

# Alpha-s 2025 Workshop

## Precise Determination of the Strong Coupling Constant from Dijet Cross Sections up to the Multi-TeV Range

K. Rabbertz



Based on:  
Phys. Rev. Lett. 135, 031903 (2025)

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João Pires<sup>10,11</sup>, Klaus Rabbertz<sup>4</sup> and Mark Sutton<sup>12</sup>

# *Outline*



- **Motivation**
- **Theory**
- **Datasets**
- **Fit Methodology**
- **Results**
- **Summary**



## Coupling constants of the SM

- Great precision for  $\alpha$  or  $G_F$
- Least known SM parameter  $\alpha_s$ 
  - ➔ Enters all LHC processes
  - ➔ Significant source of uncertainty

### Electromagnetic

$$\alpha \approx 1/137$$

$$\Delta\alpha/\alpha = 0.15 \cdot 10^{-9}$$

### Weak

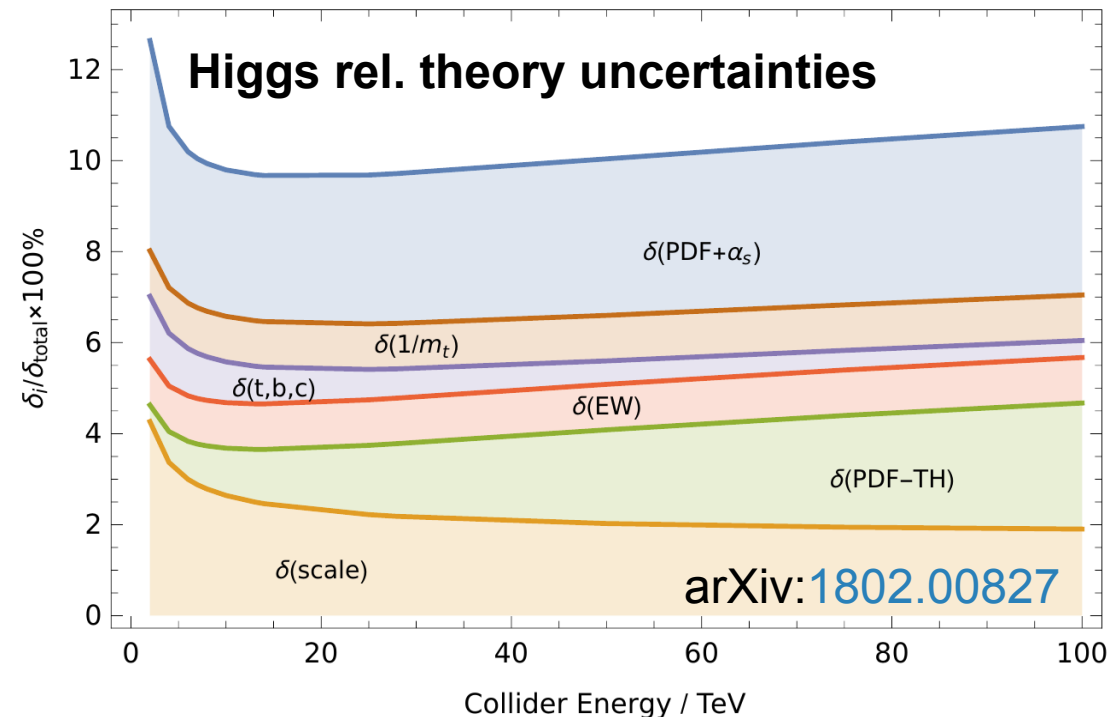
$$G_F \approx 1.17 \cdot 10^{-5} / \text{GeV}^2$$

$$\Delta G_F / G_F = 0.51 \cdot 10^{-6}$$

### Strong

$$\alpha_s \approx 0.118$$

$$\Delta\alpha_s/\alpha_s = 0.76 \cdot 10^{-2}$$



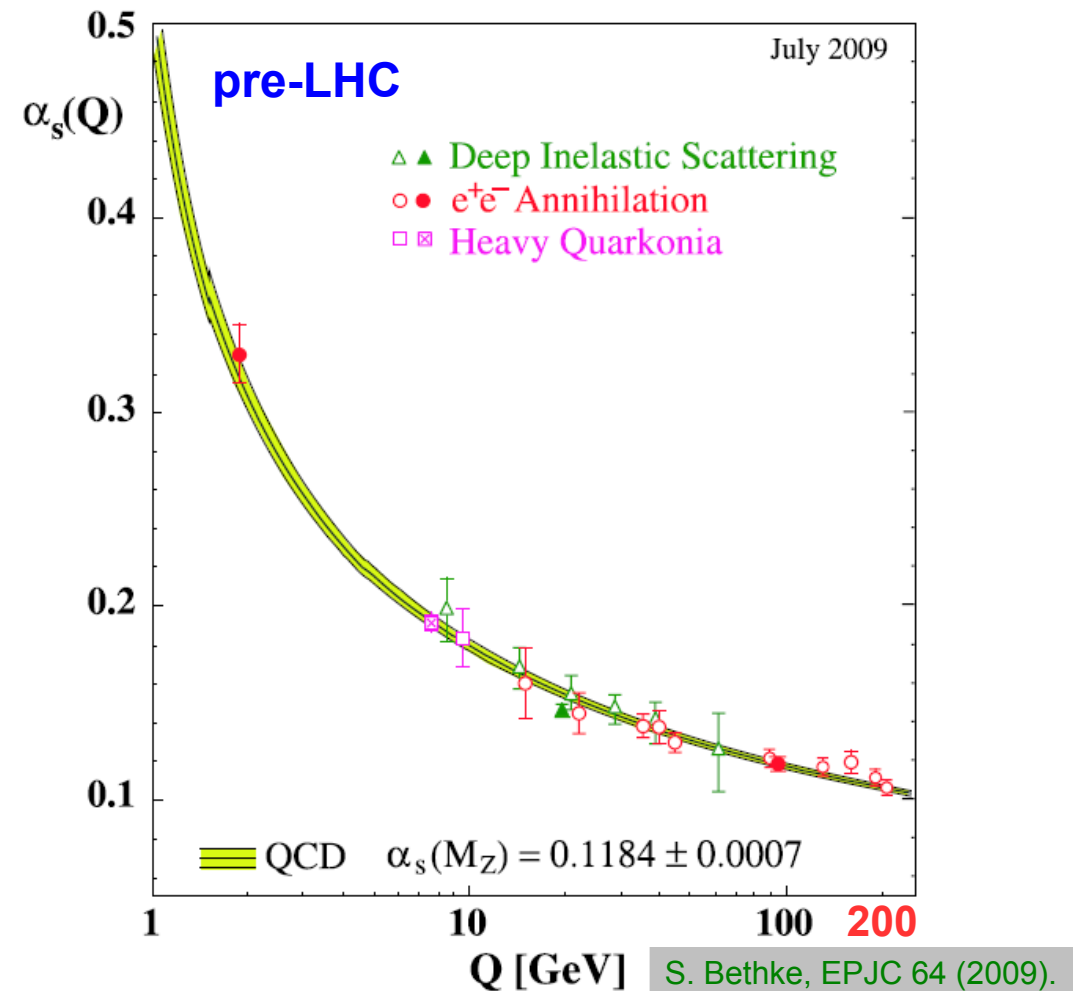


## ● Coupling constants of the SM

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- Least known SM parameter  $\alpha_s$ 
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  - ➔ Significant source of uncertainty
- Scale dependence predicted
  - ➔ Running of  $\alpha_s$  tested only up to  $\sim 200$  GeV pre LHC
  - ➔ Need highest-energy probe to further advance into asymptotic regime

## Nobel prize 2004

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \ln\left(\frac{Q^2}{\mu^2}\right)}$$





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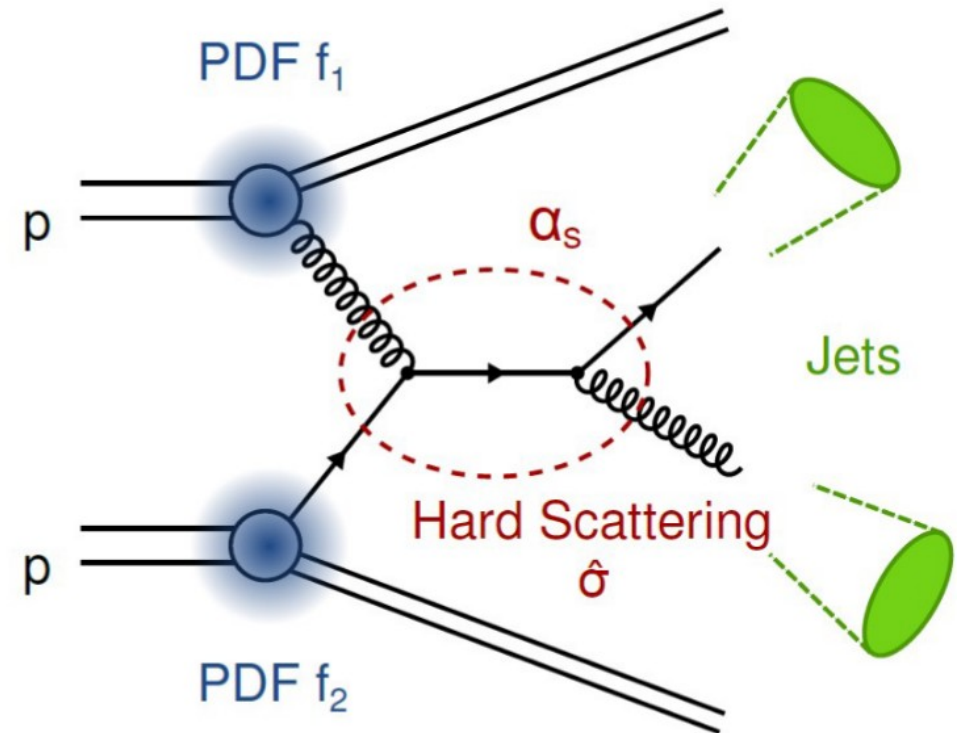
## ● Latest progress

- NNLO accuracy for predictions of jet cross-sections

M. Czakon et al., JHEP10 (2019) 262;  
M. Czakon et al., Phys. Rev. Lett. 127 (2021) 152001;  
X. Chen et al., JHEP09 (2022) 025, JHEP10 (2022) 040.

→ tomorrow's talk by R. Poncelet on 3-jets & today's talks by V. Guglielmi & C. Glasman

## Dijet production at the LHC



Huge cross section of dijet production

→ large reach in energy scale

Sensitive to  $\alpha_s(m_Z)$  already at LO pert. QCD

All partonic processes present already at LO

→ good convergence



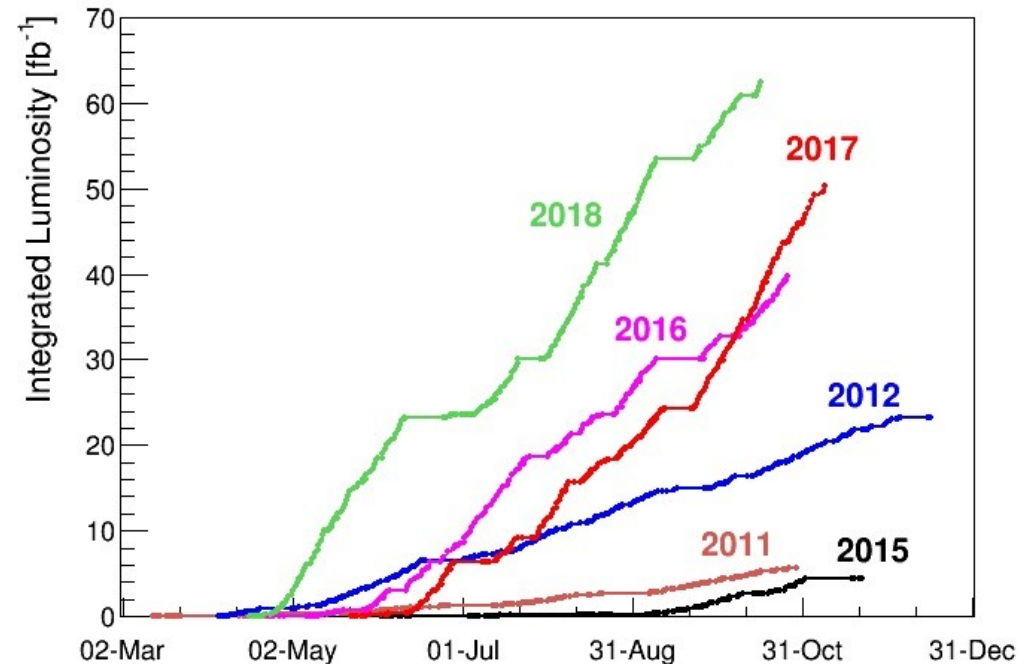
## ● Coupling constants of the SM

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## ● Latest progress

- NNLO accuracy for predictions of jet cross sections
- Huge LHC datasets at 13 TeV (and 13.6 TeV)

## LHC luminosity of Run 1 & 2



CERN

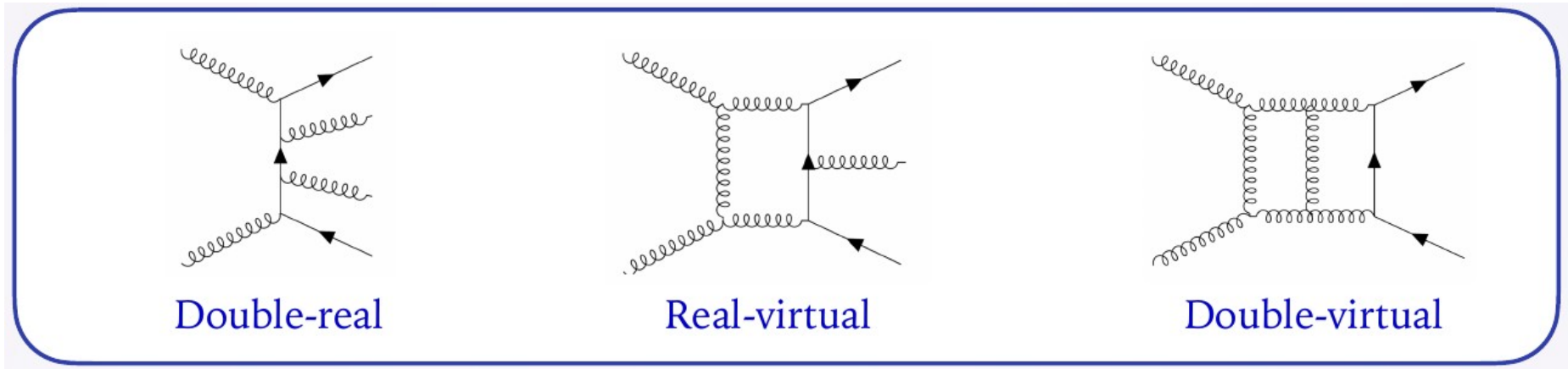
Total luminosity Run 2 at 13 TeV: **160 fb<sup>-1</sup>**

(Run 3 to finish by June 2026 ...

