A determination of $\alpha_s(M_Z)$ at a N^{3}LO$_{\rm QCD}\otimes$NLO$_{\rm QED}$ from a global PDF analysis

alphas-2025: Workshop on precision measurements of the QCD coupling constant

CNRS Centre Paul Langevin, Aussois

This talk is based upon Eur.Phys.J. **C85** (2025) 1001

Emanuele R. Nocera

Università degli Studi di Torino and INFN, Torino

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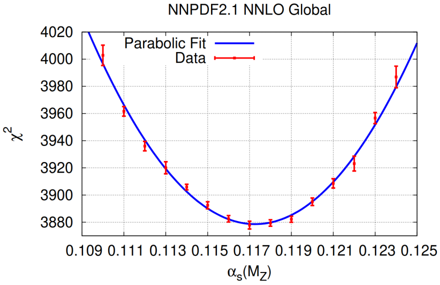
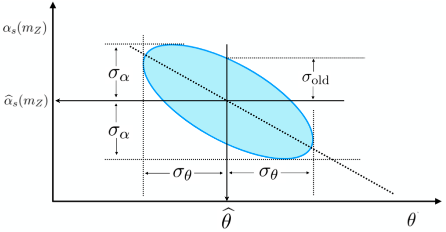
Determining PDFs and $\alpha_s(M_Z)$: NNPDF2.1



[Phys.Lett. B707 (2012)]



[Eur.Phys.J. C85 (2025) 1001]

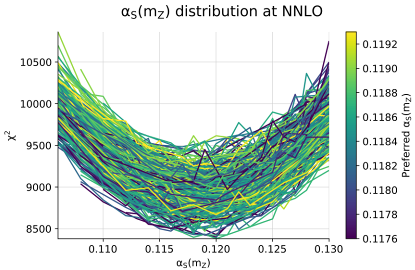
Repeat the PDF determination by varying the value of $\alpha_s(M_Z)$, which is kept fixed

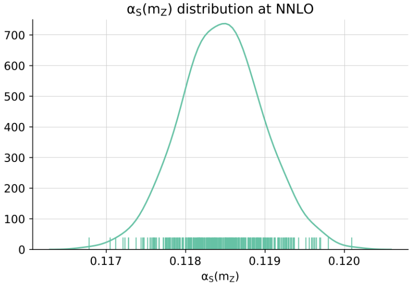
Determine the best value of $\alpha_s(M_Z)$ and its uncertainty from the $\chi^2(\alpha_s)$ profile

NNPDF2.1 [Phys.Lett. B707 (2012)] $\alpha_s^{\rm NNLO}(M_Z) = 0.1173 \pm 0.0007^{\rm exp}$

Uncertainties may be underestimated because the PDF–$\alpha_s(M_Z)$ correlation is not completely taken into account

The correlated replica method: NNPDF3.1





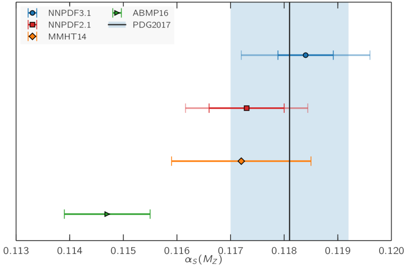
How can we take into account PDF–α_s correlations in a Monte Carlo way?

For each data sample (replica), perform a scan in α_s

Each replica has a preferred value of the α_s (the minimum of each parabola)

These preferred values form a Monte Carlo distribution

NNPDF3.1 [Eur.Phys.J. C78 (2018) 408]



$$\alpha_s^{\rm NNLO}(M_Z) = 0.1185 \pm 0.0005^{\rm exp} \pm 0.0001^{\rm meth} \pm 0.0011^{\rm th}$$

Precision and accuracy: NNPDF4.0

NNPDF2.1: $\alpha_s^{\rm NNLO}(M_Z) = 0.1173 \pm 0.0007$ (exp)

