α_s measurement prospects at the EIC and JLab@22

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Jefferson Lab (JLab)

Work done with: T. Kutz (MIT), J. R. Pybus (MIT), D. W. Upton (UVA, ODU), C. Cotton (UVa), A. Deshpande (CFNS, Stony Brook U.), W.B. Li (CFNS, Stony Brook U.), D. Nguyen (JLab, UTK), M. Nycz (UVa), X. Zheng (UVa), and the former ECCE Consortium (now part of the ePIC Collaboration), and with J.P. Chen (JLab)

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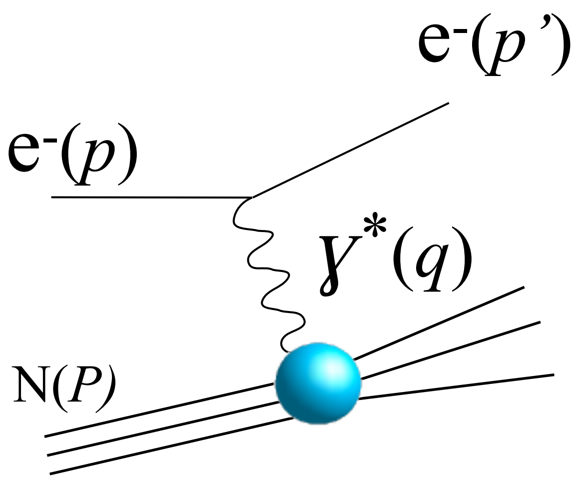
α_s from the Bjorken Sum rule

- Polarized inclusive lepton scattering
- The Bjorken Sum rule and α_s
- Measurement at EIC, $\alpha_s(M_z)$
- Measurement at JLab, $\alpha_s(M_z)$
- Search for new physics with α_s from JLab+EIC

Inclusive lepton-nucleon scattering

- $p=(E,\mathbf{p})$, $p'=(E-\nu,\mathbf{p-q})$, $q=(\nu,\mathbf{q})$
- γ^* virtual photons: q^2
- Since $q^2 <0$ here, we use $Q^2 = -q^2$.
- Bjorken scaling variable $x= Q^2 /2M\nu$.



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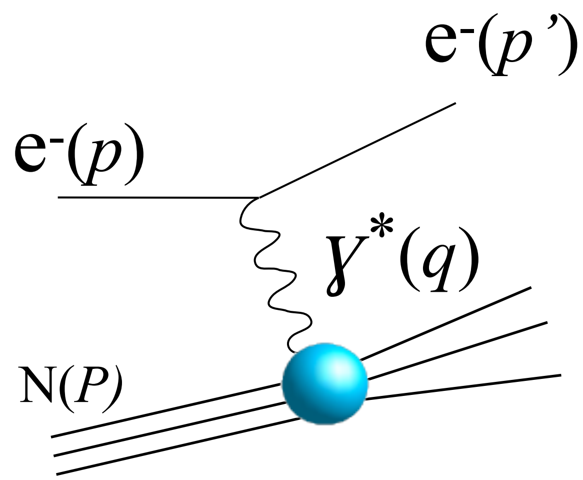

Cross section: $\sigma=\sigma_{\mathrm{Mott}}[\alpha F_1(x,Q^2)+\beta F_2(x,Q^2)+\gamma g_1(x,Q^2)+\varpi g_2(x,Q^2)]$

F_1, F_2, g_1 and g_2: structure functions

F_1 and F_2: obtained with unpolarized beam and target.

g_1 and g_2: obtained with both beam and target polarized.

Bjorken sum: $\int g_1^{\mathrm{proton}}-g_1^{\mathrm{neutron}}dx$

Considering the nucleon inclusive scattering, α_s can be extracted from:

- Q^2-evolution of structure functions/PDFs. Complex task: involves DGLAP global fit, non-perturbative inputs: quark and gluon distributions (possibly higher-twists for low-Q^2 / large-x data).

- Q^2-evolution of isovector moment $\int_0^1 g_1^{p-n}(x, Q^2)dx$, i.e Bjorken sum.
  - Simpler:
    - no x-dependence, Q^2-evolution predicted by pQCD.
    - Q^2-evolution known to higher order than structure functions/PDFs/single nucleon case.
  - Non-perturbative x-dependence integrated into precisely measured axial charge g_A. ($g_A = 1.2762 \pm 0.0005$). No gluon contribution.
  - Issues:
    - unmeasurable low-x contribution
    - Measurement on polarized p and n.

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Bjorken sum rule

$$\Gamma_1^{p-n} \equiv \int g_1^{p-n} dx = \frac{1}{6} g_A$$

Nucleon
axial charge

Bjorken sum rule

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$$+ \frac{M^2}{Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right] + \ldots$$

Nucleon axial charge

pQCD radiative corrections
($\overline{MS}$ Scheme, $n_f = 3$)

Non-perturbative $1/Q^{2n}$ power corrections.
(+rad. corr.)

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⇒ Two possibilities to extract α_s:

- Do an absolute measurement of $\Gamma_1^{p-n}(Q^2)$ and solve the Bj SR for $\alpha_s(Q^2)$.
  - One α_s per Γ_1^{p-n} experimental data point.
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Possible future extractions of α_s from $\Gamma_1^{p-n}(Q^2)$

- (Jefferson Lab: EG12 (CLAS12, 11 GeV, Data partly taken in 2022-2023))
- Electron Ion Collider (EIC)
- Jefferson Lab at 22 GeV

EIC

Simulated data: $\vec{e}$-$\vec{p}$ and $\vec{e}$-$\overrightarrow{^3He}$ DIS events generated with DJANGOH event generator for 6 collision energies (5×41 GeV, 10×100 GeV & 18×275 GeV for p, 5×41 GeV/nucleon, 10×100 GeV/nucleon & 18×166 GeV/nucleon for ^3He) and longitudinal & transverse hadron polarizations settings.

Neutron information extracted from $\overrightarrow{^3He}$ ($\simeq \vec{n}$)

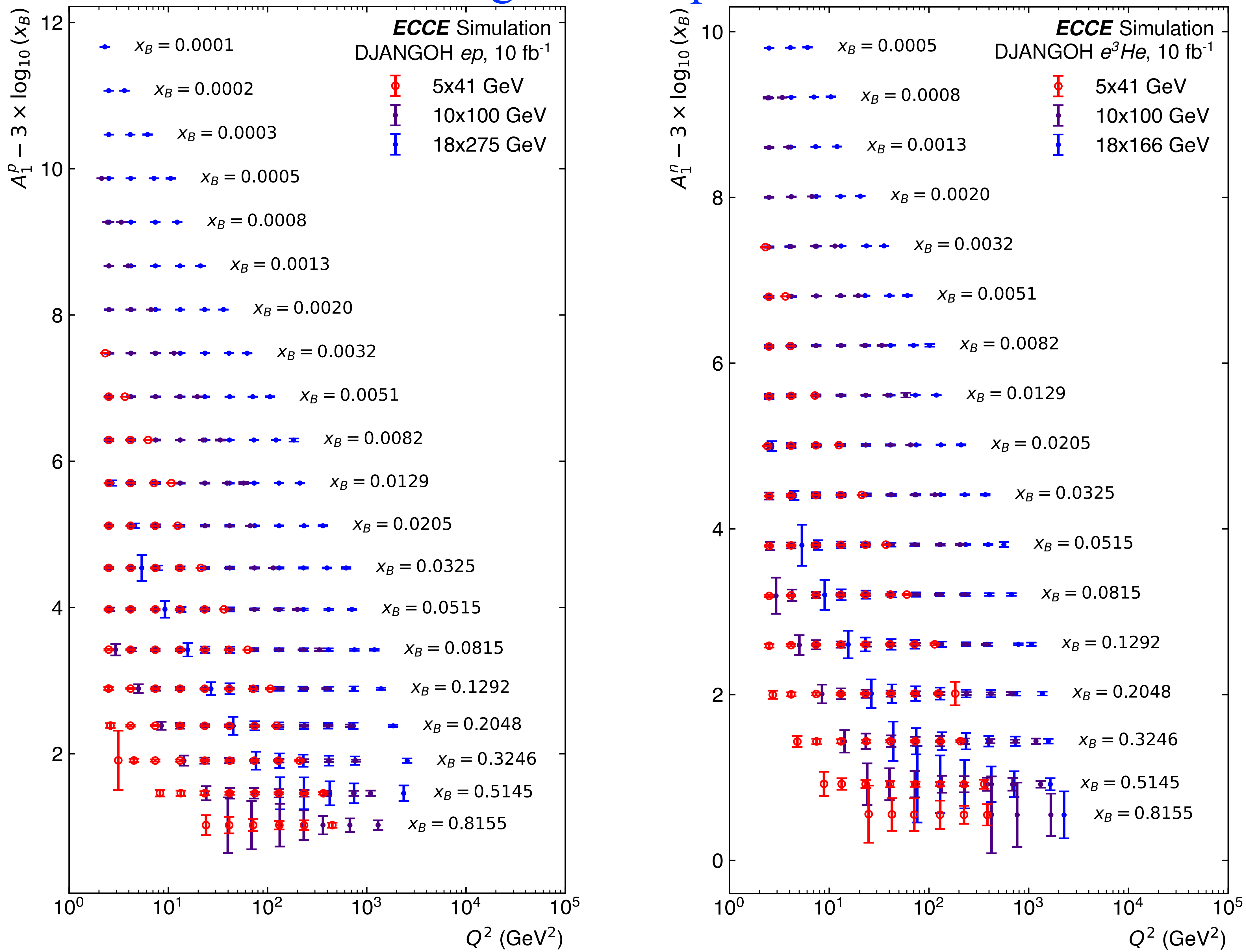
Tag two spectator protons from $\vec{e}$-$\overrightarrow{^3He}$: minimize nuclear corrections for neutron information.

Use 10 fb^{-1} luminosity.

Monte Carlo simulation of detector effects (resolution, efficiency, acceptance, radiative effects)

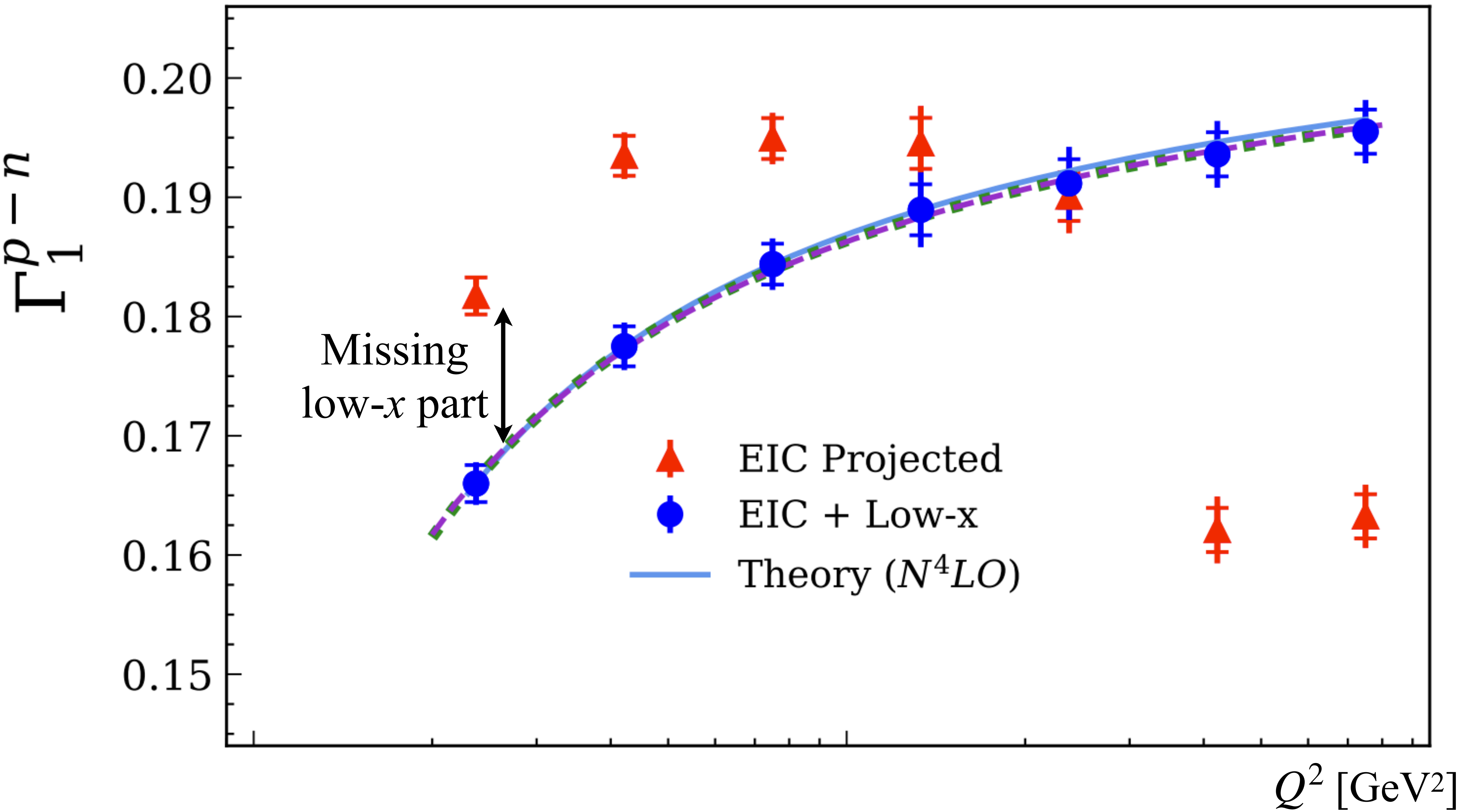
$\Rightarrow$Very realistic simulation

EIC: generated pseudo-data



$$A_1 \simeq g_1/F_1 \rightarrow g_1 \rightarrow \Gamma_1$$

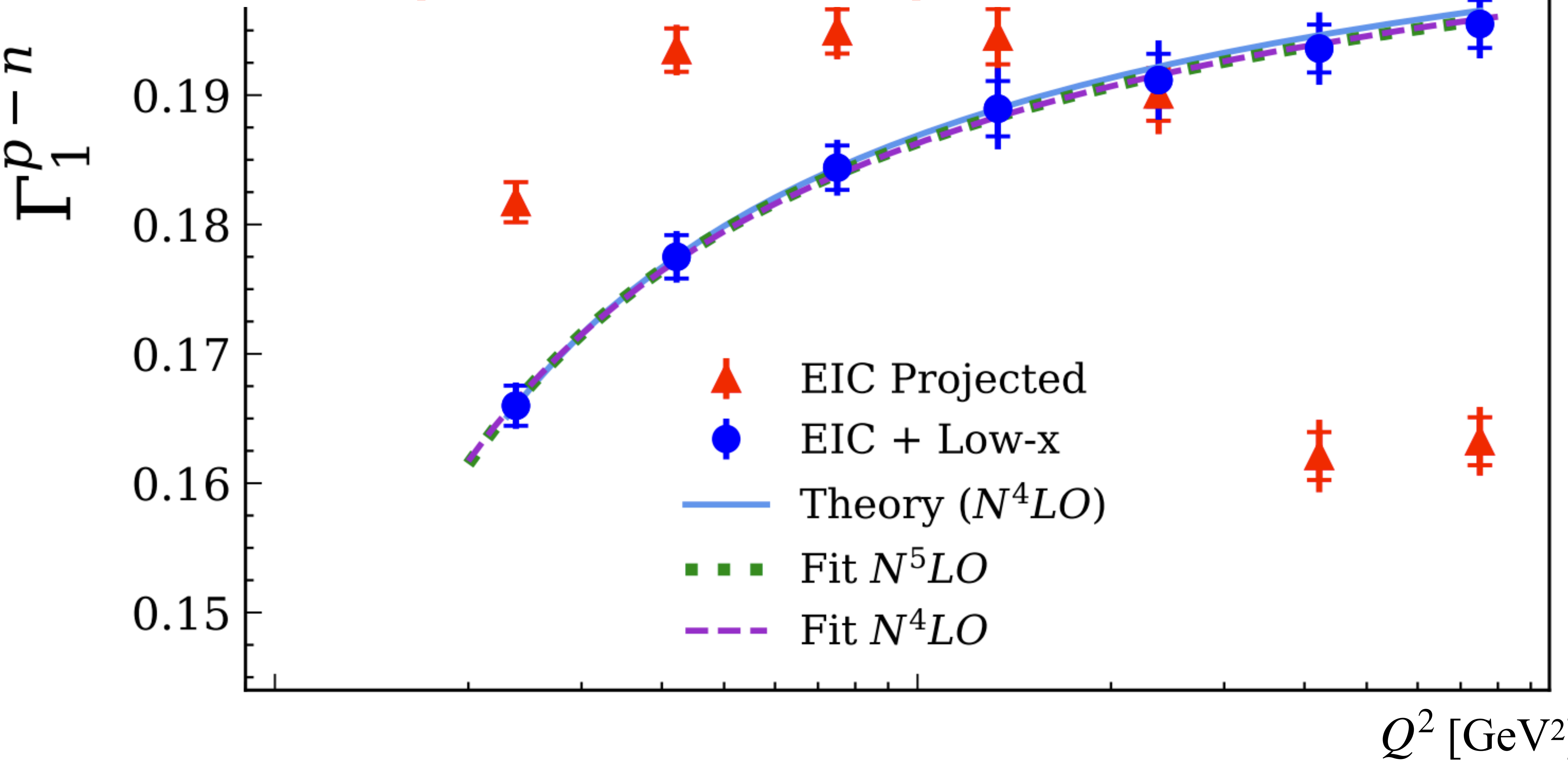
Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