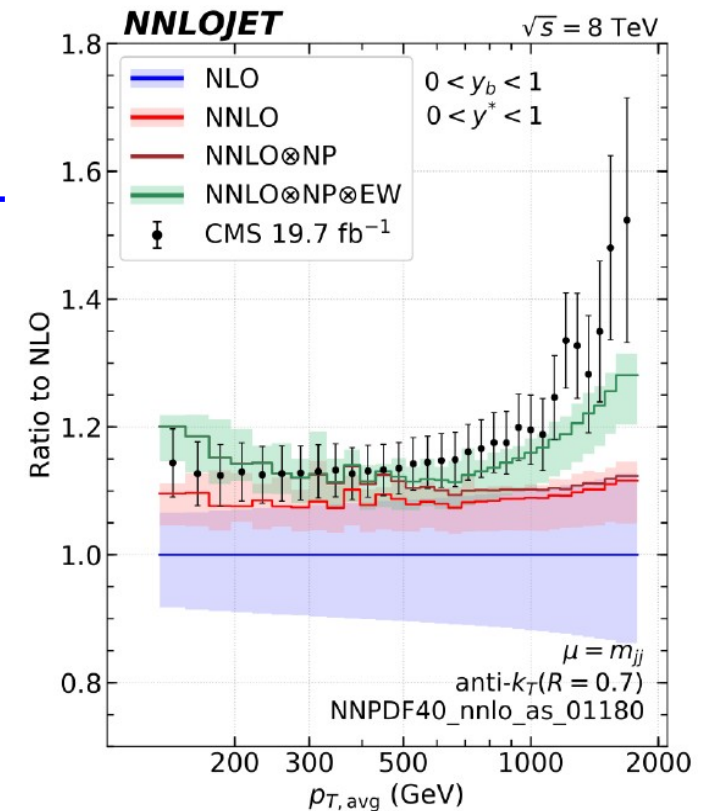
Up to now:  **$\sim 318$  fb<sup>-1</sup>** at 13.6 TeV)



# NNLO corrections for dijet process



- ➔ **NNLO parton-level cross sections** obtained with **NNLOJET**  
(pp) [arXiv:1905.09047](https://arxiv.org/abs/1905.09047), [1705.10271](https://arxiv.org/abs/1705.10271);  
(ep) [arXiv:1703.05977](https://arxiv.org/abs/1703.05977), [1606.03991](https://arxiv.org/abs/1606.03991)  
[A. Huss et al., arXiv:2503.22804](https://arxiv.org/abs/2503.22804).
- ➔ **Reduced** renormalization and factorization scale dependence
- ➔ Include **sub-leading colour contributions**  
→ used in **this work** for the first time, in  $\alpha_s$  determination with LHC data ([arXiv:1907.12911](https://arxiv.org/abs/1907.12911), [2204.10173](https://arxiv.org/abs/2204.10173), [2208.02115](https://arxiv.org/abs/2208.02115))

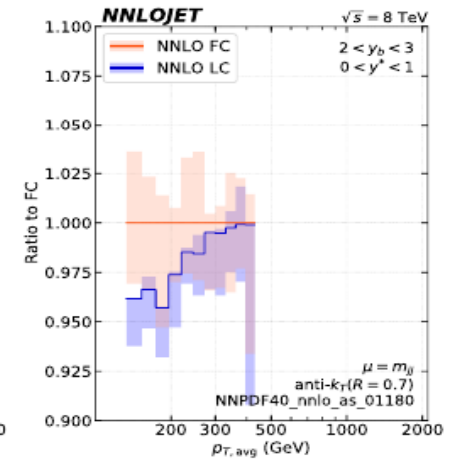
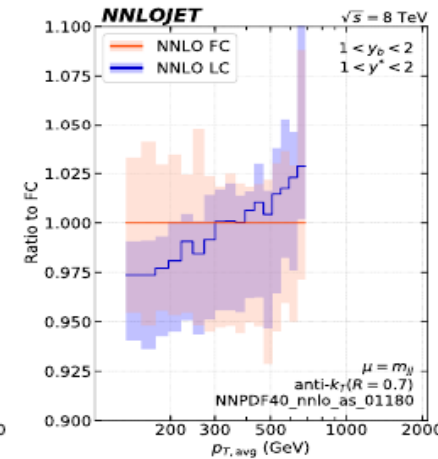
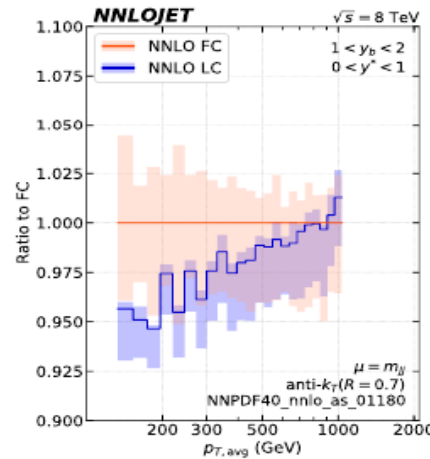
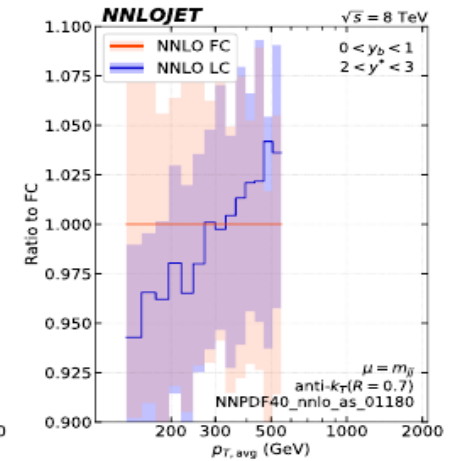
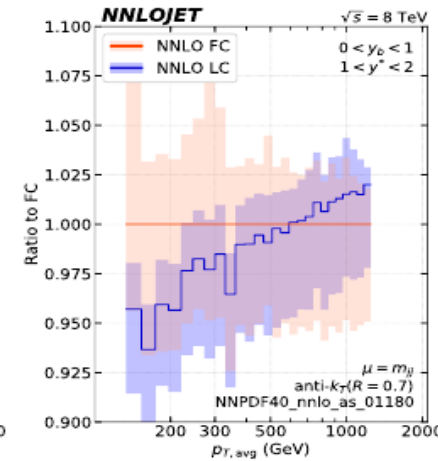
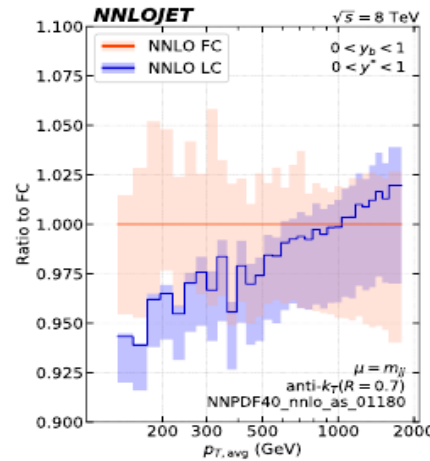
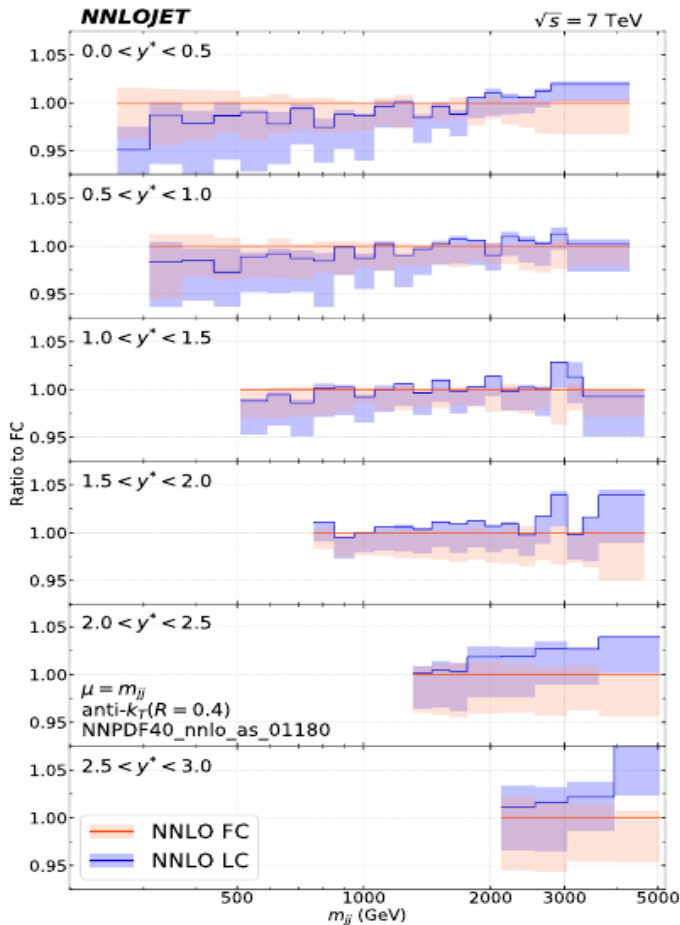




# Sub-leading colour contributions

- ➔ Sub-leading colour corrections are small  
→ but can be important for dijet production and precision determinations of  $\alpha_s$

used in this work for the first time in  $\alpha_s$  determination with **LHC jet data**



**ATLAS 7TeV and CMS 8 TeV NNLO dijet predictions: leading color vs. full color (arXiv:2204.10173)**



- Fit algorithm for  $\alpha_s$  requires (re-)calculation of cross sections for varying  $\alpha_s(M_Z)$ , PDFs, scales
- NNLOJET interfaced to APPLfast library for fast recomputation
- APPLfast: Generic interface between NNLOJet and both APPLgrid and fastNLO
- ➕ pQCD coefficients stored in interpolation grids with
  - ➔ typically sub-permille accuracy in reproducing full calculation and
  - ➔ percent level statistical precision of numerical integrations in NNLOJet
- New fastNLO grids have been submitted to Ploughshare and are available in both formats (2 CMS grids waiting for more server disk space ...)

Britzger et al., EPJC 79 (2019) 845, EPJC 82 (2022) 930.

APPLgrid, Carli et al., Eur. Phys. J. C, 2010, 66, 503.  
fastNLO, Britzger et al., arXiv:1208.3641.



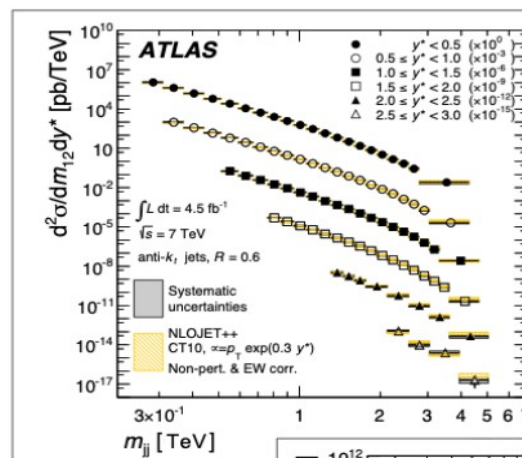
<https://ploughshare.web.cern.ch/>



# LHC dijet data

► Abundant production of jets, typically well described by pQCD calculations

Data	$\sqrt{s}$ [TeV]	$d\sigma$	$R$	$\mathcal{L}$
ATLAS	7	$\frac{d^2\sigma}{dm_{jj}dy^*}$	0.6	$4.5 \text{ fb}^{-1} \pm 1.8\%$
CMS	7	$\frac{d^2\sigma}{dm_{jj}dy_{\text{max}}}$	0.7	$5.0 \text{ fb}^{-1} \pm 2.2\%$
CMS	8	$\frac{d^3\sigma}{d(p_T)_{1,2}dy^*dy_b}$	0.7	$19.7 \text{ fb}^{-1} \pm 2.6\%$
ATLAS	13	$\frac{d^2\sigma}{dm_{jj}dy^*}$	0.4	$3.2 \text{ fb}^{-1} \pm 2.1\%$
CMS	13	$\frac{d^2\sigma}{dm_{jj}dy_{\text{max}}}$	0.8	$33.5 \text{ fb}^{-1} \pm 1.2\%$
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arXiv:1312.3524

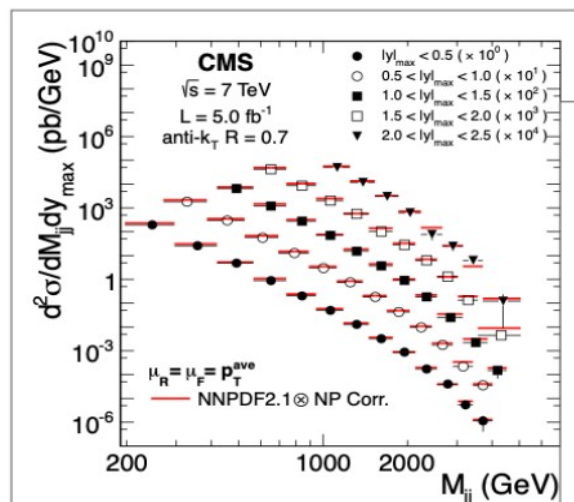
$$y^* = |y_1 - y_2|/2$$

$$y_b = |y_1 + y_2|/2$$

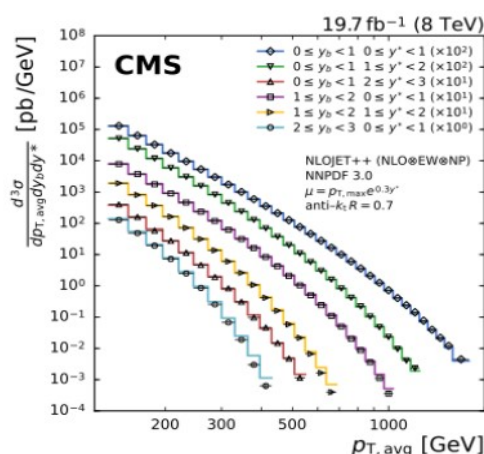
arXiv:1711.02692

used for additional fit variant

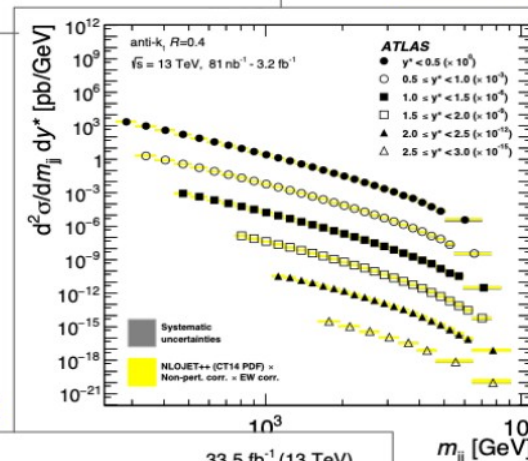
(2D and 3D cannot be used simultaneously due to unknown correlations)



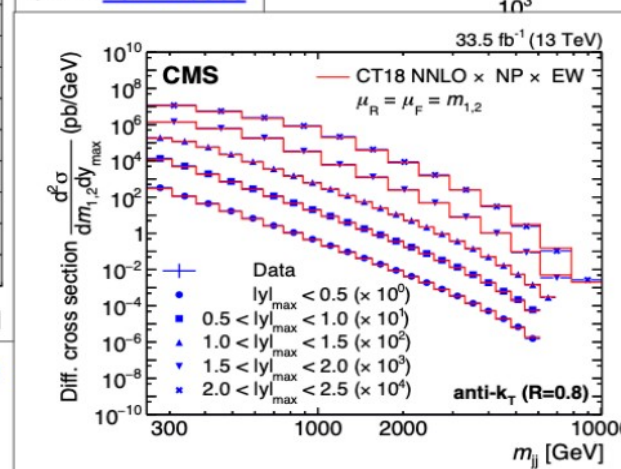
arXiv:1212.6660



arXiv:1705.02628



arXiv:2312.16669



selection:  $y_{\text{max}}$  or  $y^* = |y_1 - y_2|/2 < 2$ ;  $y_b = |y_1 + y_2|/2 < 1$ :