NNPDF3.1: $\alpha_s^{\rm NNLO}(M_Z) = 0.1185 \pm 0.0012$ (exp+th+meth)

NNPDF4.0: $\alpha_s^{\rm aN^3LO_{QCD}\otimes NLO_{QED}}(M_Z) = 0.1194^{+0.0007}_{-0.0014}$ (exp+th+meth)

A 1% precision on $\alpha_s(M_Z)$ requires command of many same order concurrent effects originating from experimental data, theoretical predictions, and fitting methodology

New since NNPDF3.1: NNPDF4.0 and beyond		
Data	More than 50 new data sets, mostly from the LHC	[Eur.Phys.J. C82 (2022) 428]
Theory	MHOUs	[Eur.Phys.J. C84 (2024) 517]
	$\rm NNLO_{QCD}\otimes NLO_{QED}$	[Eur.Phys.J. C84 (2024) 540]
	$\rm aN^3LO_{QCD}$	[Eur.Phys.J. C84 (2024) 659]
	$\rm aN^3LO_{QCD}\otimes NLO_{QED}$	[arXiv:2406.01779]
	higher twists/power corrections	[arXiv:2511.14387]
	heavy quarks	[in preparation]
Methodology	Optimisation and hyperoptimisation	[Eur.Phys.J. C79 (2019) 676]
	Closure tests	[Eur.Phys.J. C82 (2022) 330]
	Covariance matrix method	[Eur.Phys.J. C81 (2021) 830]

Discussion in reverse:
Methodology, Theory, Data

1. Methodology

The correlated replica method

Implement Bayesian parameter estimation by means of a theoretical covariance matrix

$S_{ij} = \beta_i \beta_j$	correlated uncertainty covariance matrix
$T \to T + \lambda\beta$	nuisance parameter λ on prediction T
$P(T \vert D, \lambda) \propto \exp\left[-\frac{1}{2}(T + \lambda\beta - D)^T C^{-1}(T + \lambda\beta - D\right]$	Probability of prediction T given the data D and the nuisance parameter λ
$P(\lambda) \propto \exp\left(-\frac{\lambda^2}{2}\right)$	Uncertainty on nuisance parameter
$P(T \vert D) \propto \exp\left[-\frac{1}{2}(T - D)^T (C + S)^{-1}(T - D)\right]$	Probability of prediciton T given the data
$P(\lambda \vert T, D) \propto \exp\left[-\frac{1}{2}Z^{-1}(\lambda - \bar{\lambda}(T, D))^2\right]$	Posterior distribution of the nuisance parameter λ

with $\bar{\lambda}(T, D) = \beta^T (C + S)^{-1}(D - T)$, $Z = 1 - \beta^T (C + S)^{-1}\beta$, and C exp. cov. mat.

Treat $\alpha_s - \alpha_s^0$ as nuisance parameter with prior $P(\alpha_s)$ centered about α_s^0

Closure tests

Assume true underlying PDFs and $\alpha_s(M_Z)$ value

Generate data distributed according to the experimental covariance matrix

Run the correlated replica method and the theoretical covariance method on these data

Do statistics on N_r "runs of the universe" turning Bayesian into frequentist

$$\langle\alpha_s\rangle = \frac{\sum_{j=1}^{N_r} \frac{\alpha_s^{(j)}}{\left(\sigma_\alpha^{(j)}\right)^2}}{\sum_{j=1}^{N_r} \frac{1}{\left(\sigma_\alpha^{(j)}\right)^2}}$$

weighted mean over N_r runs

$$\langle\sigma_\alpha\rangle = \frac{1}{\sqrt{\sum_{j=1}^{N_r} \frac{1}{\left(\sigma_\alpha^{(j)}\right)^2}}}$$

weighted uncertainty

$$P = \frac{\frac{1}{N_r}\sum_{j=1}^{N_r}\left(\alpha_s^{(j)} - \alpha_s^0\right)}{\langle\sigma_\alpha\rangle/N_r}$$

pull

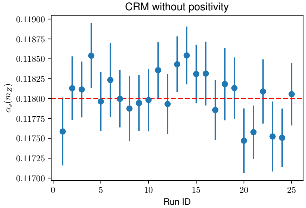
$$R_{bv} = \sqrt{\frac{1}{N_r}\sum_j^{N_r}\left(\frac{\alpha_s^j - \alpha_s^0}{\alpha_s^j}\right)^2}$$

