

$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + if_{abc} A_\mu^b A_\nu^c$
and $D_\mu \equiv \partial_\mu + it^a A_\mu^a$



QCD coupling at future colliders

$\alpha_s(2025)$ workshop

Centre Paul Langevin (Aussois)

14th–19th Dec. 2025

David d'Enterria (CERN)



Based on: ESPP Update 2026, Open Symposium Venice, June 2025
ESPP Physics Briefing Book: [arXiv:2511.03883](https://arxiv.org/abs/2511.03883) [hep-ex]

Precision QCD at future facilities

Input received: $\mathcal{O}(20)$ future projects

Plus recent **community docs**:
 $\alpha_s(2024)$, QCD at Belle-II, latt-QCD,...

	Facility, Experiments	Colliding systems, c.m.s. energy	Timeline	Precision QCD
HI-LHC	HL-LHC: ALICE 3, ATLAS & CMS pII, LHCb U2, LHCspin	pp 14 TeV AA 5.5 TeV pA 8.8 TeV	> 2035 (ALICE 3, LHCb U2)	$\alpha_s(m_Z^2)$, $\alpha_s(Q^2)$, m_t , m_W
	HL-LHC: FPF	LHC collisions, neutrino-nucleon	> 2031	
	SPS: NA60+, NA61	pA, AA, 6-17 GeV	> 2030 (NA60+)	
	FAIR SIS-100: CBM	pA, AA, 2.5-5 GeV	> 2028	
	MUonE	$\mu(160 \text{ GeV})$ -e	> 2030	$g-2$ (hadronic)
	HIE-ISOLDE upgrades	Radioactive ion beams	> 2029	
Belle/ STCF	KEK: Belle II upg.	ee 10 GeV	> 2035	$\alpha_s(m_\tau^2)$
	STCF	ee 2-7 GeV	> 2033	$\alpha_s(m_\tau^2)$
EIC	EIC	ep, eA 28-140 GeV	> 2036	$\alpha_s(m_Z^2)$, $\alpha_s(Q^2)$
LHeC	LHeC	ep, eA 1.2 TeV	> 2043	$\alpha_s(m_Z^2)$, $\alpha_s(Q^2)$
FCC	FCC	ee 90-365 GeV pp 85 TeV AA 33.5 TeV pA 53.4 TeV	> 2047 > 2074	$\alpha_s(m_Z^2)$, $\alpha_s(Q^2)$, m_t , Γ_t , m_W
LC	LCF CLIC	ee 0.25-1 TeV ee 0.38-1.5 TeV	> 2050	$\alpha_s(m_Z^2)$, $\alpha_s(Q^2)$, m_t , Γ_t , m_W
LEP3	LEP3	ee 91-230 GeV	> 2047	$\alpha_s(m_Z^2)$
	Muon Collider	$\mu\mu$ 3-10 TeV	> 2050	$\alpha_s(m_Z^2)$, $\alpha_s(Q^2)$

Precision benchmarks:

- 1) $\alpha_s(m_Z)$ & its Q^2 dependence
- 2) QCD & precision top mass (m_{top})
- 3) QCD & precision W mass (m_W)

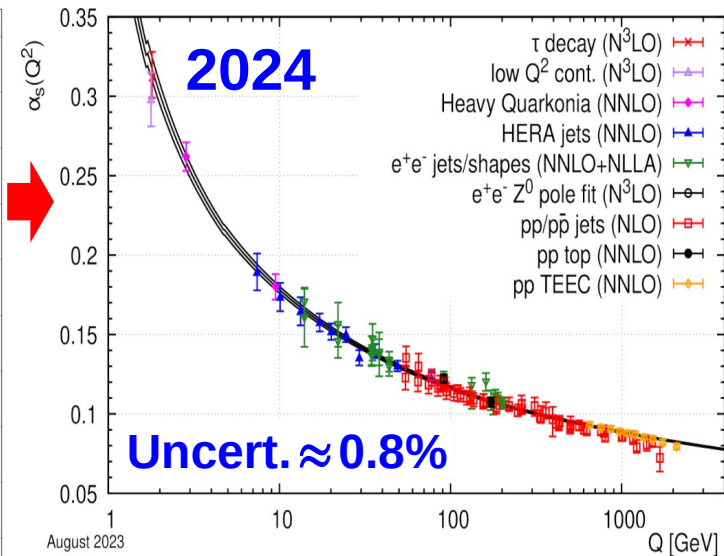
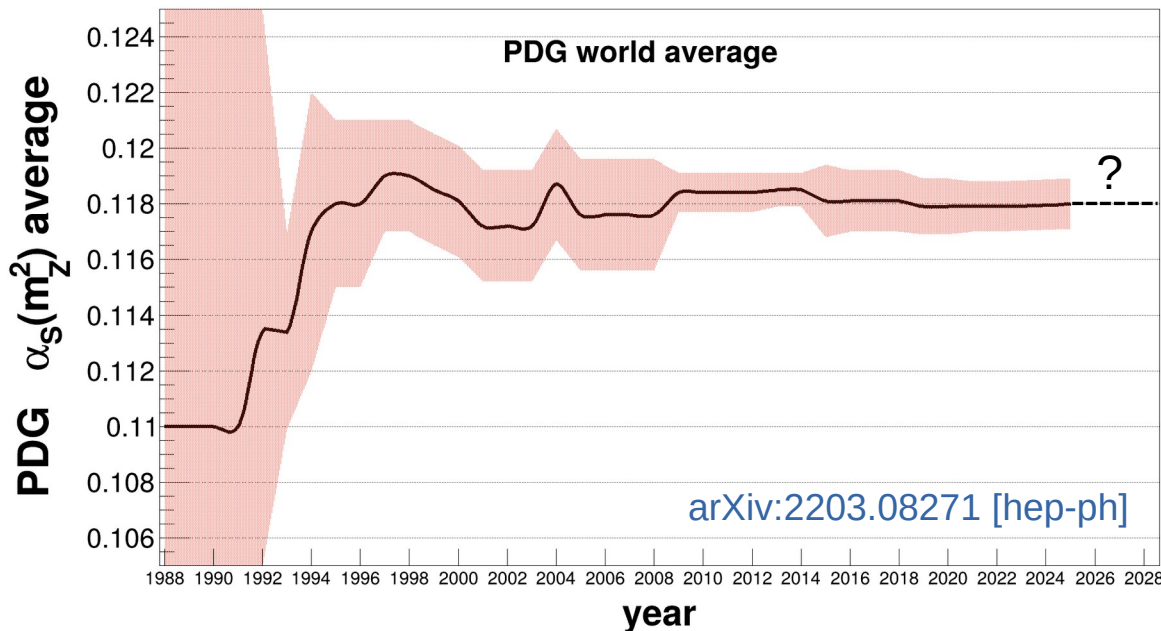
QCD & Higgs physics