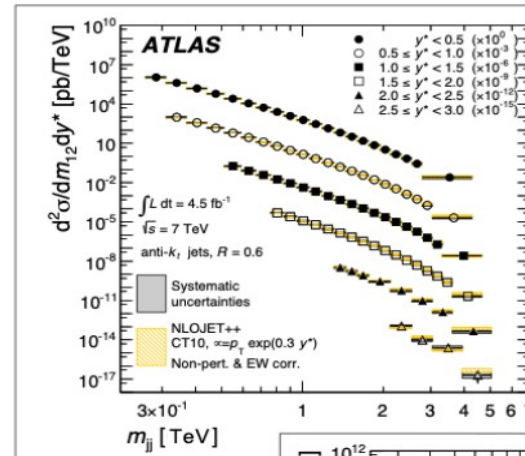
367 LHC cross sections used in  $\alpha_s$  determination



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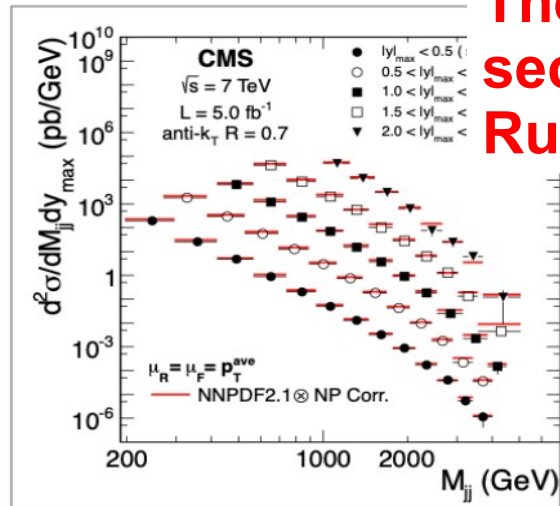
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$$y^* = |y_1 - y_2|/2$$

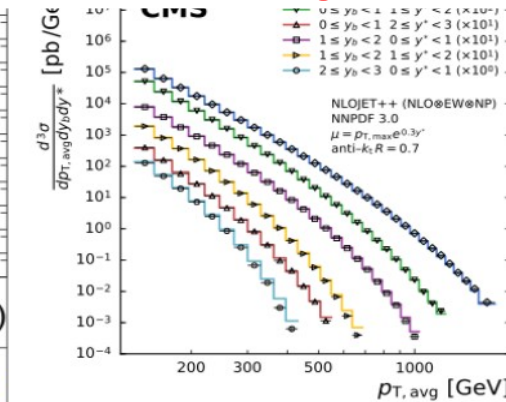
$$y_b = |y_1 + y_2|/2$$

arXiv:1711.02692

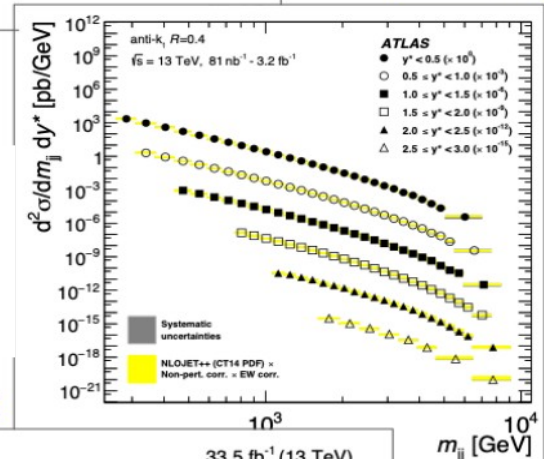
These are all LHC dijet cross sections available at the time: Run 1 and only part of Run 2!



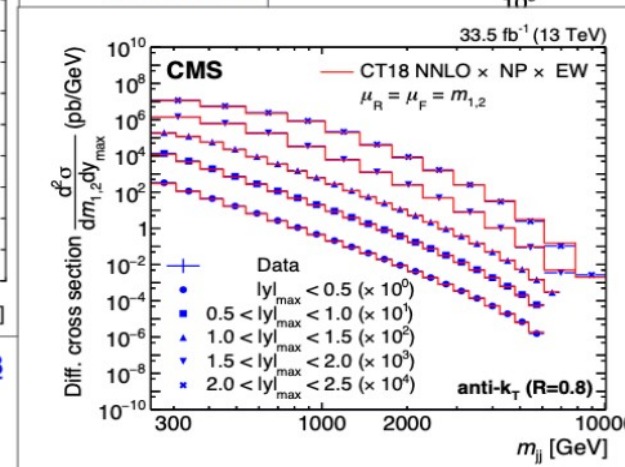
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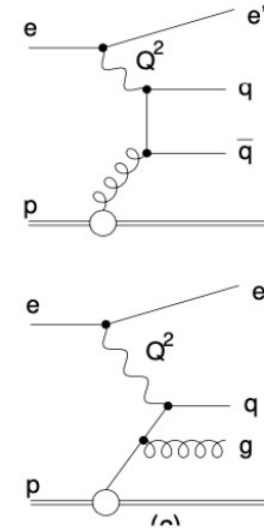
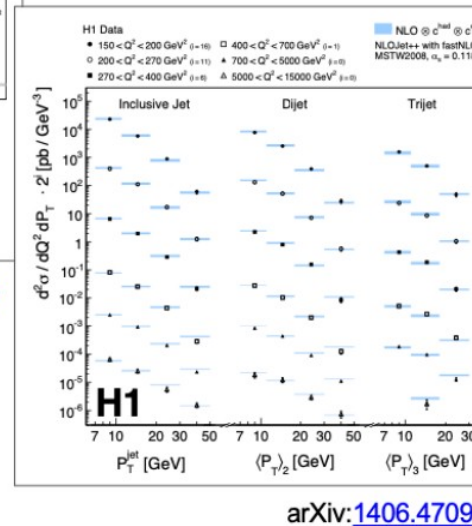
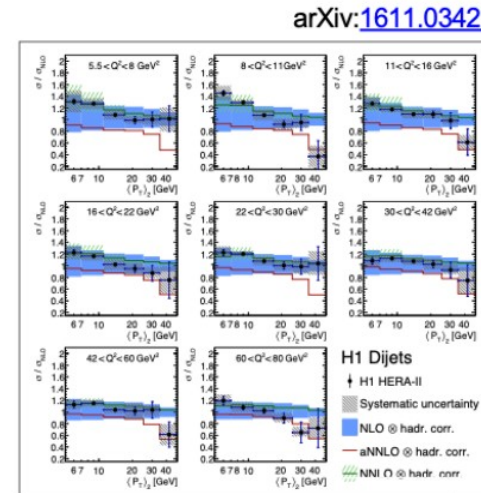
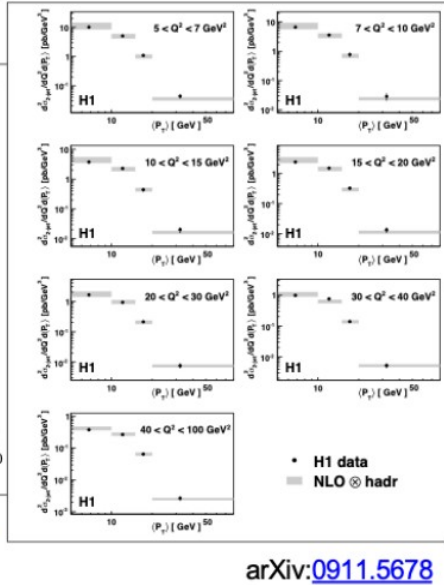
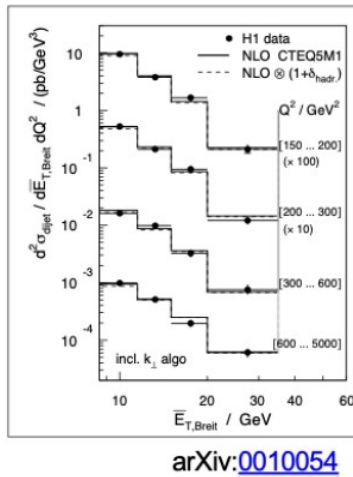


# HERA dijet data

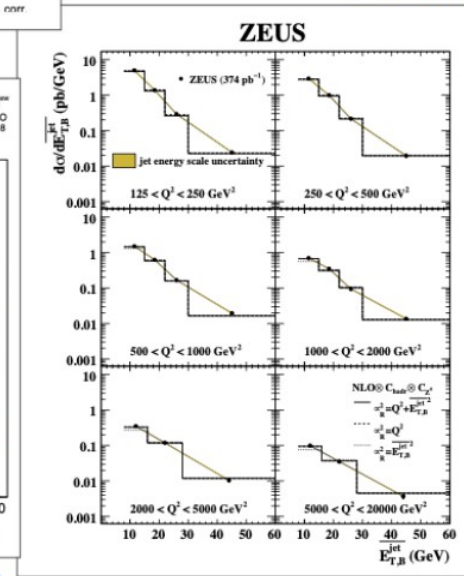
- Analysis further extended including data for dijet production in neutral-current DIS taken at the HERA  $ep$  collider

Data set	$\sqrt{s}$ [GeV]	Cuts
H1 300 GeV high- $Q^2$ [15]	300	-
H1 HERA-I low- $Q^2$ [16]	320	$\mu > 2m_b$
H1 HERA-II low- $Q^2$ [19]	320	$\mu > 2m_b$
H1 HERA-II high- $Q^2$ [18]	320	-
ZEUS HERA-I+II high- $Q^2$ [17]	320	$\langle p_T \rangle_{1,2} > 15$ GeV

$k_t$  with  $R=1.0$



arXiv:1010.6167



- Used in previous  $\alpha_s$  extractions at NLO and/or NNLO  $\rightarrow$  well understood data and theory

119 HERA cross sections used in  $\alpha_s$  determination



## Determination of $\alpha_s(M_Z)$ in $\chi^2$ minimisation (MIGRAD)

$$\chi^2 = \sum_{i,j} \log \frac{S_i}{\sigma_i} (V_{\text{exp}} + V_{\text{NP}} + V_{\text{NNLOstat}} + V_{\text{PDF}})_{ij}^{-1} \log \frac{S_j}{\sigma_j}$$

$\zeta_i$	LHC or HERA jet data
$\sigma_i$	NNLO theory
$V$	covariance matrices

## Cross section sensitive to $\alpha_s$

$$d\sigma = \sum_{a,b} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} f_a(x_1, \mu_F) f_b(x_2, \mu_F) d\hat{\sigma}_{ab}(\mu_R, \mu_F) \cdot C_{\text{NP}} \cdot C_{\text{EW}}$$

↑ pdfs
↪ matrix element (ME) / partonic cross section
↖ non-perturbative (NP) and electroweak (EW) correction factors

where both the pdfs and ME depend on  $\alpha_s$ :

$$d\hat{\sigma}_{ab}(\alpha_s) = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 d\hat{\sigma}_{ab,\text{LO}} + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 d\hat{\sigma}_{ab,\text{NLO}} + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^4 d\hat{\sigma}_{ab,\text{NNLO}} + \mathcal{O}(\alpha_s^5(\mu))$$

$$f_a(x, \mu, \alpha_s) = (\Gamma(\mathcal{P}, \mu, \mu_0, \alpha_s) \otimes f_{\mu_0})_a \quad \text{where } \Gamma \text{ are the DGLAP kernels evaluated at 3-loop with 5 active flavours using Apfel++}$$

## Starting scale of PDF evolution with DGLAP set to $\mu_0 = 90 \text{ GeV}$

## x dependence of $f(x, \mu_0)$ obtained from PDF4LHC21

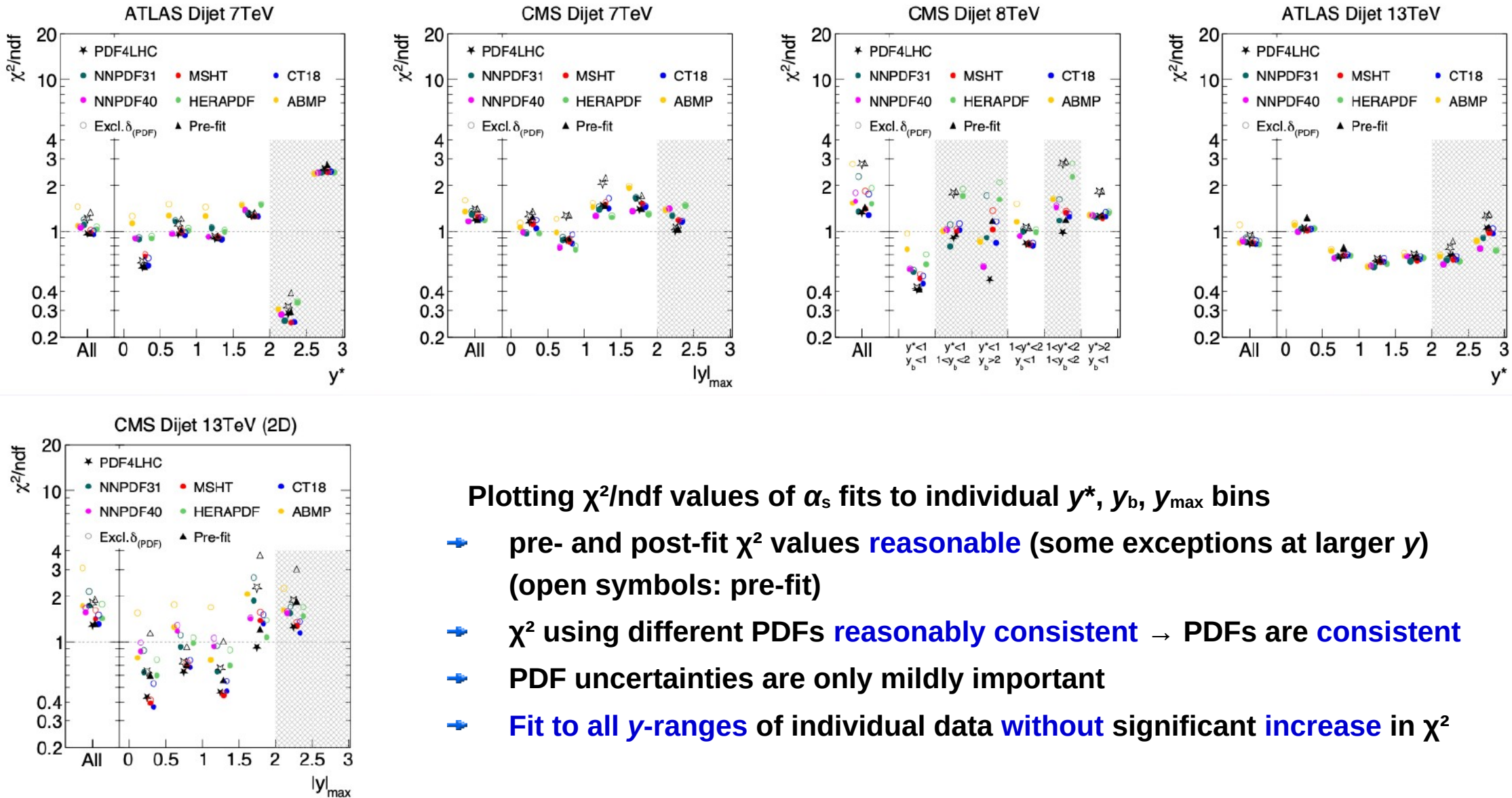


- $\alpha_s(m_Z)$  fit uncertainties: (fit,PDF)
- (Encoded in covariance matrices  $V$  and consist of...)
  - **Experimental (exp)**: including stat. and syst. correlations as reported by the experiments
  - **Non-perturbative (NP)**: provided by the LHC collaborations (using different MC models)
  - **NNLO statistical**: arising from MC integration in NNLOJET (typically at percent level or less)
  - **PDF uncertainties**: obtained from the relevant PDF set and evaluated at  $\mu_0$
- Starting scale of the PDF evolution: ( $\mu_0$ )
  - Varied by **factors of 0.5 and 2** about the nominal value of  $\mu_0=90$  GeV
  - Also covers uncertainty arising from  $\alpha_s(M_Z)$  variation in the original PDF
- Scale uncertainties: ( $\mu_R, \mu_F$ )
  - **Renormalization  $\mu_R$  and factorization scale  $\mu_F$**  varied by factors of 0.5, 1 or 2: **7-point variation**
  - Nominal scales: **LHC  $\mu_R=\mu_F=m_{jj}$** ; **HERA  $\mu_R=\mu_F=\sqrt{Q^2+\langle p_T \rangle^2}_{1,2}$**
  - Scale variations for LHC and HERA considered uncorrelated (added in quadrature for HERA+LHC fits)