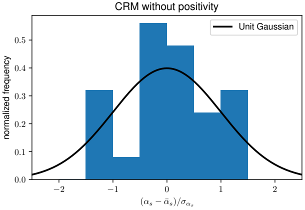
Bias/variance: mean square deviation w.r.t. truth vs unc.

Closure tests at work: $\alpha_s^0(M_Z) = 0.118$

CRM: 250 MC replicas $\times$ 12 values of α_s, $0.114 \leq \alpha_s \leq 0.123$; 25 runs of the universe

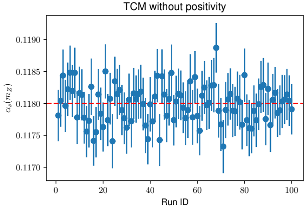
$\langle\alpha_s\rangle = 0.117984 \quad \langle\sigma_\alpha\rangle/\sqrt{N_r} = 0.000041 \quad P = 0.39 \quad R_{bv} = 0.71 \pm 0.05$

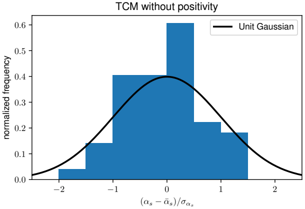




TCM: 550 MC replicas; 100 runs of the universe

$\langle\alpha_s\rangle = 0.118029 \quad \langle\sigma_\alpha\rangle/\sqrt{N_r} = 0.000077 \quad P = 0.38 \quad R_{bv} = 0.80 \pm 0.09$



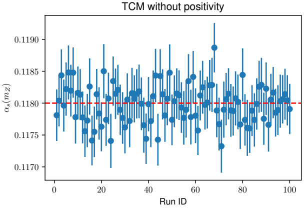


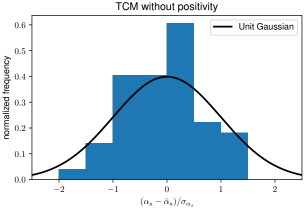
CRM and TCM in perfect agreement; methodological uncertainties validated

The problem of positivity

TCM **w/o positivity**: 550 MC replicas; 100 runs of the universe

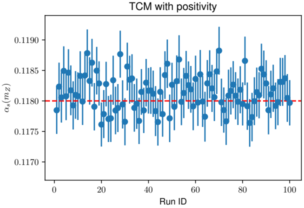
$\langle \alpha_s \rangle = 0.118029 \quad \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000077 \quad P = 0.38 \quad R_{bv} = 0.80 \pm 0.09$

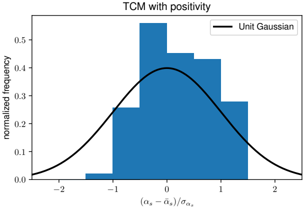




TCM **w/ positivity**: 550 MC replicas; 100 runs of the universe

$\langle \alpha_s \rangle = 0.118132 \quad \langle \sigma_\alpha \rangle / \sqrt{N_r} = 0.000039 \quad P = 3.4 \quad R_{bv} = 0.80 \pm 0.06$





Biased central value (non-Gaussian behaviour); extra unc. on real-data determination

The problem of multiplicative uncertainties

Collider data: multiplicative uncertainties in experimental covariance matrix

D'Agostini bias: using experimental covariance matrix in maximum likelihood estimate leads to an underestimate of the true value [Nucl.Instrum.Meth. A346 (1994) 306]

NNPDF solution: t_0 method [JHEP 05 (2010) 075]

Replace the experimental covariance matrix with the t_0 covariance matrix and iterate

$$\mathrm{cov}_{ij,\mathrm{exp}} \to \mathrm{cov}_{ij,t_0} = \delta_{ij}(\sigma^u D)^2 + \textstyle\sum_c^{N_{\mathrm{add}}}(\sigma^c D)_i(\sigma^c D)_j + \sum_c^{N_{\mathrm{mult}}}(\sigma^c T_0)_i(\sigma^c T_0)_j$$

New problem: theory predictions depend on α_s. Should we vary T_0 with α_s or not?

