Quark masses and α_s from global quarkonia fits @ N^{3}LO

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Taskforce J.M: Mena-Valle and P.G. Ortega based on JHEP 07 (2025) 249 + wip

α_s - 2025 workshop, Assois 15-11-2025

Outline

- Motivation
- Theory review
- Data
- Results
- Question to myself
- Conclusions

Motivation

Measuring strong coupling in pQCD

General strategy: finding an observable that fulfils these requirements:

- Small non-perturbative power corrections
- Known to high order in pQCD, with good convergence
- Very sensitive to the strong coupling
- Typical energy scale not too high
- Known experimentally with high accuracy

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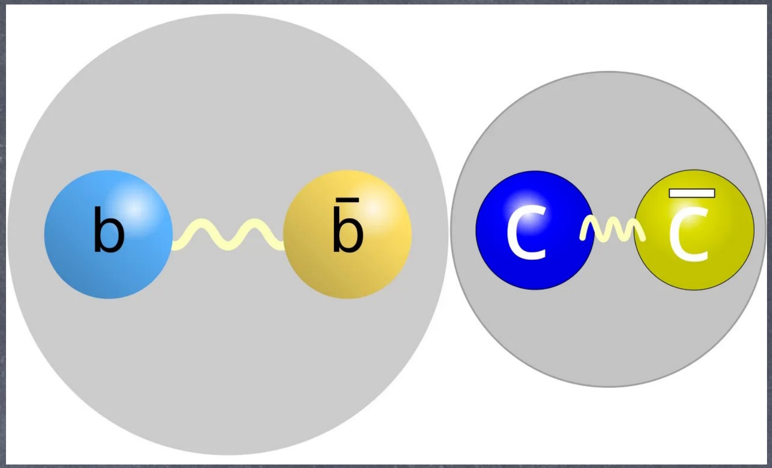
Not so easy to find a single example that satisfies all of these...

Let's see how far we get with quarkonia spectrum

Theory review

Quarkonium in a nutshell



Bound states of a heavy quark-antiquark pair

Can be treated in pNRQCD

Will be used for $\bar{b}b$ $\bar{c}c$ and $\bar{b}c$ states

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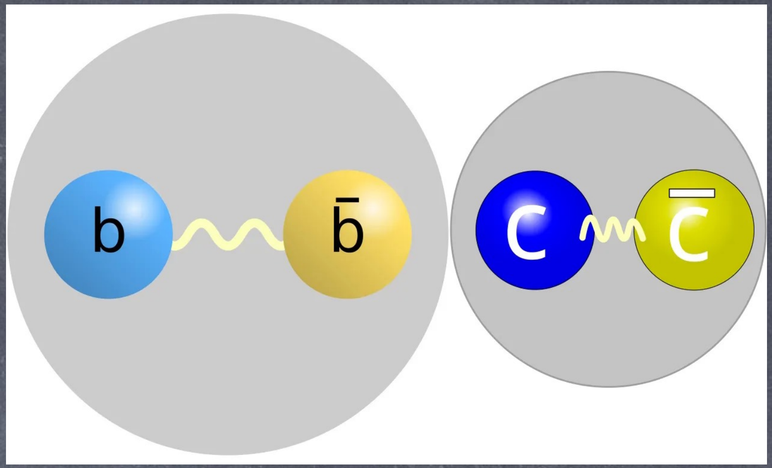

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Obtained solving Schrödinger equation in perturbation theory

Static potential + relativistic corrections + ultrasoft effects

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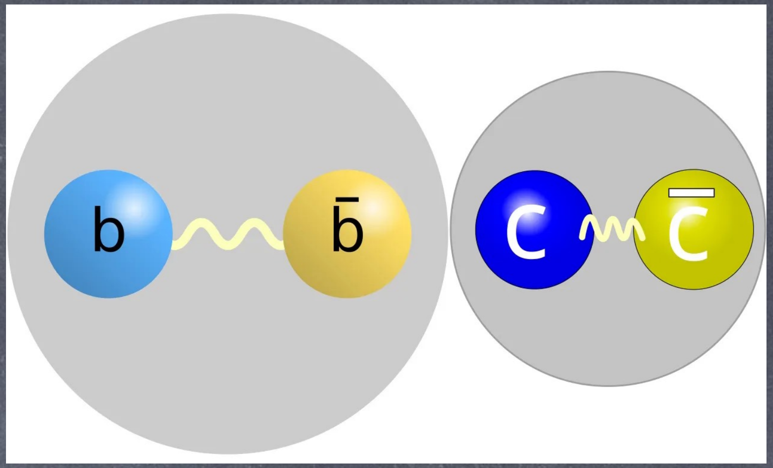

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First look at the expression (pole scheme)

$$E_X(\mu, n_\ell) = 2\, m_Q^{\rm pole}\left\{1 - \frac{\left[C_F\, \alpha_s^{(n_\ell)}(\mu)\right]^2}{8n^2} \sum_{i=0}^{\infty} \left[\frac{\alpha_s^{(n_\ell)}(\mu)}{4\pi}\right]^i \varepsilon^{i+1} P_i(L_{n_\ell})\right\}$$

Strong coupling enters as α_s^2, sensitivity not great

Full for formula for quarkonium

Pole scheme

[Kiyo, Sumino NP B889 (2014) 156-191]

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$$L_{n_\ell} = \log\left[\frac{n\mu}{C_F \alpha_s^{(n_\ell)}(\mu) m_Q^{\rm pole}}\right] + H_{n+\ell}$$

residual dependence on mass

harmonic number: depends on n and l

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$$L_{n_\ell} = \log\left[\frac{n\mu}{C_F\alpha_s^{(n_\ell)}(\mu)m_Q^{\rm pole}}\right] + H_{n+\ell}$$ residual dependence on mass

harmonic number: depends on n and l

Argument of logs non-trivial: includes $\alpha_s(\mu)$

typical scale is then $\mu \sim \dfrac{C_F\,\alpha_s\,m}{n} \ll m$ depends on n

$$P_i(L) = \sum_{j=0}^{i} c_{ij}\,L^j$$ polynomial (simply RG-invariance)

