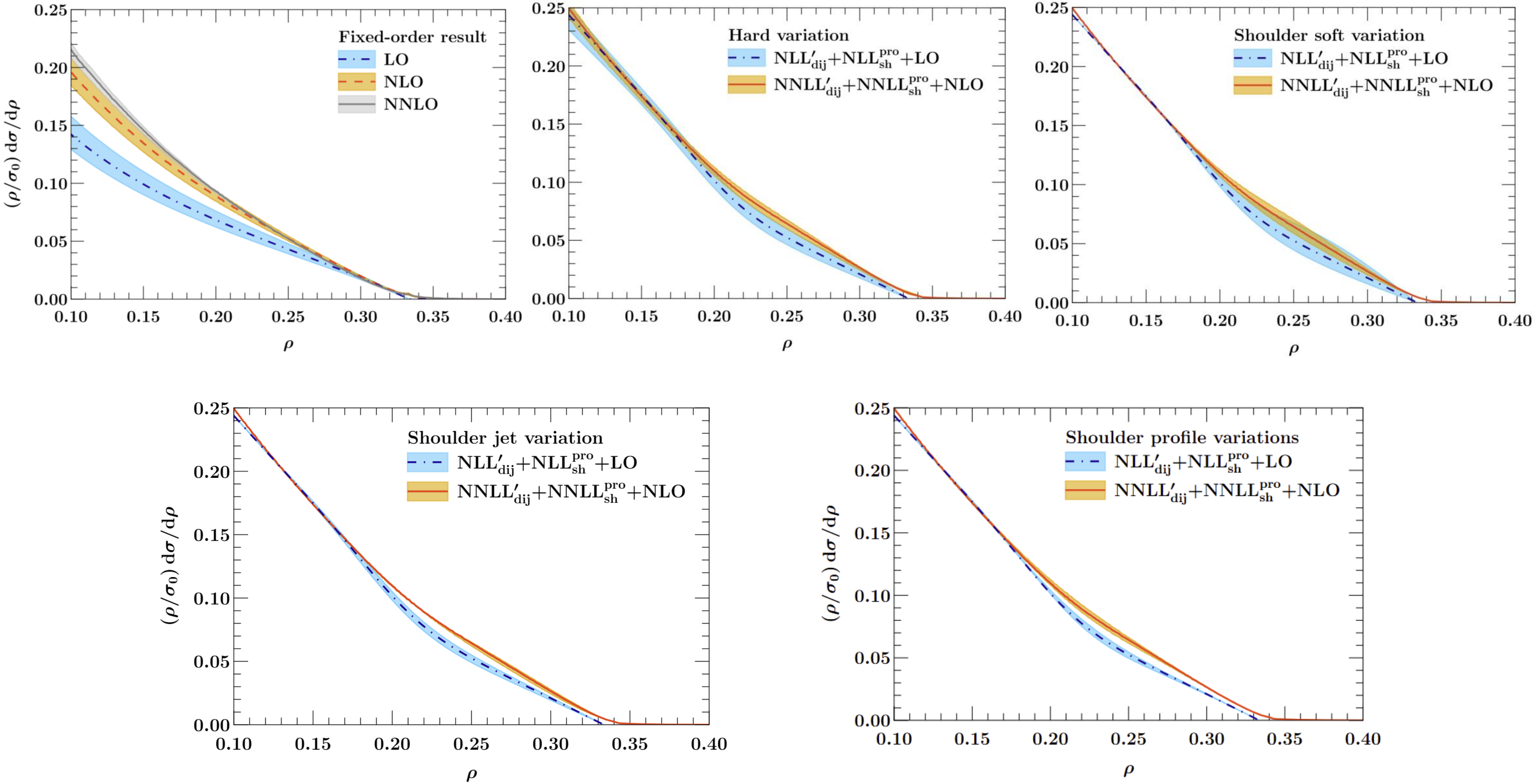
$$g(\rho) = \begin{cases} 0 & 0 \le \rho < \rho_{L1} \\ \zeta(0,0,1,0,\rho_{L1},\rho_{L2},\rho) & \rho_{L1} \le \rho < \rho_{L2} \\ 1 & \rho_{L2} \le \rho < \rho_{R1}\,, \\ \zeta(1,0,0,0,\rho_{R1},\rho_{R2},\rho) & \rho_{R1} \le \rho < \rho_{R2} \\ 0 & \rho_{R2} \le \rho < 0.5 \end{cases}$$