TCM **varying** T_0 **with** α_s: 25 runs of the universe

$$\langle\alpha_s\rangle = 0.119450 \quad \langle\sigma_\alpha\rangle/\sqrt{N_r} = 0.000077 \quad P = 19 \quad R_{bv} = 3.80 \pm 0.16$$

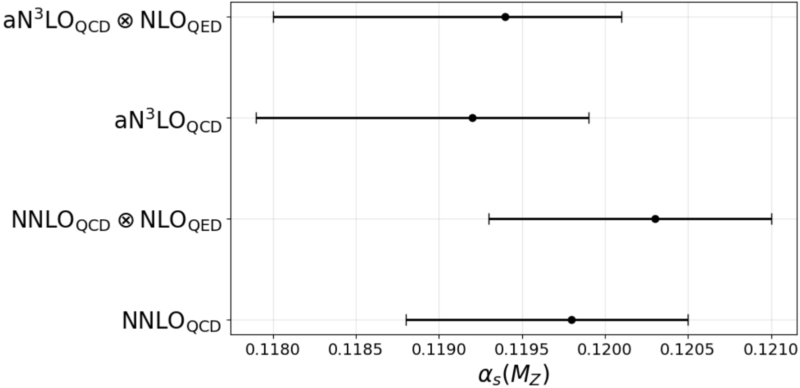
TCM **with fixed** T_0: 25 runs of the universe

$$\langle\alpha_s\rangle = 0.118152 \quad \langle\sigma_\alpha\rangle/\sqrt{N_r} = 0.000070 \quad P = 2.2 \quad R_{bv} = 0.97 \pm 0.11$$

The problem was previously undetected because:
– hadron collider data (for which multiplicative uncertainties are dominating) used to carry less weigth in the fit
– performing many runs of the universe was computationally not feasible

2. Theory

MHOUs, NLO QED, aN^3LO QCD



All results include MHOUs, needed for perturbative consistency

Impact of QED (photon PDF and DGLAP) of the order of 0.5%

Impact of aN^3LO QCD of the order of 1%

BEST VALUE (aN^3LO$_{\rm QCD}$$\otimes$NNLO$_{\rm QED}$): $\alpha_s(M_Z) = 0.1194^{+0.0007}_{-0.0014}$

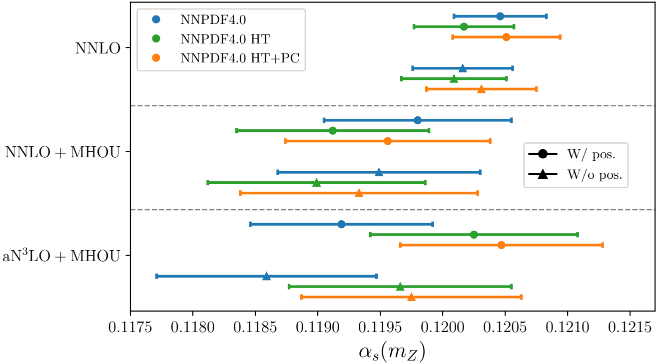
Naive extraction: $\alpha_s(M_Z) = 0.1193 \pm 0.0005$ (uncertainty underestimated)

Higher twist and other power corrections

Higher twist (DIS) and power corections (jets) may be non negligible
even if kinematic cuts are applied to the fitted data

Model corrections with a prior $H(x)$ associated to an additional theoretical cov. matrix

Repeat the TCM determination to obtain jointly α_s and $H(x)$



At NNLO, higher twist and power corrections balance out; slight increase of uncertainties