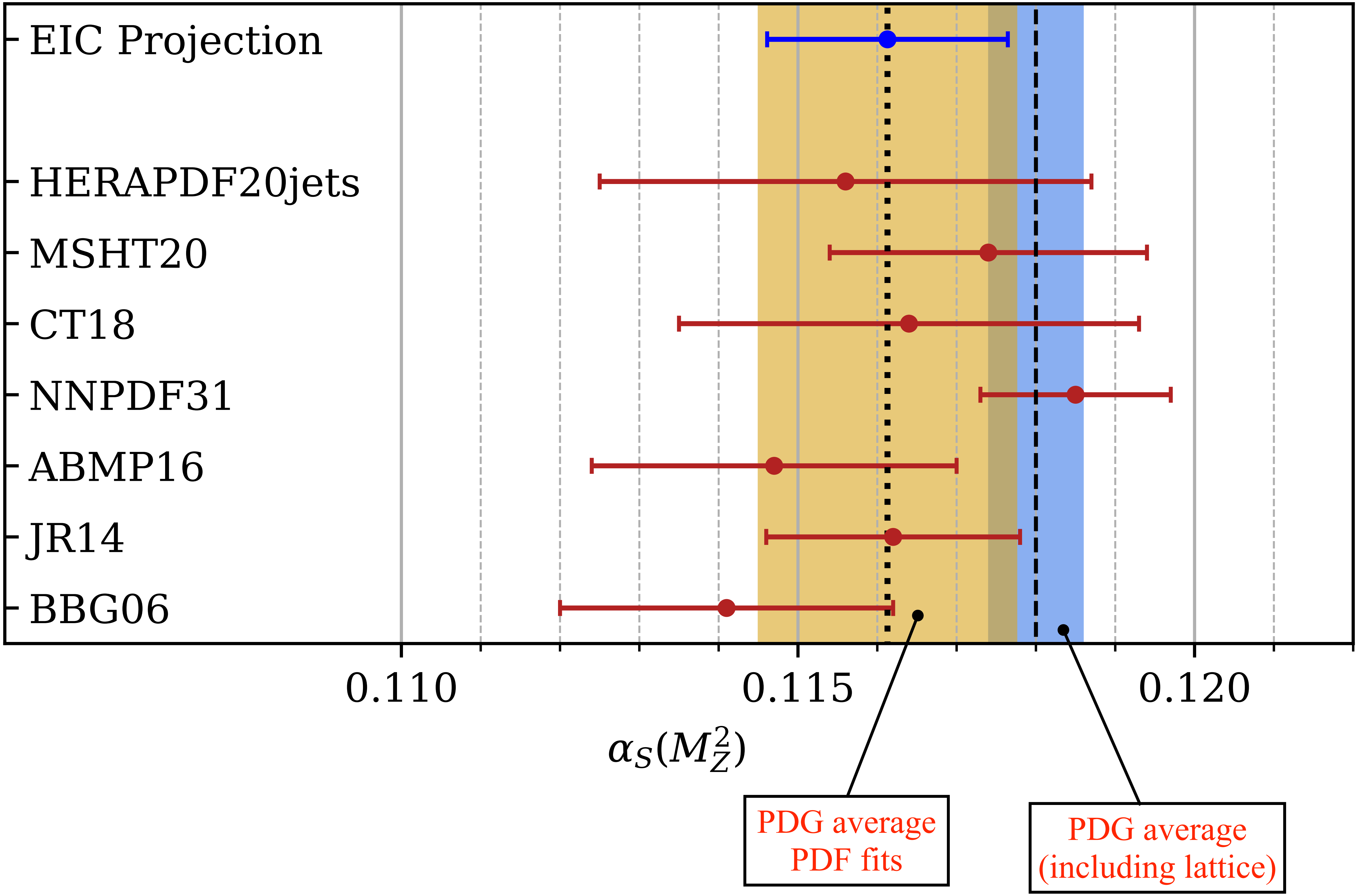
Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$

Fit yields: $\Delta\alpha_s(M_Z)/\alpha_s(M_Z)$=1.3%

$1.31\,\% = 0.83\,\%$ (exp.) $\oplus$ $0.64\,\%$ (truncation) $\oplus$ $0.78\,\%$ (polarimetries)



Compared to other DIS results and world average (from PDG)



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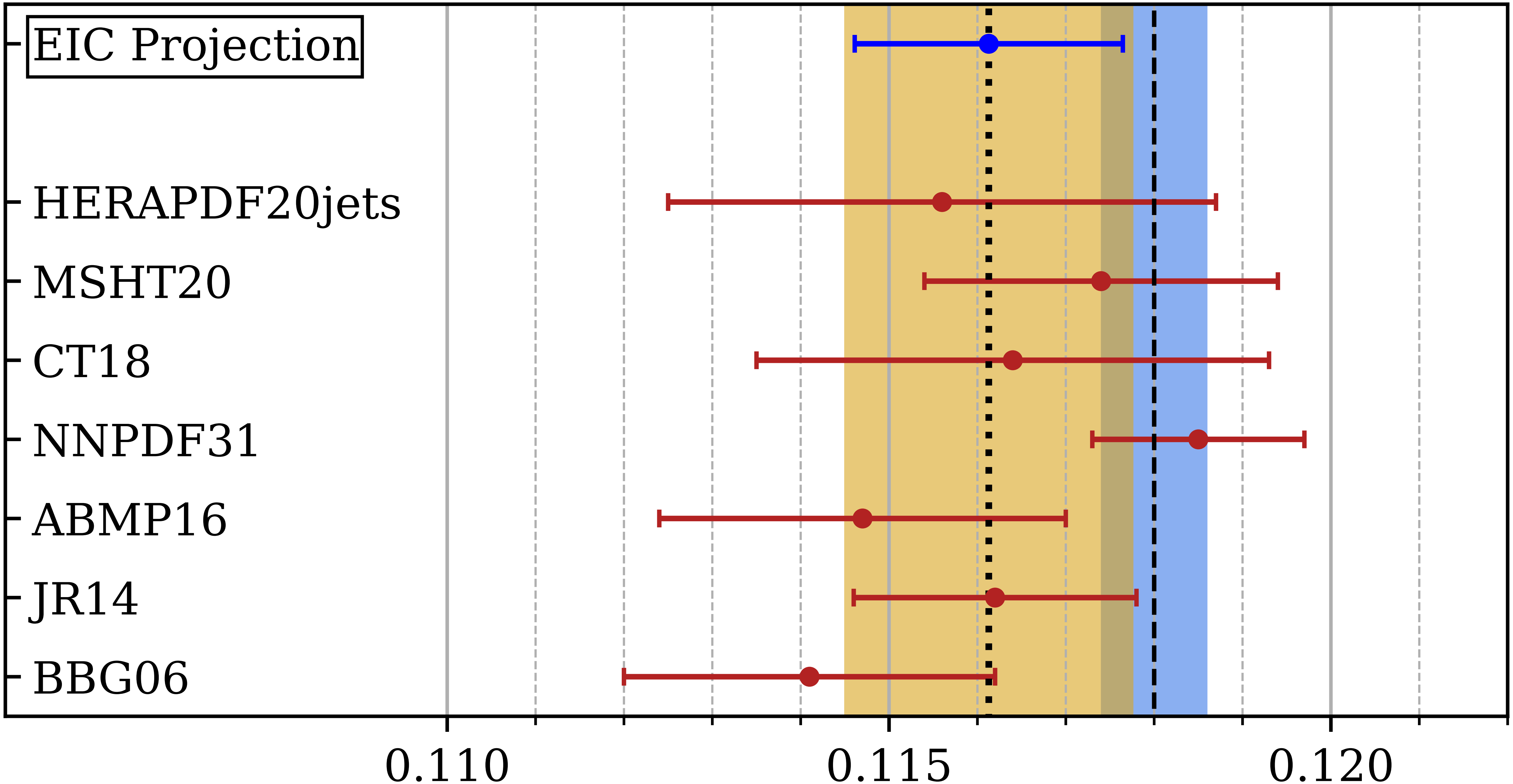

- Realistic simulation shows that EIC can yield a competitive measurement. Kutz *et al.*, PRD 110, 074004, 2024
- Just one method. Other extractions will be available, e.g.:
  - Global fits (unpolarized and polarized) Harland-Lang, *et al.* arXiv:2512.06092
  - Inclusive neutral current reactions (EIC+HERA). S. Cerci, *et al.* EPJC, 83(11):1011, 2023: $\Delta\alpha_s(M_Z)/\alpha_s(M_Z)$=0.4%

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- (Jefferson Lab: EG12 (CLAS12, 11 GeV, Data partly taken in 2022-2023))
- Electron Ion Collider (EIC)
- **Jefferson Lab at 22 GeV**

Bjorken sum rule at JLab@22 GeV

- Statistical uncertainties will be negligible:
  - JLab is a high-luminosity facility;
  - A JLab@22 GeV program would include polarized DVCS and TMD experiments. Those imply long running times compared to those needed for inclusive data gathering;
  - High precision data already available from 6 GeV and 12 GeV for the lower Q^2 bins and moderate x.
  - 6 GeV CLAS EG1dvcs data: required statistics for DVCS and TMD experiments $\Rightarrow$statistical uncertainties < 0.1% on the Bjorken sum.

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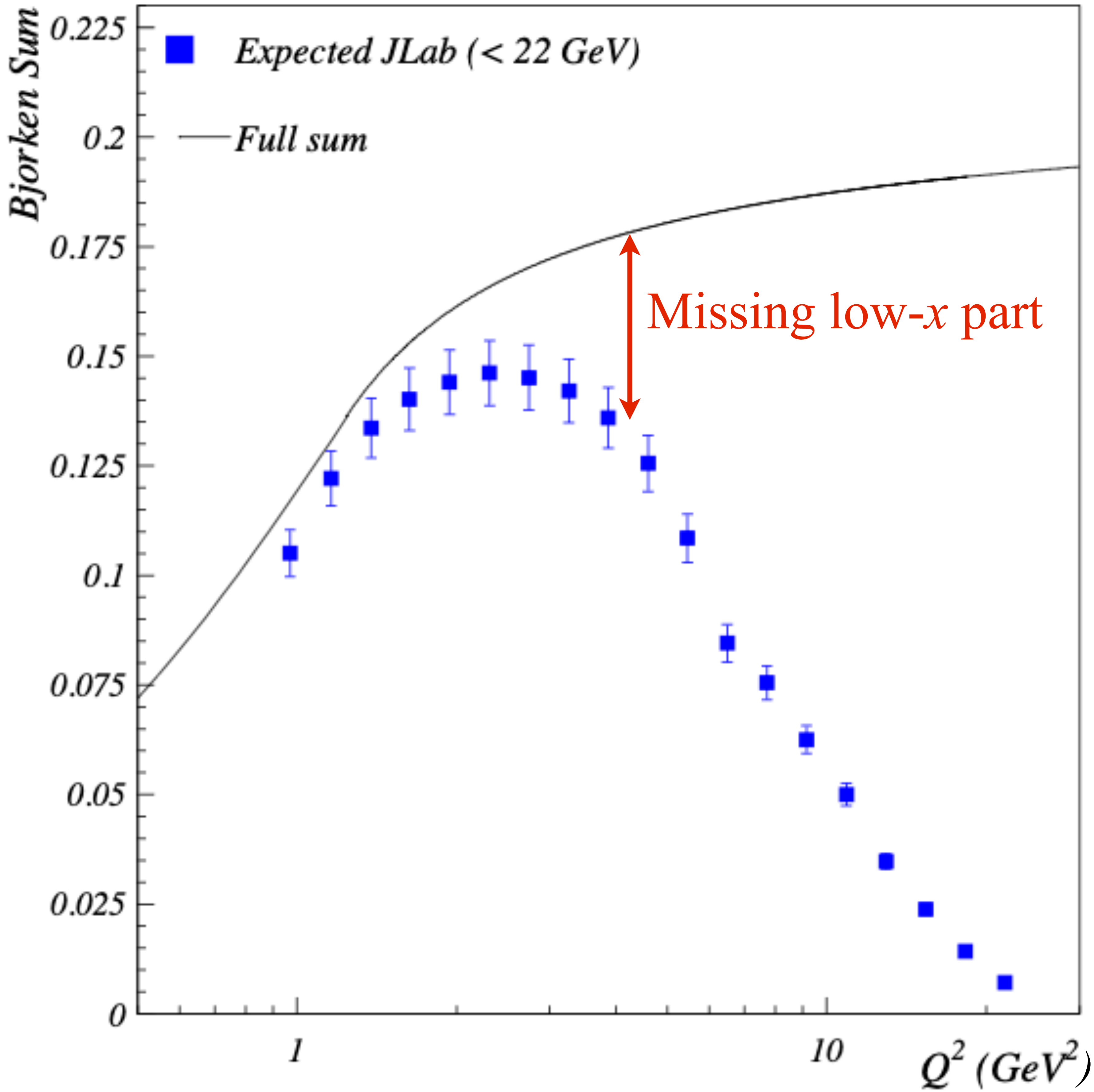
- Use 5% for experimental systematics (i.e. not including the uncertainty on unmeasured low-x).