# Consistency study

## Comprehensive study assessing consistency of NNLO predictions and data



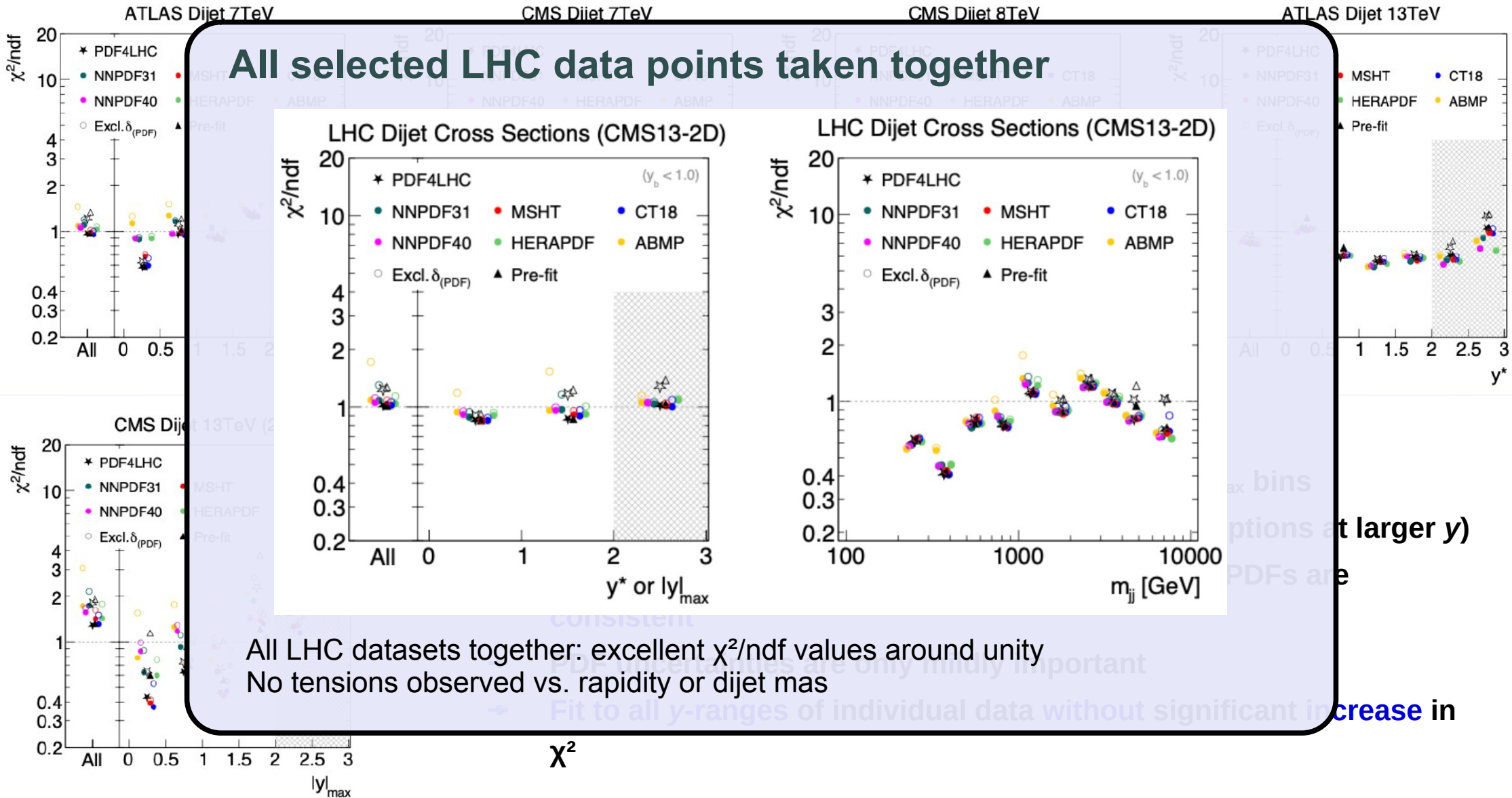
### Plotting $\chi^2/\text{ndf}$ values of $\alpha_s$ fits to individual $y^*$ , $y_b$ , $y_{\text{max}}$ bins

- ➡ pre- and post-fit  $\chi^2$  values **reasonable** (some exceptions at larger  $y$ ) (open symbols: pre-fit)
- ➡  $\chi^2$  using different PDFs **reasonably consistent** → PDFs are **consistent**
- ➡ PDF uncertainties are only mildly important
- ➡ **Fit to all  $y$ -ranges** of individual data **without significant increase in  $\chi^2$**



# Consistency study

## Comprehensive study assessing consistency of NNLO predictions and data





# Fit results

- Perform fits to
  - each individual data set
  - all LHC data sets (with either CMS 2D or 3D)

$$\alpha_s(m_Z) (\text{fit, PDF}) (\mu_0) (\mu_R, \mu_F)$$

- $\alpha_s$  from individual data sets

- all fits with good  $\chi^2$
- exp.+PDF uncertainties:  $\pm 0.0020$  to  $\pm 0.0033$
- data sets with larger integrated luminosities or higher  $\sqrt{s}$  yield smaller uncertainties
- $\alpha_s$  determination using all 5 LHC dijet datasets yields reduced experimental uncertainties (benefits from independent measurements, extended kinematic ranges and multiple  $\sqrt{s}$ )

Data set	$\chi^2/n_{\text{dof}}$	$\alpha_s(m_Z)$
ATLAS 7 TeV	74.7/ 77	0.1193 (33) (4) ( 6)
ATLAS 13 TeV	87.7/106	0.1145 (32) (4) (16)
CMS 7 TeV	50.7/ 45	0.1151 (39) (1) ( 9)
CMS 8 TeV	37.0/ 56	0.1173 (25) (1) (11)
CMS 13 TeV (2D)	71.6/ 78	0.1209 (25) (2) (20)
CMS 13 TeV (3D)	137.7/112	0.1181 (20) (1) (15)
LHC dijets (CMS13-2D)	335.3/366	0.1178 (14) (1) (17)
LHC dijets (CMS13-3D)	397.9/400	0.1172 (14) (1) (14)



# Fit results

## ● Perform fits to

- ➔ each individual data set
- ➔ all LHC data sets (with either CMS 2D or 3D)

$$\alpha_s(m_Z) (\text{fit, PDF}) (\mu_0) (\mu_R, \mu_F)$$

## ● $\alpha_s$ from individual data sets

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## ● $\alpha_s$ including HERA + LHC data

- ➔ good consistency between LHC and HERA
- ➔ further reduced experimental uncertainties
- ➔ increased scale uncertainty (since HERA is at lower scale)

Data set	$\chi^2/n_{\text{dof}}$	$\alpha_s(m_Z)$
HERA	92.8/118	0.1177 (14) (1) (34)
LHC+HERA (CMS13-2D)	428.4/485	0.1180 (10) (1) (22)
LHC+HERA (CMS13-3D)	491.0/519	0.1177 (10) (1) (24)



# Comparison to PDG24

## LHC dijet data

→ excellent consistency with world average

## Main $\alpha_s(m_Z)$ results

→ all 5 LHC dijet data sets taken together (using CMS 13TeV 2D)

$$\alpha_s(m_Z) = 0.1178 (14)_{(\text{fit,PDF})} (1)_{(\mu_0)} (17)_{(\mu_R, \mu_F)}$$

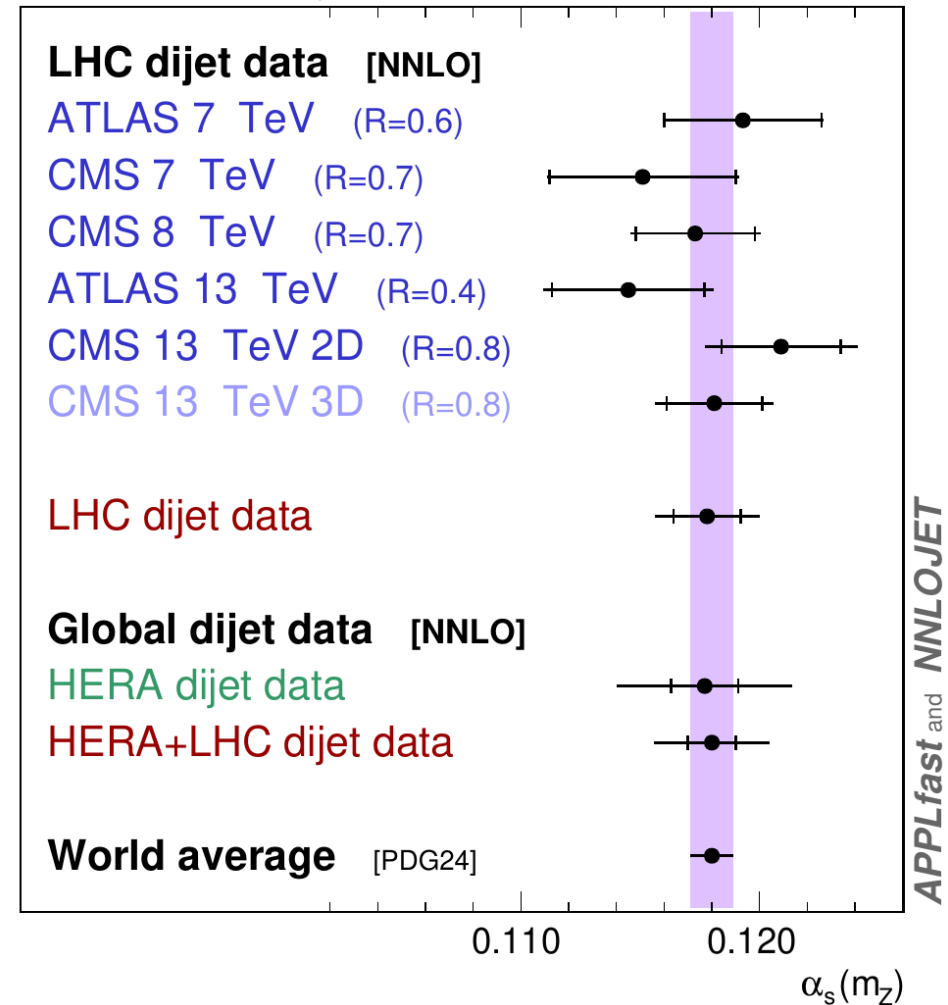
→  $\alpha_s(m_Z)$  using HERA dijet data

$$\alpha_s(m_Z) = 0.1177 (14)_{(\text{fit,PDF})} (1)_{(\mu_0)} (34)_{(\mu_R, \mu_F)}$$

→  $\alpha_s(m_Z)$  using LHC and HERA dijet data

$$\alpha_s(m_Z) = 0.1180 (10)_{(\text{fit,PDF})} (1)_{(\mu_0)} (22)_{(\mu_R, \mu_F)}$$

$\alpha_s$  from Dijet Cross Sections in NNLO



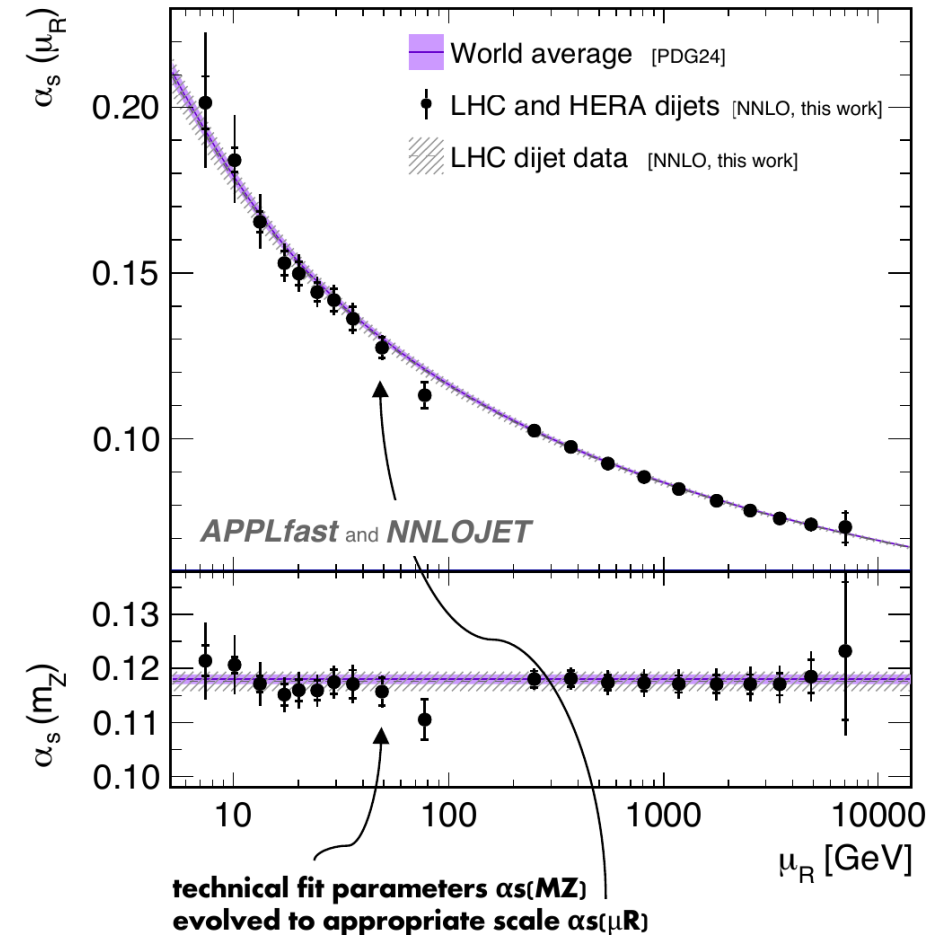


# Running coupling

- Test the **running** of  $\alpha_s$ 
  - ➔ Each cross-section measurement is assigned a single **representative  $\mu_R$**  value
    - group the data into **20 distinct  $\mu_R$  intervals**
  - ➔ **Single fit to all dijet data**, where for each of the individual ranges a separate  $\alpha_s(m_Z)$  value is used in the prediction
  - ➔ Assumes running to be valid within the very **limited range** covered by the interval (plus small extent in DGLAP from  $\mu_0=90$  to  $\mu_F=\mu_R$ )