At N^{3}LO, higher twist dominate, possibly because they are less contaminated by MHOUs

Heavy quark masses

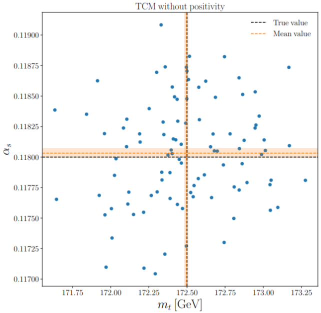
The value of α_s is in principle sensitive to the value of heavy quark masses

For charm, the problem is very much reduced in NNPDF,
because the charm PDF is fitted along with light quark PDFs

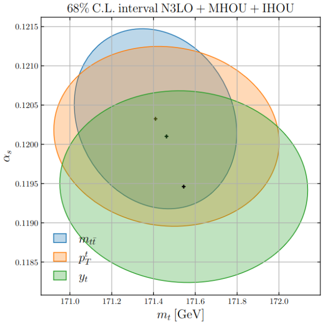
For top, we repeated the α_s determination for the following values:
$m_t = 172.5$ GeV (default) $m_t = 170.0$ GeV $m_t = 175.0$ GeV
Variations are about four times the PDG pole mass uncertainty $\Delta m_t = 0.7$ GeV
$\Delta\alpha_s = 0.0004$ (NNLO) and $\Delta\alpha_s = 0.0001$ (aN^{3}LO), increasing with increasing m_t

The TCM allows one to determine α_s and m_t jointly [Plots by courtesy of Jaco ter Hoeve]

CLOSURE TEST PRELIMINARY

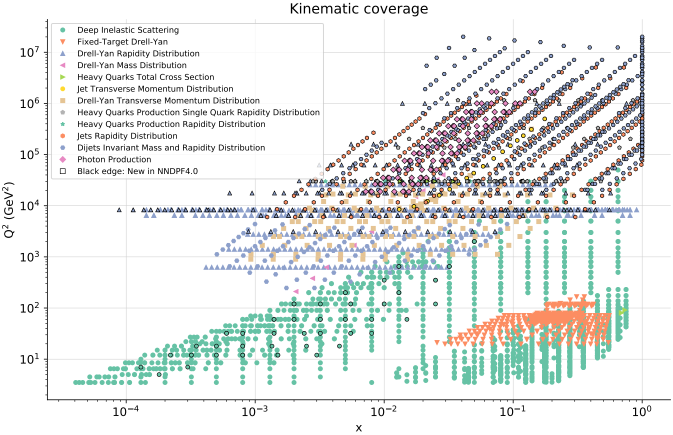


REAL DATA PRELIMINARY



3. Data

From NNPDF3.1 to NNPDF4.0: impact of data



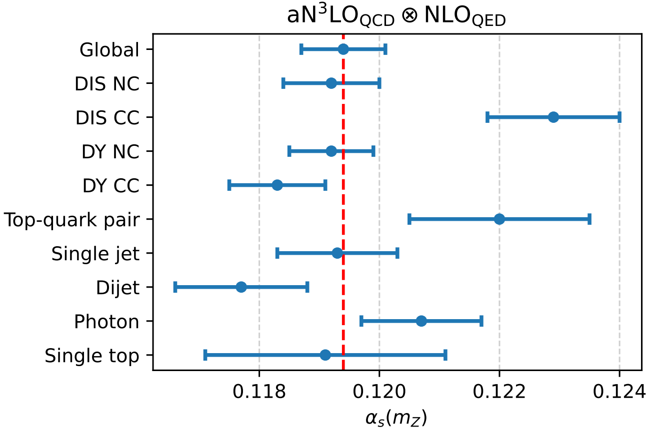
NNPDF3.1: $\alpha_s^{\rm NNLO}(M_Z) = 0.1185 \pm 0.0005$ (exp)

NNPDF4.0 (NNPDF3.1 data): $\alpha_s^{\rm NNLO}(M_Z) = 0.1186 \pm 0.0005$ (exp)

NNPDF4.0: $\alpha_s^{\rm NNLO,MHOU}(M_Z) = 0.1198 \pm 0.0008$ (exp+th)

A substantial increase in α_s central value is due to new LHC data

What is the relative contribution of each data set?

