α_s from a combination of LatticeQCD methods

ALPHA Collaboration

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Based on:

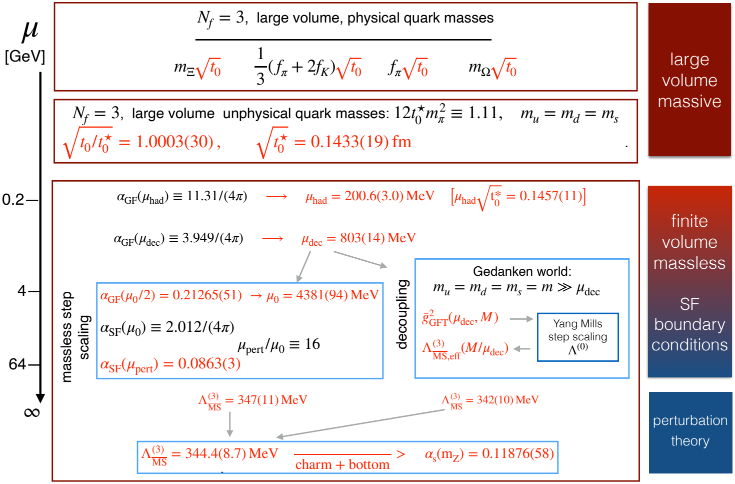
- (ALPHA) M. Dalla Brida R. Höllwieser F. Knechtli, T. Korzec A. Ramos, S. Sint, R. Sommer.
  The strength of the interaction between quatrks and gluons.
  [arXiv:2501.06633].

CSIC Consejo Superior de Investigaciones Científicas | Vniversitat de València | IFIC Institut de Física Corpuscular | Excelencia Severo Ochoa

THE STRATEGY

- Input: Hadron spectrum: $M_\pi, M_K, M_\Xi, f_K, f_\pi, M_\Omega, \ldots$
- Technical intermediate scale $\sqrt{t_0^\star}$
- "Solve" $N_f = 3$ QCD non perturbatively
  - From 200 MeV to EW scale
  - Match QCD with YM
- Use PT from EW scale on: $\Lambda^{(3)} = 344.4(8.7)$ MeV
- Use PT to cross c/b thresholds: $\alpha_s(M_Z) = 0.11876(58)$. [0.47%] precision

- Conservative error
- Statistical errors dominate



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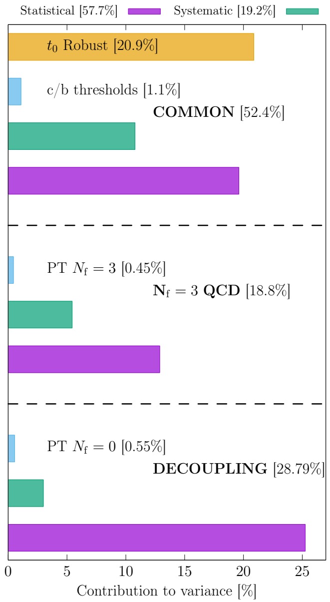


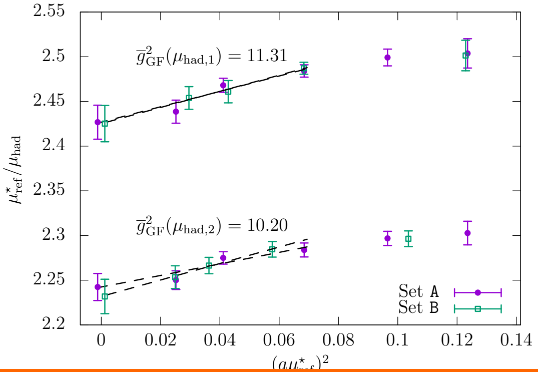
Error budget

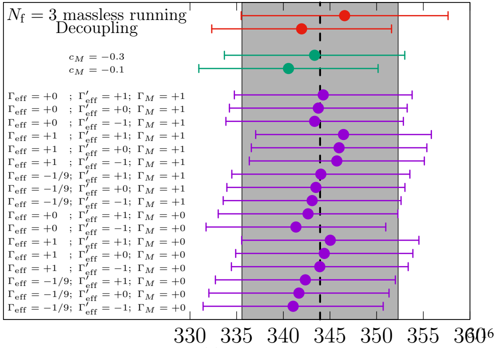
Types of error

Statistical: From MC sampling. Well undestood ($\mathcal{O}(1/\sqrt{N})$)