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c_{i0} depend on all four quantum numbers. N³LO includes α_s^5

in c_{30} first appearance of usoft effects: $\log(\alpha_s)$

Charm mass effects in bottomonium

$$E_X(\mu, n_\ell, m_Q^{\text{pole}}, \underbrace{m_q^{\text{pole}}}_{\substack{\text{massive lighter}\\ \text{quark}}}) = \underbrace{E_X(\mu, n_\ell, m_Q^{\text{pole}})}_{\substack{\text{massless result}\\ \text{known exactly}}} + \underbrace{\varepsilon^2 \delta E_X^{(1)} + \varepsilon^3 \delta E_X^{(2)}}_{\substack{\text{corrections}\\ \text{complicated}}}$$

[Eiras, Soto PLB491 (2000), 101-110], [Hoang hep-ph/0008102]

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n_ℓ scheme: massless limit manifest, decoupling limit not well defined

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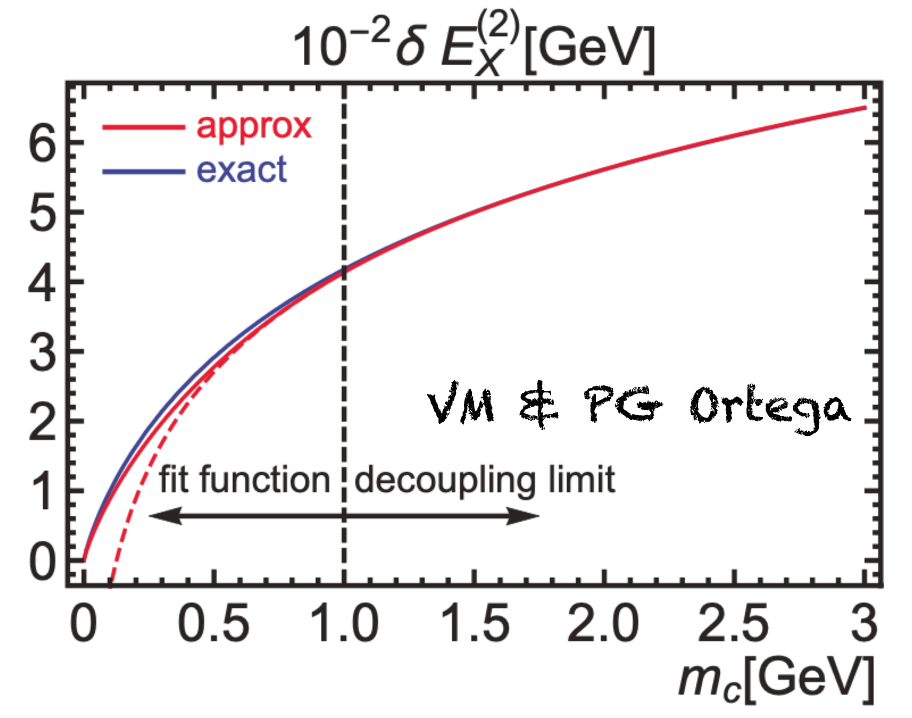
[Brambilla, Sumino, Vairo PRD65 (2002) 043001]

decoupling limit good approximation



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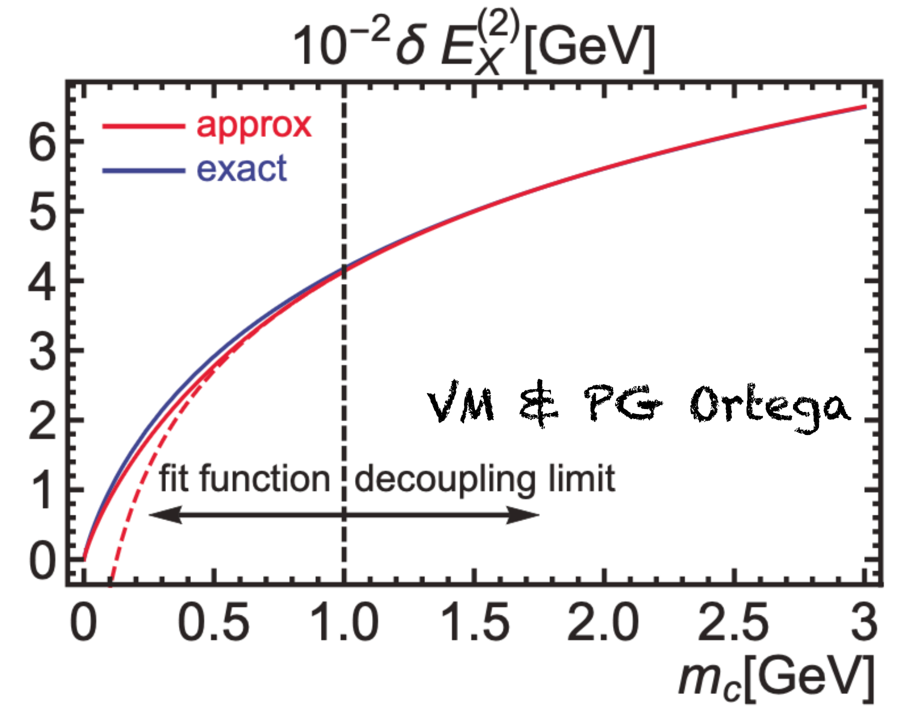