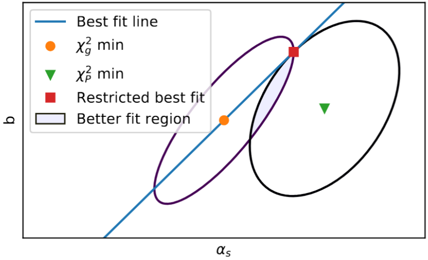


Values cannot be understood as the best-fit values associated to that process and in particular the global α_s value does not correspond to their weighted mean

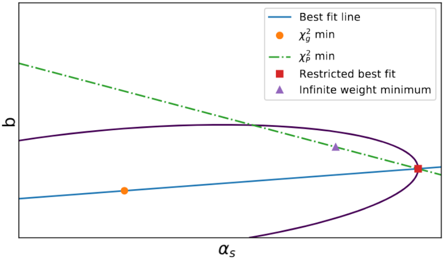
DIS CC, top-quark pair, direct photon prefer a larger value of α_s whereas DY CC and dijet prefer a smaller value of α_s

α_s determinations without PDF refitting are biased

Data set that constrains α_s and PDFs



Data set that constrains only α_s



[Eur.Phys.J. C80 (2020) 182]

g: global data set P: single process

toy model in which PDFs depend on a single parameter b

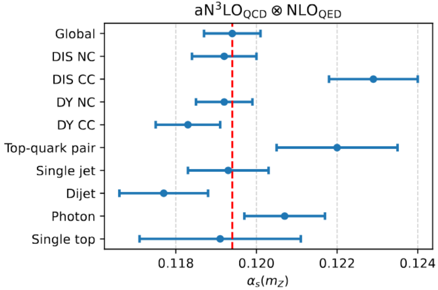
$\alpha_{s,0}^{rP}$ (restricted fit) different from $\alpha_{s,0}^{g}$ (global fit) and from $\alpha_{s,0}^{P}$ (single process fit)

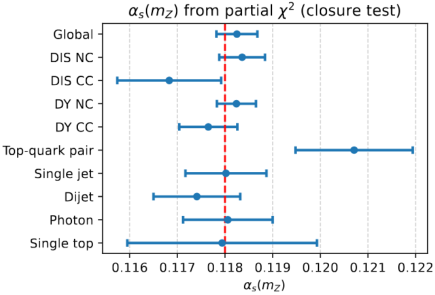
if P does not constrain PDFs, there are flat directions of χ^2_P in the (α_s, b) space

the minimum of χ^2_g along the χ^2_P segment is selected by minimising the weighted $\chi^2_w = \chi^2_g + w\chi^2_P$ with w large

Can we explicitly check this behaviour in a closure test?

α_s determinations without PDF refitting are biased





[arXiv:2511.22561]

Performing a simultaneous determination of α_s and PDFs from different (sub)sets of data generally gives different results because the data fluctuate within their uncertainties

Check this statement in a closure test (in which $\alpha_s = 0.118$ by construction)

$$\alpha_s^{t\bar{t}}(M_Z^2) = 0.1207 \pm 0.0012 \quad \alpha_s(M_Z^2) = 0.1183 \pm 0.0004 \quad \alpha_s^{t\bar{t}}(M_Z^2)_{\rm wgt} = 0.1201 \pm 0.0012$$

A sign of tension has reduced to a statistical fluctuation (as it should by construction)

There exist points in (α_s, PDF) space that provide a better fit to the global dataset while providing an equally good fit to partial datasets, that correspond to a value of α_s closer to the global value than the naive partial α_s