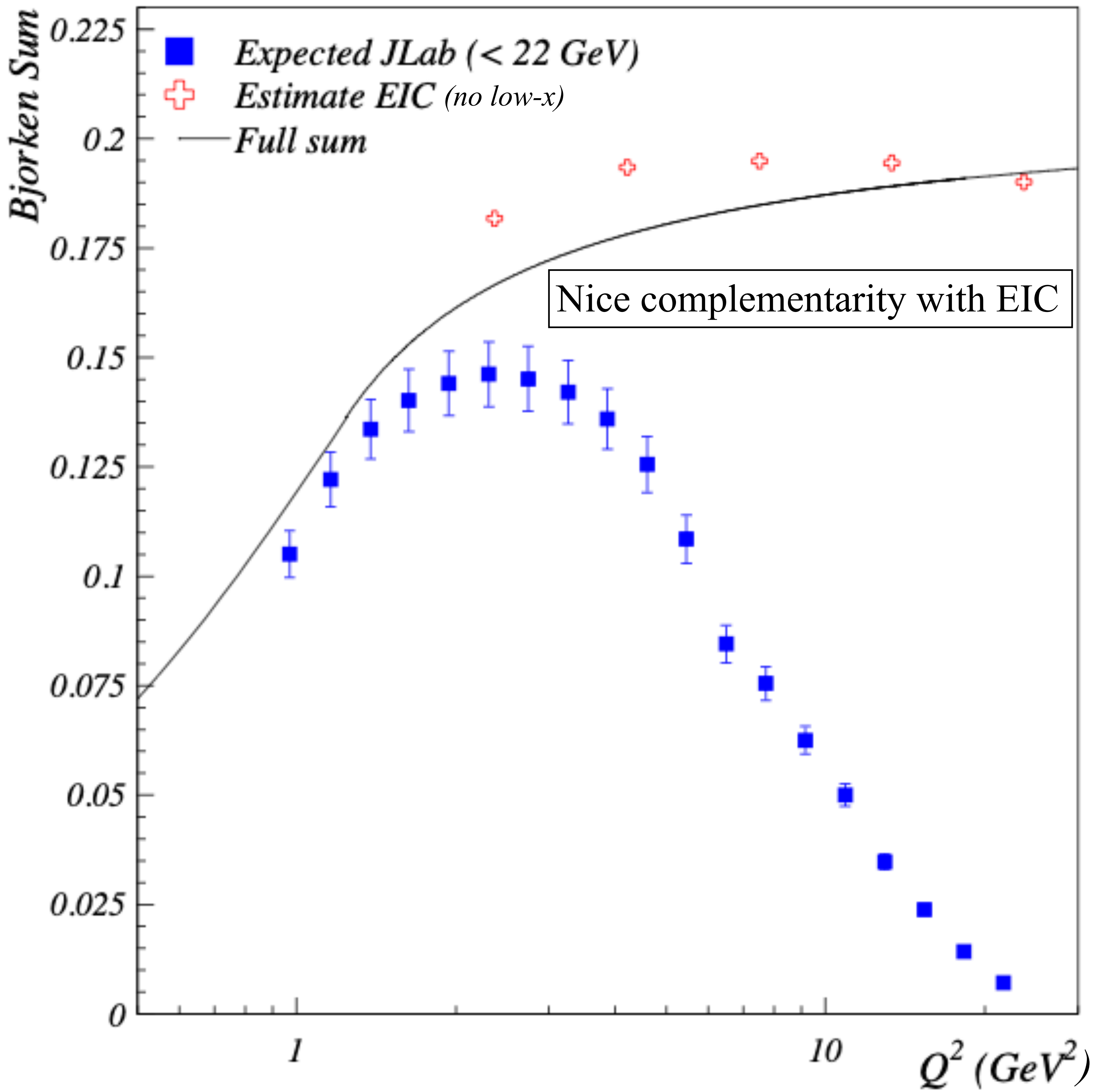
- Nuclear corrections:
  - D: negligible assuming we can tag the ~spectator proton
  - ^{3}He: 2% (5% on n, which contribute to 1/3 to the Bjorken sum: 5%/3≃2%)
- Polarimetries: Assume ΔP_{e}-ΔP_N = 3%.
- Radiative corrections: 1%
- F_1 to form g_1 from A_1: 2%
- g_2 contribution to longitudinal asym: Negligible, assuming it will be measured.
- Dilution/purity:
  - Bjorken sum from P & D: 4%
  - Bjorken sum from P & ^{3}He: 3%
- Contamination from particle miss-identification: Assumed negligible.
- Detector/trigger efficiencies, acceptance, beam currents: Neglected (asym).

Adding in quadrature: ~5%

Under these assumptions:



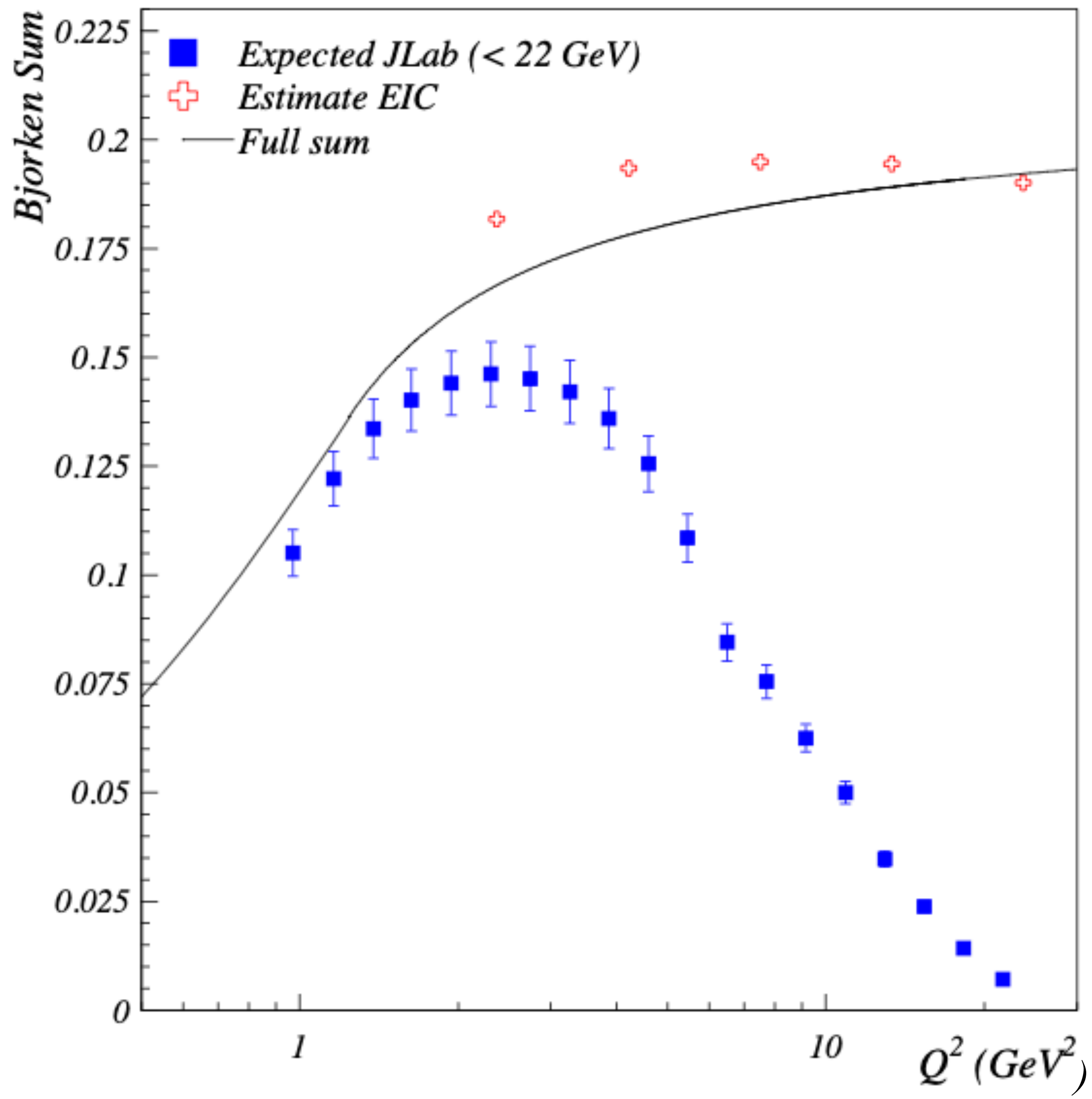
Comparison with EIC



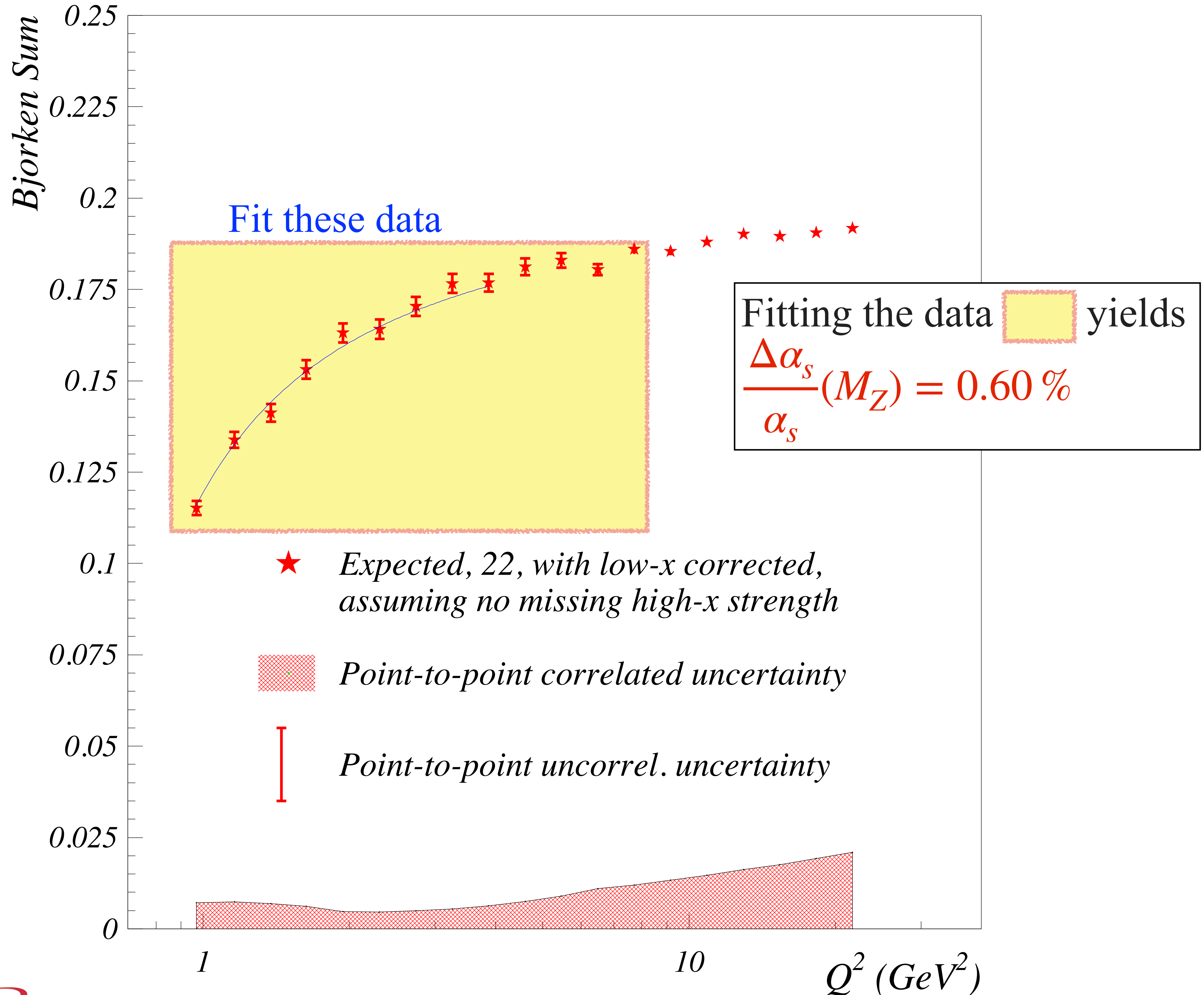
Low-x uncertainty

- For the Q^2 bins covered by EIC, global fits will be available up to the lowest x covered by EIC.
  $\Rightarrow$ assume 10% uncertainty on that missing (for the JLab measurement) low-x part.
  Assume 100% for the very small-x contribution not covered by EIC.

- For the 5 lowest Q^2 bins not covered by EIC:
  - Bin #5 close to the EIC coverage $\Rightarrow$ Constrained extrapolation, assume 20% uncertainty on missing low-x part.
  - Bin #4, assume 40% uncertainty, Bin #3, assume 60%, Bin #2, assume 80%, Bin #1, assume 100%.



Extraction of $\alpha_s(M_Z)$



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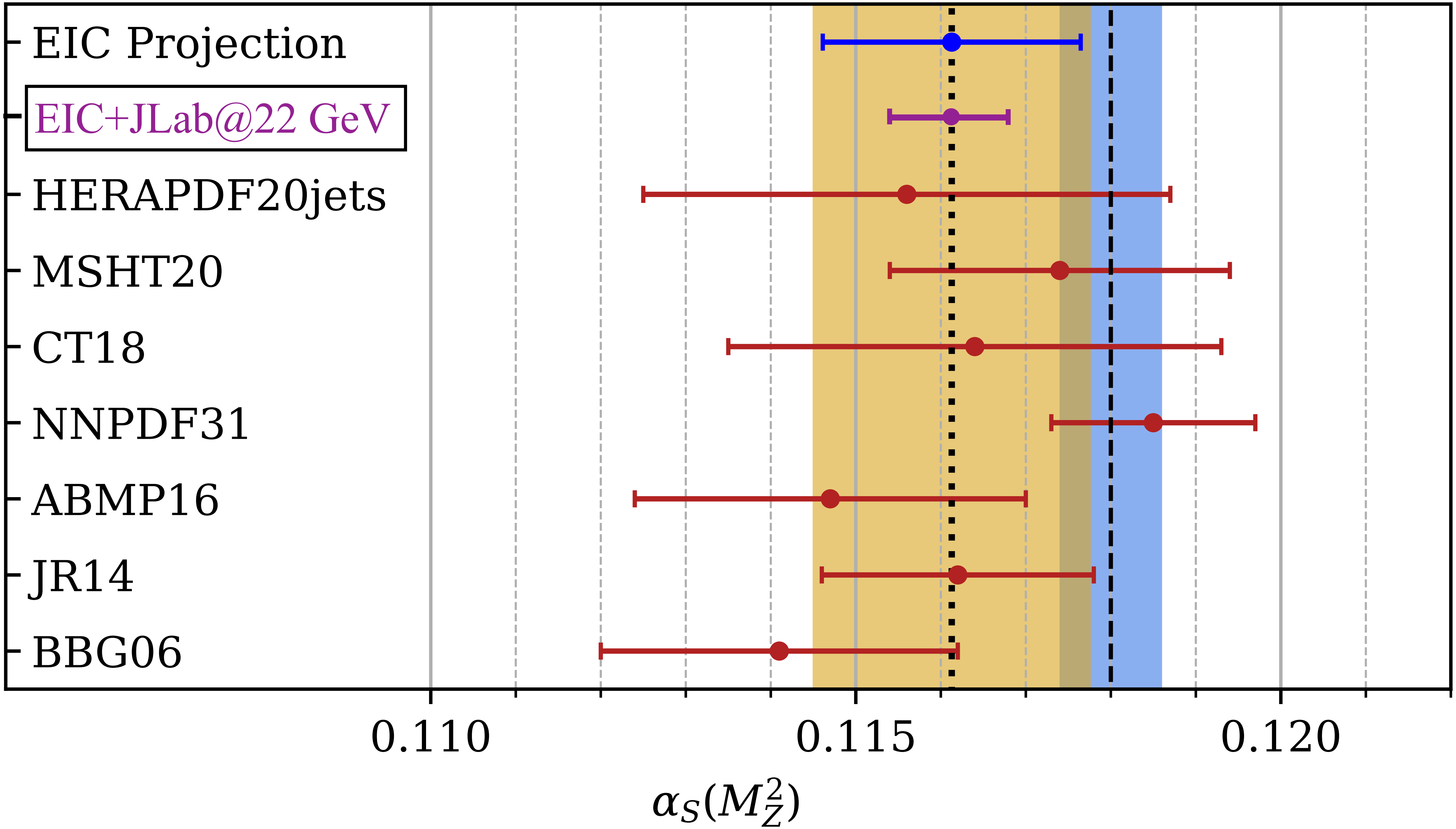

EIC+JLab@22 GeV can yield a compelling 0.6% determination of $\alpha_s(M_Z)$ from the Bjorken sum rule.

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Nucleon axial charge

pQCD radiative corrections ($\overline{MS}$ Scheme, $n_f = 3$)

Non-perturbative $1/Q^{2n}$ power corrections. (+rad. corr.)