Theory **calculations wish-list**:

pQCD, latt-QCD,
 MC parton-showers

The strong coupling α_s

- Determines **strength of the strong interaction** among quarks & gluons.
- **Single free parameter of QCD** in the $m_q = 0$ limit.
- Determined at a **ref. scale** ($Q=m_Z$), decreases as $\alpha_s \approx \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \approx 0.2$ GeV



$$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009$$

▶ **Least precisely known** of all interaction **couplings** !

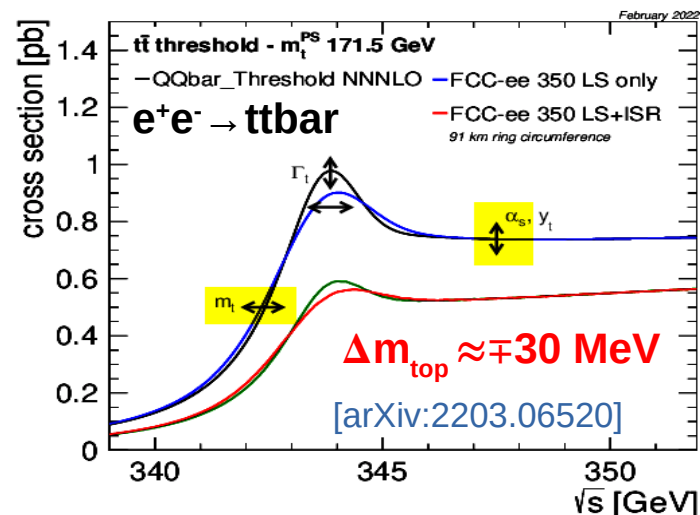
$$\Delta\alpha \approx 10^{-10} \ll \Delta G_F \approx 10^{-7} \ll \Delta G \approx 10^{-4} \ll \Delta\alpha_s \approx 0.8\%$$

α_s impact beyond QCD

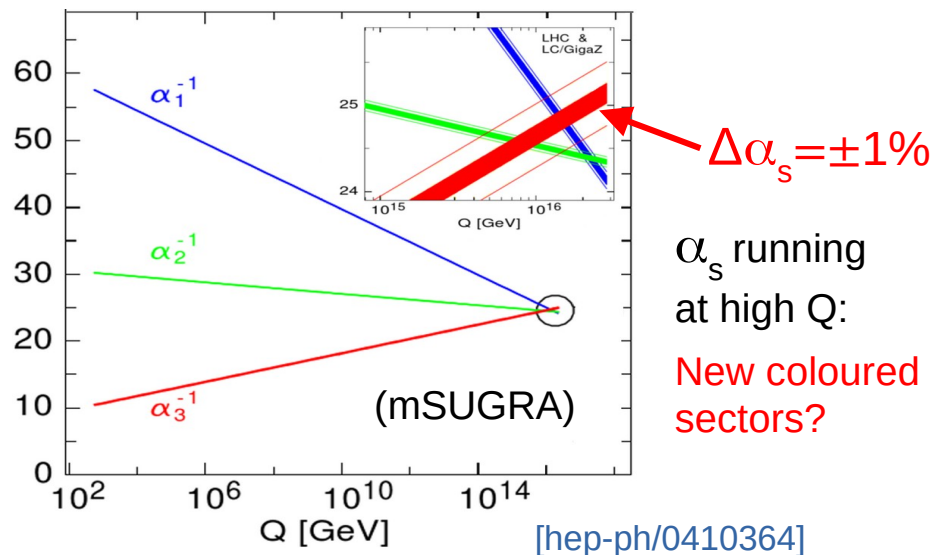
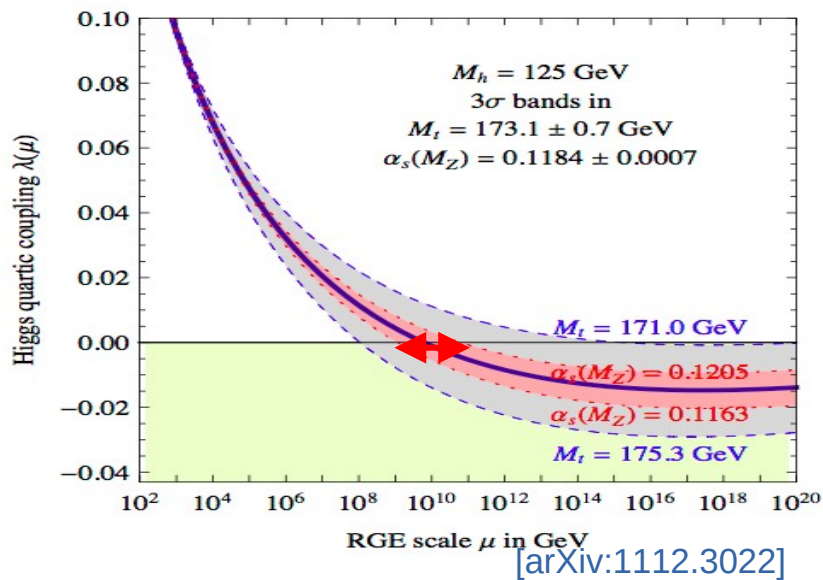
- Parametric uncertainty for key Higgs, EWPO, top quark calculations:

Process	σ (pb)	$\delta\alpha_s$ (%)	PDF + α_s (%)	Scale (%)
ggH	49.87	± 3.7	-6.2 +7.4	-2.61 + 0.32
ttH	0.611	± 3.0	± 8.9	-9.3 + 5.9