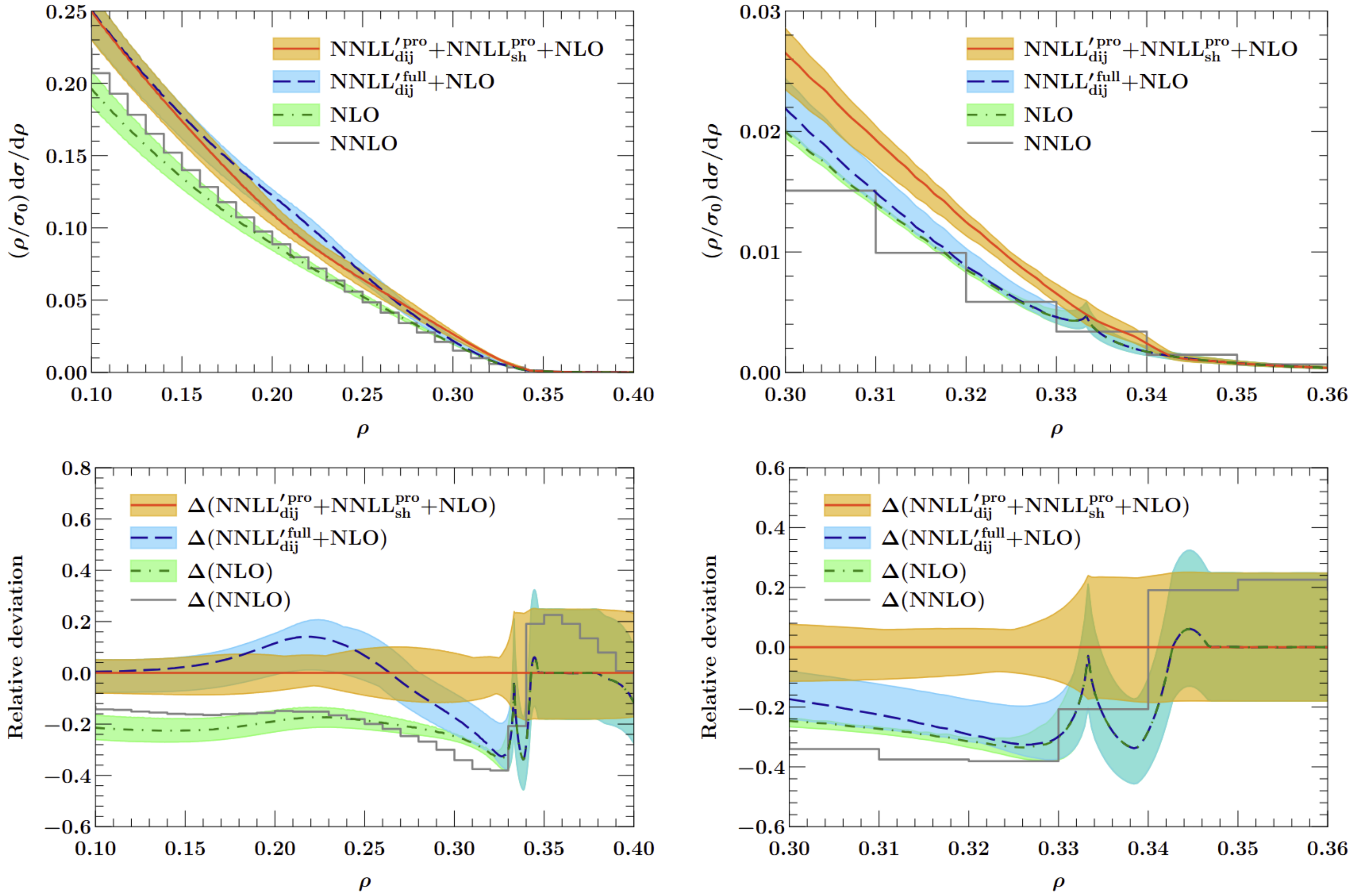
Shoulder Scales

Parameter	Default	Range
v_h	-1	$[-2,\ 0]$
v_s	0	$[-1,\ 1]$
v_j	0.5	$[0.4,\ 0.6]$
ρ_{L1}	0.20	$[0.17,\ 0.23]$
ρ_{L2}	0.28	$[0.25,\ 0.31]$
ρ_{R1}	$1/3$	–
ρ_{R2}	0.342	–