Need to implement charm mass corrections in renormalon subtraction

Bc masses

[Peset, Pineda, Stahlhoffen JHEP 2016]

pole scheme

$$E_{B_c}(\mu, n_\ell) = m_b^{\text{pole}} + m_c^{\text{pole}} - \frac{C_F^2[\alpha_s^{n_\ell}(\mu)]^2 m_r^{\text{pole}}}{2n^2} \sum_{n=0}^{3} \left[\frac{\alpha_s(\mu)}{\pi}\right]^i P_i(L_\mu)$$

reduced mass $m_r^{\text{pole}} = \dfrac{m_c^{\text{pole}} m_b^{\text{pole}}}{m_c^{\text{pole}} + m_b^{\text{pole}}}$

$$L_\mu = \log\left(\frac{n\mu}{C_F \alpha_s^{(n_\ell)}(\mu) m_r^{\text{pole}}}\right) + H_{n+\ell}$$

very similar expression

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In this EFT the bottom and charm quarks are not dynamic

n_ℓ = 3 above. Also for the short-distance masses

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Charmonium masses

Same formula as bottomonium w/o massive lighter corrections

MSR mass

[Hoang, Jain, Scimemi, Stewart (2008)]

[Hoang, Jain, Lepenik, VM, Preisser Scimemi, Stewart (2008)]

$\overline{\mathrm{MS}}$ mass

$$m_p - \overline{m} = \overline{m} \sum_{n=1} \left[\frac{\alpha_s^{(n_\ell+n_h)}(\overline{m})}{4\pi} \right]^n a_{n0}(n_\ell, n_h)$$

ambiguity does not depend on $\overline{m}$ or n_h

[Beneke Phys. Lett. B344 (1995) 341-347]

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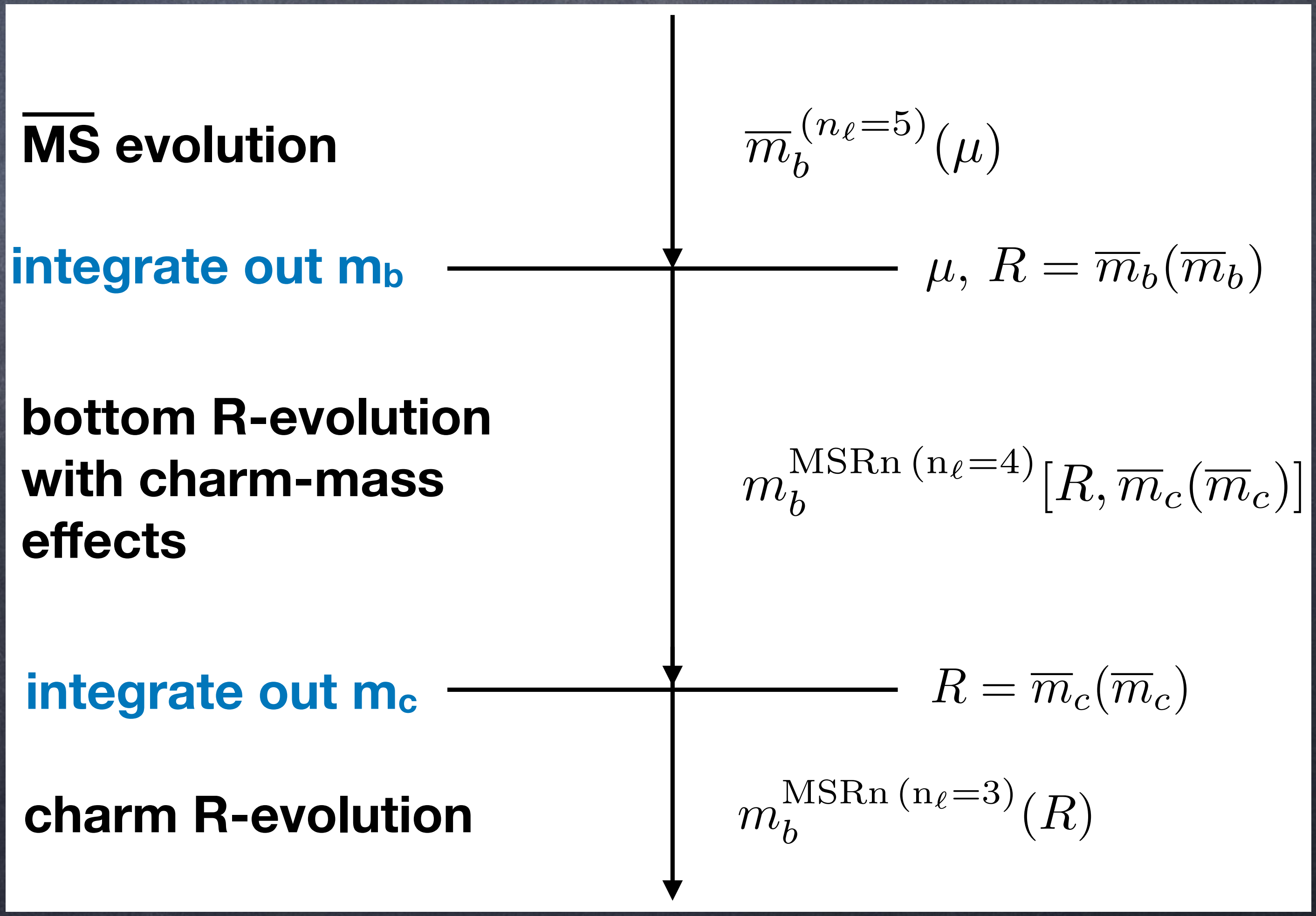
Including finite charm mass effects in MSR bottom mass