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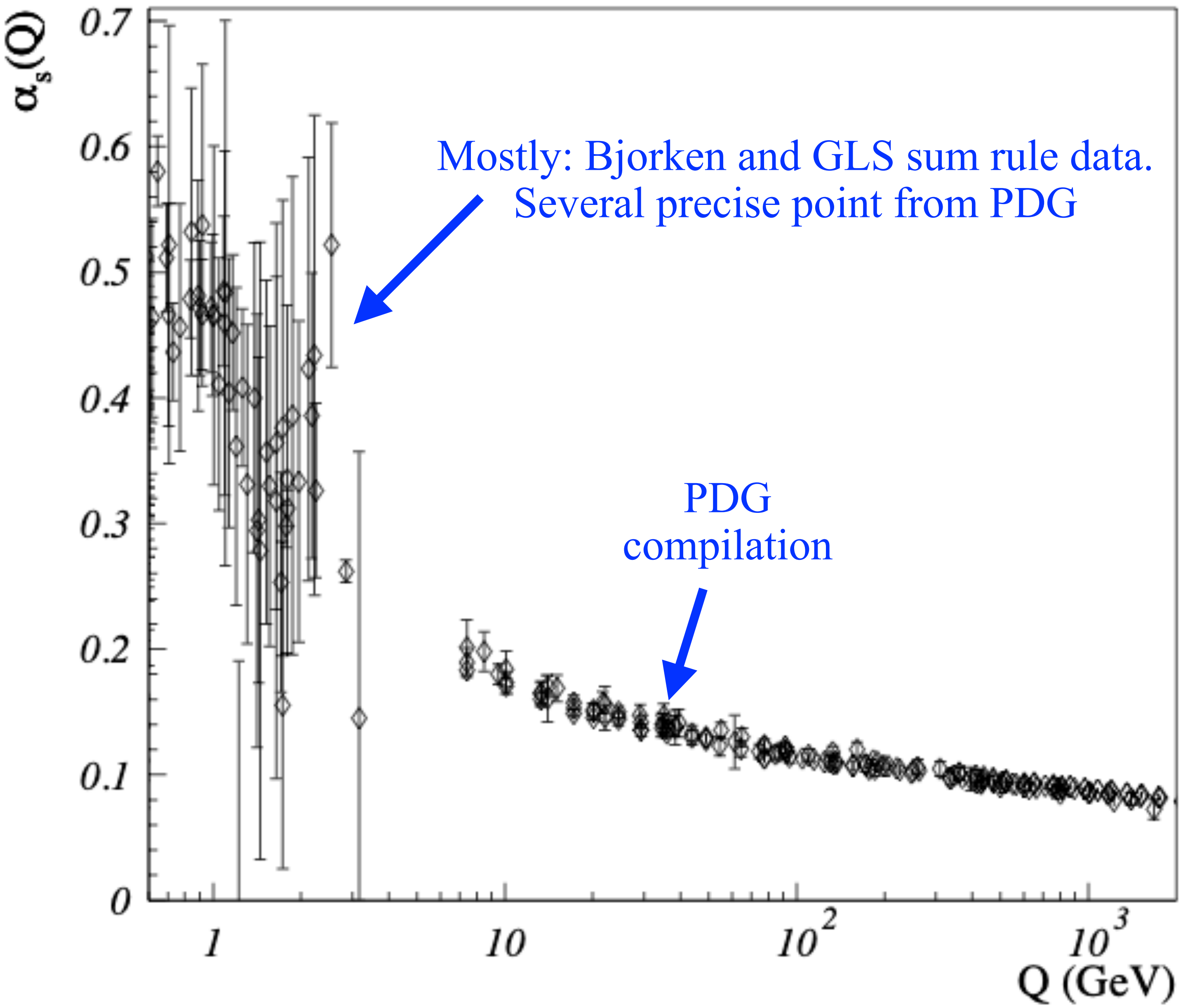
World data on $\alpha_s(Q)$



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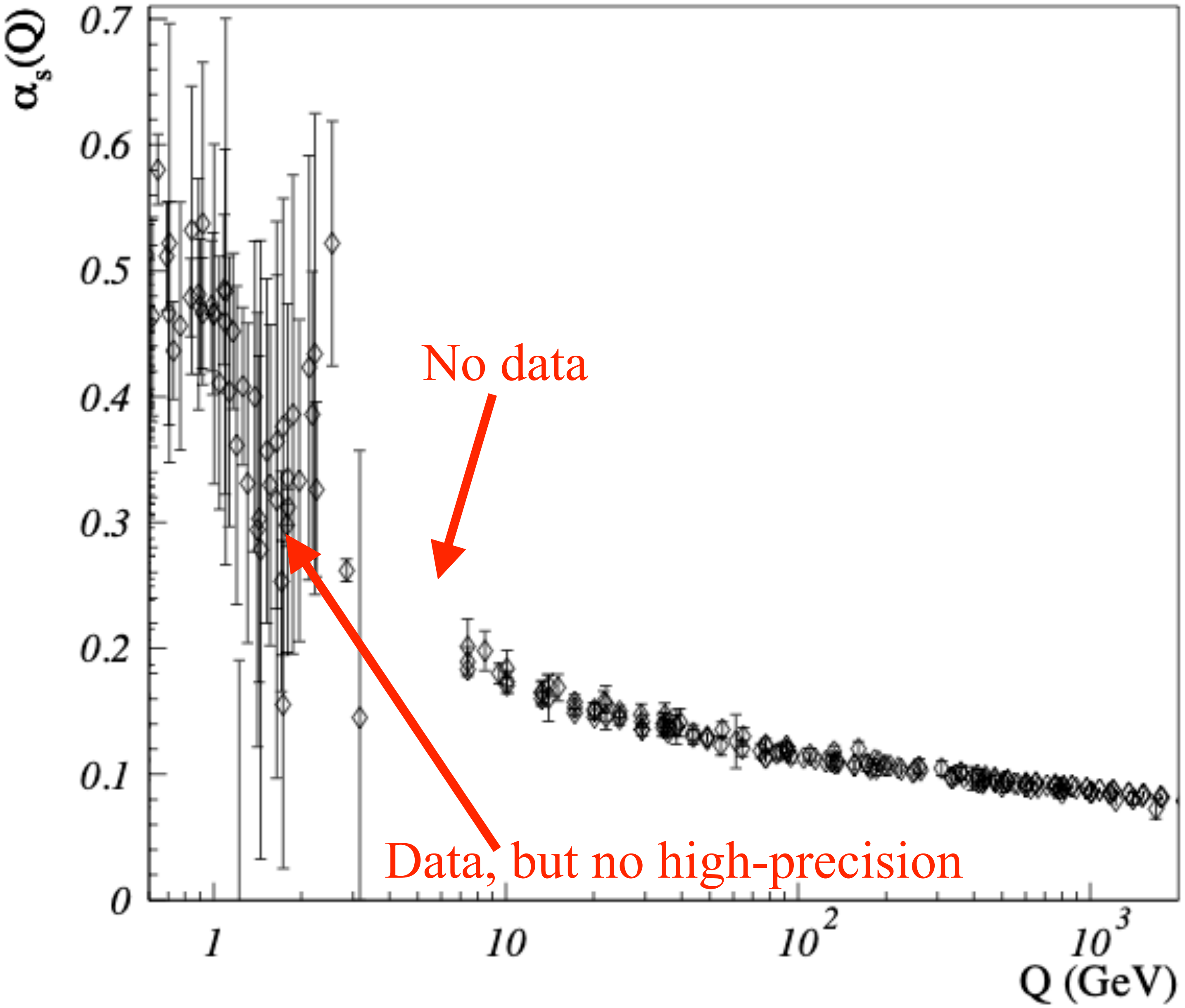
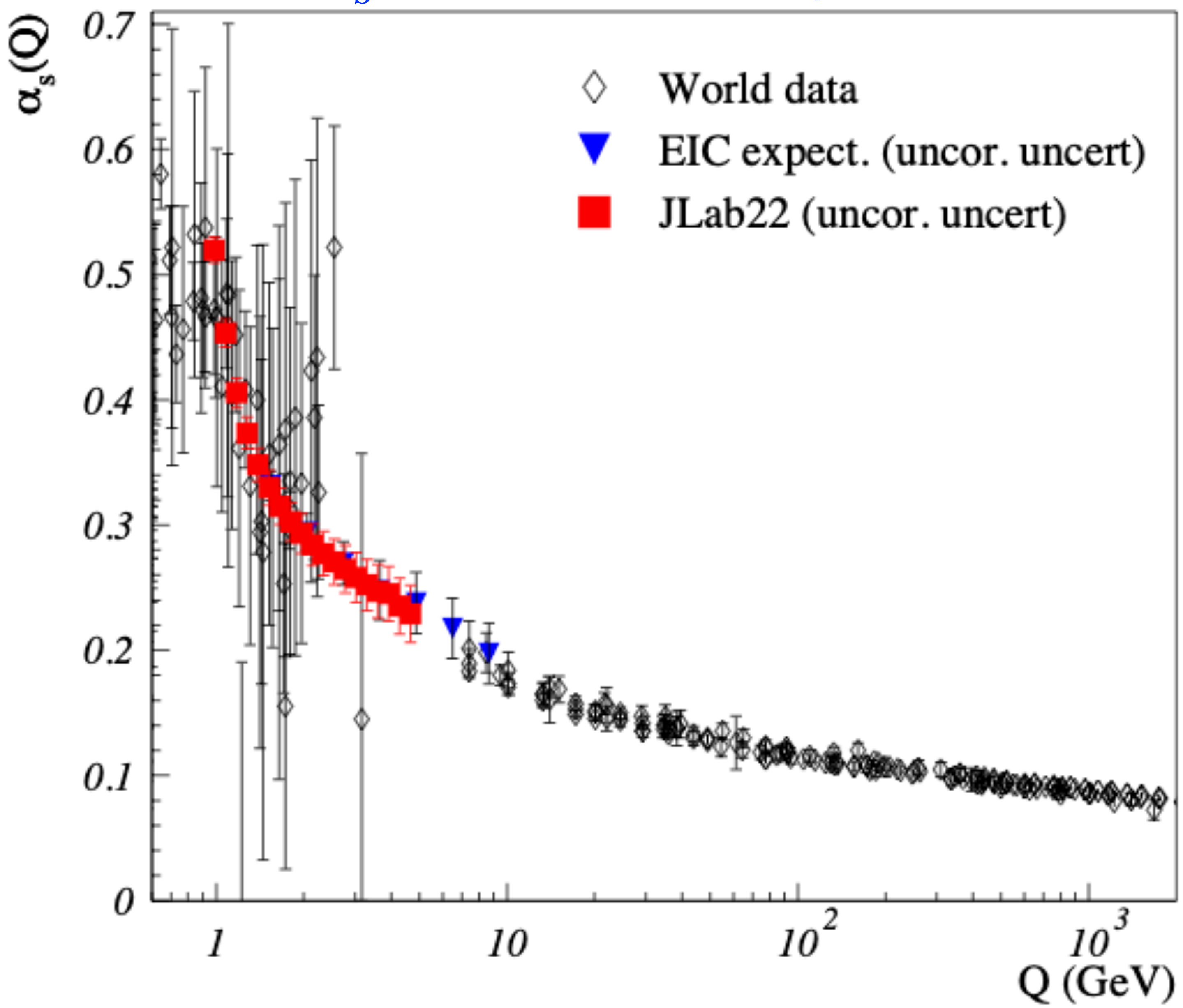


$\alpha_s(Q)$ from JLab@22+EIC



Single experiment: Correlated uncertainty irrelevant for Q-behavior of $\alpha_s \Rightarrow$ 5% uncertainty → ~1%.

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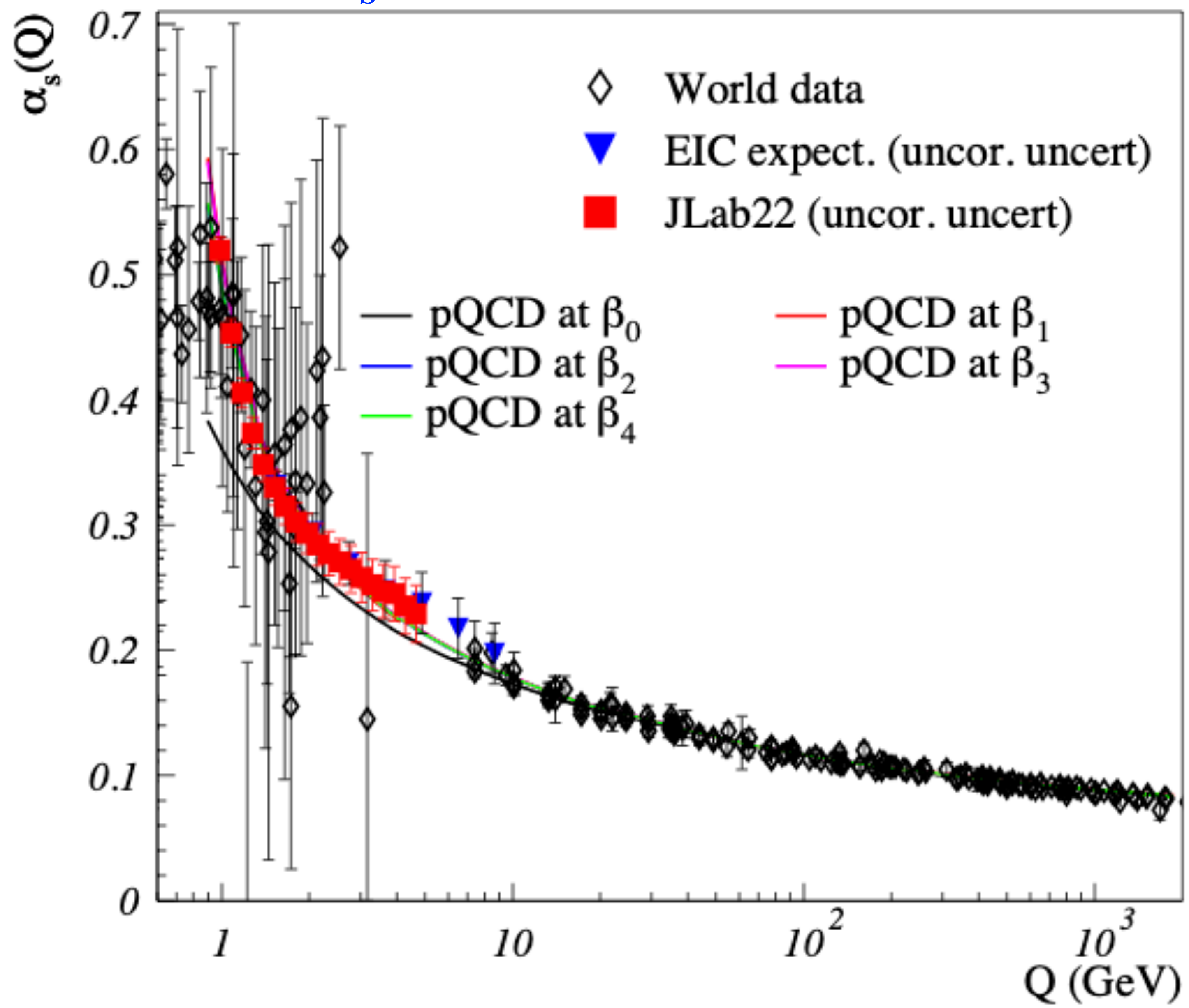

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Constraint on BSM from $\alpha_s(Q)$ from JLab@22+EIC

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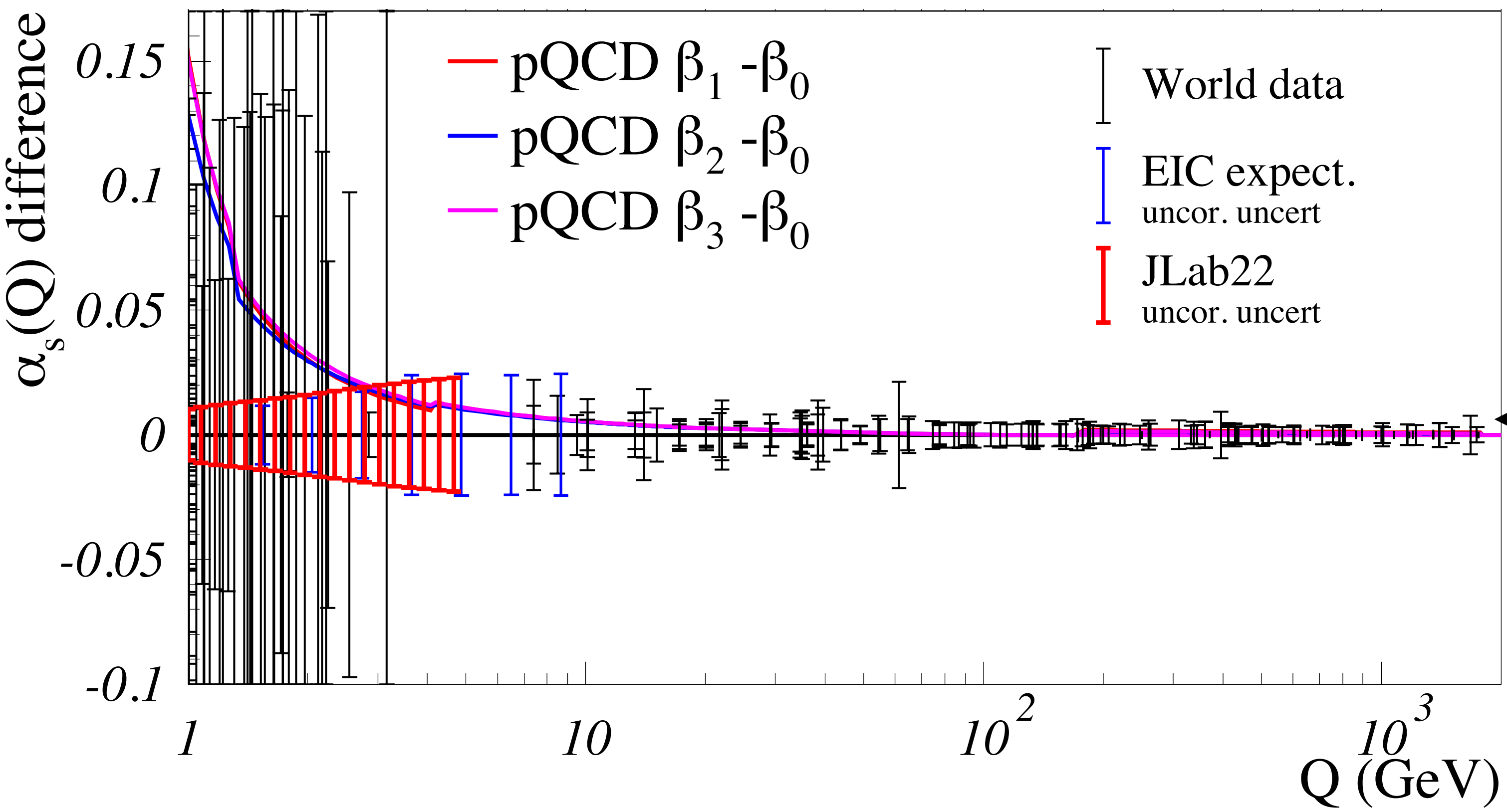