Scale Variations



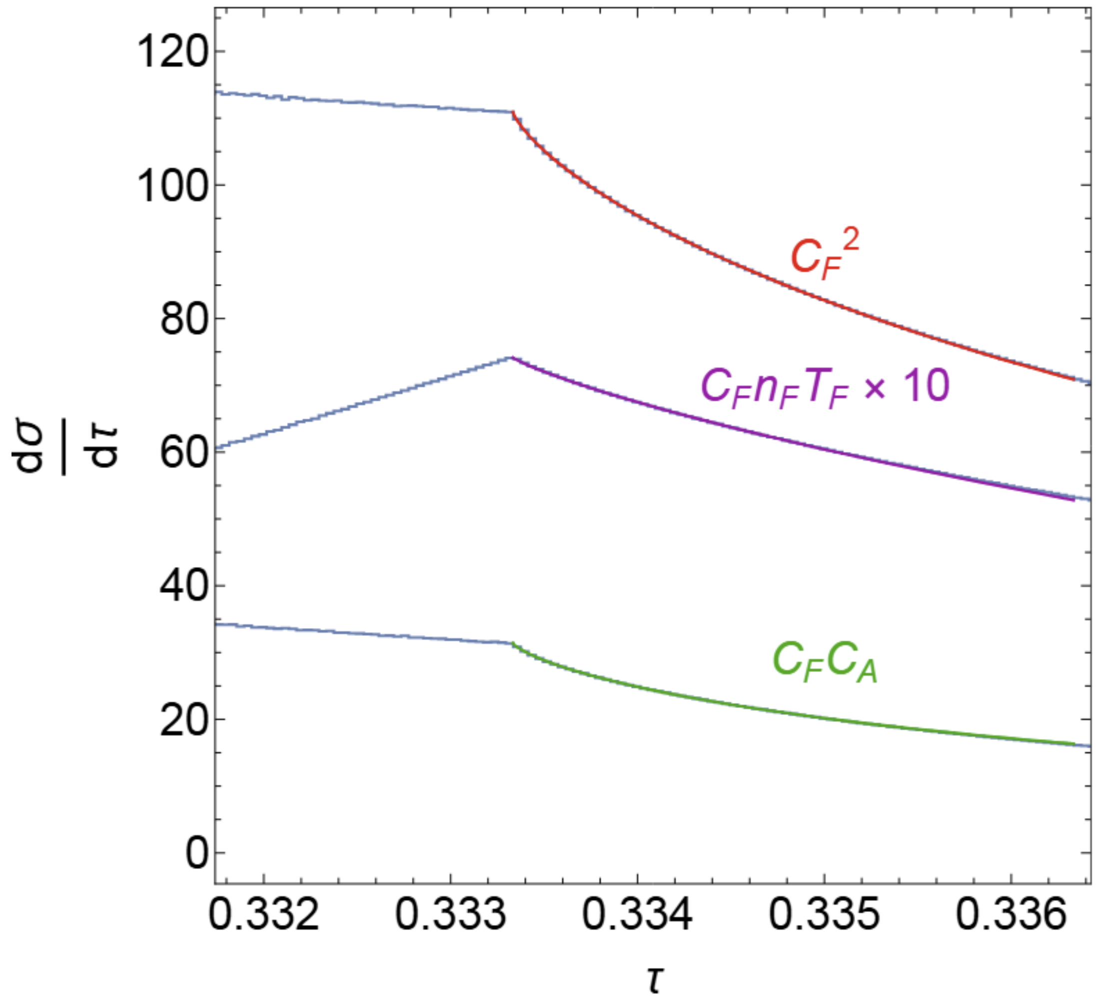
Relative Contributions



Sudakov Shoulders Thrust

$$\frac{1}{\sigma_{\rm LO}}\frac{d\sigma^{\rm sing}}{dt} = \frac{\alpha_s}{4\pi}\left\{-6(2C_F + C_A)t\ln^2 t + \Big[6C_F(1-4\ln 3) + C_A(1-12\ln 3) + 4n_f T_F\Big]t\ln t\right\} + \mathcal{O}(\alpha_s^2)$$

$$t \equiv \tau - \tfrac{1}{3}$$



Sudakov Shoulders HJM Slope

$$\frac{1}{\sigma_{\mathrm{LO}}}\frac{d\sigma_i}{d\rho} = 2\ \mathrm{Re}\left[\int_0^\infty \frac{dz}{2\pi} K(z,r)\widetilde{\sigma}_i(z)\right]$$

Integration kernel to transform between spaces

$$K(z,r) = \frac{1}{z^2}\left[1 - e^{-izr} + r(1 - e^{iz})\right]$$

Choice set to get LO cross section $r\theta(r)$ when $\tilde{\sigma}(z) = 1$

Factorization controls difference between slopes in singular parts

Set sum of slopes (non-singular) is obtained from FO