Small Systematic: From models to extrapolate to continuum/interpolate/.... Values scatter below statistical.

Significant systematic: From models to extrapolate to continuum/interpolate/.... Result show some tension





Scale setting: The case with significant systematics

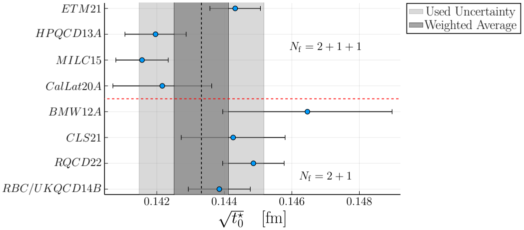
Technical scale $t_0^\star$

- Different collaborations determine dimensionless:

$$\sqrt{t_0^\star} \times (M_\pi, M_K, M_\Xi, M_\Omega, f_\pi, f_K, \dots)$$

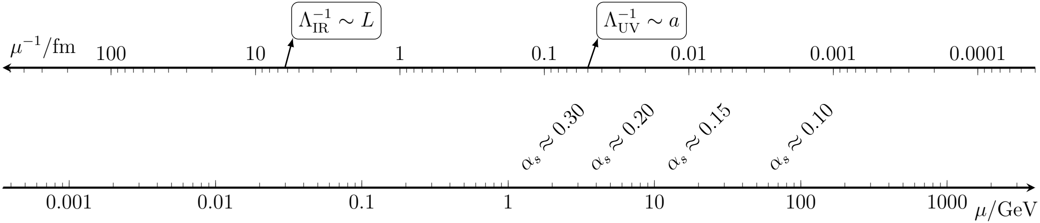
- Use all results entering FLAG average
- $\chi^2/\text{dof} = 2.8$
- **Robust** error: all "precise" results covered

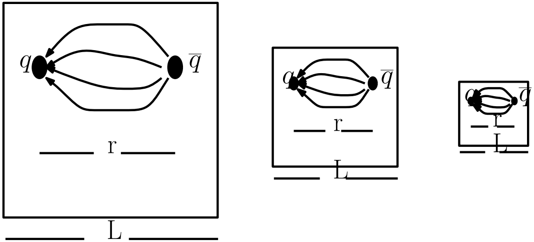
$$\sqrt{t_0^\star} = 0.1433(7)_{\text{stat}}(4)_{\text{sys}}(17)_{\textbf{robust}}(19)_{\text{tot}}\ \text{fm}\,.$$



- Only case of significant systematic in our work
- **Robust** error band covers all (precise) central values
- Effect propagated in all quantities.
- Our error 2.5× larger than "standard" (i.e. FLAG/PDG) error inflation
- Small effect in final error of α_s: $58 \times 10^{-4} \to 51 \times 10^{-4}$

Lattice QCD typical scales





Finite size scaling [L üscher, Weisz, Wolff '91]

- Safe continuum extrapolations
- Arbitrarily high energy scales explored
- Step scaling function:

$$\sigma(u) = \alpha(Q/2)\Big|_{\alpha(Q)=u}$$

"Easy" to compute on the lattice

- **Dedicated approach**

Example: massless running in $N_\mathrm{f} = 3$ QCD [ALPHA '17]

Gradient flow scheme [ALPHA; Phys.Rev.D 95 (2017)]