$\overline{\text{MS}}$ evolution $\overline{m}_b^{(n_\ell=5)}(\mu)$

integrate out m_b $\mu,\ R = \overline{m}_b(\overline{m}_b)$

bottom R-evolution with charm-mass effects $m_b^{\text{MSRn}\,(n_\ell=4)}[R, \overline{m}_c(\overline{m}_c)]$

integrate out m_c $R = \overline{m}_c(\overline{m}_c)$

charm R-evolution $m_b^{\text{MSRn}\,(n_\ell=3)}(R)$

Mateu-Ortega implementation

exact HQ symmetry

MSR mass

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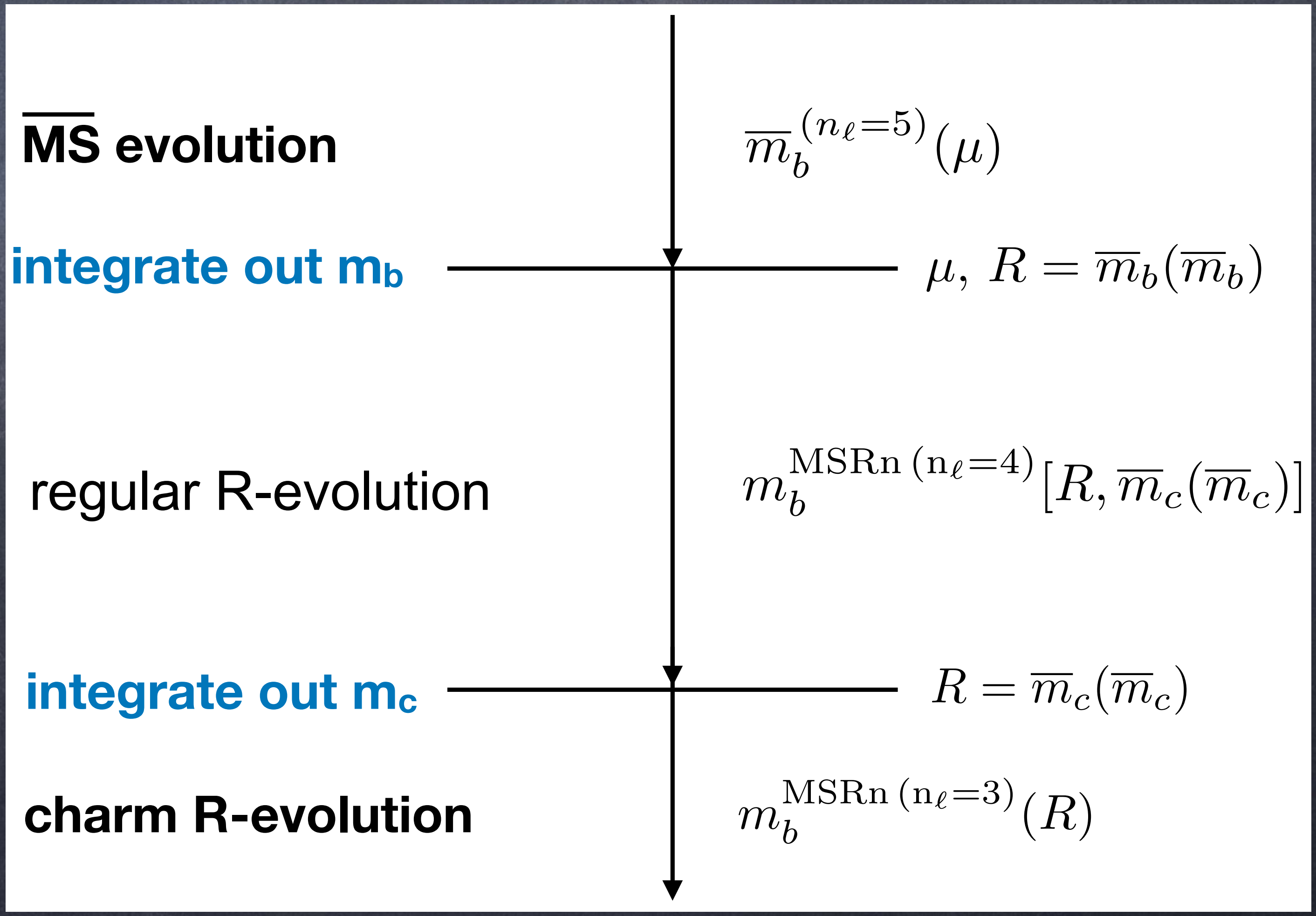
Including finite charm mass effects in MSR bottom mass

[Hoang, Lepenik, VM (2022)]

REvolver implementation

Different matching

approximate HQ symmetry



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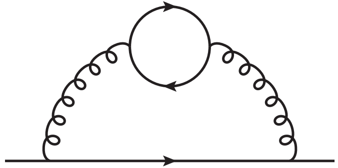
integrate out m_c — $R = \overline{m}_c(\overline{m}_c)$

charm R-evolution $m_b^{\text{MSRn}\,(n_\ell=3)}(R)$

MSR mass: charm effects

$$m_p - \overline{m}_Q = \delta\overline{m}_Q + \overline{m}_Q \Delta^{\overline{\mathrm{MS}}}_{\overline{m}_q}(\overline{m}_Q, \xi)$$



$$\xi = \frac{\overline{m}_q}{\overline{m}_Q}$$

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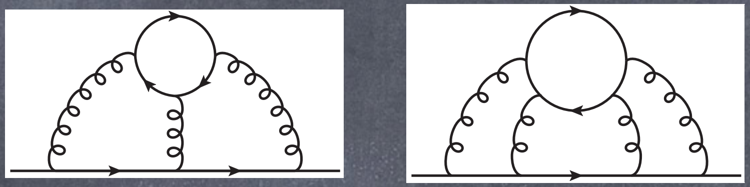

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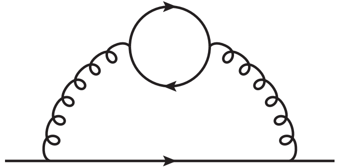