Partial width	intr. QCD	para. m_q	para. α_s
$H \rightarrow b\bar{b}$	$\sim 0.2\%$	1.4%	0.4%
$H \rightarrow c\bar{c}$	$\sim 0.2\%$	4.0%	0.4%
$H \rightarrow gg$	$\sim 3\%$	$< 0.2\%$	3.7%



- Impacts physics approaching Planck scale: EW vacuum stability, GUT:



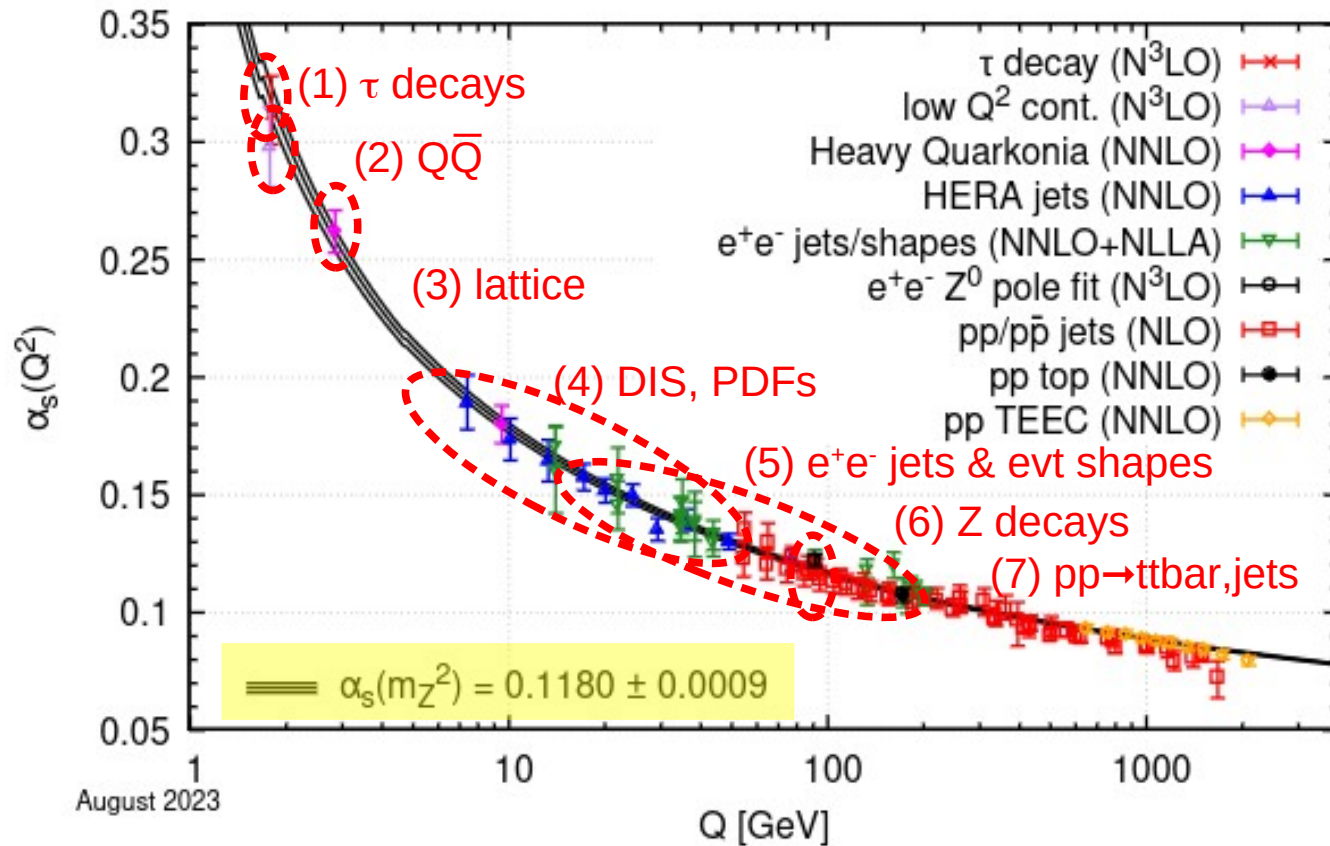
α_s running at high Q:

New coloured sectors?

α_s determination (today)

- Current precision: $\Delta\alpha_s = \pm 0.8\%$

From combination of 7 experimental observables compared to $N^{2,3}LO$ pQCD predictions:



- Running coupling probed at the LHC up to $Q \approx 5$ TeV

α_s future (our community report)

■ Expected improvements:

“The strong coupling constant: state of the art and the decade ahead”, JPG 51 (2024) 090501

Method	Relative $\alpha_s(m_Z^2)$ uncertainty	
	Current theory & exp. uncertainties sources	Near (long-term) future theory & experimental progress
(1) Lattice	0.7% Finite lattice spacing & stats. N ^{2,3} LO pQCD truncation	≈ 0.3% (0.1%) Reduced latt. spacing. Add more observables Add N ^{3,4} LO, active charm (QED effects) Higher renorm. scale via step-scaling to more observ.
(2) τ decays	1.6% N ³ LO CIPT vs. FOPT diffs. Limited τ spectral data	< 1% Add N ⁴ LO terms. Solve CIPT–FOPT diffs. Improved τ spectral functions at Belle II
(3) $Q\bar{Q}$ bound states	3.3% N ^{2,3} LO pQCD truncation $m_{c,b}$ uncertainties	≈ 1.5% Add N ^{3,4} LO & more ($c\bar{c}$), ($b\bar{b}$) bound states Combined $m_{c,b} + \alpha_s$ fits
(4) DIS & PDF fits	1.7% N ^{2,(3)} LO PDF (SF) fits Span of PDF-based results	≈ 1% (0.2%) N ³ LO fits. Add new SF fits: $F_2^{p,d}$, g_i (EIC) Better corr. matrices. More PDF data (LHeC/FCC-eh)
(5) e^+e^- jets & evt shapes	2.6% NNLO+N ^(1,2,3) LL truncation Different NP analytical & PS corrs. Limited datasets w/ old detectors	≈ 1.5% (< 1%) Add N ^{2,3} LO+N ³ LL, power corrections Improved NP corrs. via: NNLL PS, grooming New improved data at B factories (FCC-ee)
(6) Electroweak fits	2.3% N ³ LO truncation Small LEP+SLD datasets	(≈ 0.1%) N ⁴ LO, reduced param. uncs. ($m_{W,Z}$, α , CKM) Add W boson. Tera-Z, Oku-W datasets (FCC-ee)
(7) Hadron colliders	2.4% NNLO(+NNLL) truncation, PDF uncs. Limited data sets ($t\bar{t}$, W, Z, e-p jets)	≈ 1.5% N ³ LO+NNLL (for color-singlets), improved PDFs Add more datasets: Z p_T , p-p jets, σ_i/σ_j ratios,...
World average	0.8%	≈ 0.4% (0.1%)