- Determine lattice version of SSF

$$\Sigma(u, L/a) = \alpha(Q/2)\Big|_{\alpha(Q)=u,\mathrm{fixed}L/a}$$

Use $8 \to 16, 12 \to 24, 16 \to 32, (20 \to 40, 24 \to 48, 32 \to 64)$ at fixed (g_0, am_0)

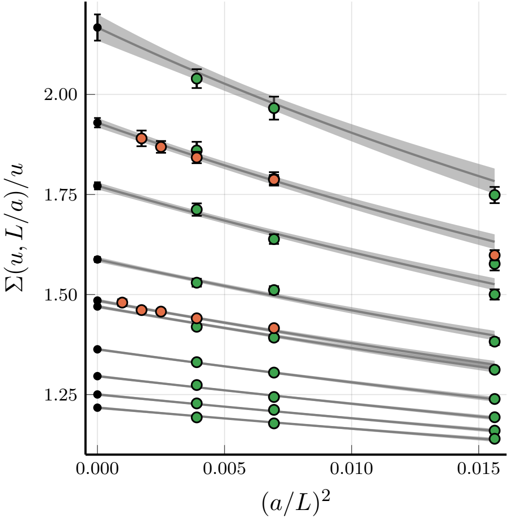
- Continuum limit

$$\sigma(u) = \lim_{a/L\to 0} \Sigma(u, L/a)\,.$$

- Use to determine β-function

$$\log(2) = \int_u^{\sigma(u)} \frac{\mathrm{d}x}{\beta(x)}\,.$$

- **Continuum limit under control**
- Cover energy range 200 MeV $\to$ 4 GeV.



Example: massless running in $N_f = 3$ QCD [ALPHA '17]

Gradient flow scheme [ALPHA; Phys.Rev.D 95 (2017)]

- Define

$$\alpha_{GF}(\mu_{had}) = 11.31/(4\pi)$$

with

$$\mu_{had} \times \sqrt{t_0^\star} = 0.1457(11)_{stat}(1)_{sys}(11)_{tot}$$

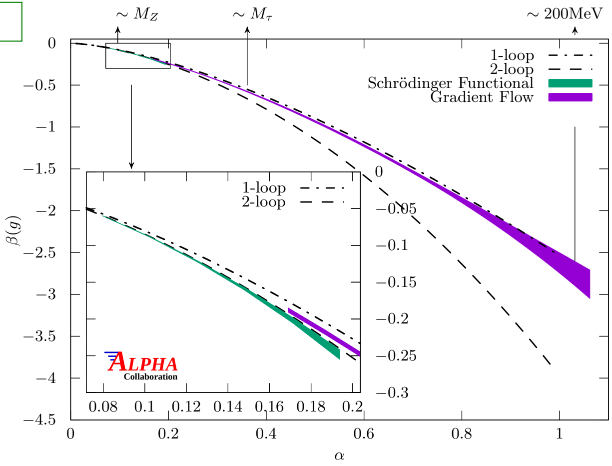
- Using $\alpha_{GF}(\mu_0/2) = 0.21265(51)$

$$\frac{\mu_0}{\mu_{had}} = 21.85(30)_{stat}(17)_{sys}(34)_{tot}$$

and

$$\mu_0 = 4385(71)_{stat}(36)_{sys}(51)_{robust}(94)_{tot}\ \text{MeV}\ .$$

- Reached scale of cutoff of typical lattice simulations with full control over continuum extrapolation!

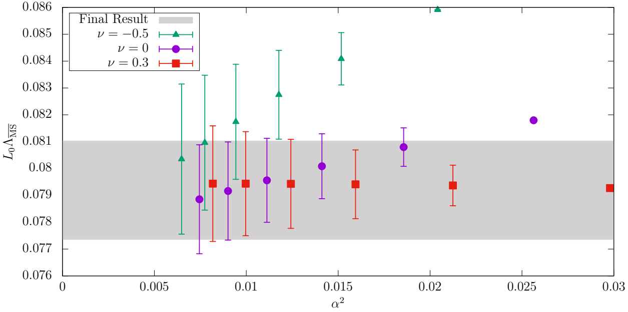


Determination of α_s

- One can use PT directly at $\mu_0 \approx 4$ GeV

$$\Lambda^{(3)}_{\overline{\rm MS}} = 347.6(6.3)\,\text{MeV}$$

- [0.34%] error in α_s…
- …But what about PT
  - Missing orders
  - Power corrections
- Continue to high energies **[ALPHA' 2019]**
  - Improved $\mathcal{O}(a)$



