Accelerators for Newcomers

Based on slides prepared by D.Brandt

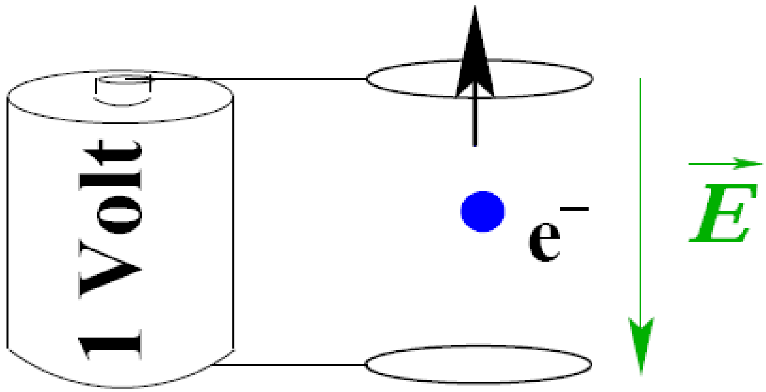
Why this Introduction?

- During this school, you get into contact with accelerators at CERN
- but some of you are completely new to the field of accelerator physics.
- It seems therefore justified to start with the introduction of a few very basic concepts, which will be used throughout the course.

This is a completely intuitive approach (no mathematics) aimed at highlighting the physical concepts, without any attempt to achieve any scientific derivation.

Some generalities …

Units: the electronvolt (eV)



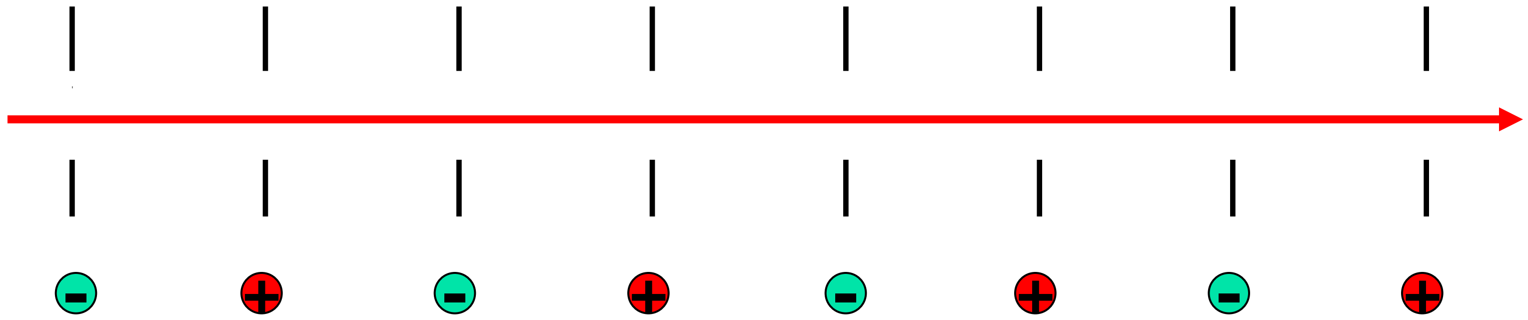
The electronvolt (eV) is the energy gained by an electron travelling, in vacuum, between two points with a voltage difference of 1 Volt. $1\ \text{eV} = 1.602\ 10^{-19}$ Joule

We also frequently use the electronvolt to express masses from $E=mc^2$: $1\ \text{eV}/c^2 = 1.783\ 10^{-36}$ kg

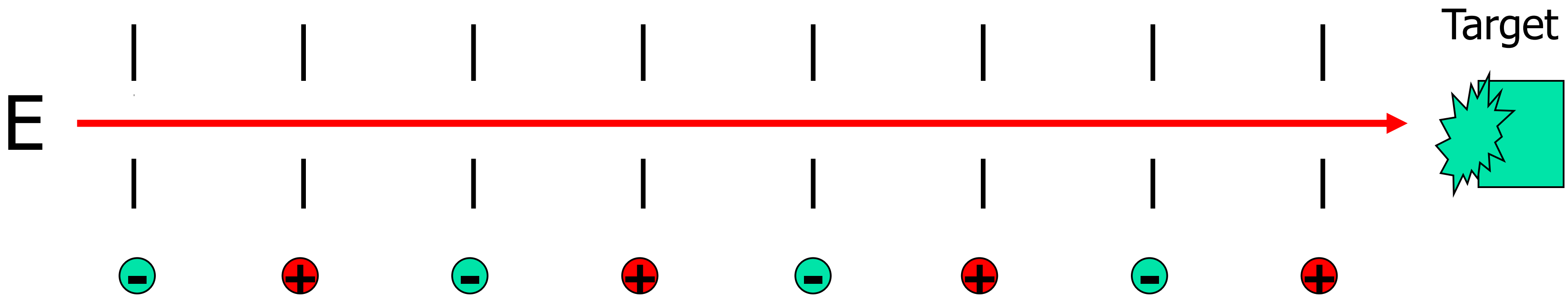
What is a Particle Accelerator?