World data: very precise at large Q, but multi-loop effect is too small

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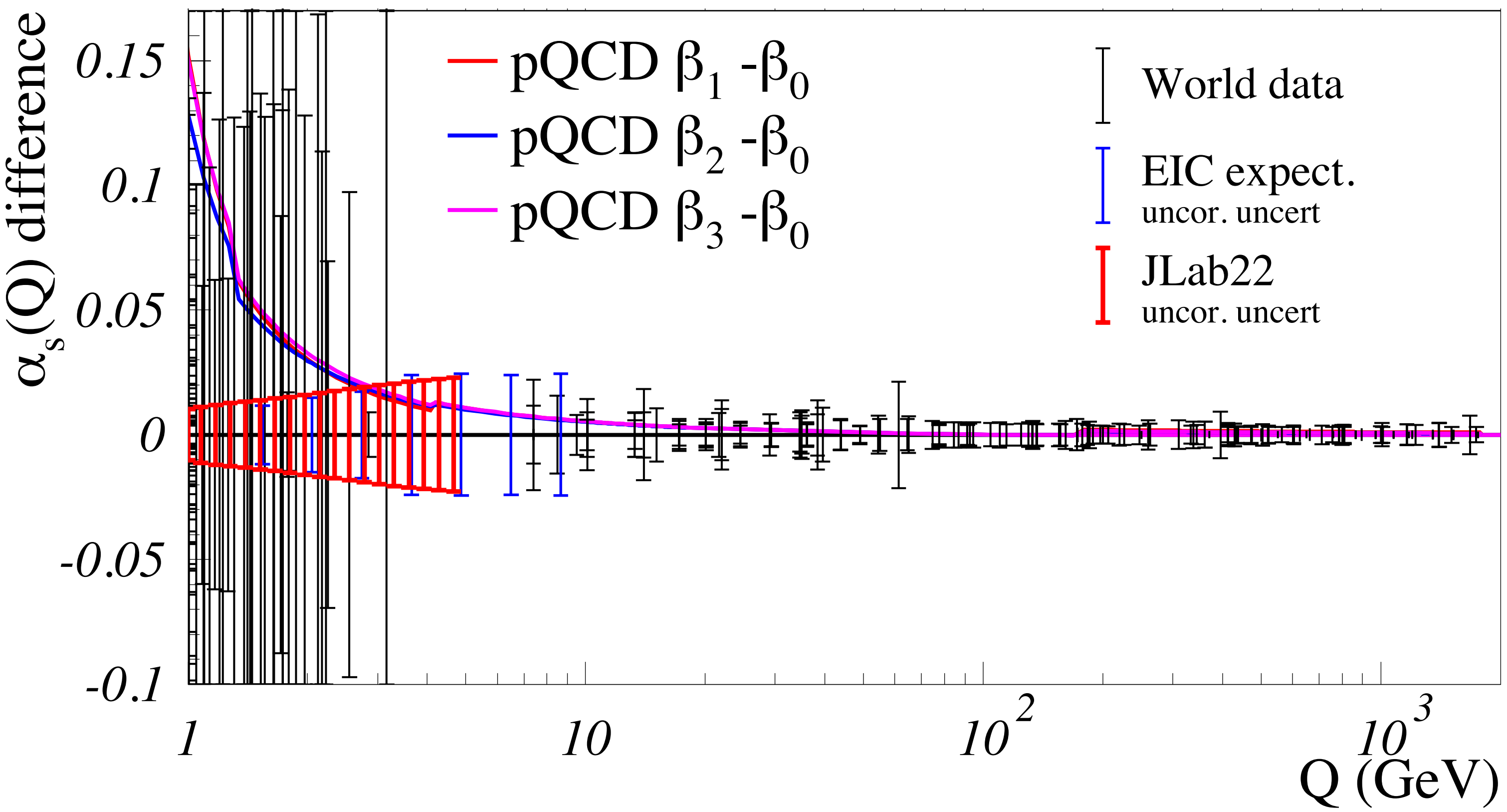

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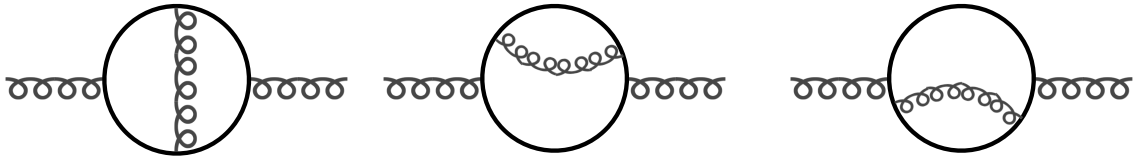

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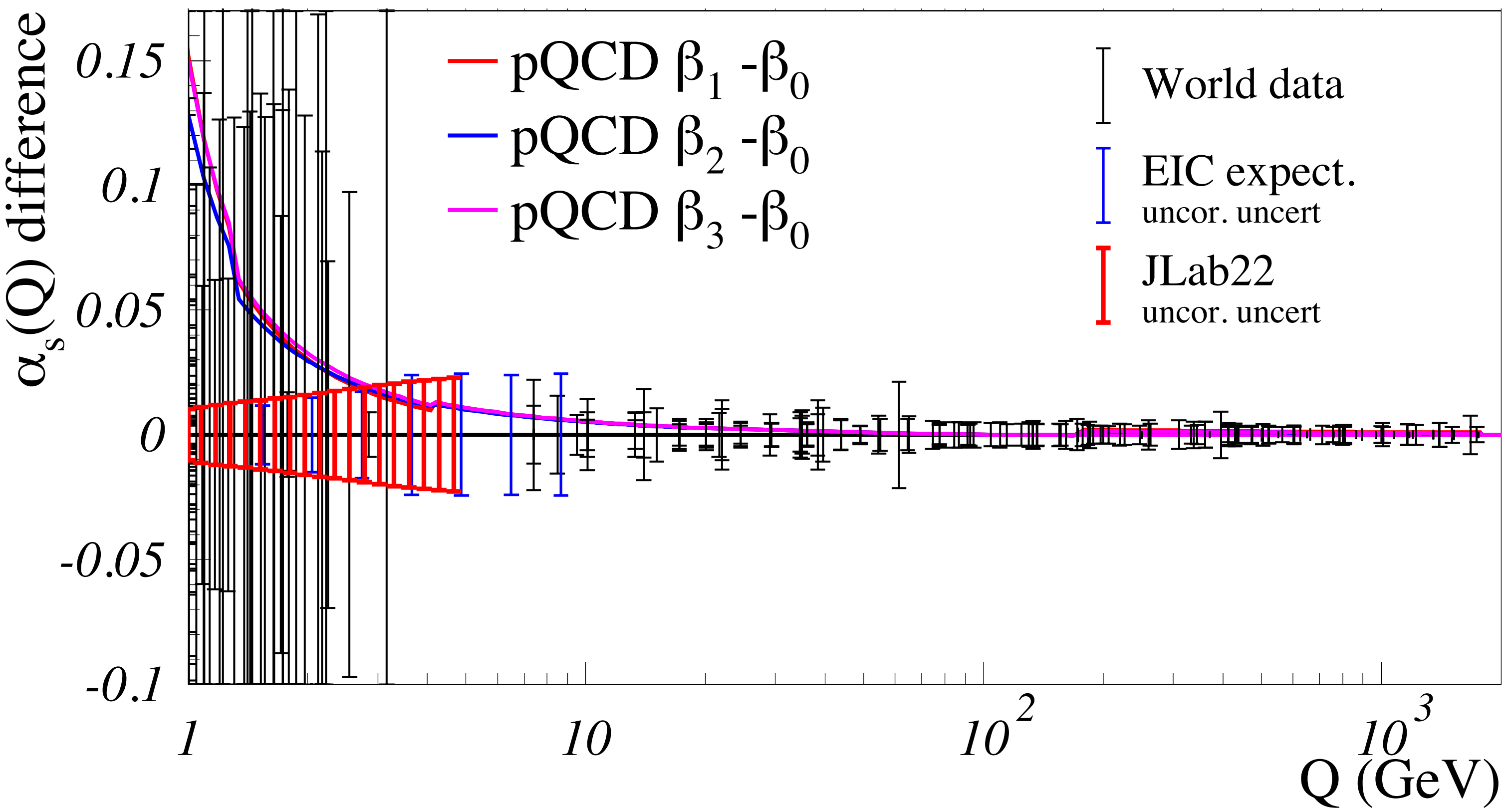

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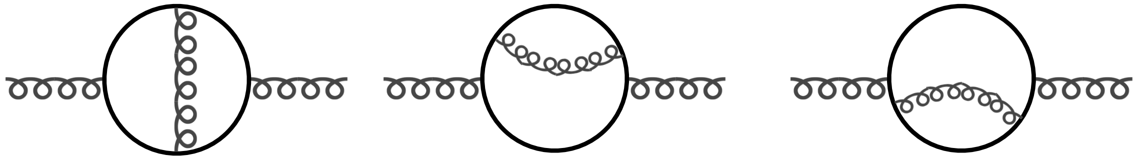
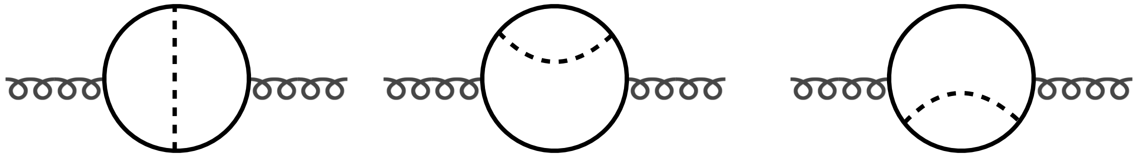


New physics appears at NLO.
BSM boson with quark coupling



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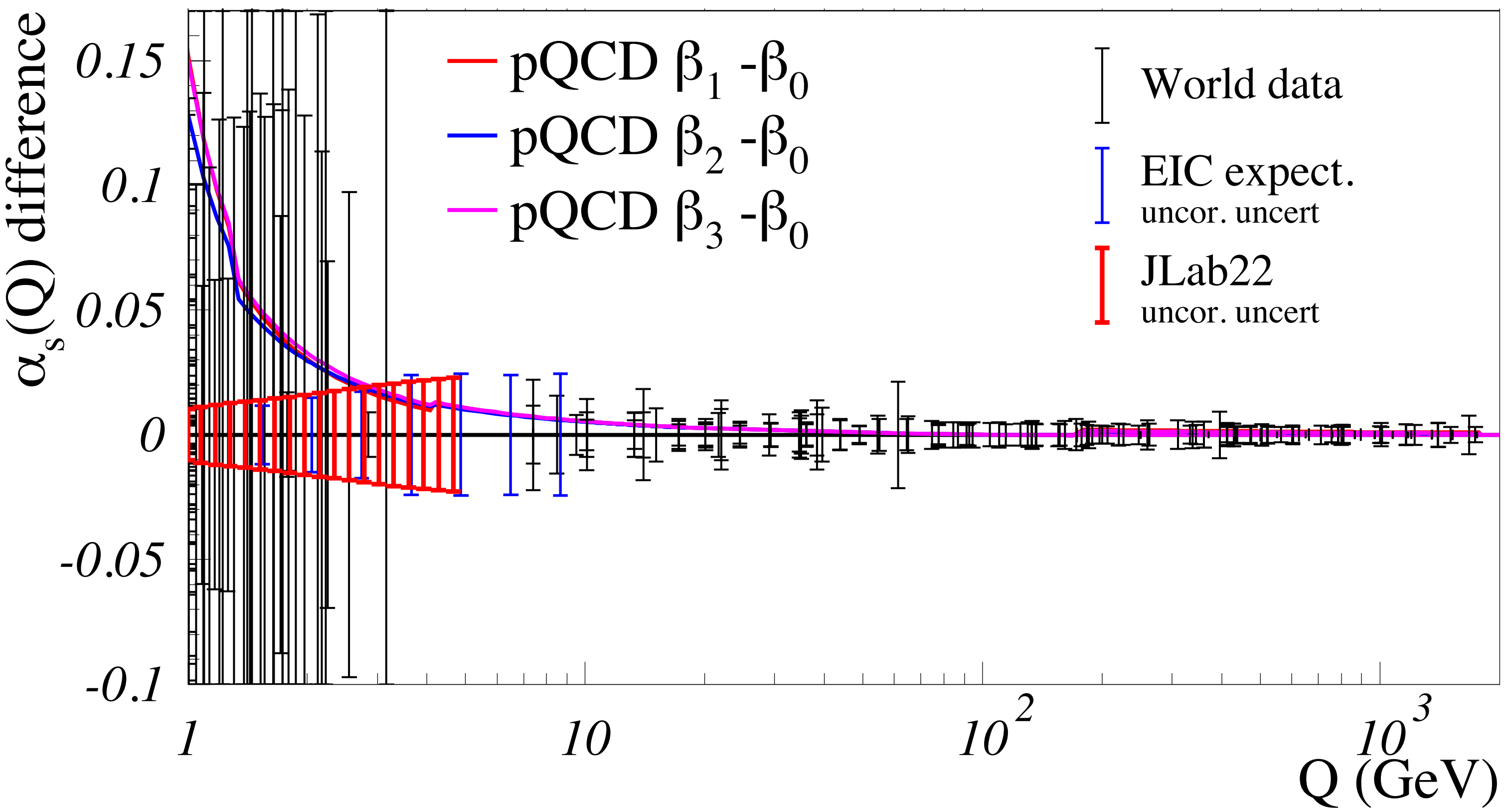


10–90 GeV: interesting window for BSM search:

- No resonances (heaviest hadronic M<10 GeV, then Z^0 pole ~90 GeV)
  ⇒ many precise techniques for BSM search unavailable.
- α_s run: one of the cleanest QCD-based probes (especially when sum rule are involved). Available at low Q where other clean QCD probes (W/Z hadronic width, event shapes) are not available.
- Q as low as possible (still within pQCD) is beneficial.
  - α_s running steepest ⇒ most sensitive to deviation from pQCD;
  - Larger lever-arm M_{BSM}→Q;
  - α_s is larger.