ALPHA approach

Use PT at genuinely high energy: 70 GeV

$$\Lambda^{(3)}_{\overline{\rm MS}} = 347(11)\,\text{MeV}$$

Check with a one-parameter family of observables! ν

Alternative method: Decoupling

$$\frac{\Lambda^{(3)}_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{dec}}} = \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\mu'_{\mathrm{dec}}} \times \frac{1}{P\left(z\Lambda^{(3)}_{\overline{\mathrm{MS}}}/\mu_{\mathrm{dec}}\right)} + \mathcal{O}(\alpha^4(m^\star)) + \mathcal{O}\left(\frac{\mu_{\mathrm{dec}}}{M}\right) + \mathcal{O}\left(\frac{\mu^2_{\mathrm{dec}}}{M^2}\right)$$

- $z = M/\mu_{\mathrm{dec}}$: Quark masses
- Work in finite volume $T \times L^3$ with Dirichlet bcs. in time (SF). ($\mu \sim 1/L$): "Only" two scales.
- Use Gradient Flow couplings

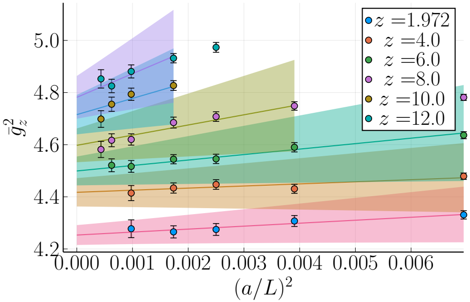
$$\bar{g}^2(\mu) = \mathcal{N}^{-1}(c, a/L)\, t^2 \langle E(t)\rangle\Big|_{\mu^{-1}=\sqrt{8t}=cL}.$$

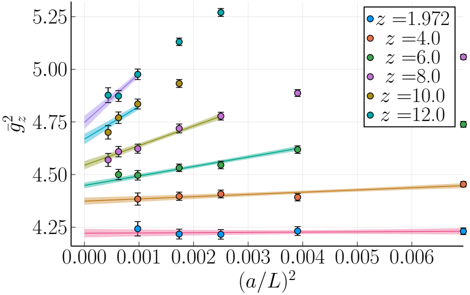
- Fix $\bar{g}^2(\mu_{\mathrm{dec}})|\Big|_{N_f=3,M=0,T=L} = 3.95$. This defines μ_{dec}
- Small volume $\Longrightarrow$ We can simulate heavy quarks (i.e. $a \sim 30 - 50\,\mathrm{GeV}^{-1}$)
- Matching condition ($\{N_f = 3, M\} \leftrightarrow \{N_f = 0\}$) between massive scheme and effective theory

$$\bar{g}^2(\mu_{\mathrm{dec}}(M))\Big|_{N_f=3,M,T=2L} = \bar{g}^2(\mu_{\mathrm{dec}})\Big|_{N_f=0,T=2L}.$$

Matching: QCD in a finite volume!

THE CONTINUUM EXTRAPOLATION OF MASSIVE COUPLINGS





Previous determination [ALPHA '23]

- Most error: estimate of $b_g - b_g^{1-\text{loop}}$
- This is a systematic!
- **Dominant error in** $\bar{g}^2(\mu, M)$

NP determination of b_g [ALPHA '24]

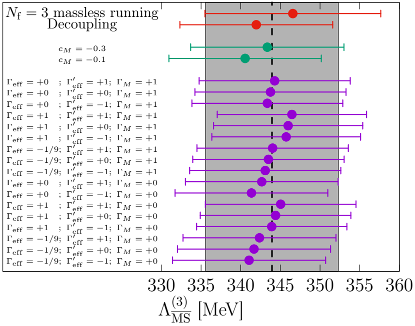
- Precise of continuum values
- **Subdominant** effect in α_s
- But removes largest systematic!