- ➢ a machine to accelerate some particles ! How is it done ?

- ➢ Many different possibilities, but rather easy from the general principle:



Ideal linear machines (linacs)



Available Energy : $E_{c.m.} = m \cdot (2+2\gamma)^{1/2} = (2m.(m+E))^{1/2}$

with $\gamma = E/E_0$

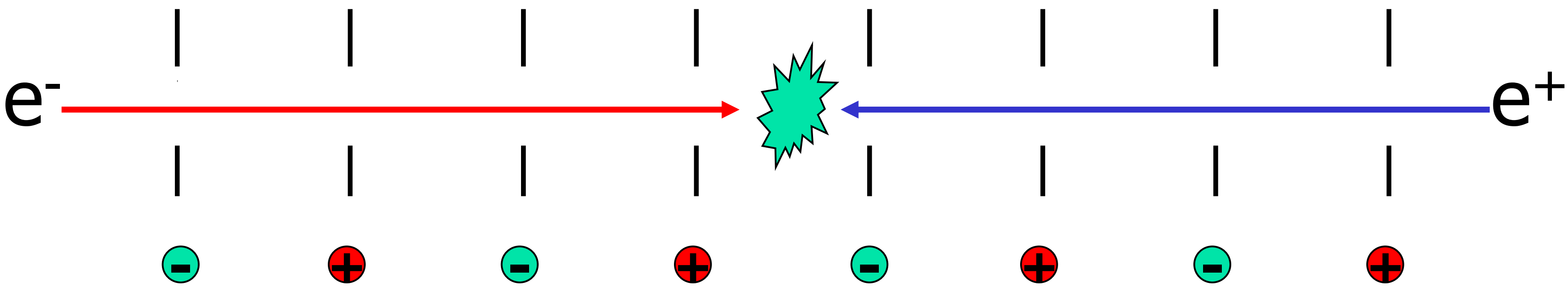
Advantages: Single pass

High intensity

Drawbacks: Single pass

Available Energy

Improved solution for $E_{c.m.}$



Available Energy : $E_{c.m.} = 2m\gamma = 2E$

with $\gamma = E/E_0$

<u>Advantages</u>: High intensity

<u>Drawbacks</u>: Single pass

Space required

Watch out !

The difference between fixed target and colliding mode deserves to be considered in some detail:

Fixed target mode:

$E_{c.m.} \propto (2mE)^{1/2}$

$\Leftrightarrow$

Colliding mode:

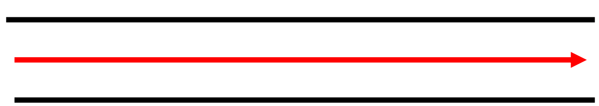
$E_{c.m.} \propto 2E$

What would be the required beam energy to achieve $E_{c.m.}$=14 TeV in fixed target mode ?

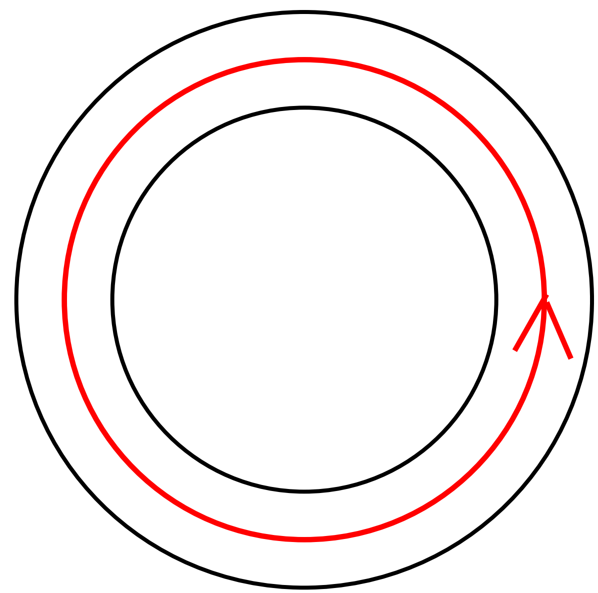
Keep particles: circular machines

Basic idea is to keep the particles in the machine for many turns.