## Results

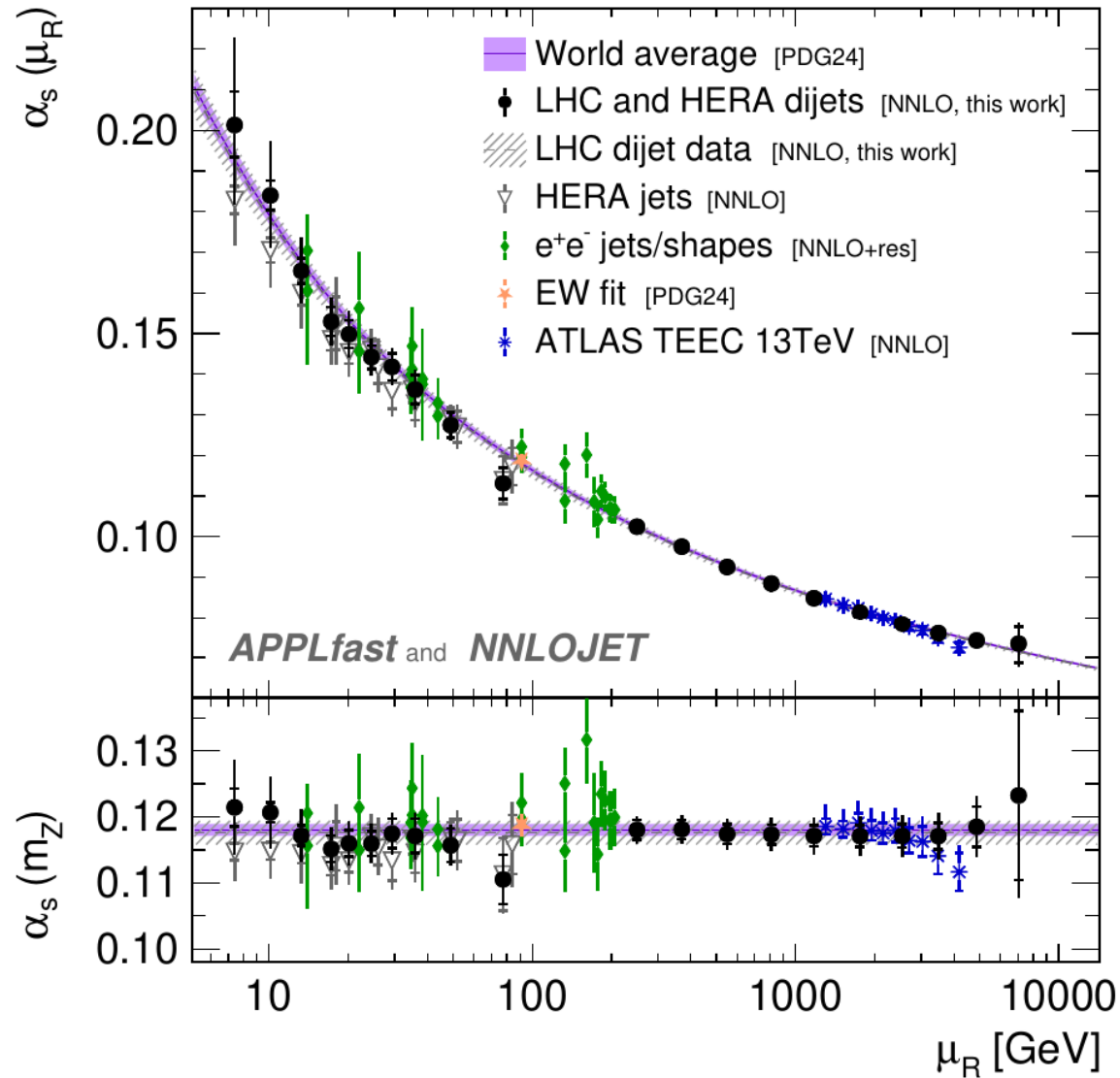
- ➔ **Consistent with the expectation from the RGE running** at all scales, from about **7 GeV to 7 TeV**
- ➔ Theo. and experimental uncertainties of **similar size** (albeit scale uncertainty always dominates)



Scales up to 7 TeV probed for the first time



# Comparison to other data



**Huge range in scale  $\mu_R$  covering 3 orders of magnitude with rather small uncertainties**



- Strong coupling  $\alpha_s$  determined from dijet data for the first time based on **complete NNLO** calculations (including sub-leading color contributions)
- Using **multiple LHC dijet datasets** from ATLAS and CMS at **7, 8 and 13 TeV**,  $\alpha_s(m_Z)$  is determined to

$$\alpha_s(m_Z) = 0.1178 (22)_{(\text{tot})}$$

- Including dijet cross sections from HERA, we performed a **comprehensive** and precise test of the **QCD RGE running of  $\alpha_s$** , probing a range of scales from **7 GeV to 7 TeV**
- Only LHC Run 1 and partial data of Run 2 used so far  
→ can be extended substantially with full Run 2 & Run 3 data

**Thank you for your attention!**

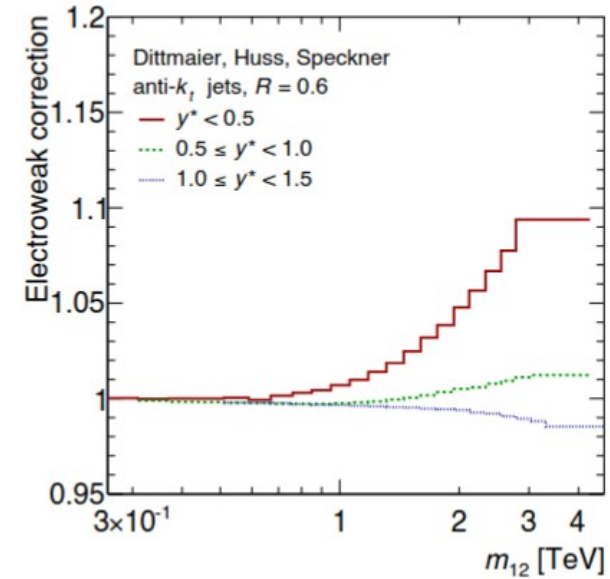
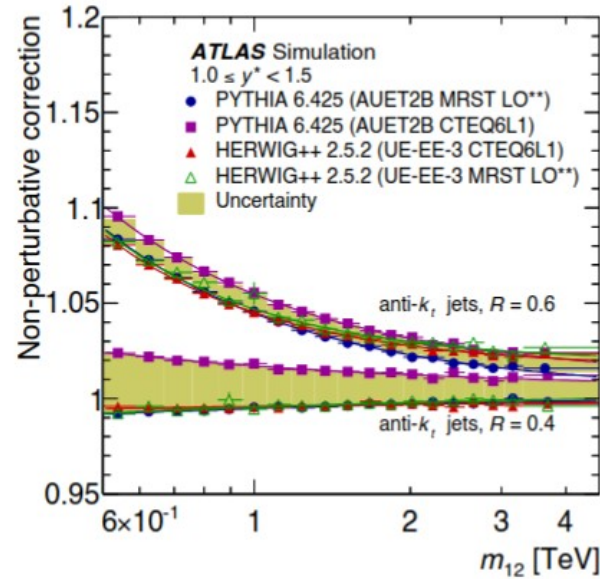


# *Backup*

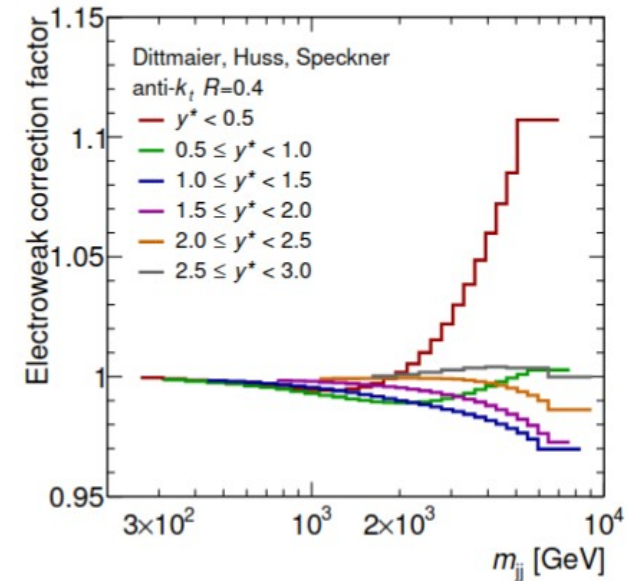
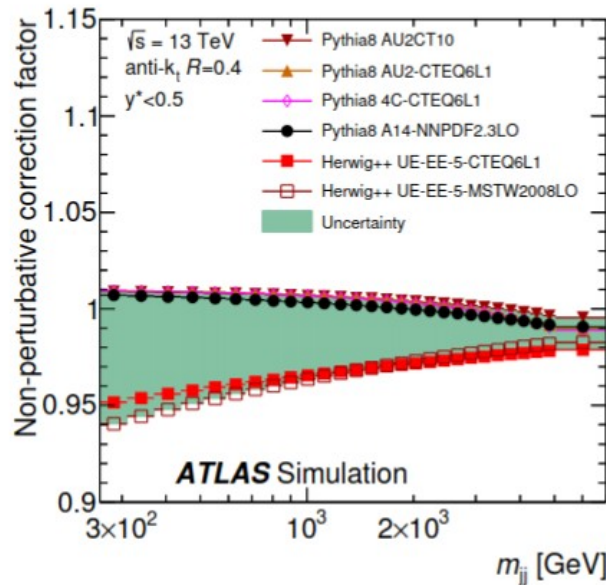


# EW and NP corrections (ATLAS)

7 TeV



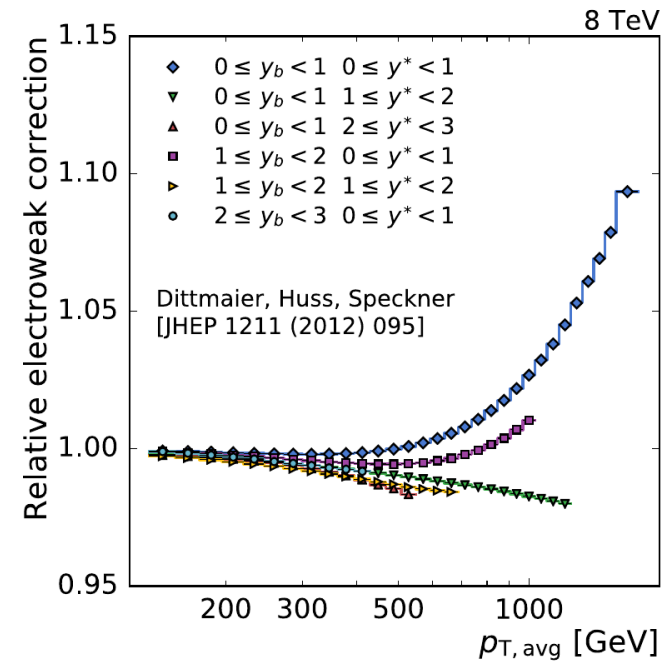
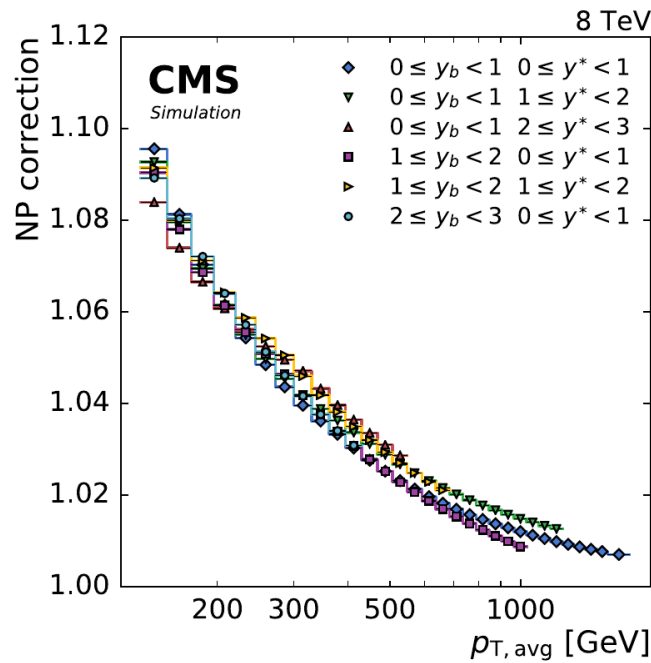
13 TeV



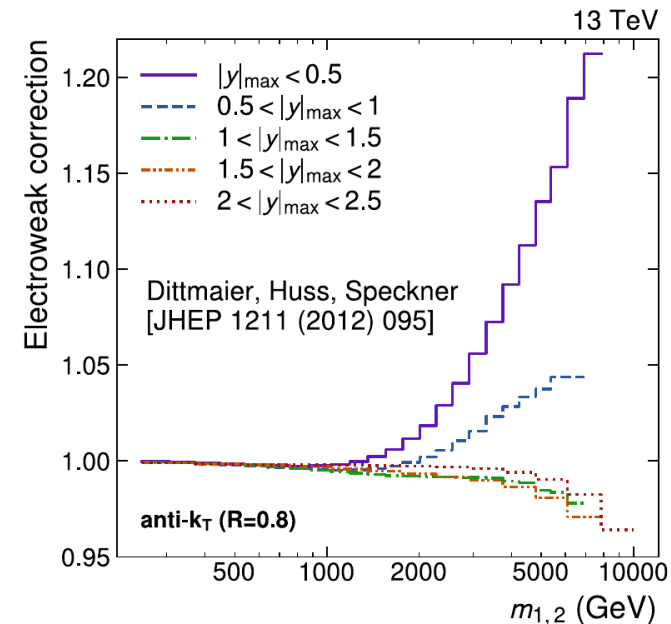
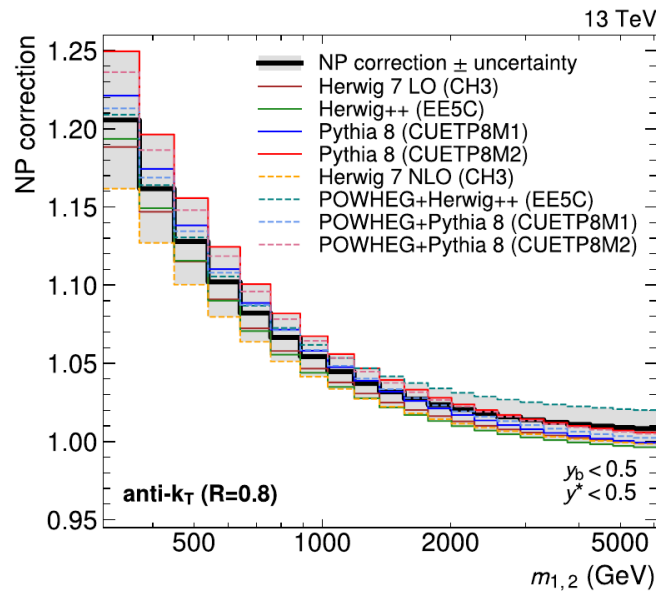