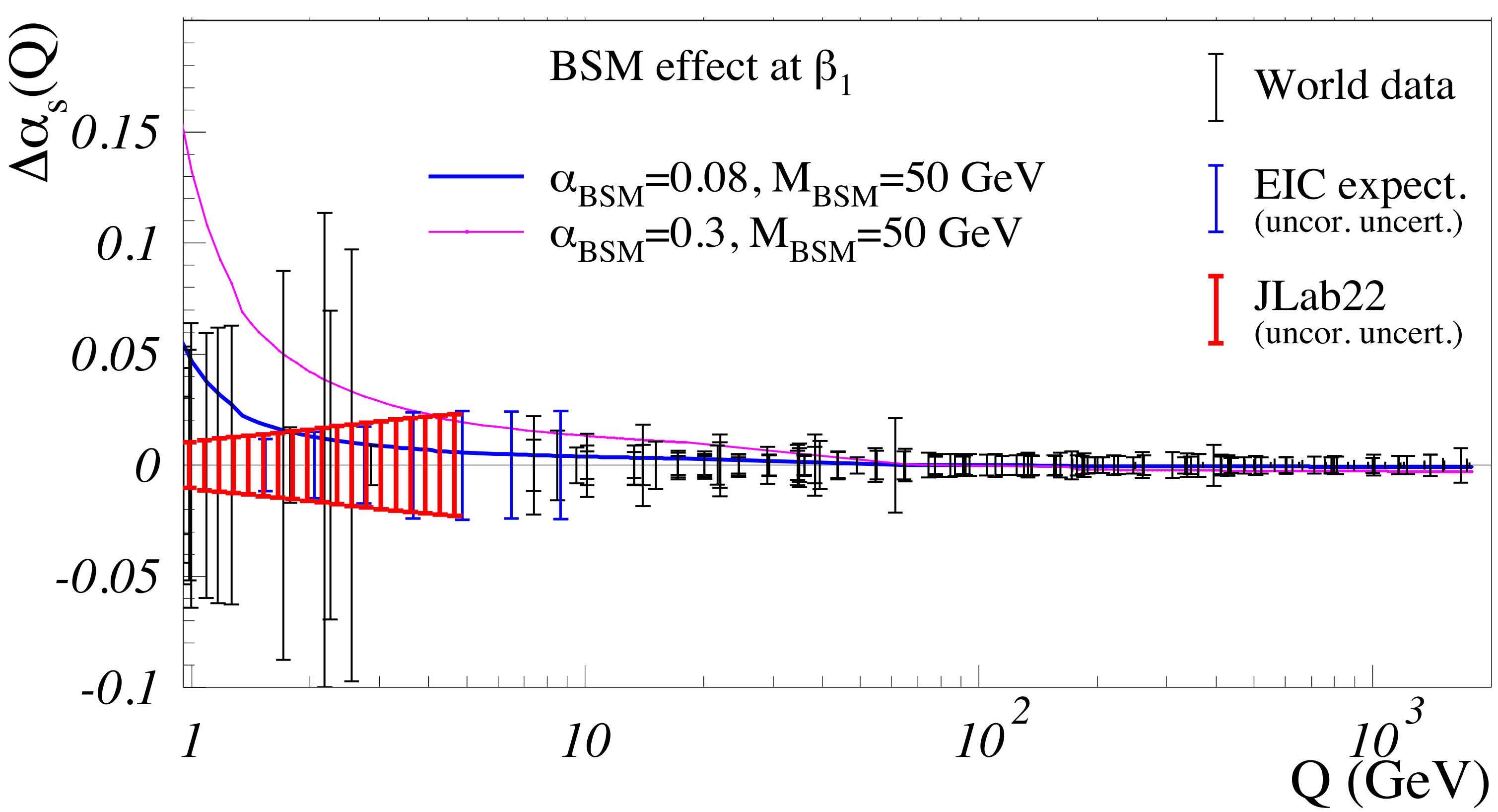
Constraint on BSM from $\alpha_s(Q)$ from JLab@22+EIC

Illustrative BSM model:

- coupling to quark only (coupling to leptons more efficiently checked with EW probes: colliders, hyperfine-structure, & g–2 experiments. Coupling to gluons would affected pQCD at LO)
- boson must be massive, $M_{BSM} \gg \Lambda_{QCD}$ (strict limits on long-range force from 5th force searches, beam-dump experiments, hadron decays, Big-Bang nucleosynthesis, stellar cooling and cosmic microwave background anisotropies)
- flavor-preserving, universal coupling (most experiments sensitive to flavor-changing BSM bosons but much less to flavor-preserving case due to GIM mechanism suppression)
- no direct coupling or mixing with Higgs boson (direct coupling or mass mixing ⇒ BSM boson couples to all Standard-Model fields. Tight limits already exist)

Effect on spin-0 BSM meson on $\alpha_s(Q)$:

Machacek &. Vaughn, Nucl. Phys. B 222 (1983)



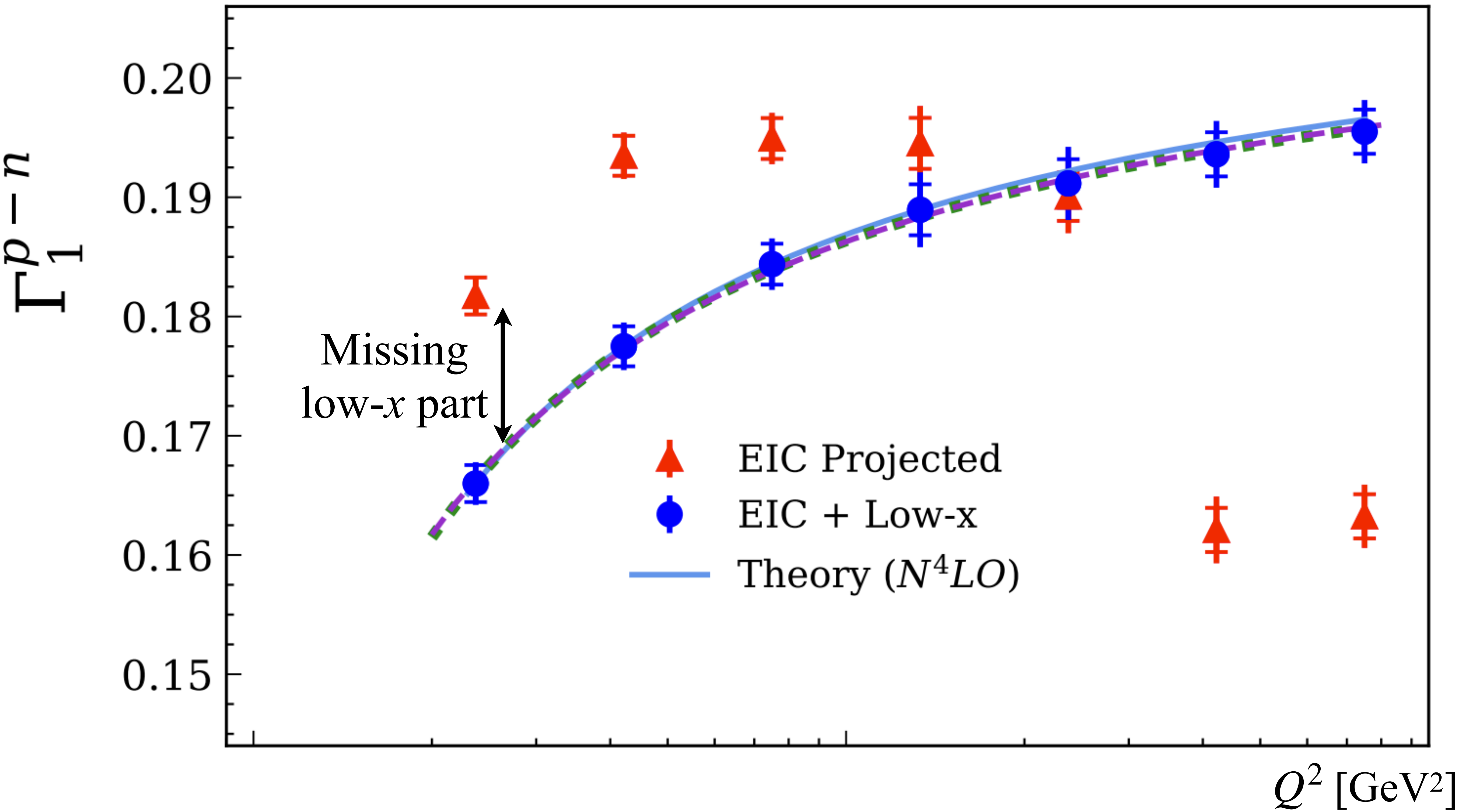
JLab@22: starts to be sensitive (χ^2/ndf $\simeq$ 3) to BSM boson for a coupling as low as α_{BSM} = 0.04.

Summary

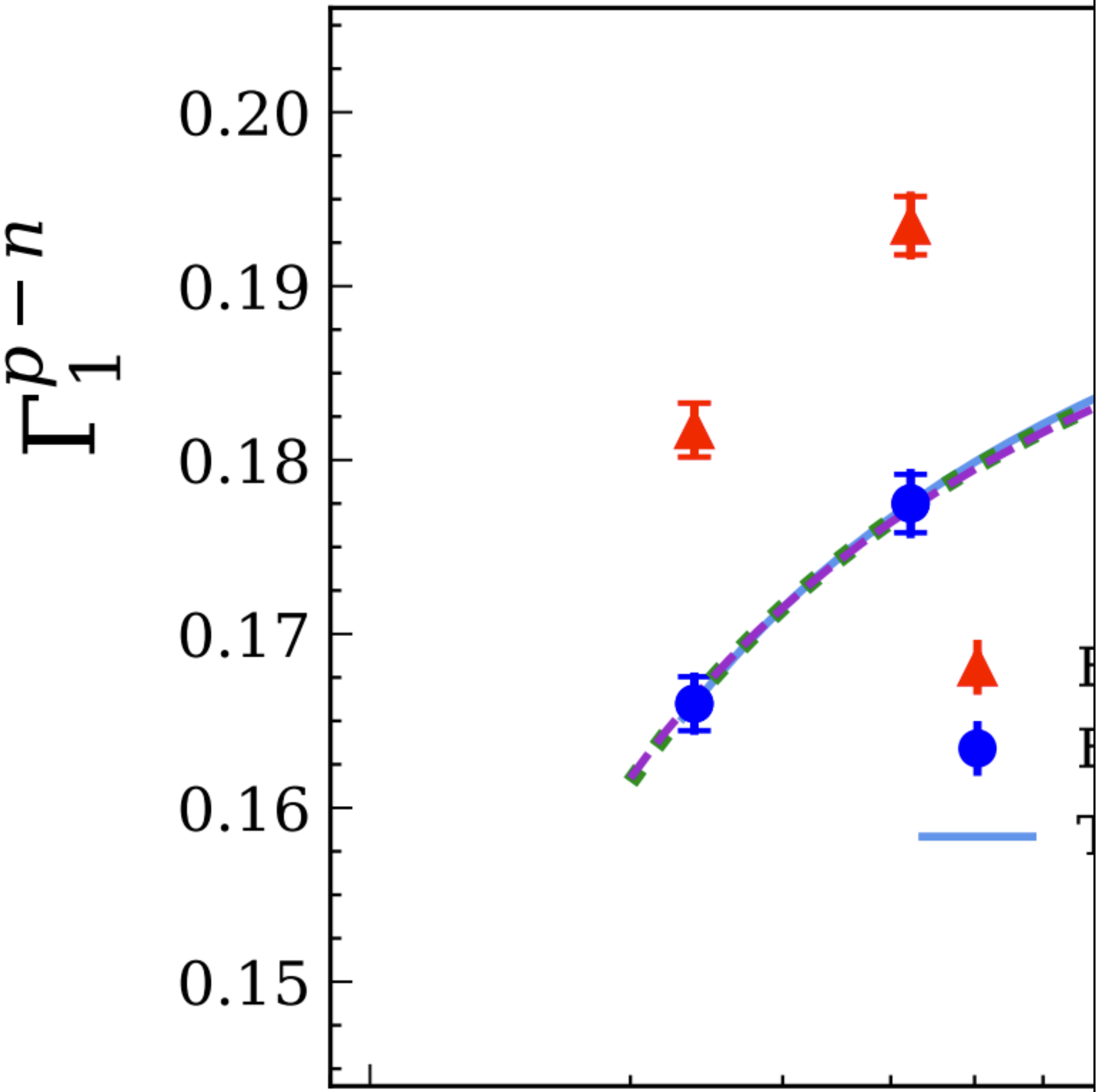
- Bjorken sum rule: simple (⇒clean) and competitive method to determine α_s.
- Realistic simulation: EIC (g_1 from inclusive polarized DIS reaction) can yield $\frac{\Delta\alpha_s}{\alpha_s} = 1.3\%$.
- Preliminary study shows that JLab@22 GeV can lower this to ~0.6% with same method.
- Possible further improvement:
  1. Improved knowledge of pQCD series: α_s at β_4 already available. Estimate for N^{5}LO results for Γ_1^{p-n} available.
  2. Improved perturbative methods minimizing pQCD truncation. Some have already been worked out for Γ_1^{p-n}.
- One but of several ways to determine α_s at EIC or JLab. Others, *e.g.*, global PDF fits or inclusive neutral current reactions also provide competitive measurements.
- Valuable comparison with α_s extracted from other processes: very different data (polarized DIS).
- JLab@22+EIC: clear sensitivity to β_1 ⇒ clean QCD-based BSM search in hard to test energy range.
- Sensitive to BSM boson (massive spin-0 boson with flavor-preserving universal coupling to quarks only) for $\alpha_{\mathrm{BSM}} \gtrsim 0.04$.

Supplementary slides

Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



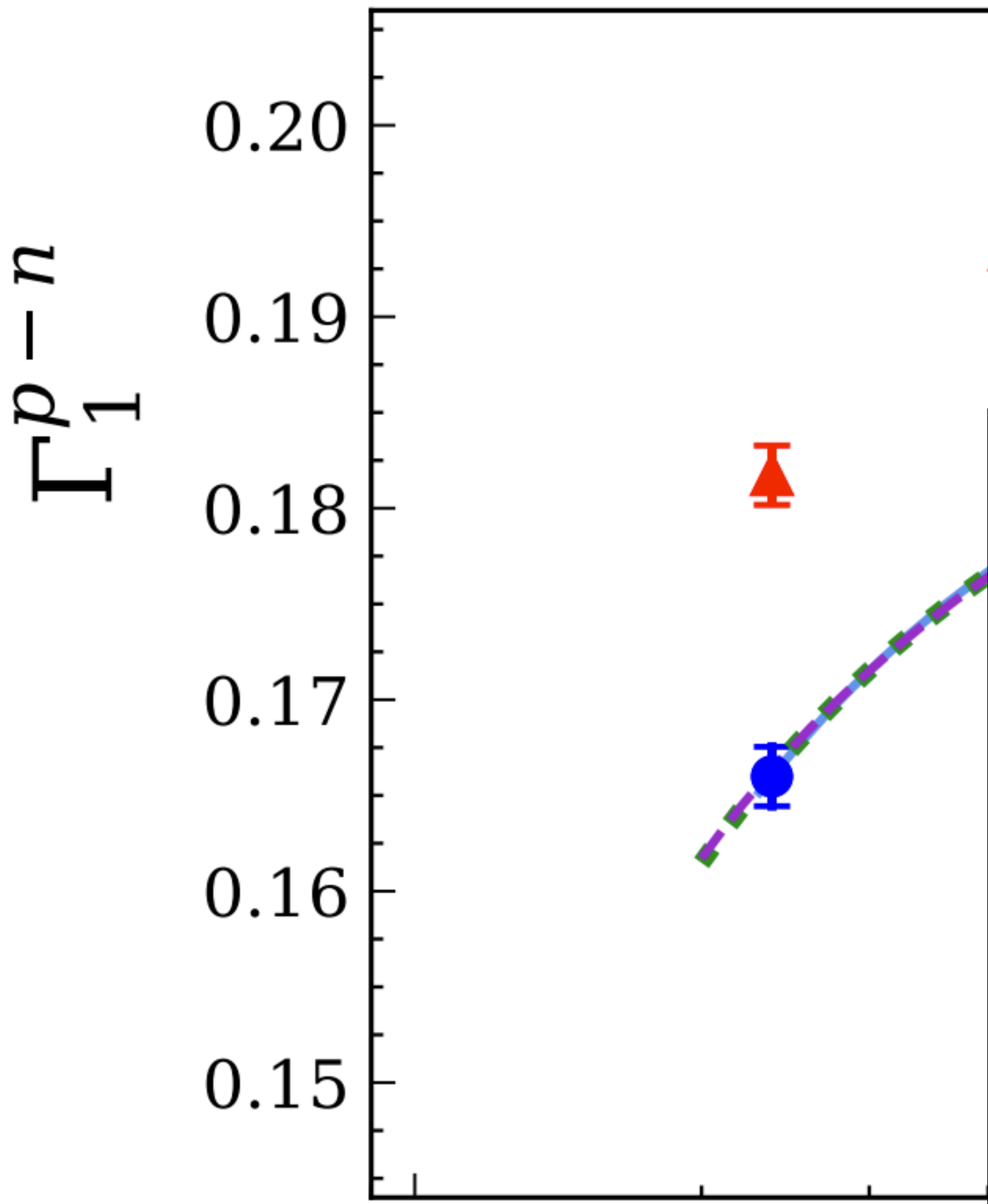
Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