Move from the linear design



To a circular one:

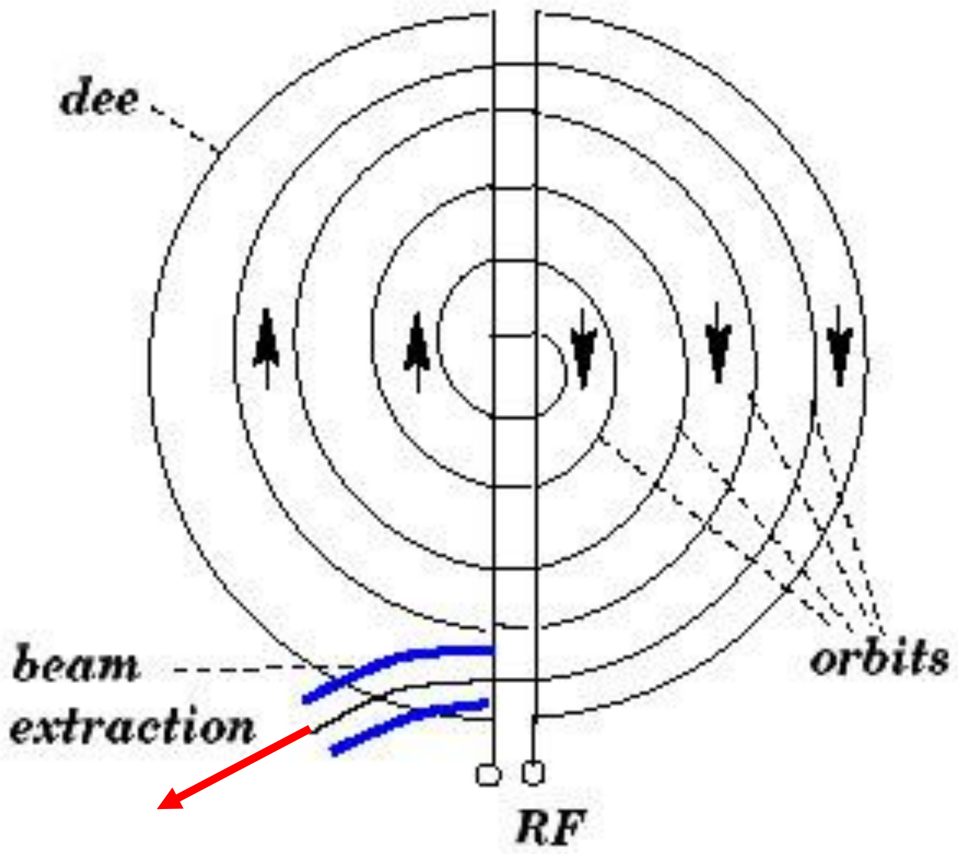


- Need Bending
- Need Dipoles!

Circular machines 1 ($E_{c.m.} \sim (mE)^{1/2}$)

fixed target:

cyclotron



huge dipole, compact design, B = constant , low energy, single pass.

Circular machines 2 ($E_{c.m.} \sim (mE)^{1/2}$)

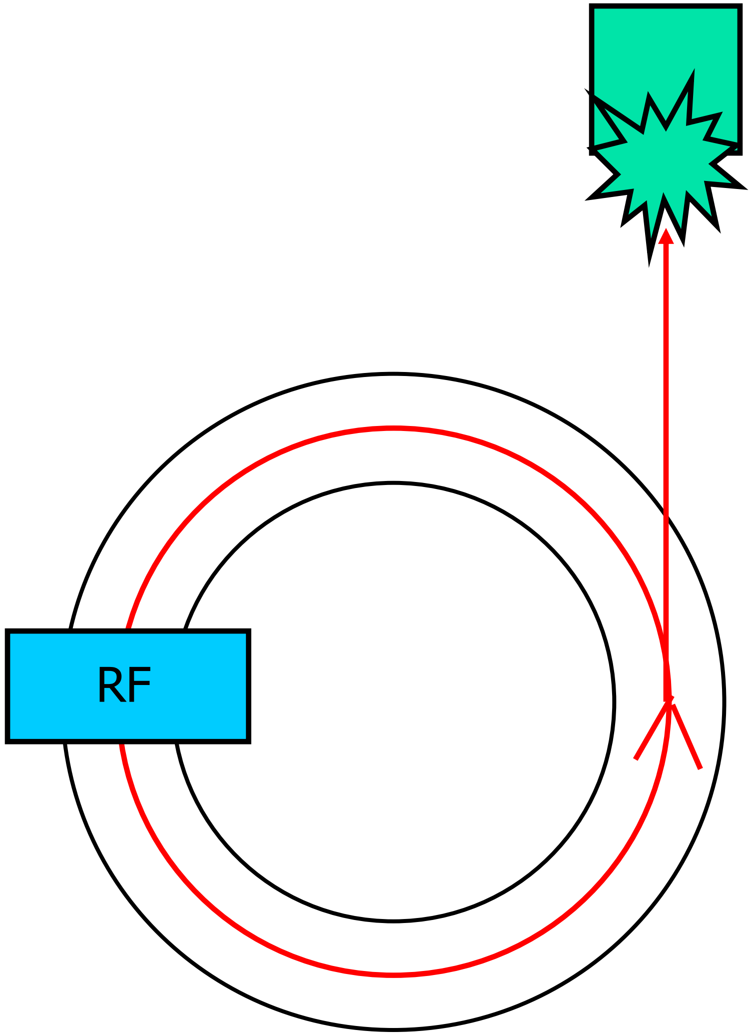
fixed target:

synchrotron

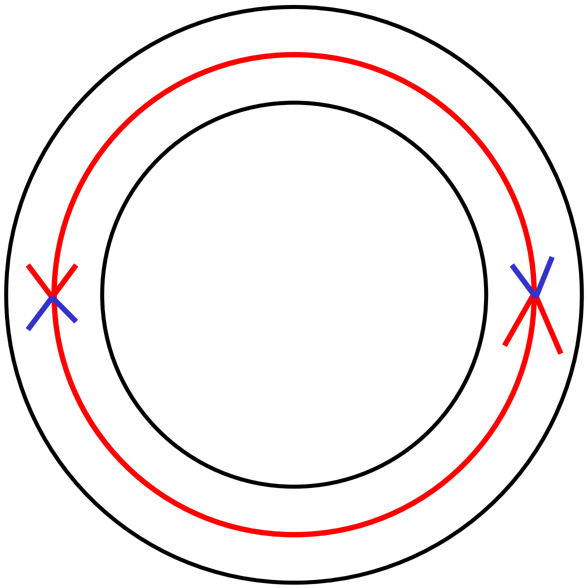
varying B

small magnets,

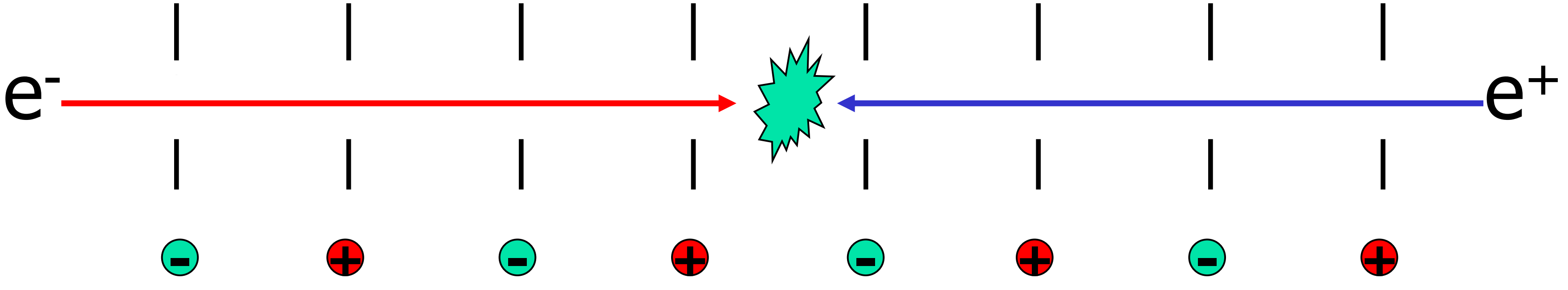
high energy



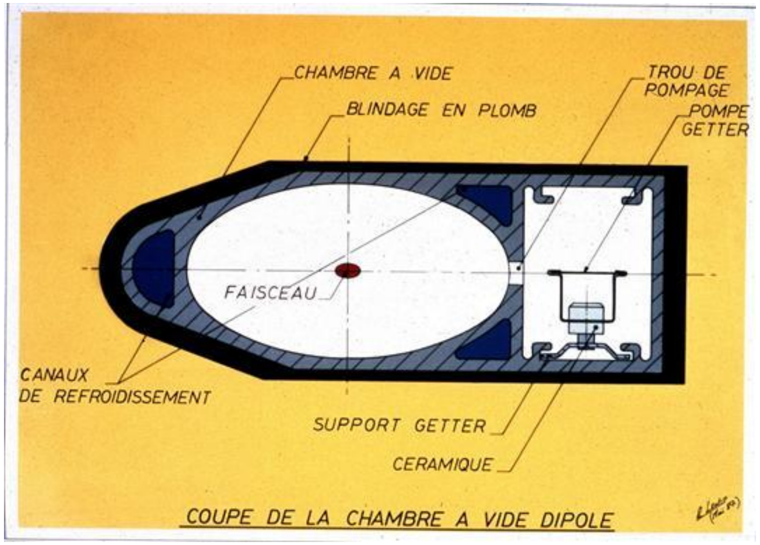
Colliders ($E_{c.m.}=2E$)