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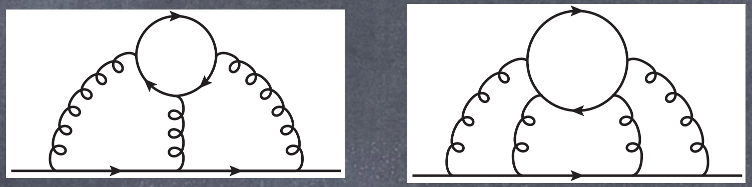

[VM, Ortega (2018)]

Mateu-Ortega

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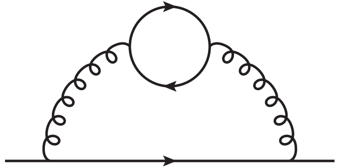

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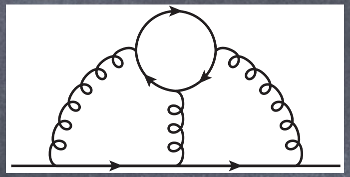

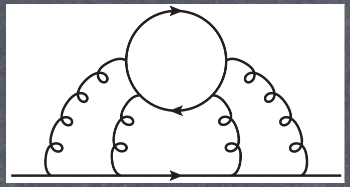

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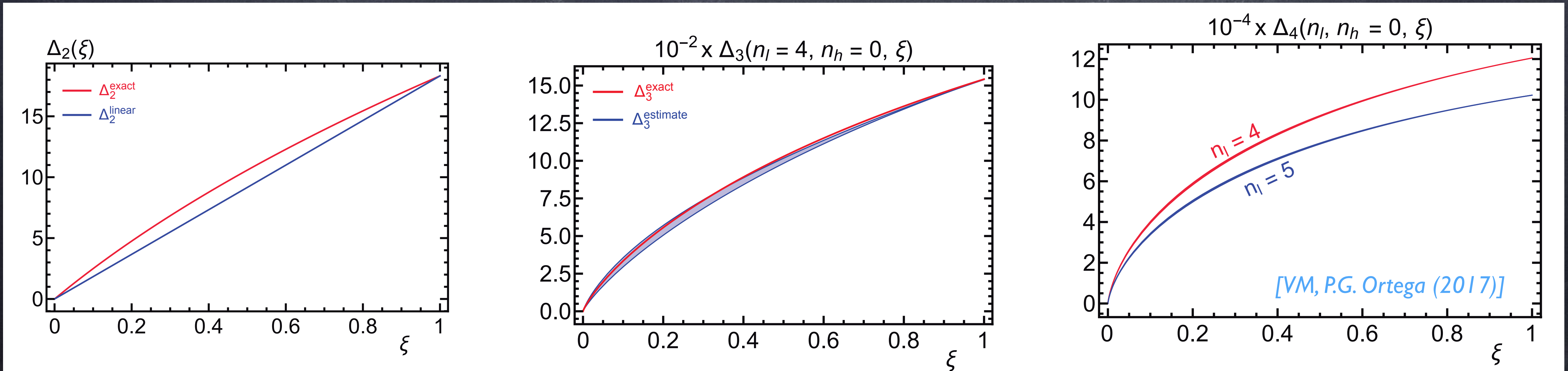
We have repeated 2018 analysis with REvolver

Can be used to predict high-order corrections



Short distance scheme

The expression has a u = 1/2 renormalon

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Express pole mass in terms of low-scale short-distance scheme

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Express pole mass in terms of low-scale short-distance scheme

We employ the MSR mass and R-evolution

- Bottomonium n_ℓ scheme: R-evolutiom with charm corrections
  or usual R-evolution
- Bottomonium $n_\ell - 1$ sch: charm MSR mass R-evolution
  consistent running and matching to $\overline{m}_b$
- Chamonium: usual MSR mass with 3 flavors
- B$_c$ charm and bottom MSR masses with 3 flavors