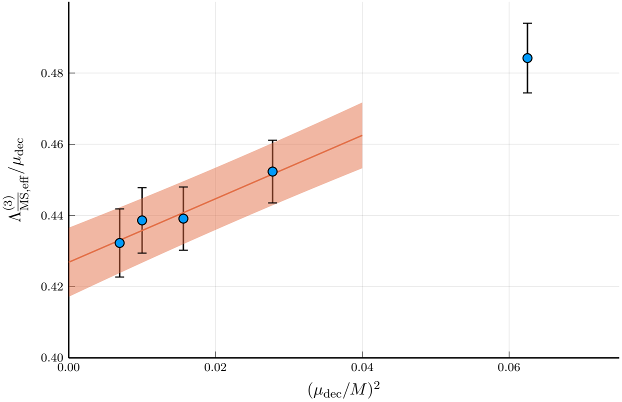
Striong coupling from decoupling

- Precise result for α_s

$$\Lambda^{(3)}_{\overline{\rm MS}} = 342(10)\ \text{MeV}\,.$$

- Different models to extrapolate to continuum
- Different modesl to extrapolate $M \to \infty$
- Use of **already existing** pure gauge results





From $\Lambda^{(3)}_{\overline{MS}}$ to $\Lambda^{(5)}_{\overline{MS}}$

$$\frac{\Lambda^{(5)}_{\overline{MS}}}{\Lambda^{(3)}_{\overline{MS}}} = P_{3,5}$$

Use RunDec to explore PT corrections in crossing c/b thresholds

loop-orders	$P^{\rm ref}_{3,4}$	$P^{\rm ref}_{4,5}$	$P^{\rm ref}_{3,5}$	$\alpha_s^{\rm ref}(m_Z)$
5/4	0.87548	0.72143	0.63160	0.11872
loop-orders	$100\times\delta P_{3,4}$	$100\times\delta P_{4,5}$	$100\times\delta P_{3,5}$	$10^5\times\Delta\alpha_s(m_Z)$
4/3	−0.2536	0.2056	−0.3313	−5.992
3/2	−0.8503	0.5758	−1.2237	−22.20
2/1	−3.8555	1.2228	−6.8235	−126.3
SI,$m^\star$	0.0	0.0		
SI,$2m^\star$	−0.4364	−0.0702		
SI,$m^\star/2$		−0.0117		
$\overline{MS}$,$\mu=\mu_h$	−0.0299	−0.0014		
$\overline{MS}$,$\mu=2\mu_h$	−0.1036	−0.0105		
$\overline{MS}$,$\mu=\mu_h/2$	0.0016	0.0119		

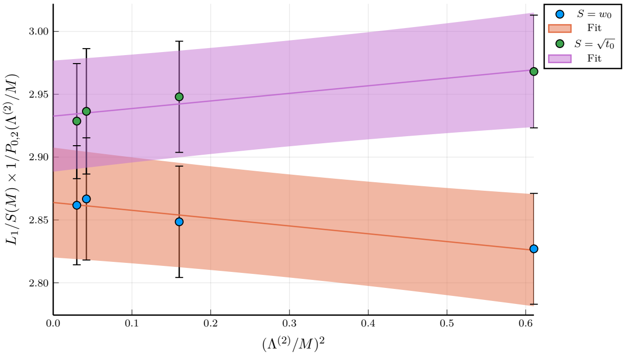
Small corrections

- PT corrections 0.3%
- NP corrections 0.1%

From $\Lambda^{(3)}_{\overline{\rm MS}}$ to $\Lambda^{(5)}_{\overline{\rm MS}}$

$$\frac{\Lambda^{(5)}_{\overline{\rm MS}}}{\Lambda^{(3)}_{\overline{\rm MS}}} = P_{3,5}$$