- Precision at HL-LHC time: $\pm 0.4\%$. At LHeC (incl. DIS+jets): $\pm 0.15\%$
- Precision after FCC-ee: $\pm 0.1\%$ (Z,W \rightarrow hadrons pseudoobserv.)
- Running coupling probed up to $Q \approx 40$ TeV (FCC-hh)

QCD coupling at EIC, Belle-II, STCF, LHeC

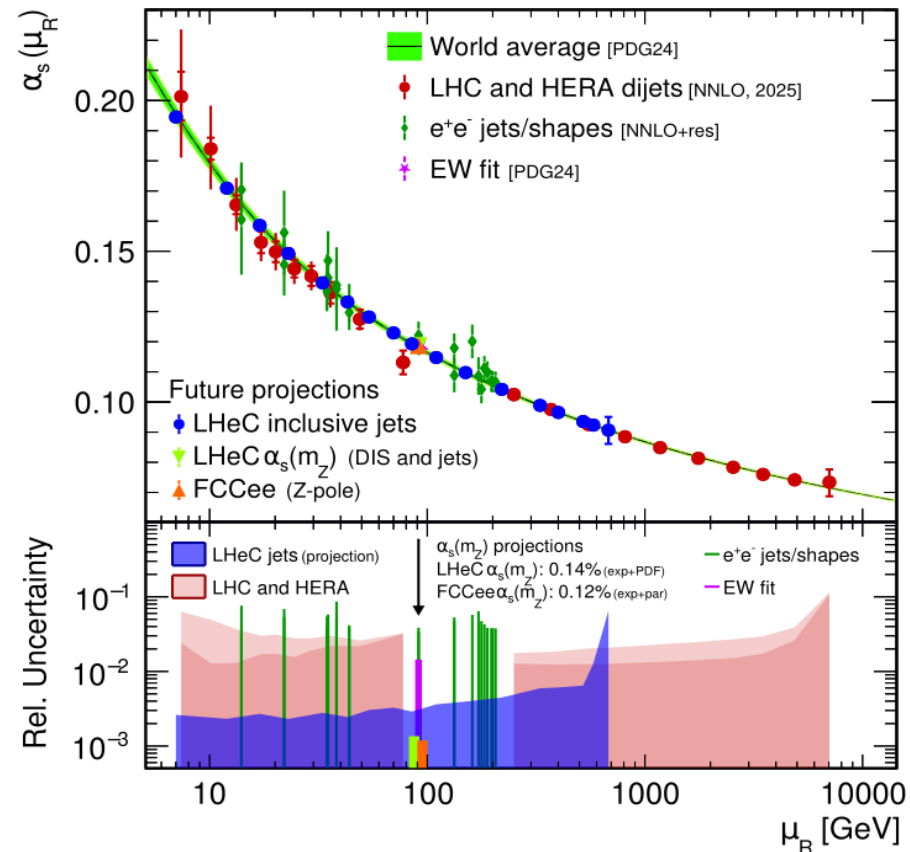
- α_s extractions at **EIC/JLab-22GeV**:
 - $\pm 1\%$ from inclusive DIS
 - $\pm 0.6\%$ from new observables (deuteron & spin-dependent SFs):
 - $\pm 0.6\%$ from **Bjorken SR** & pol. PDFs.

- α_s extractions at **Belle-II (Upgrade)**: (50 billion tau pairs in 50 ab^{-1}): Improved tau spectral functions $\ll 1\%$ (stat.) from **hadronic tau** decays.

Also possible (but not quantified):
 α_s from **R(s)** ratio over $\sqrt{s}=1-10 \text{ GeV}$
 α_s from **event shapes, EECs, FFs**.

- α_s extraction at **STCF**: $\ll 1\%$ (stat.) from **hadronic tau** decays

- α_s extractions at **LHeC**:
 - $\pm 0.2\%$ from inclusive DIS fits
 - $\pm 0.15\%$ from incl. **DIS + jets**



Running $Q = 5-500 \text{ GeV}$ (jets)

QCD coupling at e^+e^- (LC)

EW boson pseudoobservables known at N³LO in pQCD:

- The W and Z hadronic widths :

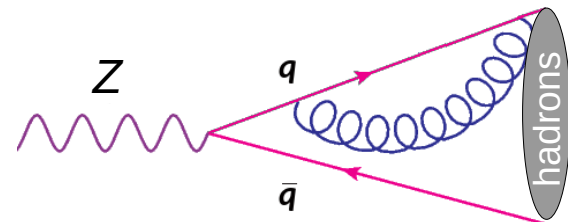
$$\Gamma_{W,Z}^{\text{had}}(Q) = \Gamma_{W,Z}^{\text{Born}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{EW}} + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

- The ratio of W, Z hadronic-to-leptonic widths :

$$R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{\text{had}}(Q)}{\Gamma_{W,Z}^{\text{lep}}(Q)} = R_{W,Z}^{\text{EW}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

- In the Z boson case, the hadronic cross section at the resonance peak in e^+e^- :

$$\sigma_Z^{\text{had}} = \frac{12\pi}{m_Z} \cdot \frac{\Gamma_Z^e \Gamma_Z^{\text{had}}}{(\Gamma_Z^{\text{tot}})^2}$$



Note: Sensitivity to $\alpha_s(m_Z)$ via $\mathcal{O}(4\%)$ higher-order corrs.