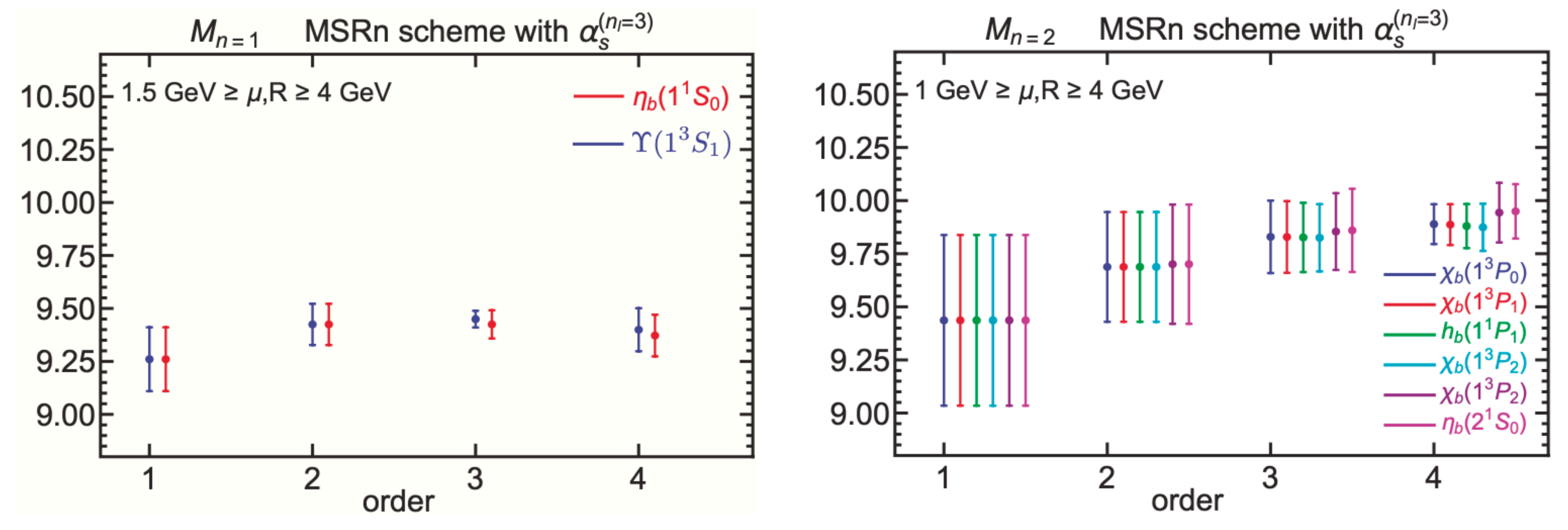
Scale setting

Bottomonium: consider only states with n = 1, 2

For different values of n we use different scale variation

$$\mu_2(\mu) = \mu, \qquad R_2(R) = R\,, \qquad 1\,\mathrm{GeV} \le \mu, R \le 4\,\mathrm{GeV}\,,$$

$$\mu_{1,3}(\mu) = 1.5\,\mathrm{GeV} + 2.5(\mu - 1\,\mathrm{GeV})/3\,, \quad R_{1,3}(\mu) = 1.5\,\mathrm{GeV} + 2.5(\mu - 1\,\mathrm{GeV})/3\,,$$



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We believe this propagates correlations correctly

Using theory nuisance parameters is an obvious improvement on this

[F. Tackmann, JHEP 08 (2025) 098]

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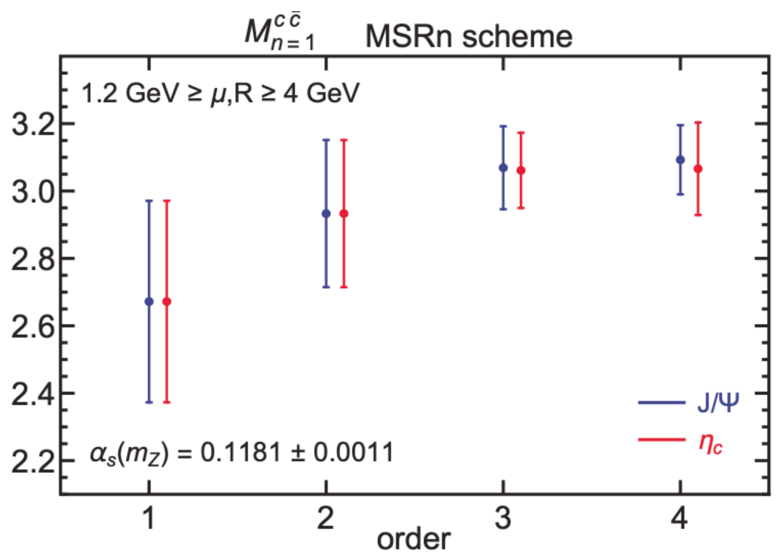
for global fits, correlated with bottomonium



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PRELIMINARY

B_c: consider only n = 1

$$\mu_{B_c} = \mu_{\mathrm{charm}}$$

for global fits, correlated with charmonium and bottomonium

Scale setting

Bottomonium: consider only states with n = 1, 2

For different values of n we use different scale variation

$$\mu_2(\mu) = \mu, \qquad R_2(R) = R\,, \qquad 1\,\text{GeV} \le \mu, R \le 4\,\text{GeV}\,,$$

$$\mu_{1,3}(\mu) = 1.5\,\text{GeV} + 2.5(\mu - 1\,\text{GeV})/3\,, \quad R_{1,3}(\mu) = 1.5\,\text{GeV} + 2.5(\mu - 1\,\text{GeV})/3\,,$$

We believe this propagates correlations correctly

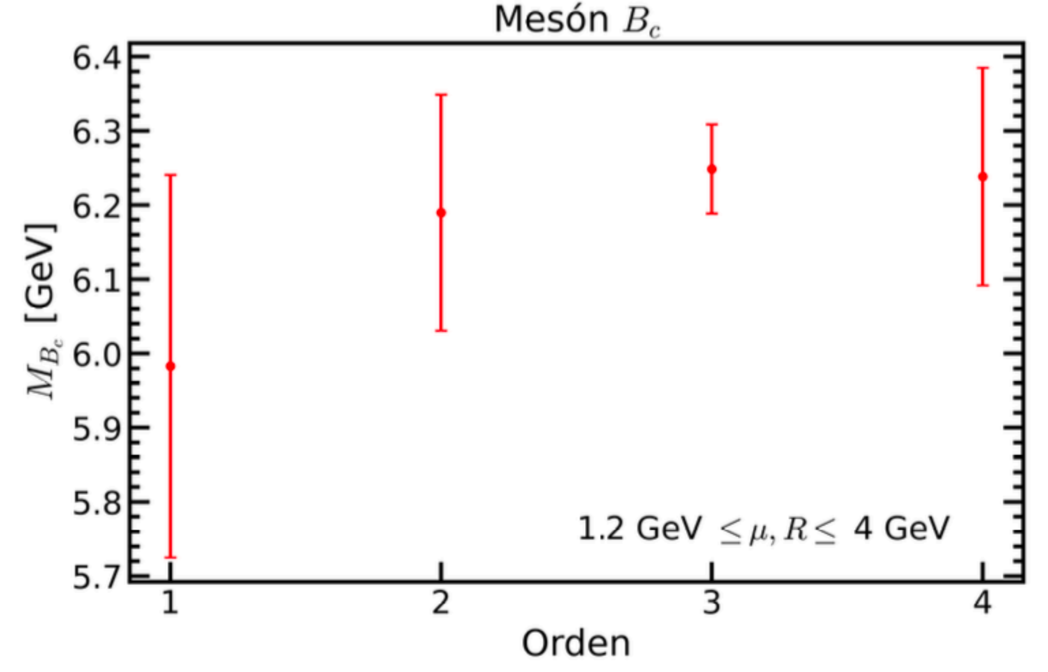
Using theory nuisance parameters is an obvious improvement on this