Use RunDec to explore PT corrections in crossing c/b thresholds



Small corrections

- PT corrections 0.3%
- NP corrections 0.1%

Final results

Final result for α_s

- Two strategies, consistent results

$$\text{Massless running:} \quad \Lambda^{(3)}_{\overline{\text{MS}}} = 347(11)\,\text{MeV}\,,$$
$$\text{Decoupling:} \quad \Lambda^{(3)}_{\overline{\text{MS}}} = 342(10)\,\text{MeV}\,.$$

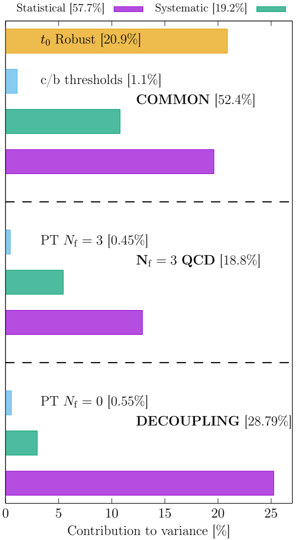
- Average

$$\Lambda^{(3)}_{\overline{\text{MS}}} = 344.4(8.7)\,\text{MeV}\,.$$

- "Crossing" c/b thresholds

$$\alpha_s(m_Z) = 0.11876(58) \quad [0.46\%]\,.$$

- Two times more precise than all pheno determinations combined!
- PT errors negligible
- Error dominated by statistics



Replication package freely available

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Installing

This code has been tested with Julia v1.11.5, and we recommend to use the same version. Nevertheless, the code should work with any version.

Download and Install julia v1.11.5. From the root directory (parent of `julia_env`), run

```
julia --project=julia_env -e "import Pkg; Pkg.instantiate()"
```

Analysis replication

The full analysis chain corresponds to running in order the following codes:

- `scale.jl`: Determination of Large Volume scales t0 / muref. Section 1.3 of the supplementary material.
- `running_le.jl`: Determination of the Nf=3 step scalign function and determination of the ratios `muref/mudec` and `muref/mu0`. Section 1.4 of the supplementary material.
- `lam_nf3.jl`: Determination of the three flavor Lambda parameter using `Nf=3` techniques. Section 1.5 of the supplementary material.
- `lam_dec.jl`: Determination of the three flavor Lambda parameter using the decoupling technique. Section 1.6 of the supplementary material.
- `alphas.jl`: Determination of the 4,5 flavor Lambda parameters and alphas with a complete description of the error budget. Section 1.8 of the supplementary material.
- `nf0_parameters.jl`: Prints the relevant `Nf = 0` parameters and their covariance. Section 1.6.2 of the supplementary material.

The programs should be run, respecting the order, inside the folder `main`:

```
main $ julia scale.jl      > scale.log
main $ julia running_le.jl > running_le.log
main $ julia lam_nf3.jl    > lam_nf3.log
main $ julia lam_dec.jl    > lam_dec.log
main $ julia alphas.jl     > alphas.log
```