Fit and procedure:

Q^2 [GeV^2]

Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



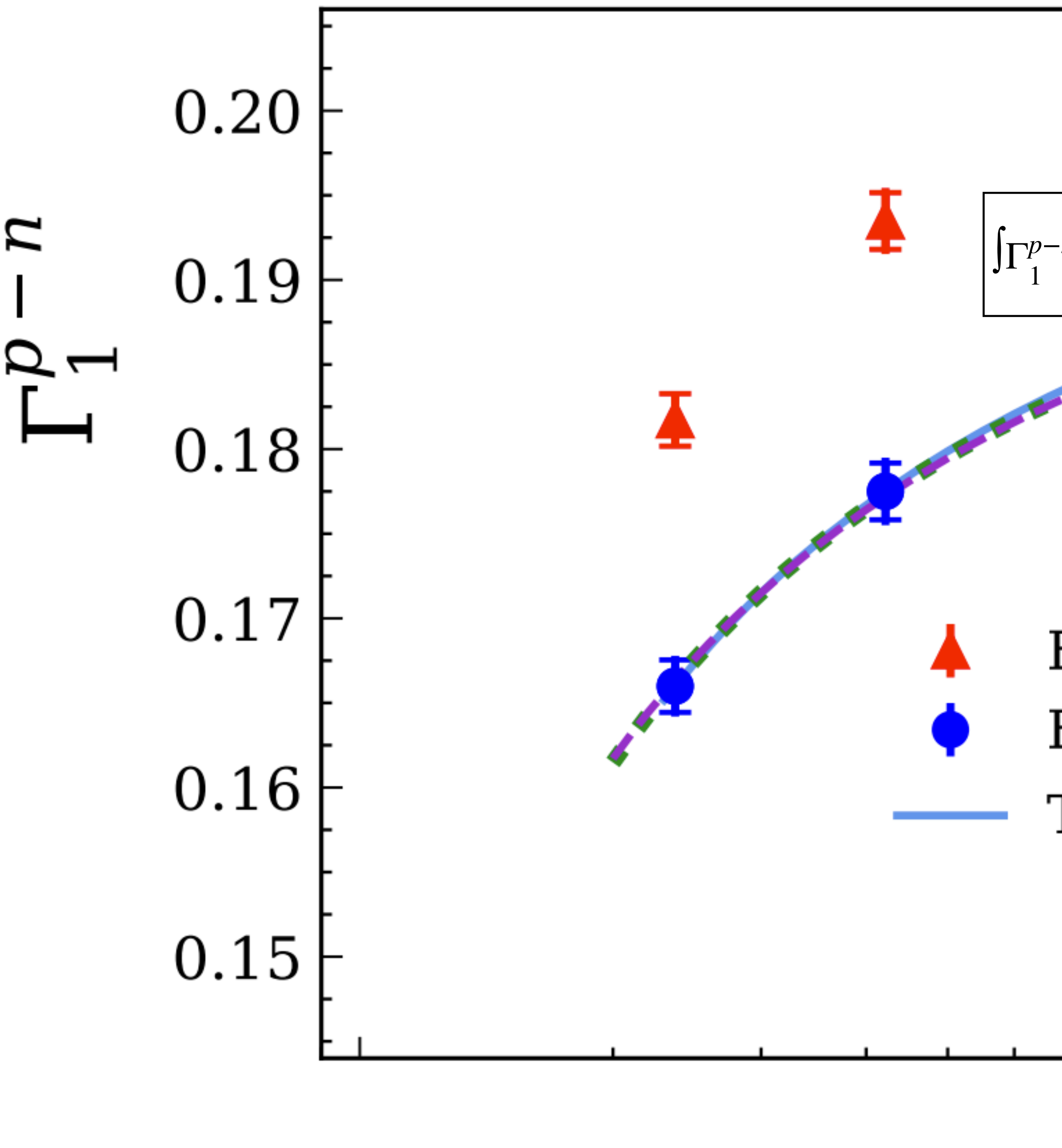
Fit and procedure:

- Main fit function: Bjorken integral approximant at N4LO with α_s at 4-loop (i.e. β_3), for main result.

$$\int \Gamma_1^{p-n} = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s}{\pi}\right)^3 - 175.7 \left(\frac{\alpha_s}{\pi}\right)^4 \right]$$

$$\alpha_s^{\overline{\rm MS}}(Q) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_s^2)} \Bigg[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln(\ln(Q^2/\Lambda_s^2))}{\ln(Q^2/\Lambda_s^2)} + \frac{\beta_1^2}{\beta_0^4 \ln^2(Q^2/\Lambda_s^2)} \left(\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1 + \frac{\beta_2 \beta_0}{\beta_1^2} \right) + \frac{\beta_1^3}{\beta_0^6 ln^3(Q^2/\Lambda_s^2)} \left(-\ln^3(\ln(Q^2/\Lambda_s^2)) + \frac{5}{2} \ln^2(\ln(Q^2/\Lambda_s^2)) + 2 \ln(\ln(Q^2/\Lambda_s^2)) - \frac{1}{2} - 3 \frac{\beta_2 \beta_0}{\beta_1^2} \ln(\ln(Q^2/\Lambda_s^2)) + \frac{\beta_3 \beta_0^2}{2\beta_1^3} \right) + \frac{\beta_1^4}{\beta_0^8 ln^4(Q^2/\Lambda_s^2)} \left(\ln^4(\ln(Q^2/\Lambda_s^2)) - \frac{13}{3} \ln^3(\ln(Q^2/\Lambda_s^2)) - \frac{3}{2} \ln^2(\ln(Q^2/\Lambda_s^2)) + 4 \ln(\ln(Q^2/\Lambda_s^2)) + \frac{7}{6} + \frac{7}{6} + \frac{3\beta_2 \beta_0}{\beta_1^2} \left(2 \ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1 \right) - \frac{\beta_3 \beta_0^2}{\beta_1^3} \left(2 \ln(\ln(Q^2/\Lambda_s^2)) + \frac{1}{6} \right) \right) \Bigg]$$

Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



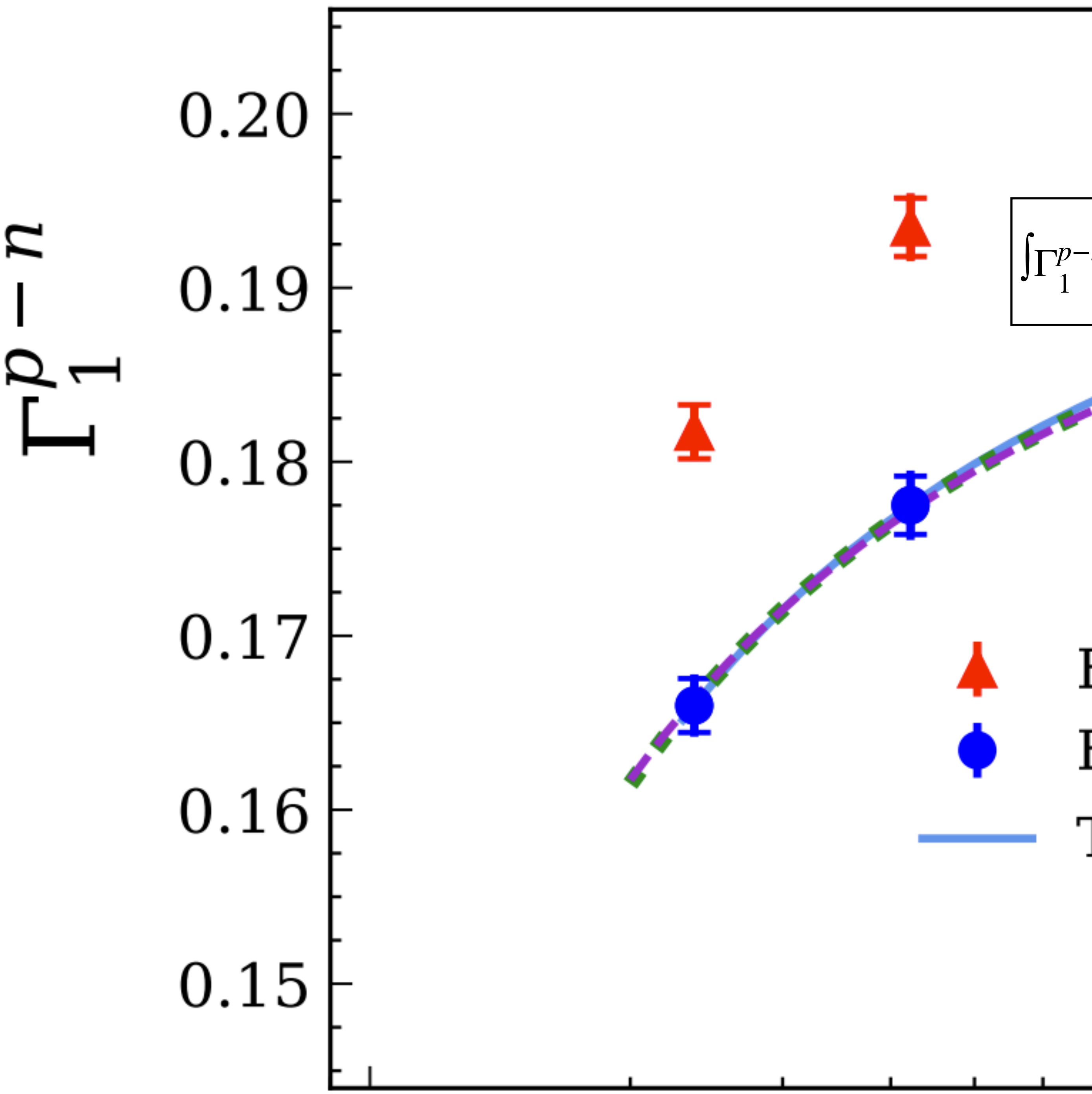
Fit and procedure:

- Main fit function: Bjorken integral approximant at N^{4}LO with α_s at 4-loop (i.e. β_3), for main result.

$$\int \Gamma_1^{p-n} = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s}{\pi}\right)^3 - 175.7 \left(\frac{\alpha_s}{\pi}\right)^4 \right]$$

- Secondary fit at N^{5}LO and α_s at 5-loop, for pQCD truncation uncertainty.

Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



Fit and procedure:

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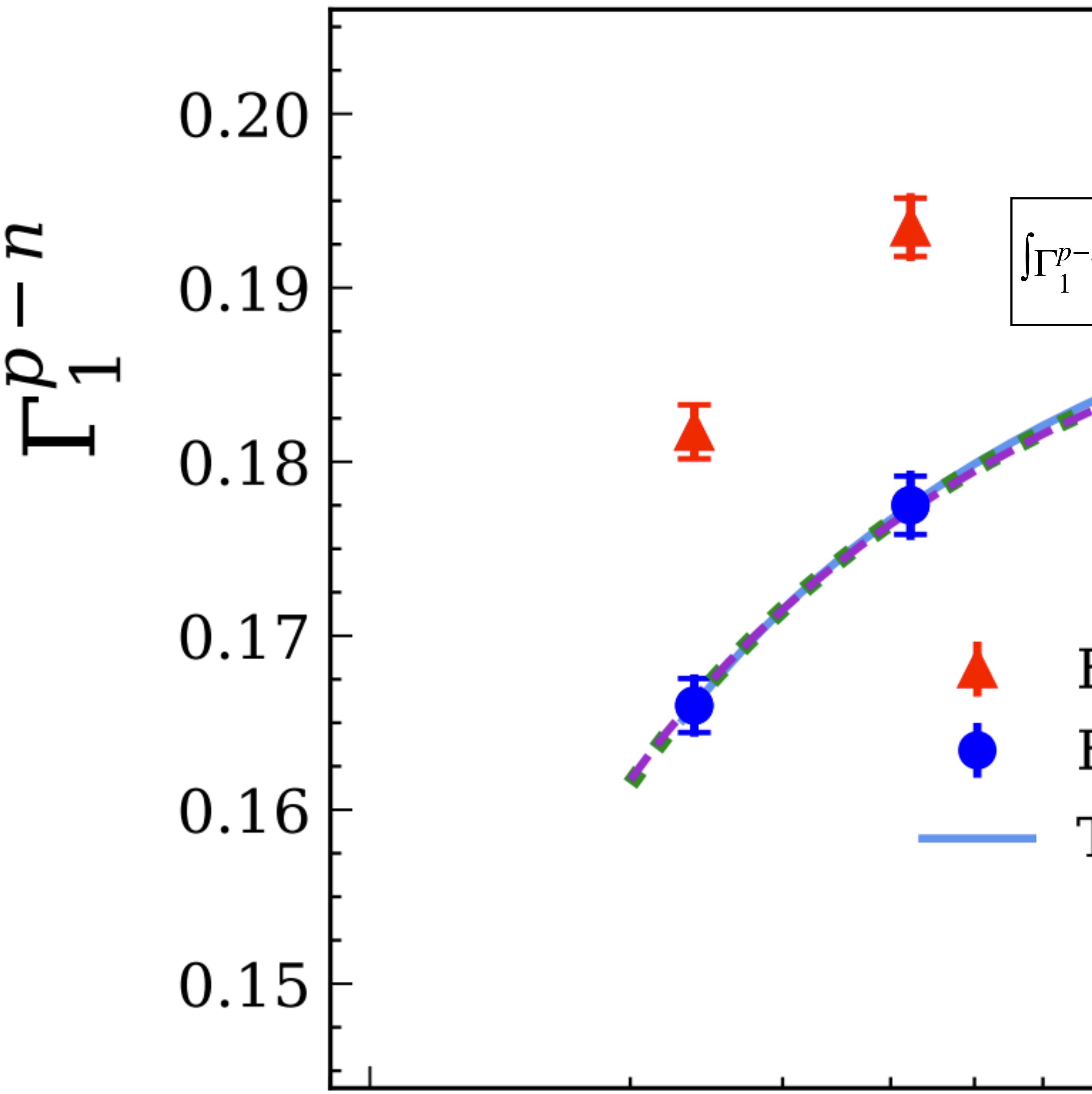
$$\int\Gamma_1^{p-n} = \frac{1}{6}g_A\left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4\right]$$

- Secondary fit at N^{5}LO and α_s at 5-loop, for pQCD truncation uncertainty.
- Systematically vary fit Q^2-range to find the optimal range minimizing the uncertainty: Low Q^2 points have high α_s sensitivity but larger pQCD truncation error. High Q^2 points have smaller α_s sensitivity but smaller pQCD error. May not be worth including the low and/or high Q^2 points. (Not worth using all data for statistics sake since stat. error is small.)
- 2-parameter fit:
  1. Λ_s is the free parameter of interest. From it, we obtain $\alpha_s(M_z)$.
  2. Twist-4: free fit parameter.

Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



Fit and procedure:

- Main fit function: Bjorken integral approximant at N⁴LO with α_s at 4-loop (i.e. β_3), for main result.

$$\int\Gamma_1^{p-n} = \frac{1}{6}g_A\left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4\right]$$

- Secondary fit at N⁵LO and α_s at 5-loop, for pQCD truncation uncertainty.
- Systematically vary fit Q^2-range to find the optimal range minimizing the uncertainty: Low Q^2 points have high α_s sensitivity but larger pQCD truncation error. High Q^2 points have smaller α_s sensitivity but smaller pQCD error. May not be worth including the low and/or high Q^2 points. (Not worth using all data for statistics sake since stat. error is small.)
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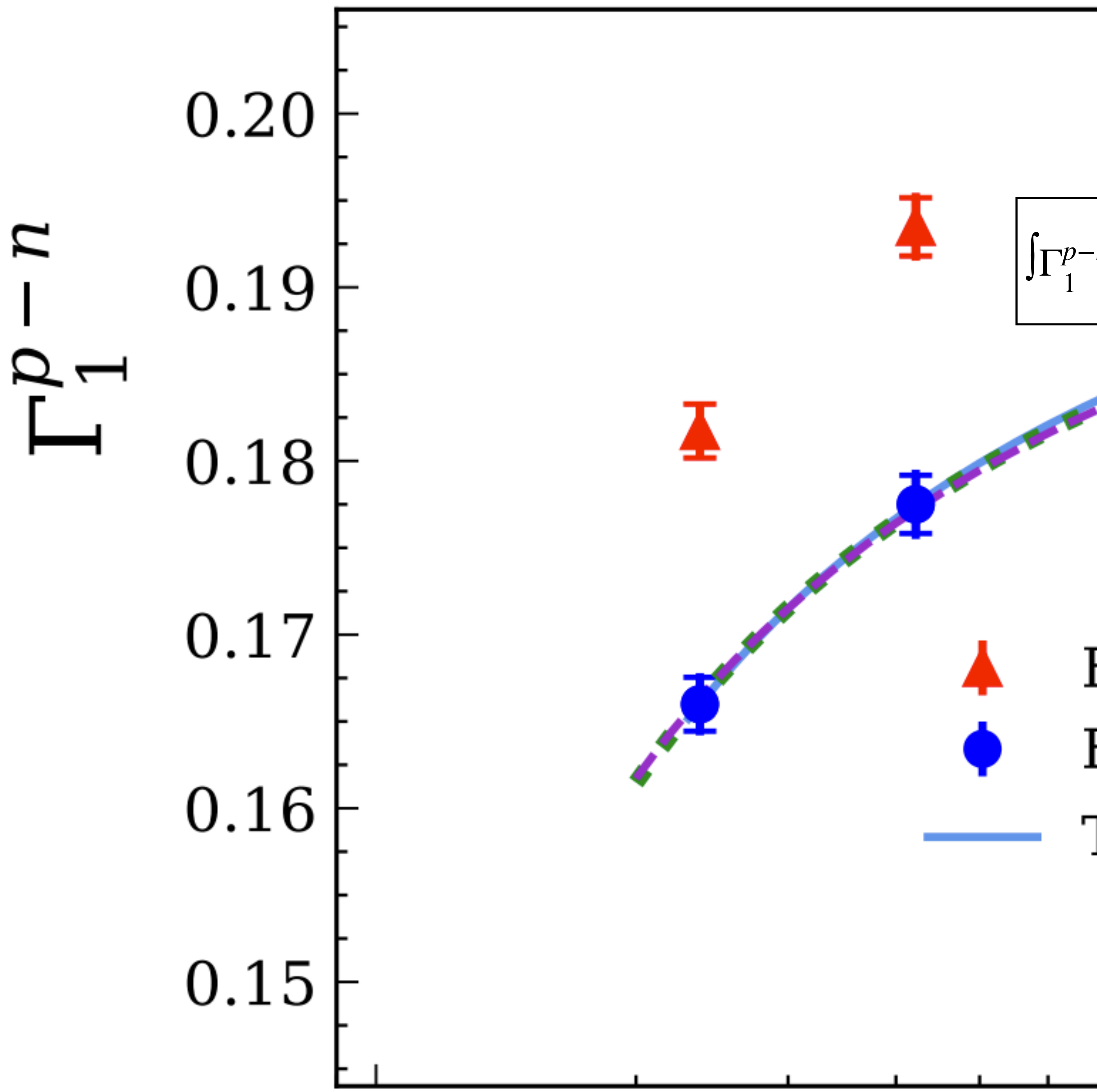
Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



Fit and procedure:

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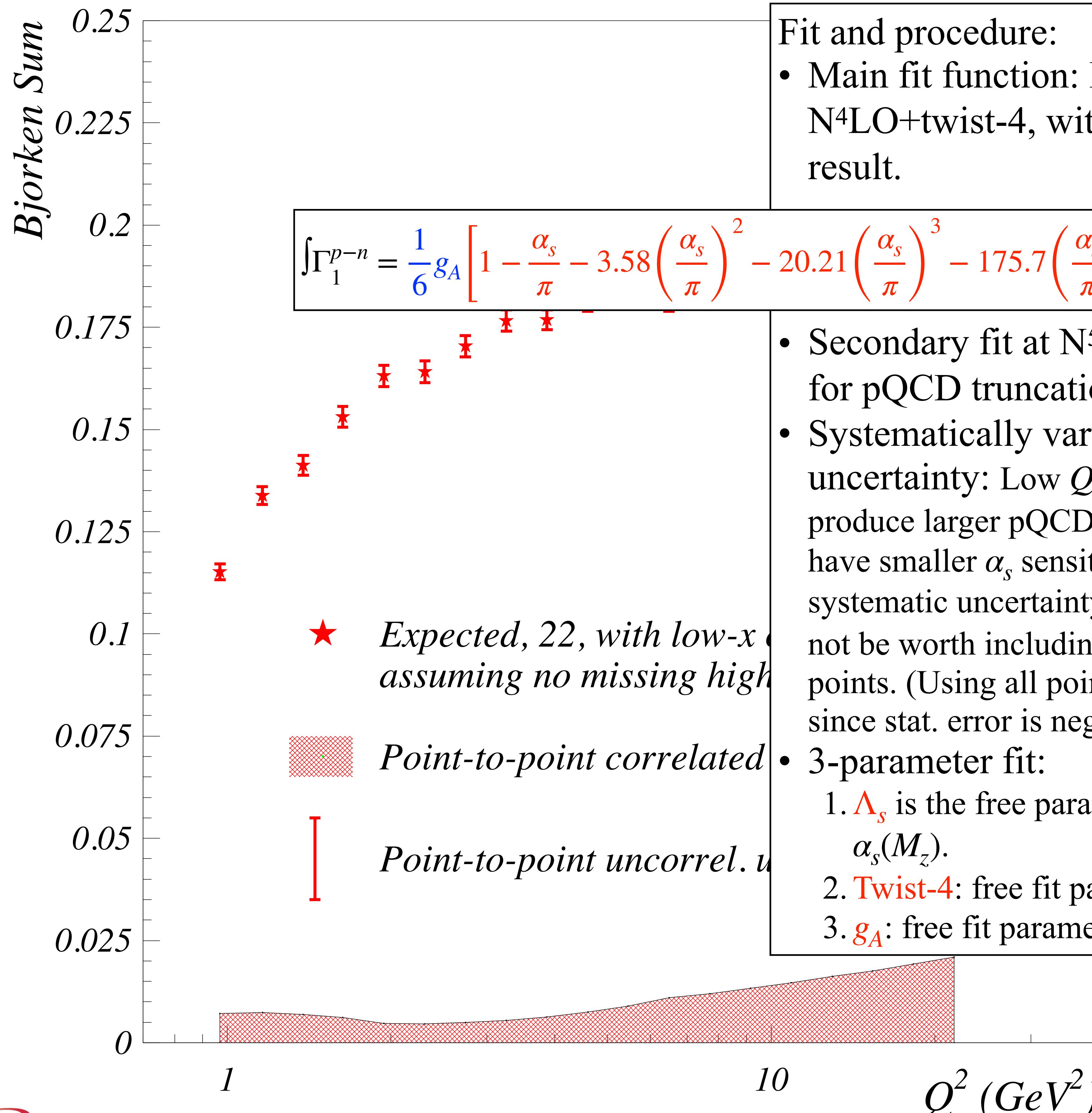
- Secondary fit at N⁵LO and α_s at 5-loop, for pQCD truncation uncertainty.
- Systematically vary fit Q^2-range to find the optimal range minimizing the uncertainty: Low Q^2 points have high α_s sensitivity but larger pQCD truncation error. High Q^2 points have smaller α_s sensitivity but smaller pQCD error. May not be worth including the low and/or high Q^2 points. (Not worth using all data for statistics sake since stat. error is small.)

  2-parameter fit:
  1. Λ_s is the free parameter of interest. From it, we obtain $\alpha_s(M_z)$.
  2. g_A. Well-known but left as a free to account for normalization uncertainties.

Extraction of $\alpha_s(M_Z)$



Fit and procedure:

- Main fit function: Bjorken sum approximant at N^{4}LO+twist-4, with α_s at 4-loop (i.e. β_3), for main result.

$$\int \Gamma_1^{p-n} = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4\right] + \frac{M^2}{Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right]$$

- Secondary fit at N^{5}LO+twist-4 and α_s at 5-loop, for pQCD truncation uncertainty.
- Systematically vary fit Q^2 range to minimize total uncertainty: Low Q^2 points have high α_s sensitivity but produce larger pQCD truncation error. High Q^2 points have smaller α_s sensitivity and larger experimental systematic uncertainty but smaller pQCD error. ⇒May not be worth including the lowest and/or highest Q^2 points. (Using all points for statistics sake is not worth it, since stat. error is negligible.)
- 3-parameter fit:
  1. Λ_s is the free parameter of interest. From it, we obtain $\alpha_s(M_z)$.
  2. Twist-4: free fit parameter.
  3. g_A: free fit parameter (for normalization adjustment)

Comparison JLab@22 GeV and EIC

EIC

- Best low-x coverage.
- No Higher-Twist uncertainties
- Smaller pQCD uncertainties.

JLab@22 GeV

- Covers region with strong Q^2-dependence: best sensitivity to α_s. (Up to 50 time more sensitive.)
- Small Higher-Twist uncertainties.
- Finer Q^2 binning (19 bins (JLab) vs 7 bins (EIC)).

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- Best low-x coverage.
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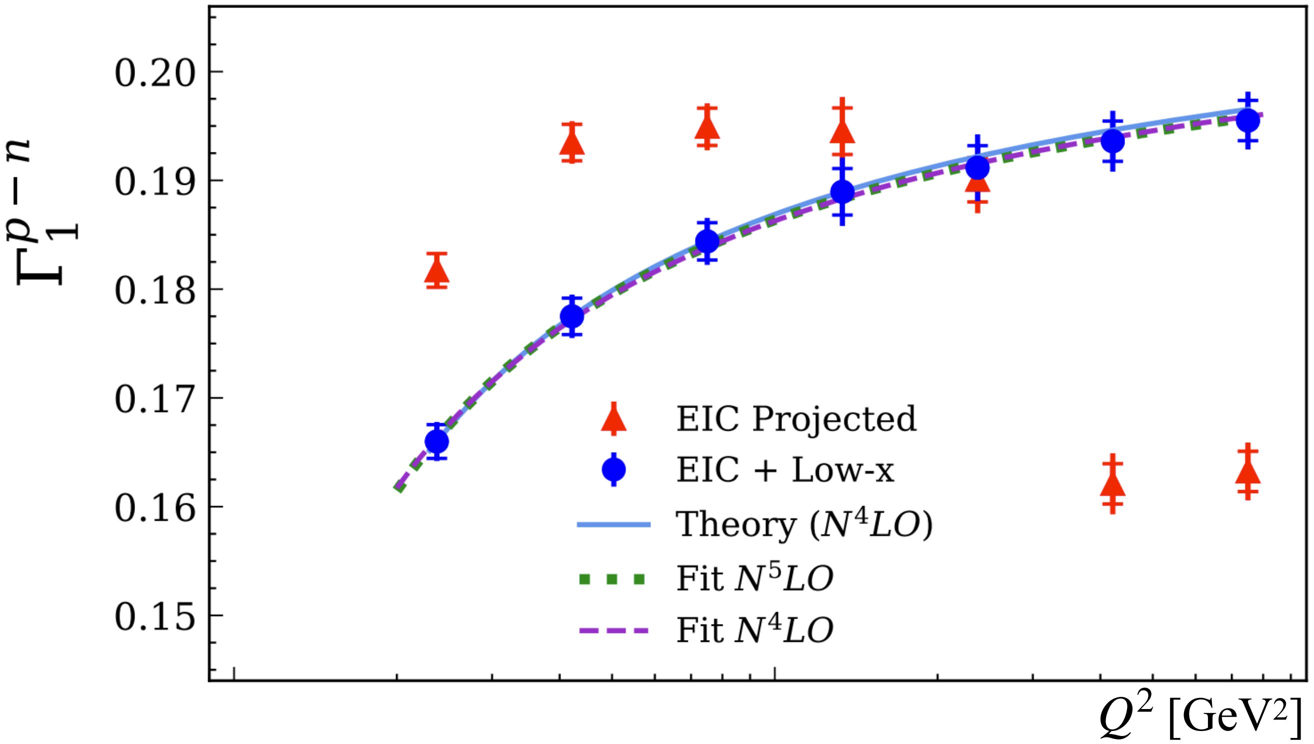
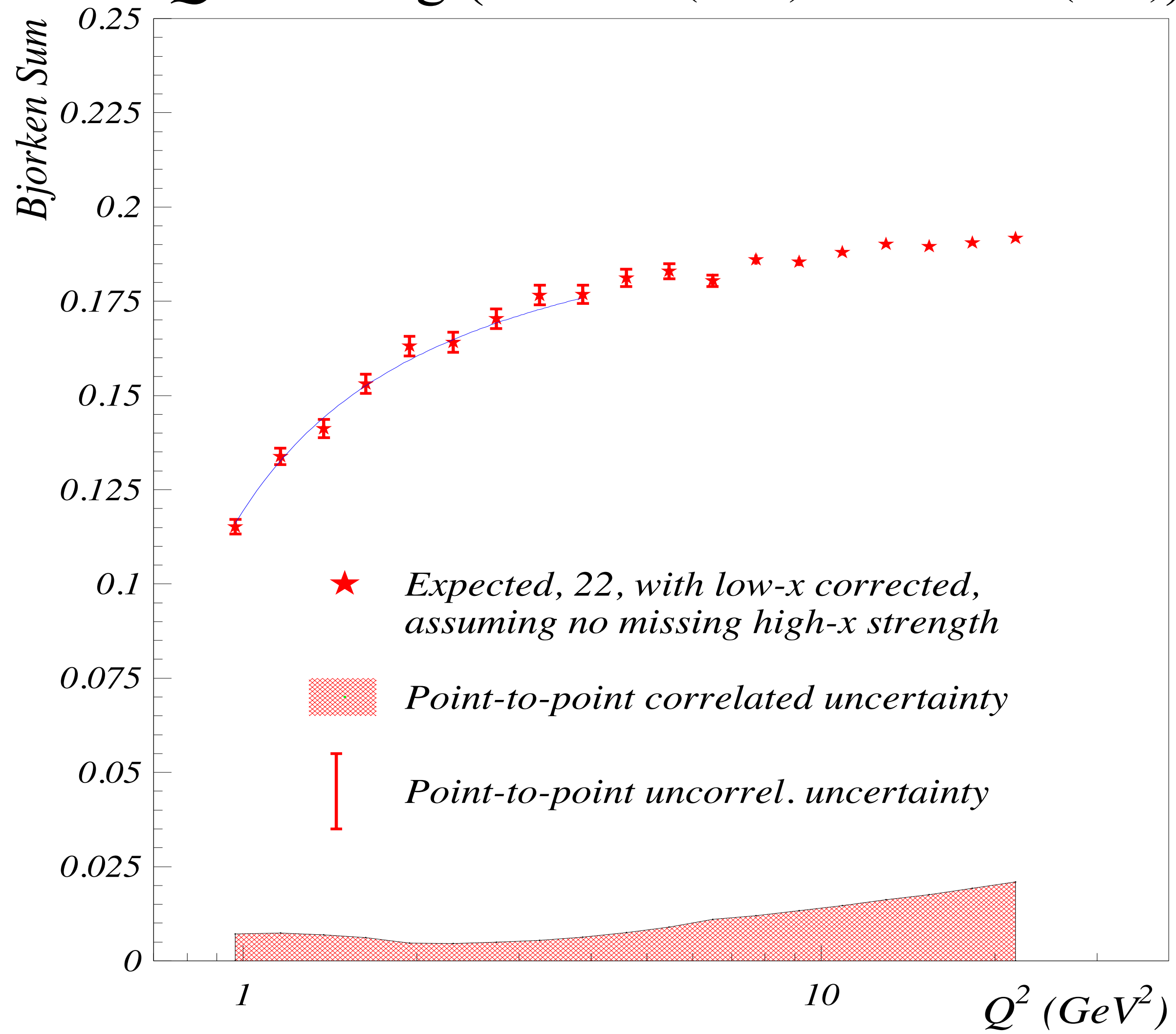


$\frac{\Delta\alpha_s}{\alpha_s} \simeq 1.3\ \%$ EIC alone.

JLab@22 GeV

- Covers region with strong Q^2-dependence: best sensitivity to α_s. (Up to 50 time more sensitive.)
- Small Higher-Twist uncertainties.
- Finer Q^2 binning (19 bins (JLab) vs 7 bins (EIC)).



$\frac{\Delta\alpha_s}{\alpha_s} \simeq 0.60\ \%$. EIC+JLab