# EW and NP corrections (CMS)

8 TeV



13 TeV





## $\mu_R$ dependence

$\mu_R^{\text{avg}}$ [GeV]	$\alpha_s(m_Z)$	$\alpha_s(\mu_R)$
7.4	0.1214 (28) (1) (66)	0.2013 (82) (4) (196)
10.1	0.1207 (15) (1) (53)	0.1840 (37) (2) (130)
13.3	0.1171 (15) (0) (37)	0.1654 (31) (0) (77)
17.2	0.1151 (20) (0) (26)	0.1530 (36) (1) (47)
20.1	0.1160 (20) (1) (27)	0.1498 (34) (1) (46)
24.5	0.1159 (18) (0) (23)	0.1442 (29) (1) (37)
29.3	0.1175 (23) (0) (22)	0.1418 (33) (0) (32)
36.0	0.1171 (26) (0) (24)	0.1362 (35) (1) (33)
49.0	0.1157 (26) (1) (16)	0.1275 (31) (1) (20)
77.5	0.1105 (37) (3) (12)	0.1131 (39) (3) (12)
250	0.1180 (15) (1) (14)	0.1025 (11) (1) (11)
370	0.1181 (15) (1) (16)	0.0975 (10) (1) (11)
550	0.1174 (15) (1) (19)	0.0925 (9) (1) (12)
810	0.1173 (15) (2) (21)	0.0885 (9) (1) (11)
1175	0.1171 (16) (2) (23)	0.0848 (8) (1) (12)
1760	0.1171 (17) (2) (25)	0.0813 (8) (1) (12)
2545	0.1171 (18) (2) (27)	0.0783 (8) (1) (12)
3490	0.1171 (20) (2) (29)	0.0760 (8) (1) (12)
4880	0.1185 (31) (3) (34)	0.0742 (12) (1) (13)
7040	0.1232 (128) (12) (37)	0.0734 (43) (4) (13)

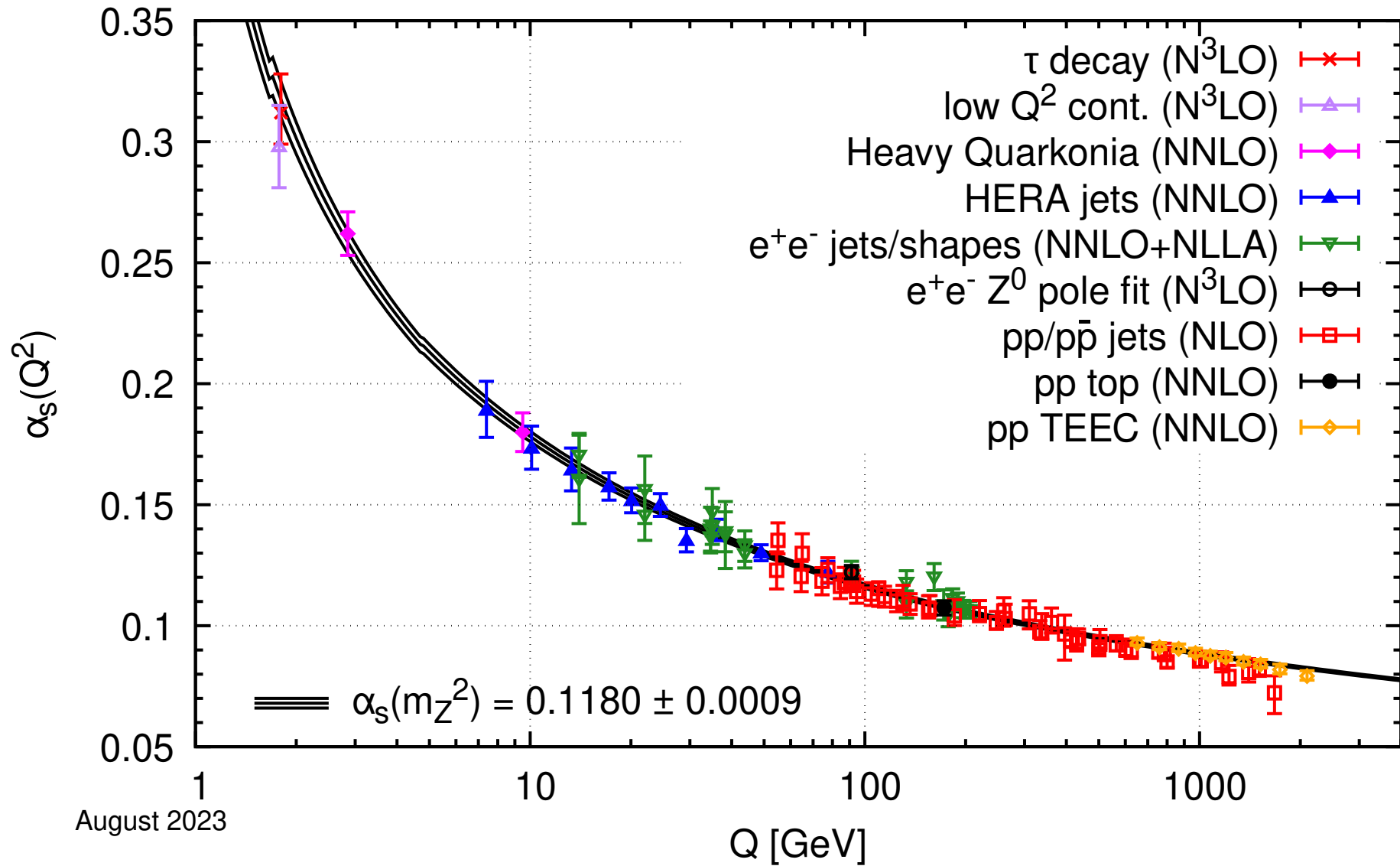
## Correlations

$\mu_R$ [GeV]	Correlations																			
7.4	–	56	29	21	19	22	15	17	16	12	2	1	–1	–2	–3	–3	–3	–1	0	
10.1	56	–	65	50	49	50	37	38	36	23	9	8	5	2	0	–2	–3	–3	–2	0
13.3	29	65	–	58	52	54	40	45	39	23	11	11	9	7	5	2	1	0	0	1
17.2	21	50	58	–	48	52	39	44	41	24	9	9	8	7	5	3	2	1	1	1
20.1	19	49	52	48	–	52	39	38	41	24	9	9	8	7	5	4	2	1	1	1
24.5	22	50	54	52	52	–	55	49	53	36	10	11	11	10	9	7	5	3	2	1
29.3	15	37	40	39	39	55	–	41	44	33	6	8	9	10	9	8	7	5	3	1
36.0	17	38	45	44	38	49	41	–	39	28	5	6	8	8	8	8	7	5	3	1
49.0	16	36	39	41	41	53	44	39	–	31	4	5	6	7	8	7	6	5	3	1
77.5	12	23	23	24	24	36	33	28	31	–	1	2	2	3	4	4	3	2	1	1
250	2	9	11	9	9	10	6	5	4	0	–	90	87	83	78	71	64	54	36	9
370	1	8	11	9	9	11	8	6	5	1	90	–	95	91	87	80	72	61	40	10
550	–1	5	9	8	9	11	9	8	6	2	87	95	–	97	93	88	80	67	45	11
810	–2	2	7	7	8	10	10	8	7	2	83	91	97	–	97	93	86	74	49	12
1175	–3	0	5	5	7	9	9	8	8	3	78	87	93	97	–	97	92	80	55	14
1760	–3	–2	2	3	5	7	8	8	7	4	71	80	88	93	97	–	96	87	62	17
2545	–3	–3	1	2	4	5	7	7	6	4	64	72	80	86	92	96	–	92	70	21
3490	–3	–3	0	1	2	3	5	5	5	3	54	61	67	74	80	87	92	–	78	27
4880	–1	–2	0	1	1	2	3	3	3	2	36	40	45	49	55	62	70	78	–	30
7040	0	0	1	1	1	1	1	1	1	1	9	10	11	12	14	17	21	27	30	–

TABLE V. Correlations of the (fit,PDF) uncertainty from the fit of 20  $\alpha_s(m_Z)$  parameters to HERA and LHC dijet data.

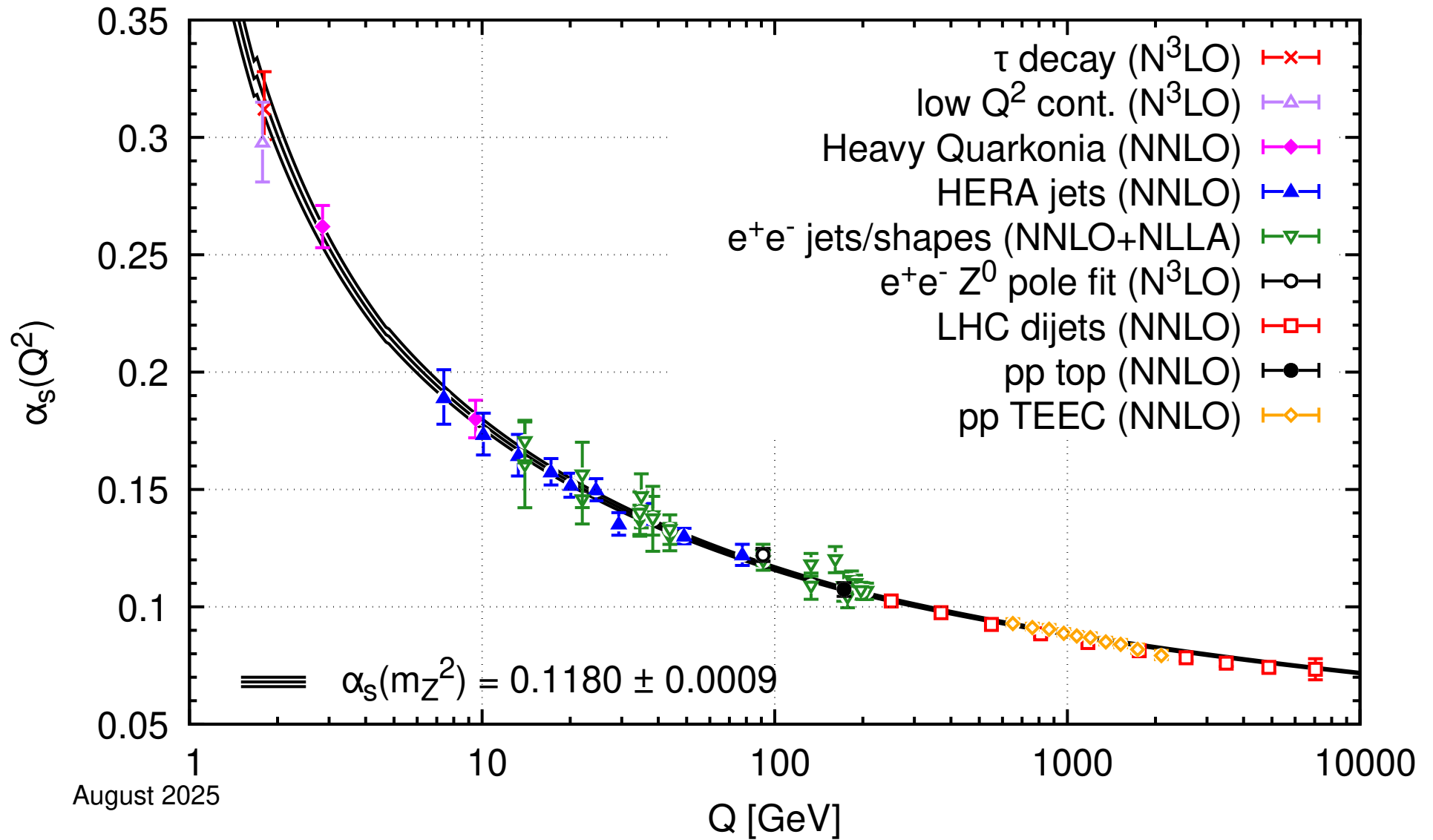


# PDG 2023 $\alpha_s$ running





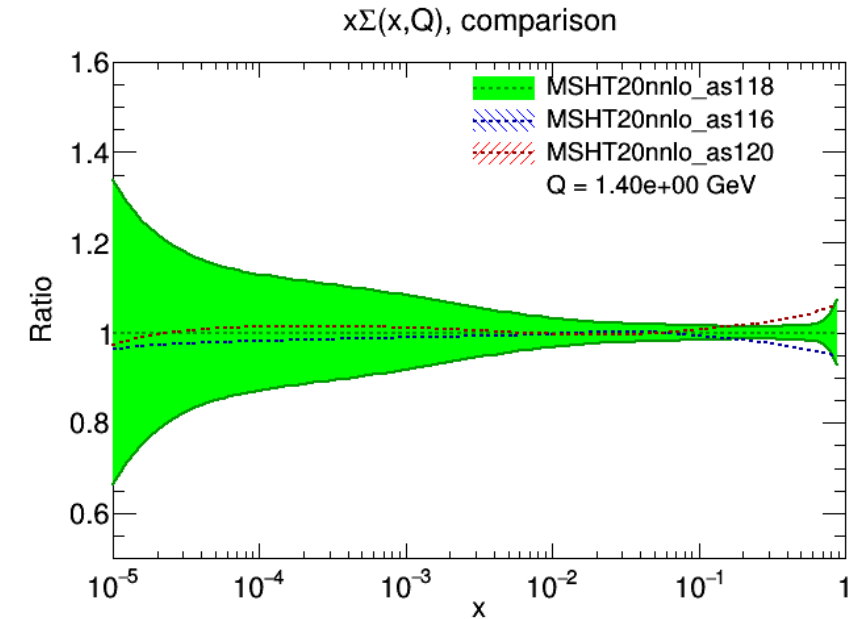
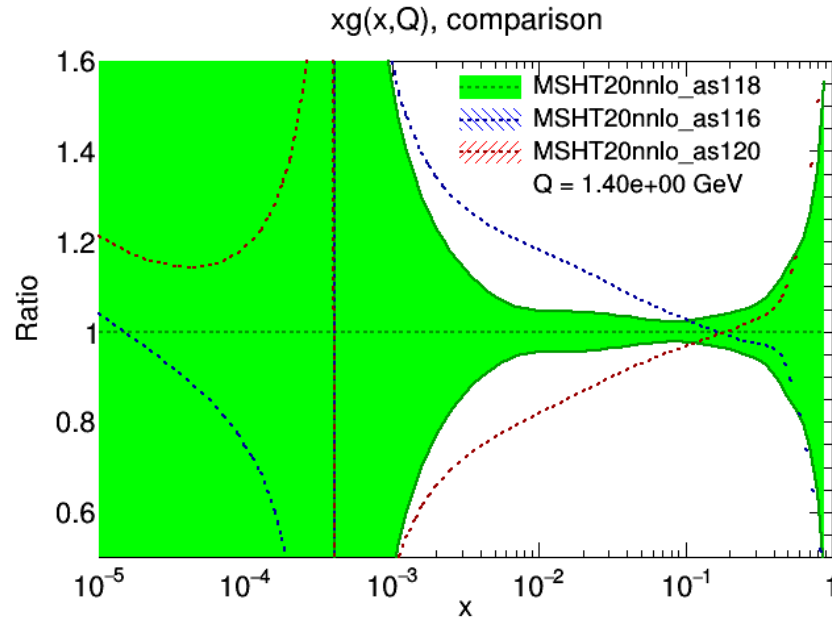
# PDG 2025 $\alpha_s$ running