Colliders with the same type of particles (e.g. p-p) require two separate chambers. The beam are brought into a common chamber around the interaction regions

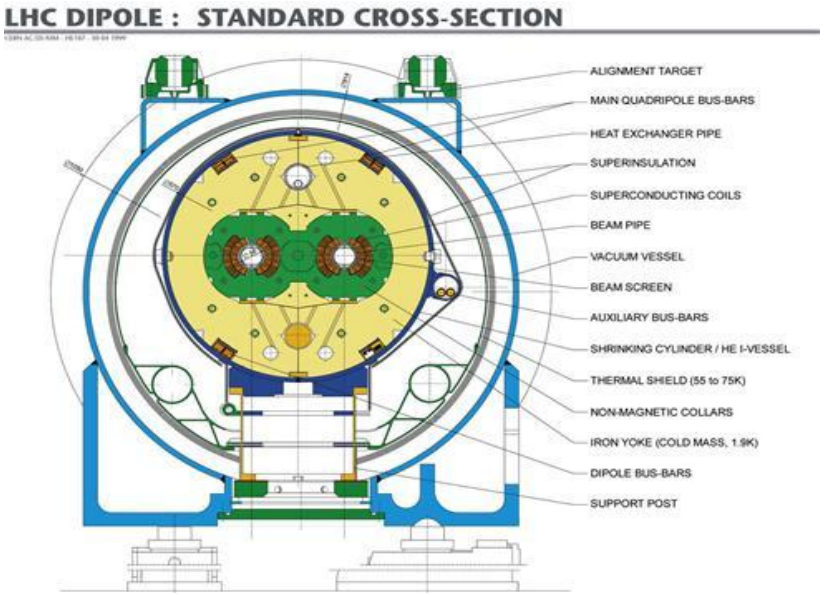


Colliders (e^+ - e^-) et (p – p)



LEP

LHC



Transverse Dynamics

$$F = e\,(E + v \times B)$$

!

Beam Dynamics (1)

<u>In order to describe the motion of the particles, each particle is characterised by:</u>

- Its azimuthal position along the machine: s
- Its momentum: p (or Energy E)
- Its horizontal position: x
- Its horizontal slope: x'
- Its vertical position: y
- Its vertical slope: y'

i.e. a sixth dimensional vector

(s, p, x, x', y, y')

Beam Dynamics (2)

- In an accelerator designed to operate at the energy E_{nom}, all particles having $(s, E_{nom}, 0, 0, 0, 0)$ will happily fly through the center of the vacuum chamber without any problem. These are "ideal particles".

- The difficulties start when:

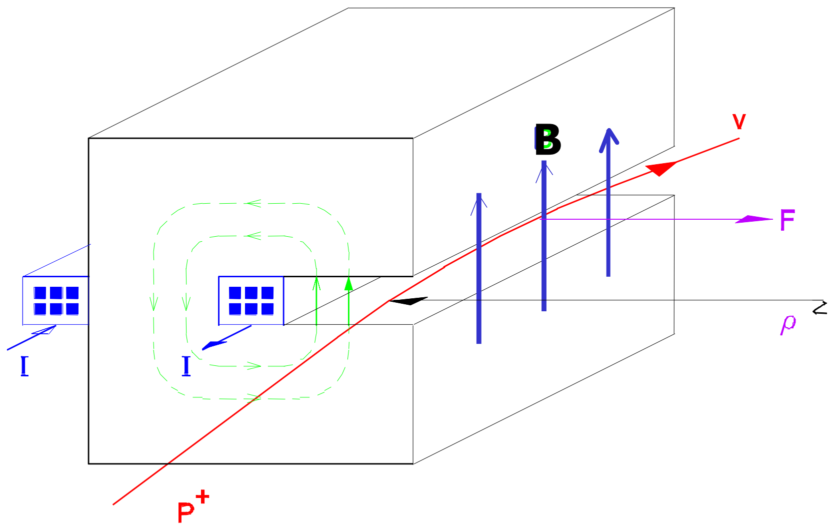
  - one introduces dipole magnets
  - the energy $E \neq E_{nom}$ or $(p\text{-}p_{nom}/p_{nom}) = \Delta p/p_{nom} \neq 0$
  - either of $x, x', y, y' \neq 0$

Basic problem:

With more than 10^{10} particles per bunch, most of them will not be ideal particles, i.e. they are going to be lost !

Purpose of this lecture: how can we keep the particles in the machine ?