[F. Tackmann, JHEP 08 (2025) 098]

Charmonium: consider only n = 1

$$1.2\,\text{GeV} \le \mu_{\text{charm}} \le 4\,\text{GeV}$$

PRELIMINARY



Bc: consider only n = 1

$$\mu_{B_c} = \mu_{\text{charm}}$$

Data

Experimental data

Charmonium $\eta_c(1S), J/\psi(1S)$

Bottomonium

$$n = 1 : \quad \{\eta_b(1S), \Upsilon(1S)\}$$

$$n = 2 : \quad \{\chi_{b0}(1P), \chi_{b1}(1P), h_b(1P), \chi_{b2}(1P), \eta_2(2S), \Upsilon(2S)\}$$

B_c only one state

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Bc only one state

We have explored different

- kinds of global fits
- schemes for finite charm mass effects
- MSR prescriptions (M0 vs REvolver)

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B_c only one state

We have explored different

- kinds of global fits
- schemes for finite charm mass effects
- MSR prescriptions (M0 vs REvolver)

In this talk discuss only analysis in which α_s is determined

Analyses

Some thoughts

One could follow same strategy as in [Boito VM JHEP 03 (2020) 094]

Take ratios of n = 1 and n = 2 bottomonium masses to loose dependence on bottom quark (only within logs)

We find this fails since α_s comes only squared

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Determine m_b from bottomonium, m_c from charmonium, α_s from B_c

strategy used in [Peset, Pineda, Segovia JHEP 09 (2018) 167]

$\Delta_{\rm RF} = M_{B_c} - M_{\eta_b}/2 - M_{\eta_c}/2$ renormalon-free, small NP

$\alpha_s^{(n_f=5)}(m_Z) = 0.1195 \pm 0.0053$ caveat: theory correlations?

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This talk: global fits [JM Mena-Valle, VM, PG Ortega, wip]

Details on fits

Construct a χ^2 function with experimental errors only

$$\chi^2\left(\overline{m}_b, \overline{m}_c, \alpha_s, \mu, R\right) = \sum_i \left[\frac{M_i^{\text{exp}} - M_i^{\text{pert}}\left(\mu, R, \overline{m}_b, \overline{m}_c, \alpha_s\right)}{\sigma_i^{\text{exp}}}\right]^2$$

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Scan over a grid of μ and R values correlated as described

$$\Delta^{\mathrm{pert}} = \frac{1}{2}\left[\max\left(\overline{m}_Q^{\mathrm{BF}}(\mu, R)\right) - \min\left(\overline{m}_Q^{\mathrm{BF}}(\mu, R)\right)\right]$$

perturbative error from max and min best-fit values

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perturbative error from max and min best-fit values

experimental error from average of fit uncertainties

inflated by $\sqrt{\chi^2_{\text{min}}/dof}$

add in quadrature

Results I

Only bottomonium [VM, Ortega JHEP] $n_\ell - 1$ scheme

$$\overline{m}_b\left(\overline{m}_b\right) = 4.219 \pm 0.0002_{\rm exp} \pm 0.062_{\rm pert}\ \text{GeV}$$

$$\alpha_s^{(n_f=5)}\left(m_Z\right) = 0.1178 \pm 0.00001_{\rm exp} \pm 0.0050_{\rm pert}$$

Remainder:

$\overline{m}_b^{\rm PDG} = 4.18^{+0.03}_{-0.02}\,\text{GeV}$

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Reproduction using REvolver (JM Mena-Valle Bachelor Thesis)

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$$\overline{m}_b\left(\overline{m}_b\right) = 4.219 \pm 0.064\,\text{GeV}$$

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excellent reproduction

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n_ℓ scheme

$$\overline{m}_b\left(\overline{m}_b\right) = 4.229 \pm 0.058\,{\rm GeV}$$

$$\alpha_s^{(n_f=5)}\left(m_Z\right) = 0.1177 \pm 0.0042$$

more accurate results

Remainder:

$\overline{m}_b^{\rm PDG} = 4.18^{+0.03}_{-0.02}\,{\rm GeV}$

Results II

Charmonium & bottomonium (JM Mena-Valle Bachelor Thesis)

$$\overline{m}_b\left(\overline{m}_b\right) = 4.228 \pm 0.055\,\text{GeV}$$

n_ℓ scheme from now on

$$\overline{m}_c\left(\overline{m}_c\right) = 1.289 \pm 0.054\,\text{GeV}$$

$$\alpha_s^{(n_f=5)}\left(m_Z\right) = 0.1178 \pm 0.0041$$

small error reduction on bottom mass and strong coupling

central values essentially unchanged

Remainder

$\overline{m}_c^{\text{PDG}} = 1.27 \pm 0.02\,\text{GeV}$ $\quad \overline{m}_b^{\text{PDG}} = 4.18^{+0.03}_{-0.02}\,\text{GeV}$

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small error reduction on bottom mass and strong coupling

central values essentially unchanged

bottomonium & B$_c$ (JM Mena-Valle Master Thesis)

$$\overline{m}_b\left(\overline{m}_b\right) = 4.234 \pm 0.053\,\text{GeV}$$

$$\alpha_s^{(n_f=5)}\left(m_Z\right) = 0.1172 \pm 0.0039$$

small error reduction on bottom mass and strong coupling

affects both central values a bit

Remainder

$$\overline{m}_c^{\text{PDG}} = 1.27 \pm 0.02\,\text{GeV} \quad \overline{m}_b^{\text{PDG}} = 4.18^{+0.03}_{-0.02}\,\text{GeV}$$