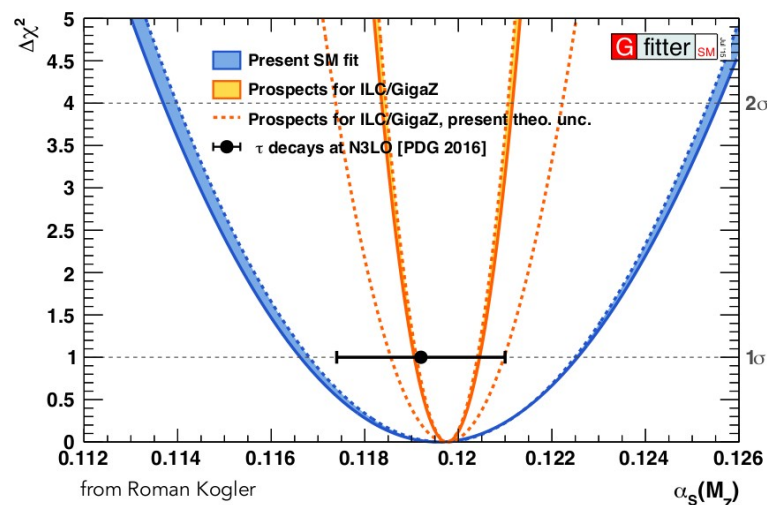
arXiv:1512.05194 [hep-ph]

LC (Giga-Z) will reach 0.6% precision on $\alpha_s(m_Z)$ ($\times 4$ better than LEP):

- Assume Z-pole stats.: 10^9 bosons
- Other uncertainties (syst., parametric) not considered (but subleading):

$$\alpha_s(m_Z) = 0.1200 \pm 0.0007$$

- Also from τ hadronic decays, evt. shapes, jet rates: $< 1\%$
- However, LC claim is that α_s should be taken from latt-QCD ($\pm 0.1\%$ precision) instead

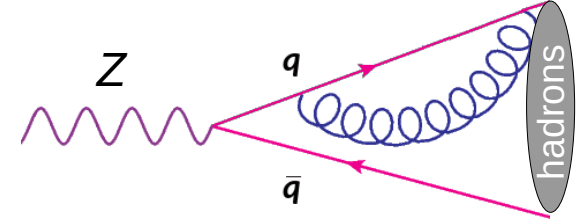


QCD coupling at e^+e^- (FCC-ee)

EW boson pseudoobservables known at N³LO in pQCD:

- The W and Z hadronic widths :

$$\Gamma_{W,Z}^{\text{had}}(Q) = \Gamma_{W,Z}^{\text{Born}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{EW}} + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$



- The ratio of W, Z hadronic-to-leptonic widths :

$$R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{\text{had}}(Q)}{\Gamma_{W,Z}^{\text{lep}}(Q)} = R_{W,Z}^{\text{EW}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

Note: Sensitivity to $\alpha_s(m_Z)$ via $\mathcal{O}(4\%)$ higher-order corrs.

- In the Z boson case, the hadronic cross section at the resonance peak in e^+e^- :

$$\sigma_Z^{\text{had}} = \frac{12\pi}{m_Z} \cdot \frac{\Gamma_Z^e \Gamma_Z^{\text{had}}}{(\Gamma_Z^{\text{tot}})^2}$$

[arXiv:2005.04545]

FCC-ee will reach 0.1% precision on $\alpha_s(m_Z)$ ($\times 20$ better than LEP):

- Huge Z pole stats. ($\times 10^5$ LEP):
- Exquisite syst./parametric precision:

$$\Delta R_Z = 10^{-3}, \quad R_Z = 20.7500 \pm 0.0010$$

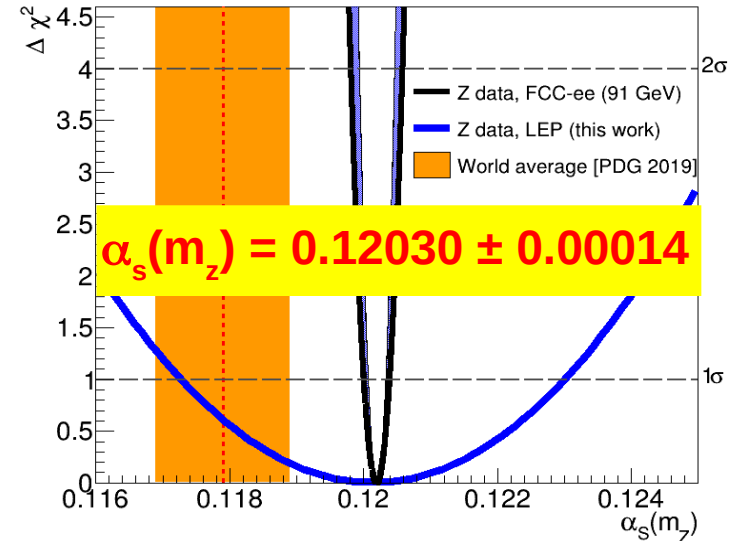
$$\Delta \Gamma_Z^{\text{tot}} = 0.1 \text{ MeV}, \quad \Gamma_Z^{\text{tot}} = 2495.2 \pm 0.1 \text{ MeV}$$

$$\Delta \sigma_Z^{\text{had}} = 4.0 \text{ pb}, \quad \sigma_Z^{\text{had}} = 41\,494 \pm 4 \text{ pb}$$

$$\Delta m_Z = 0.1 \text{ MeV}, \quad m_Z = 91.18760 \pm 0.00001 \text{ GeV}$$

$$\Delta \alpha = 3 \cdot 10^{-5}, \quad \Delta \alpha_{\text{had}}^{(5)}(m_Z) = 0.0275300 \pm 0.0000009$$

- TH uncertainty needs to be reduced by $\times 4$ from missing $\alpha_s^5, \alpha^3, \alpha\alpha_s^2, \alpha\alpha_s^2, \alpha^2\alpha_s$ terms



QCD coupling at e^+e^- (FCC-ee)