Circular machines: Dipoles



Classical mechanics:

Equilibrium between two forces

Lorentz force	Centrifugal force
$F = e.(\underline{v} \times \underline{B})$	$F = mv^2/\rho$

$$evB = mv^2/\rho$$

$$p = m_0.c.(\beta\gamma)$$

Magnetic rigidity:

$$B\rho = mv/e = p/e$$

Relation also holds for relativistic case provided the classical momentum mv is replaced by the relativistic momentum p

Why fundamental ?

Constraints:

E and ρ given ➡ Magnets defined (**B**)

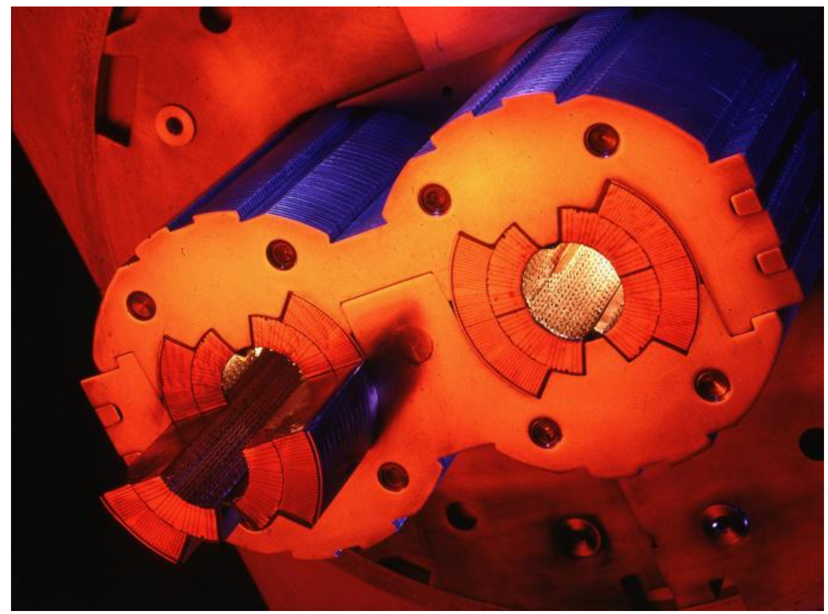
Constraints:

E and **B** given ➡ Size of the machine (ρ)

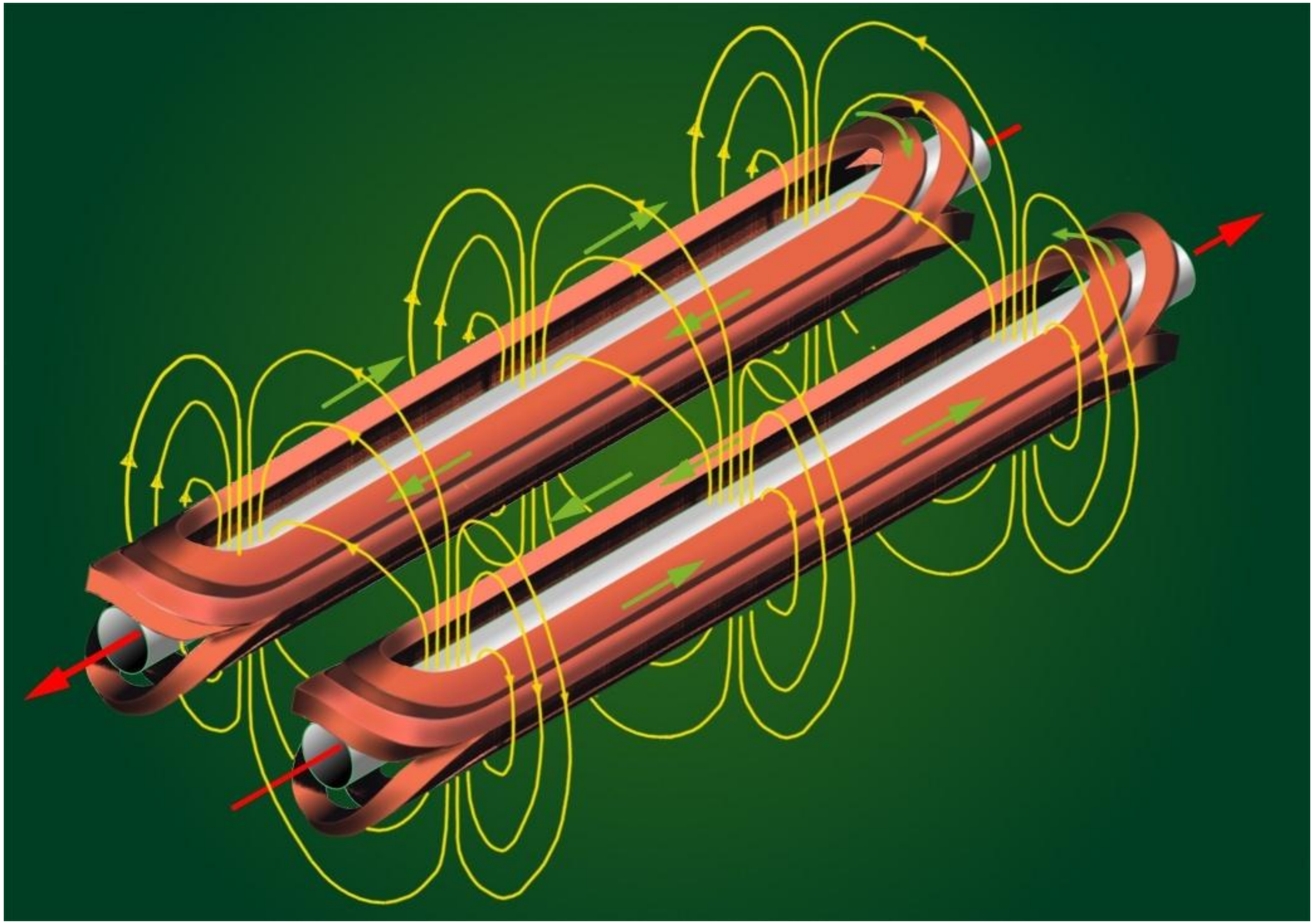
Constraints:

B and ρ given ➡ Energy defined (**E**)

Dipoles (1):



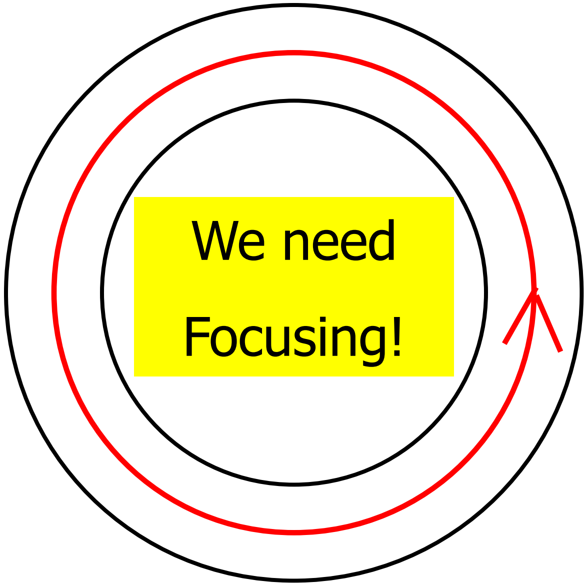
Dipoles (2):



Ideal circular machine:

- Neglecting radiation losses in the dipoles
- Neglecting gravitation

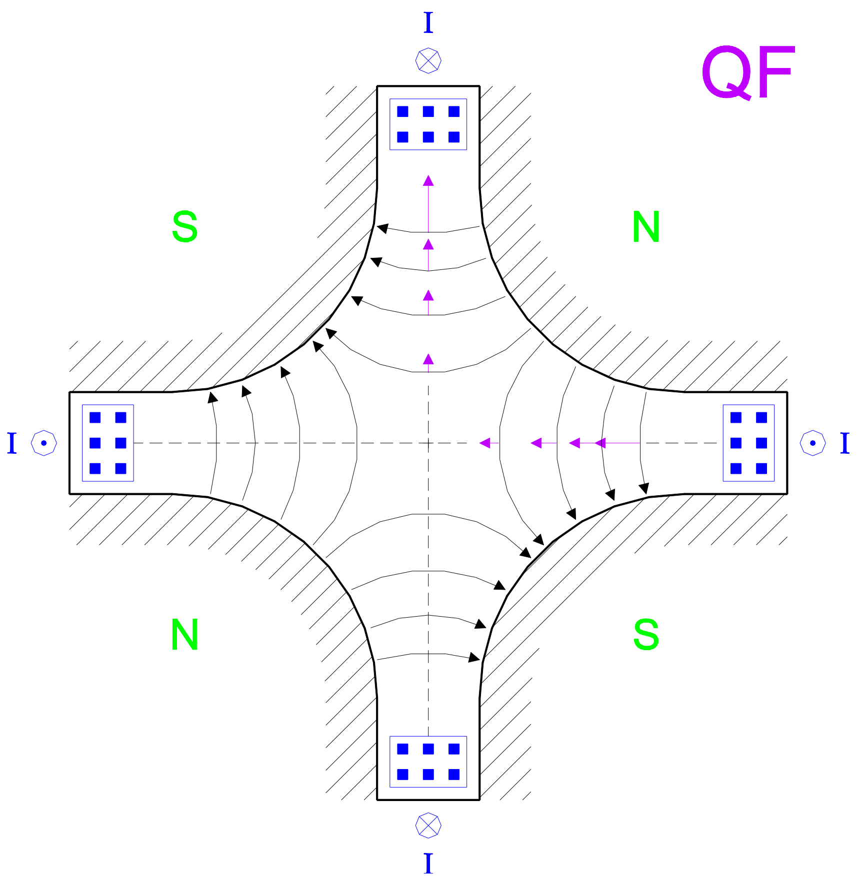
ideal particle would happily circulate on axis in the machine for ever!