Global fit

Charmonium, bottomonium & B_c (JM Mena-Valle Master Thesis)

$$\overline{m}_b\left(\overline{m}_b\right) = 4.231 \pm 0.055\,\mathrm{GeV}$$

$$\overline{m}_c\left(\overline{m}_c\right) = 1.289 \pm 0.048\,\mathrm{GeV}$$

$$\alpha_s^{(n_f=5)}\left(m_Z\right) = 0.1174 \pm 0.0042$$

small error reduction on charm mass

affects α_s central value, small increase of uncertainty

small effect on bottom mass central value

Global fit

Charmonium, bottomonium & B_c (JM Mena-Valle Master Thesis)

$$\overline{m}_b\left(\overline{m}_b\right) = 4.231 \pm 0.055\,\mathrm{GeV}$$

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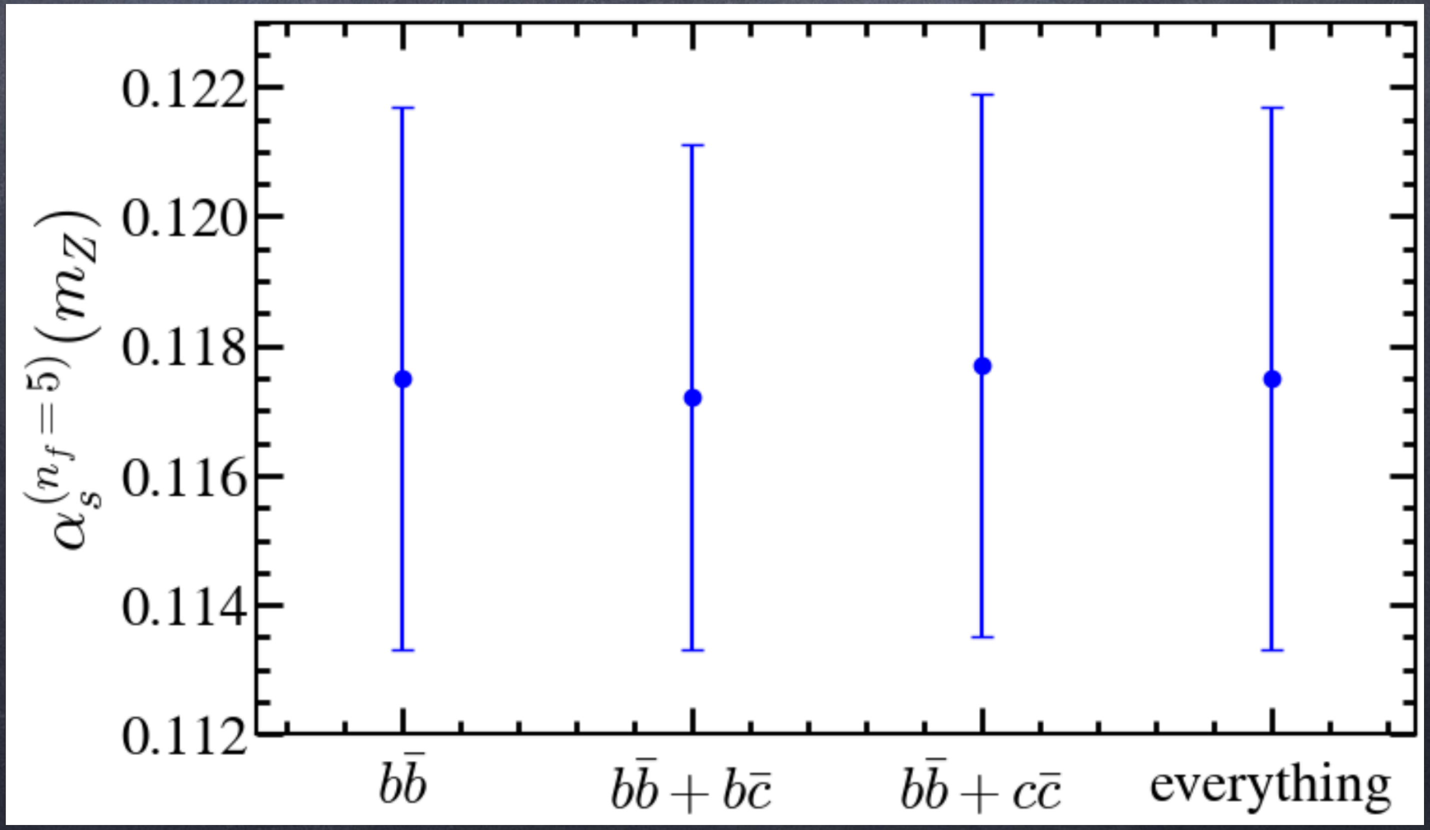
summary



Anticipating your first question

What if you fix charm and bottom masses to PDG values and fit for α_s ?

$$\alpha_s^{(n_f=5)}(m_Z) = 0.1193 \pm 0.0063_{\text{pert}} \;\pm 0.00004_{m_b} \pm 0.0025 = 0.1193 \pm 0.0068$$

significantly higher, more uncertain

Can be traced to tensions between quark masses obtained from quarkonium fits and PDG values

Remainder

$$\overline{m}_c^{\text{PDG}} = 1.27 \pm 0.02\,\text{GeV} \quad \overline{m}_b^{\text{PDG}} = 4.18^{+0.03}_{-0.02}\,\text{GeV}$$

Conclusions

- Strong coupling overly driven by bottomonium mass splitting
- B_c has little to add to the α_s determination
- Global fits do not improve the landscape significantly
- Most important contribution to α_s error budget: perturbative
- Implementing theory nuisance parameters next natural step
- Finding a new strategy to improve convergence could be instrumental

Thanks for your attention