4. Conclusions

Summary

NNPDF4.0: $\alpha_s^{\mathrm{aN^3LO_{QCD}\otimes NLO_{QED}}}(M_Z) = 0.1194^{+0.0007}_{-0.0014}$ (exp+th+meth)

A 1% precision on $\alpha_s(M_Z)$ requires command of many same order concurrent effects originating from experimental data, theoretical predictions, and fitting methodology

The NNPDF4.0 α_s determination investigates many of these effects w.r.t NNPDF3.1
New LHC data prefer a higher value of $\alpha_s(M_Z)$ by slightly more than 1%

Caution is required when interpreting the pull of single data sets

N^{3}LO QCD corrections decease the value of $\alpha_s(M_Z)$ by slightly less than 1% w.r.t NNLO, whereas NLO QED corrections increase it by about 0.5%

Higher twist and power corrections may increase the value of $\alpha_s(M_Z)$ by an extra 1%
Heavy quark mass effects are comparatively sub-dominant

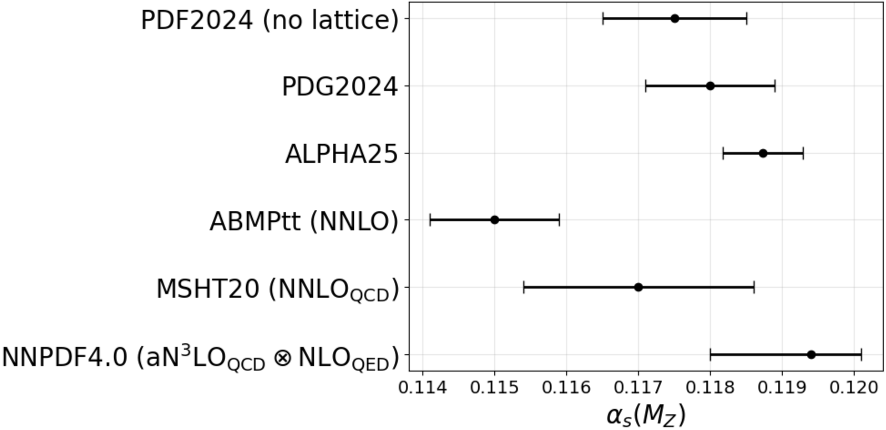
Inclusion of MHOUs is crucial to preserve perturbative compatibility
They increase the uncertainty by about a third (less than half the NLO-NNLO shift)

The impact of the methodological uncertainties has been thoroughly validated by closure testing a frequentist (CRM) and a Bayesian (TCM) determination

Subtleties related to positivity and multiplicative uncertainties have been revealed

Data availability

All PDF sets are available, in LHAPDF format, through the NNPDF web site
https://nnpdf.mi.infn.it/nnpdf4-0-alphas/



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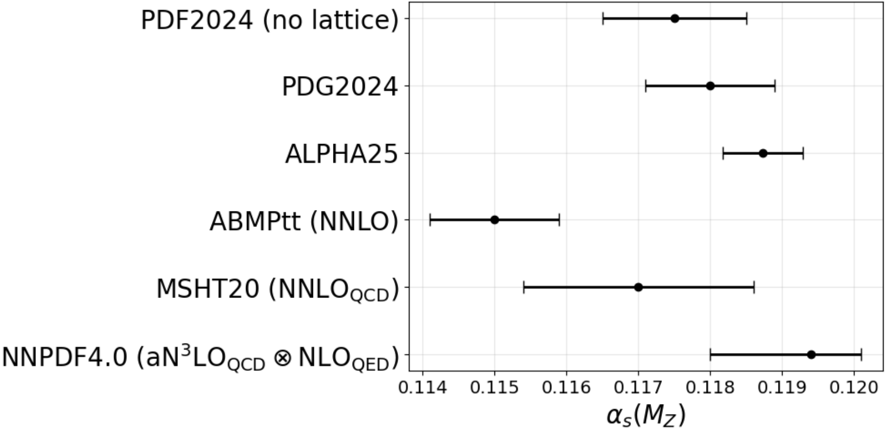

Thank you