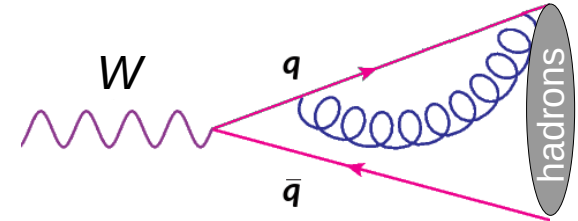
EW boson pseudoobservables known at N³LO in pQCD:

- The W and Z hadronic widths :

$$\Gamma_{W,Z}^{\text{had}}(Q) = \Gamma_{W,Z}^{\text{Born}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{EW}} + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

- The ratio of W, Z hadronic-to-leptonic widths :

$$R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{\text{had}}(Q)}{\Gamma_{W,Z}^{\text{lep}}(Q)} = R_{W,Z}^{\text{EW}} \left(1 + \sum_{i=1}^4 a_i(Q) \left(\frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$



Note: Sensitivity to $\alpha_s(m_Z)$ via $\mathcal{O}(4\%)$ higher-order corrs.

[arXiv:2005.04545]

FCC-ee will reach 0.2% precision on $\alpha_s(m_W)$ ($\times 300$ better than LEP):

- Huge W pole stats. ($\times 10^4$ LEP-2).
- Exquisite syst./parametric precision:

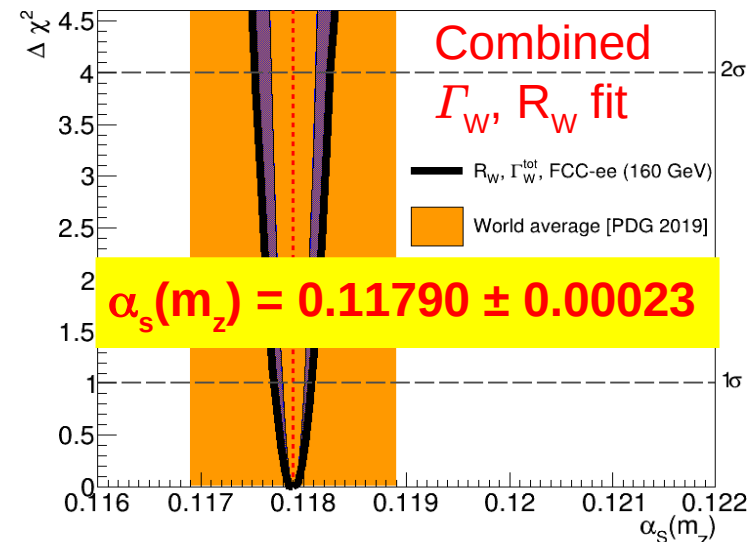
$$\Gamma_W^{\text{tot}} = 2088.0 \pm 1.2 \text{ MeV}$$

$$R_W = 2.08000 \pm 0.00008$$

$$m_W = 80.3800 \pm 0.0005 \text{ GeV}$$

$$|V_{cs}| = 0.97359 \pm 0.00010 \quad \leftarrow \mathcal{O}(10^{12}) \text{ D mesons}$$

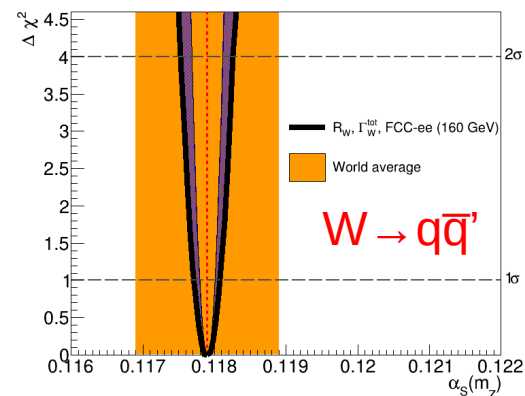
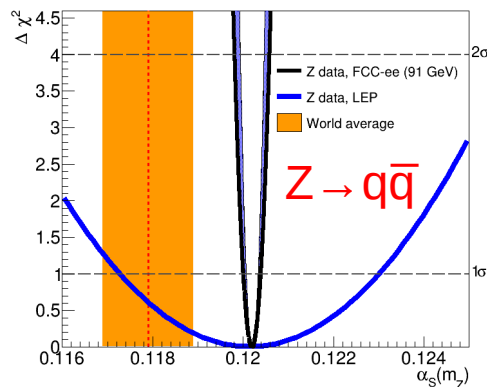
- TH uncertainty needs to be reduced by $\times 10$ from missing $\alpha_s^5, \alpha^2, \alpha^3, \alpha\alpha_s^2, \alpha\alpha_s^2, \alpha^2\alpha_s$ terms



QCD coupling α_s at FCC-ee,hh & LEP3

■ α_s extractions at FCC-ee:

- $\pm 0.1\%$ from **Z** pseudoobservables
- $\pm 0.2\%$ from **W** hadronic decays
- $\ll 1\%$ from **tau** hadronic decays
- $\ll 1\%$ from evt. **shapes** & **jet** rates



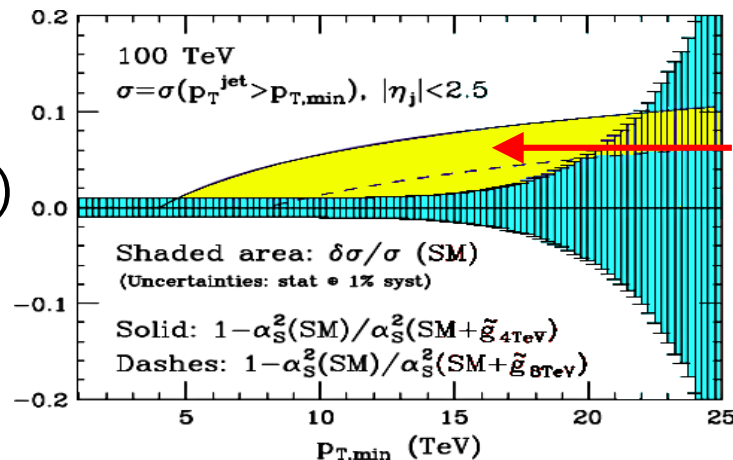
■ α_s extractions at LEP3:

Same physics measurements as FCC-ee modulo **smaller samples** ($\times 4$ less Z) & **worse syst.** (e.g. beam energy calibration/spread) uncertainties.