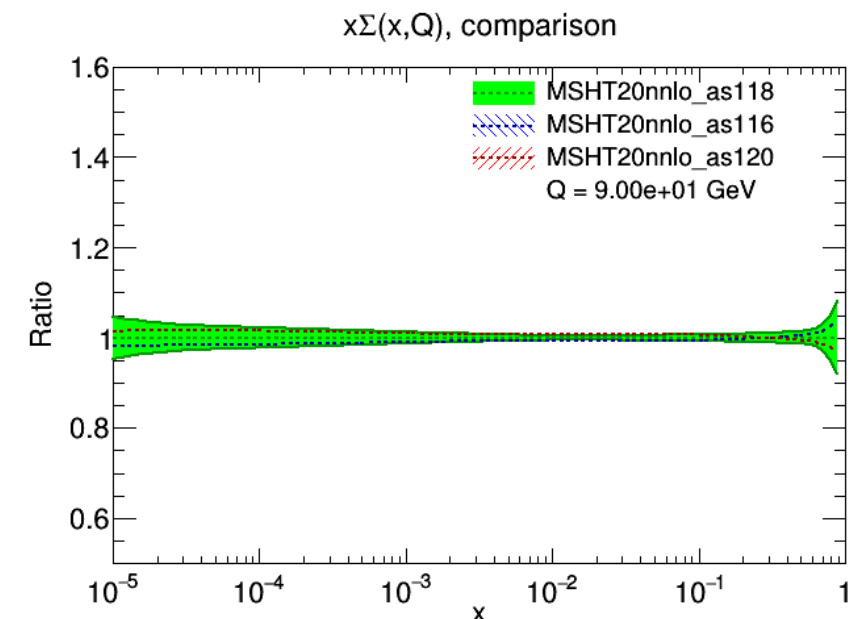
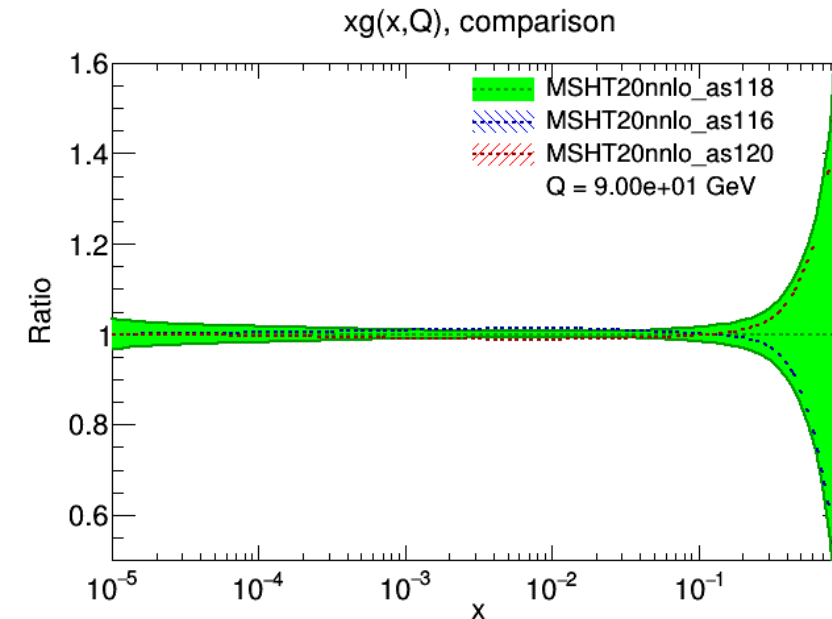


# PDF dependence

$\mu_F = 1.4 \text{ GeV}$



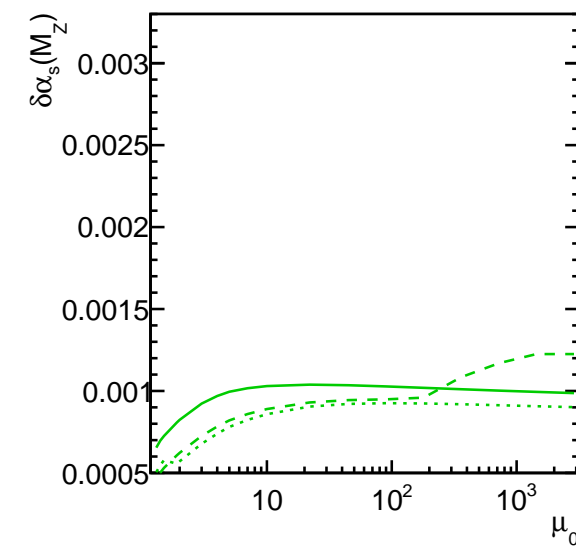
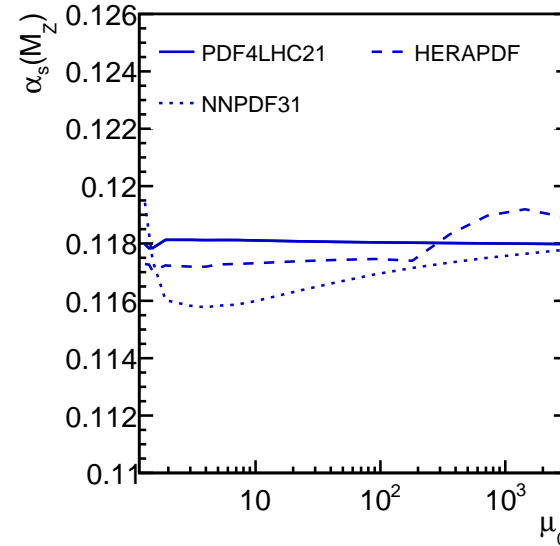
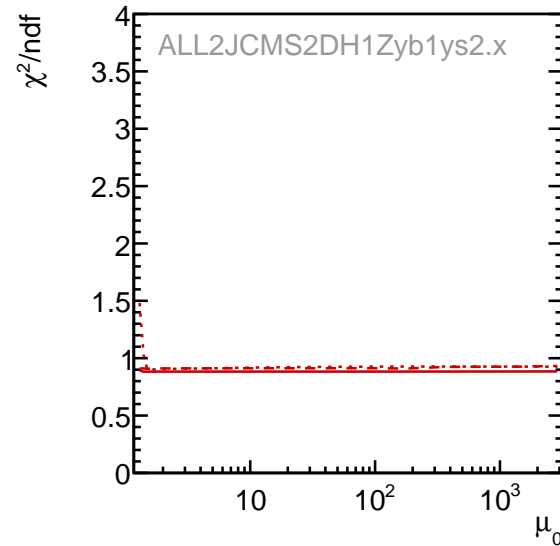
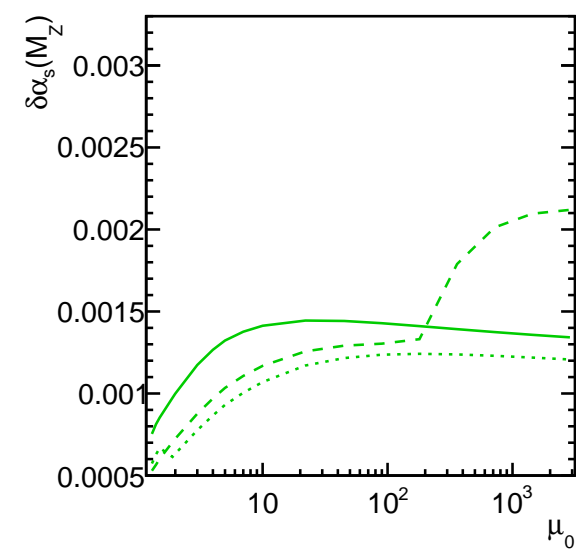
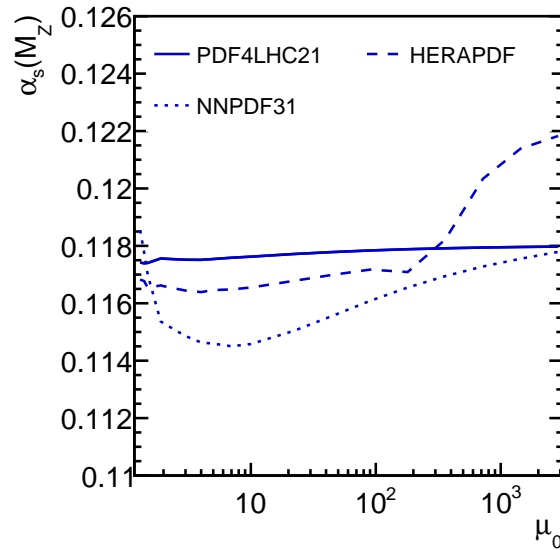
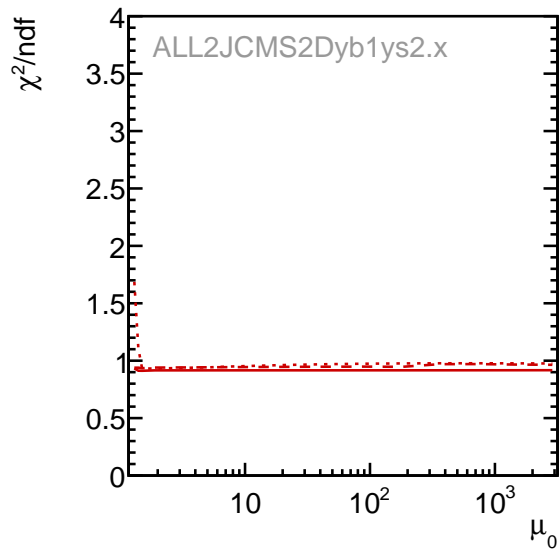
$\mu_F = 90 \text{ GeV}$





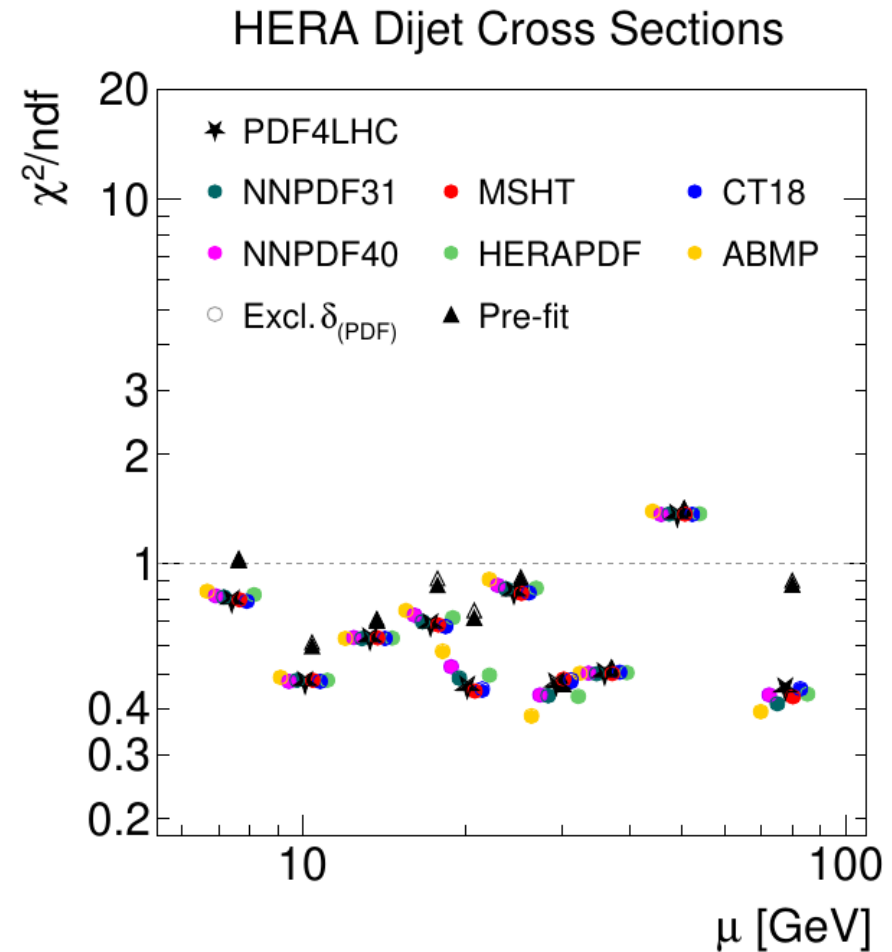
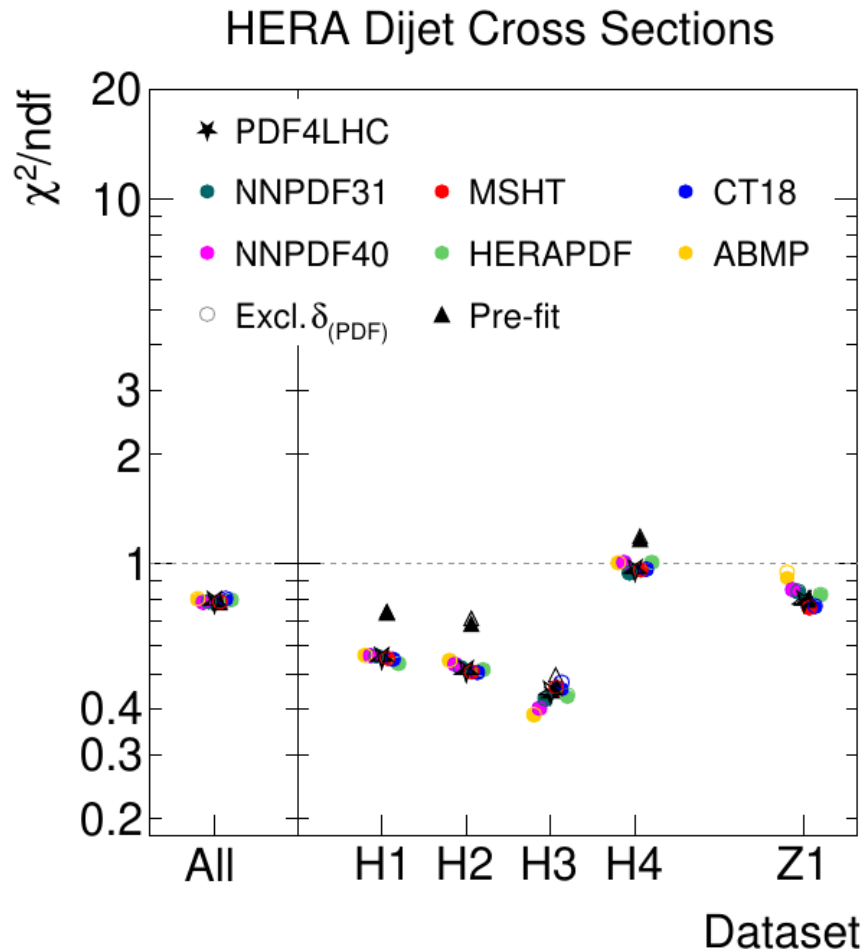
# $\mu_0$ dependence