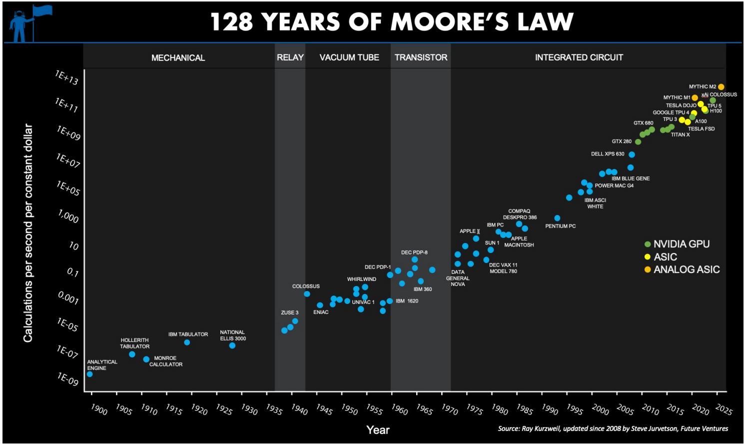
Replication package freely available

```
  016    3.202943051353 +/- 5.2266e-03           4.263036510391 +/- 1.8050e-02
  016    2.735900148089 +/- 3.5270e-03           3.485130963024 +/- 1.2394e-02
  016    2.389968335729 +/- 3.0204e-03           2.934814663809 +/- 7.9179e-03
  016    2.125706344543 +/- 2.4645e-03           2.536041782789 +/- 7.0626e-03
  020    5.857000000000 +/- 1.1000e-02          10.965000000000 +/- 8.6347e-02
  020    3.949300000000 +/- 6.3000e-03           5.755000000000 +/- 2.4492e-02
  024    5.877000000000 +/- 1.4000e-02          11.090000000000 +/- 1.1493e-01
  024    3.949200000000 +/- 6.2000e-03           5.770000000000 +/- 2.6576e-02
  032    3.949000000000 +/- 9.7000e-03           5.844000000000 +/- 3.2743e-02
 ## END Dataset


## Fitting beta function beta(x) with 1/P(x)
 # P(x) is polynomial degree 2
 # Adding extra a^4 weights to fit
 # csq / <csq>: 16.640325211620894 / 65.50667297047977
Continuum parameter 1: 16.140351683265884 +/- 0.6111074341673388
Continuum parameter 2: 0.1817488677136792 +/- 0.2161256737544785
Continuum parameter 3: -0.010678773286612774 +/- 0.017156156721400476
Covariance:
   3.734522960946e-01   -1.284723814076e-01    9.750645674202e-03
  -1.284723814076e-01    4.671030685583e-02   -3.659509358346e-03
   9.750645674202e-03   -3.659509358346e-03    2.943337134493e-04
## END fitting beta function
## Systematic in scale factor muhad/mudec
   - All L/a:          4.00032(0.026)_stat(0.014)_sys(0.0)_robust(0.03)_tot
   - L/a > 8:          4.02117(0.032)_stat(0.013)_sys(0.0)_robust(0.035)_tot
   - Systematic:       0.020849510565578377
## END Systematic in scale factor


 ## Scale factors with systematic
 # Scale factor between mudec/muhad: 4.00032(0.026)_stat(0.025)_sys(0.0)_robust(0.036)_tot
 # scale mudec [MeV]: 802.565(8.8)_stat(5.7)_sys(9.3)_robust(14.0)_tot
 # mudec x sqrt(t0):  0.583138(0.0057)_stat(0.0041)_sys(2.6e-18)_robust(0.0071)_tot
 # Scale factor between mu0/muhad:   21.8572(0.3)_stat(0.17)_sys(0.0)_robust(0.34)_tot
 # scale mu0 [MeV]: 4385.1(71.0)_stat(36.0)_sys(51.0)_robust(94.0)_tot
```

Community effort!



- Step Scaling
- Preconditioning
- Large Volume simulations
- SF in QCD
- High order PT computations
- …
- GF couplings
- Decoupling

Conclusions

- Precise result for α_s from a combination of LQCD techniques
  - Step scaling in QCD ($N_f = 3$)
    - Finer lattices
    - Better $\mathcal{O}(a)$ systematic understanding
    - ...
  - Decoupling and pure gauge running
    - Removed systematic from b_g
  - Exahustive analysis of c/b contributions
    - Both PT and NP!
- Error budget: Statistical errors dominate
- Replication package
  - Numbers in paper can be reproduced
  - Error sources are tracked down
  - Easy to update (i.e. t_0, pure gauge)
- Input is low energy spectral quantities
  - No new physics
- Should be used to improve our chances to discover NP