- $\pm 0.2\%$ (stat) from **Z** pseudoobservables
- $\pm 0.9\%$ (stat) from **W** hadronic decays
- $< 1\%$ from **tau** hadr. decays, evt. **shapes** & **jet** rates

■ α_s extractions at FCC-hh:

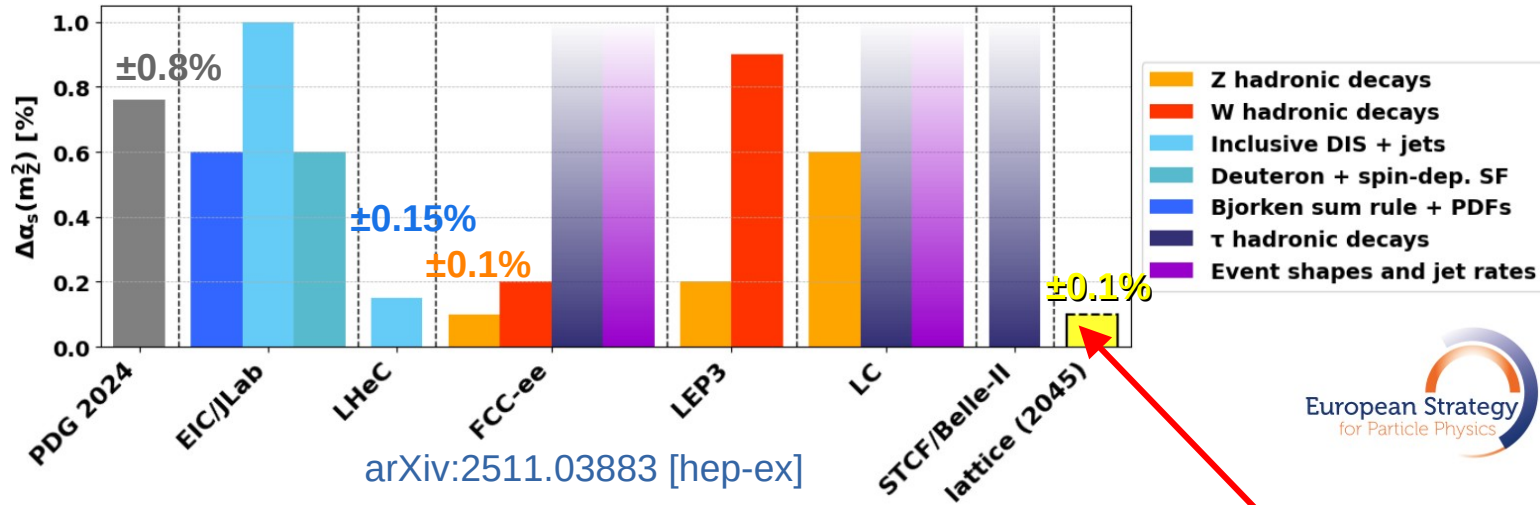
Test α_s **running** (asymptotic freedom) with multi-TeV jets **up to $Q \approx 50$ TeV**



New BSM coloured sectors?

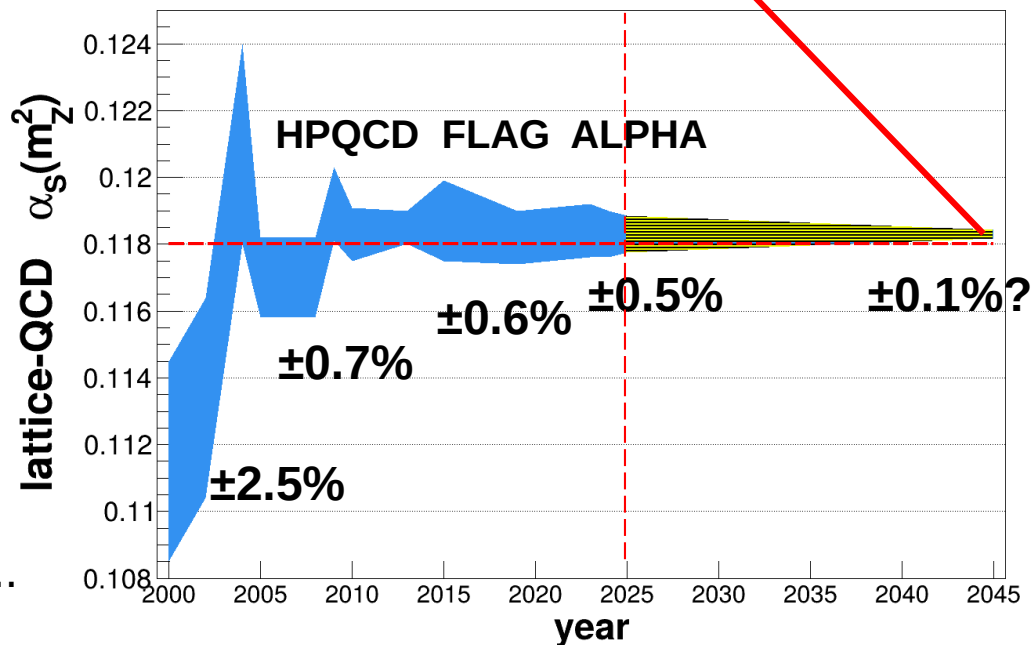
Summary: QCD coupling α_s

■ **Forecast**
precision
at future
facilities:



■ **Lattice-QCD:**

- **Step-scaling & decoupling** are state-of-the-art techniques.
- **Stat. uncert.** $\propto (\text{computing power})^2$ reducible by $\sim 1/2$ every 10 yrs
- **pQCD** observables needed at higher accuracy: α_s^n
- **Syst. uncerts.** will start playing role: QED, $m_{u,d}$, dynamical charm mass,...