## (Extreme) Variation of $\mu_F$ starting scale $\mu_0$





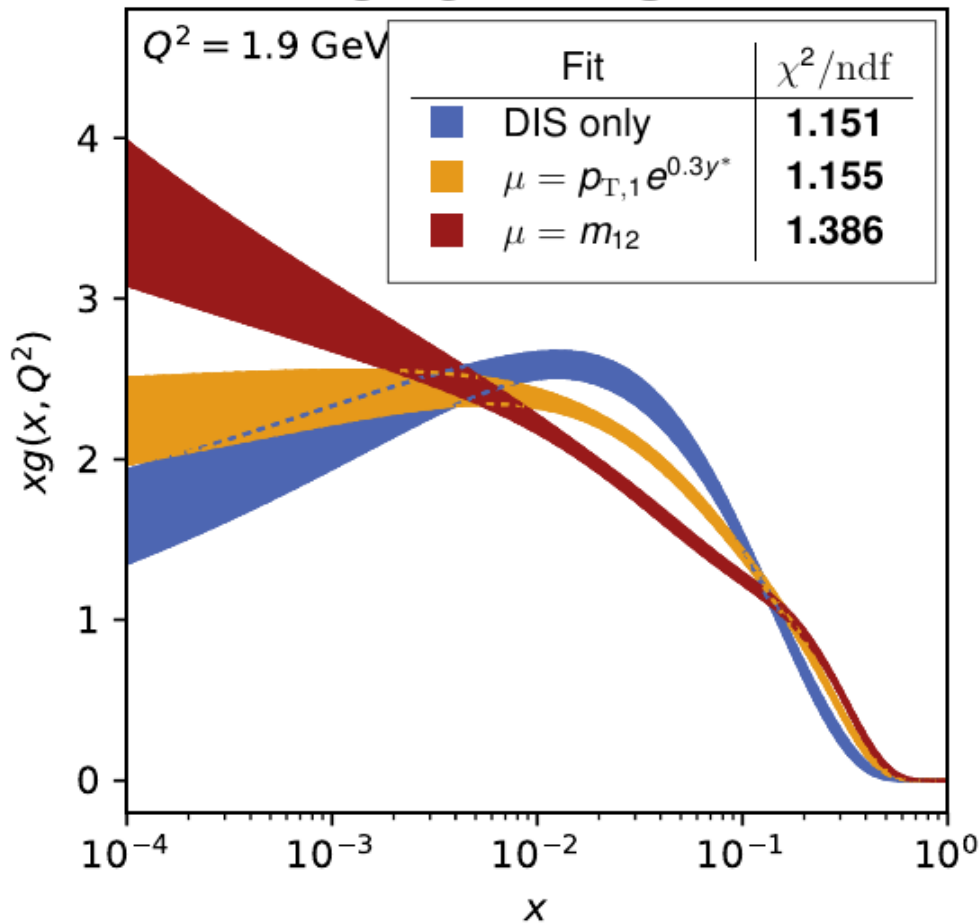
# Consistency study for HERA data



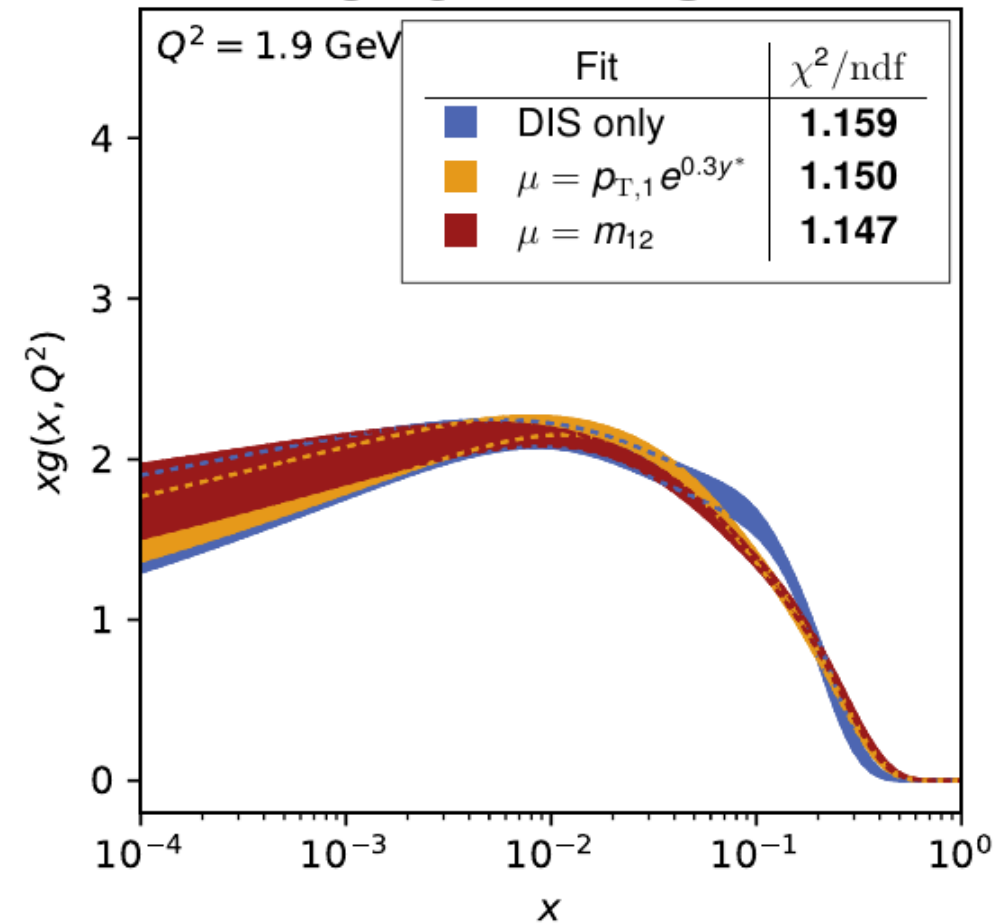


# PDF only fit: NLO vs. LC-NNLO

## 8 TeV NLO



## 8 TeV NNLO

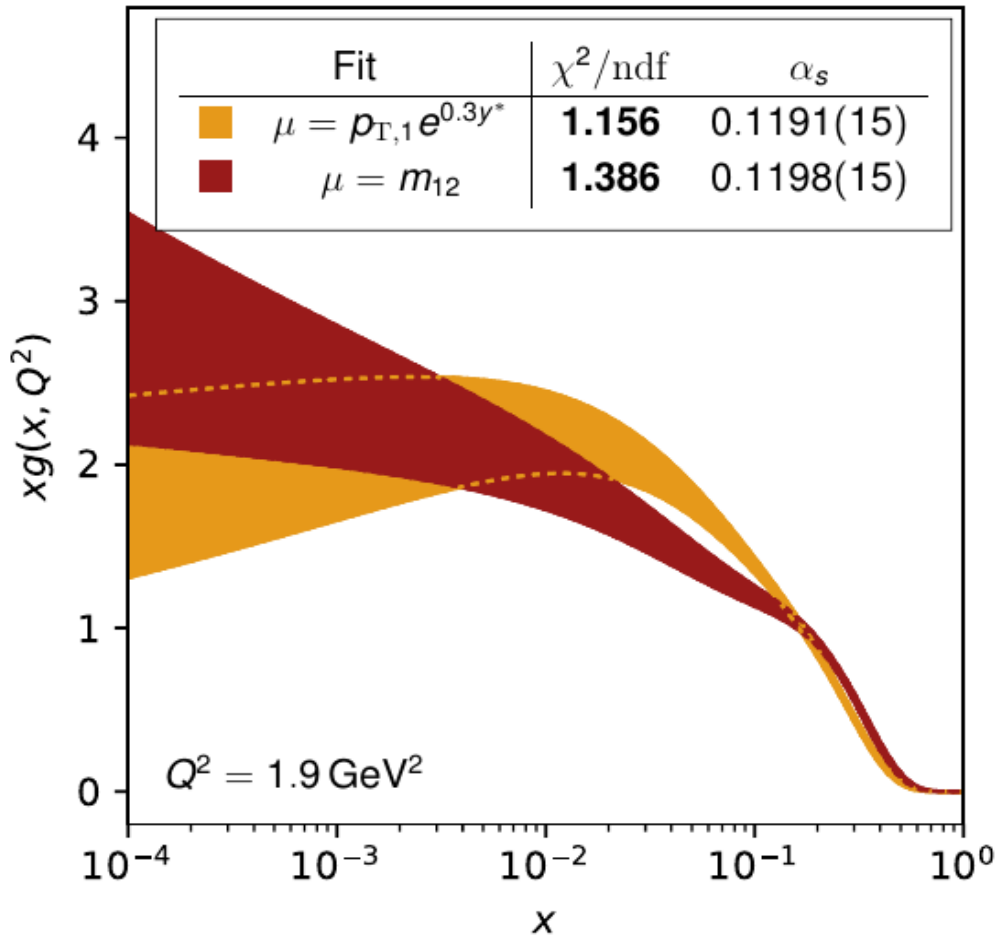


- NLO: significant differences between scales (**ptmax** vs. **dijet mass**)
- NNLO: both central scales agree very nicely
- $m_{12}$  scale definition much smaller  $\chi^2$  in NNLO

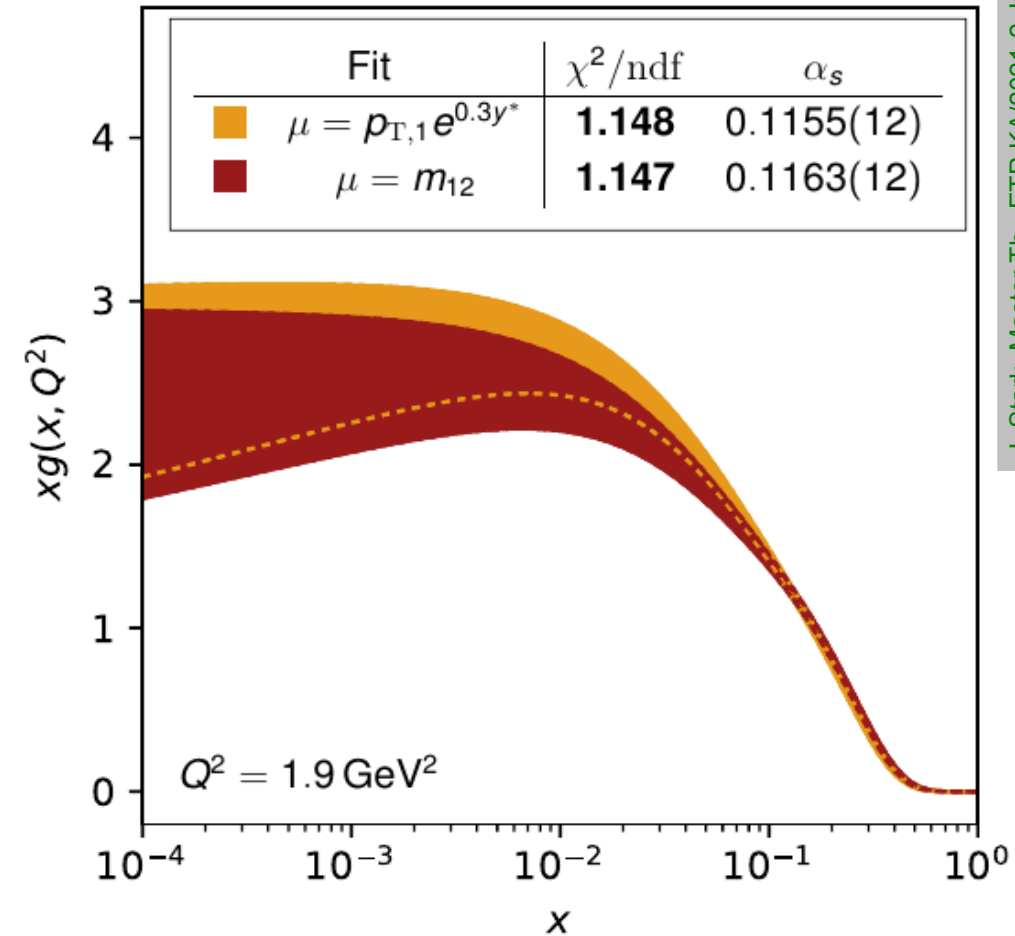
From J. Stark, Master Thesis ETP-KA/2021-2, KIT.  
Presented at CERN Theory Workshop, KR, May 2025.



## 8 TeV NLO



## 8 TeV NNLO



- no scale uncertainties shown here
- larger exp. uncertainty due to correlation of gluon PDF with  $\alpha_s$
- as before nice agreement between different scales at NNLO



## Fitted $\alpha_s(M_Z)$ values

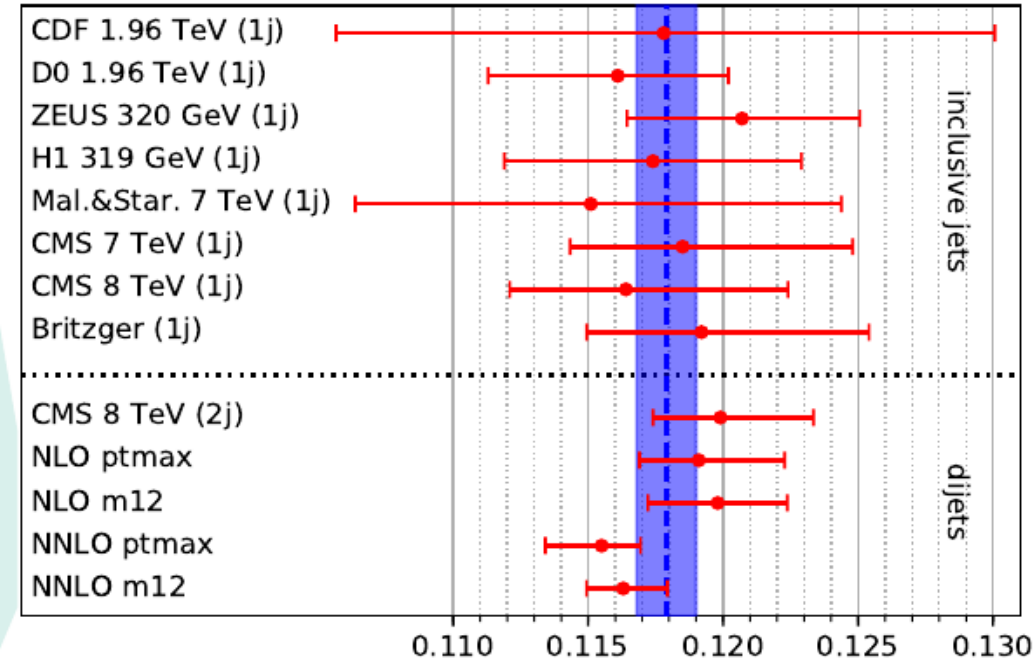
NLO	$\mu = p_{T,1} e^{0.3y^*}$	$0.1191 \pm 0.0015(\text{exp})_{-0.0016}^{+0.0028}(\text{scale})$
	$\mu = m_{12}$	$0.1198 \pm 0.0015(\text{exp})_{-0.0021}^{+0.0021}(\text{scale})$
NNLO	$\mu = p_{T,1} e^{0.3y^*}$	$0.1155 \pm 0.0012(\text{exp})_{-0.0017}^{+0.0008}(\text{scale})$
	$\mu = m_{12}$	$0.1163 \pm 0.0013(\text{exp})_{-0.0004}^{+0.0010}(\text{scale})$

⚠ Only experimental and scale uncertainties

- as expected, smaller  $\alpha_s$  values at NNLO
- scale uncertainties: maximal envelope of 6 scale variations
- experimental and especially scale uncertainties smaller at NNLO

Remark: It was not aimed for a new, fully flexible fit with xfitter

→ no parameterisation or model uncertainties.



Comparison with other values obtained from jet cross sections and the world average (blue)