Theory calculations “wish list”

- Summary table for precision-QCD wish-list (pQCD, lattice, MC PS):

Observable	Missing higher-order & power-suppressed corrections
$\alpha_s(m_Z)$ in e^+e^-	Hadronic Z width $\mathcal{O}(\alpha_s^5), \mathcal{O}(\alpha^3), \mathcal{O}(\alpha_s\alpha^3), \mathcal{O}(\alpha_s^2\alpha^2)$
	Hadronic W width $\mathcal{O}(\alpha_s^5), \mathcal{O}(\alpha^2), \mathcal{O}(\alpha^3), \mathcal{O}(\alpha_s\alpha^2), \mathcal{O}(\alpha_s\alpha^3), \mathcal{O}(\alpha_s^2\alpha^2)$
	Hadronic τ width $\mathcal{O}(\alpha_s^5)$
	Hadronic event shapes (Z, W, H decays) N^3 LO differential, $N^{3,4}$ LL resummation, power corrections
	Inclusive jet rates 3-jet cross sections at N^3 LO, 4-jets at N^2 LO, 5-jets at NLO
$\alpha_s(m_Z)$ in latt.	Lattice QCD results $\mathcal{O}(\alpha_s^6)$ β -function; $\mathcal{O}(\alpha_s^5)$ heavy quark decoupling; $\mathcal{O}(\alpha_s^4)$ static potential
	(α_s extractions; quark masses m_c, m_b) $\mathcal{O}(\alpha_s^3)$ lattice perturbation theory matching (lattice coupling to $\alpha_s^{\overline{MS}}$ etc.)
m_W, m_{top} in e^+e^-	$\sigma(e^+e^- \rightarrow W^+W^-)$ vs. \sqrt{s} EW N^2 LO: $\mathcal{O}(\alpha^2)$, Mixed EW-QCD: $\mathcal{O}(\alpha_s\alpha^2), \mathcal{O}(\alpha_s^2\alpha)$
	$\sigma(e^+e^- \rightarrow t\bar{t})$ vs. \sqrt{s} NRQCD: $\mathcal{O}(\alpha_s^5)$, Non-resonant: $\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^3)$ differential; QED: $\mathcal{O}(\alpha^3)$ at NNLL
QCD in Higgs	$H \rightarrow b\bar{b}$ width N^4 LO (massive b-quark); N^4 LO differential (massless b-quark)
	$H \rightarrow gg$ width N^5 LO (heavy-top limit), N^4 LO (massive top)
$e^+e^-, e-p,$ p-p PS	N^4 LO differential, N^3 LO differential (massive top)
	MC simulations for $e^+e^- \rightarrow X$ processes $N^{2,3}$ LO matched to $N^{2,3}$ LL PS. Per mille control of non-perturbative QCD effects (hadronization, CR,...)
$\alpha_s(m_Z)$ in DIS	$ep \rightarrow$ hadrons (PDF and α_s determination) $N^{3,4}$ LO evolution equations and inclusive cross sections
	$ep \rightarrow$ jets (α_s determination) N^3 LO cross sections

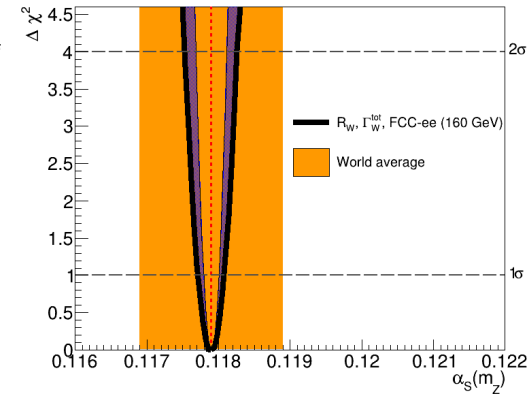
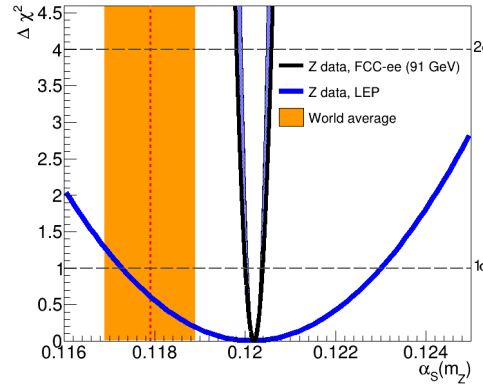
arXiv:2511.03883 [hep-ex]

- Future Higgs/EW/top factories (in particular FCC-ee) impose **very strong requirements on theory developments to match expected exp. precision.**

Backup slides

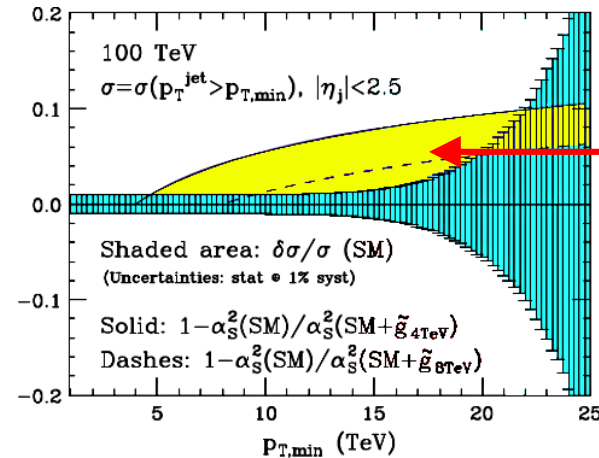
QCD coupling α_s at FCC-ee/FCC-hh/MuColl

- α_s measurements at FCC-ee:
 - $\pm 0.1\%$ from Z pseudoobservables
 - $\pm 0.2\%$ from W hadronic decays
 - $\ll 1\%$ from tau hadronic decays
 - $\ll 1\%$ from evt. shapes & jet rates



- α_s measurements at FCC-hh:

Test α_s running (asymptotic freedom) with multi-TeV jets up to $Q \approx 50$ TeV



New BSM coloured sectors

[arXiv:1607.01831]

- α_s measurements at MuColl:

Running up to $Q \approx 10$ TeV (EECs)

Measurements of neutrino SFs at a far-forward detector.