Unfortunately: real life is different!

Gravitation: Δy = 20 mm in 64 msec!	
Alignment of the machine	Limited physical aperture
Ground motion	Field imperfections
Energy error of particles and/or $(x, x')_{inj} \neq (x, x')_{nominal}$	
Error in magnet strength (power supplies and calibration)	

Focusing with quadrupoles



$$F_x = -g.x$$

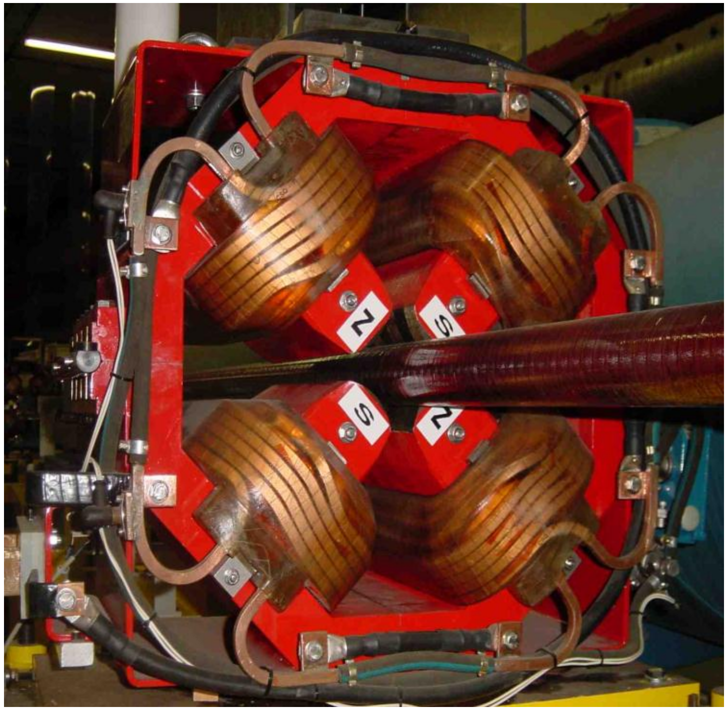
$$F_y = g.y$$

Force increases linearly with displacement.

Unfortunately, effect is opposite in the two planes (H and V).

Remember: this quadrupole is focusing in the horizontal plane but defocusing in the vertical plane!

Quadrupoles:



Focusing properties …

A quadrupole provides the required effect in one plane…

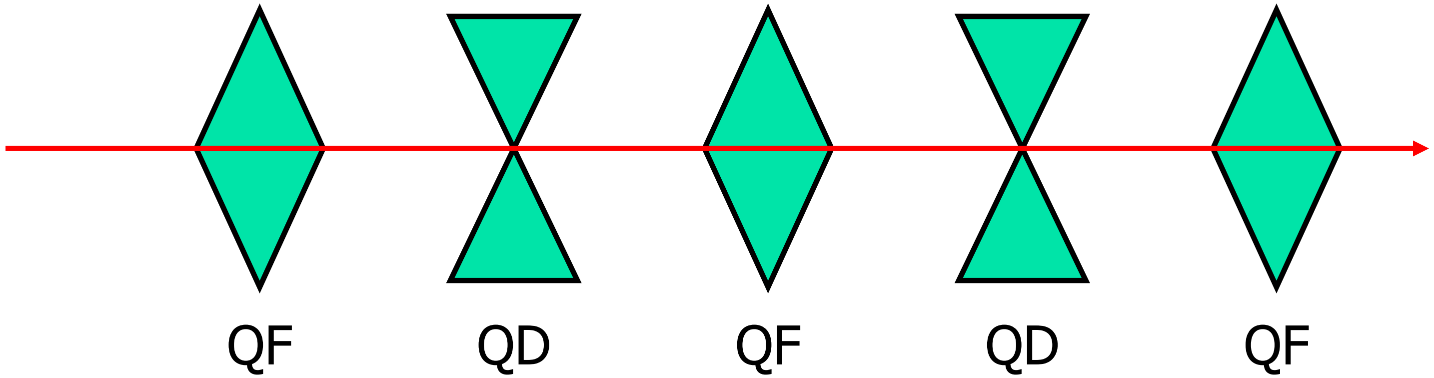
but the opposite effect in the other plane!

Is it really interesting ?

Alternating gradient focusing

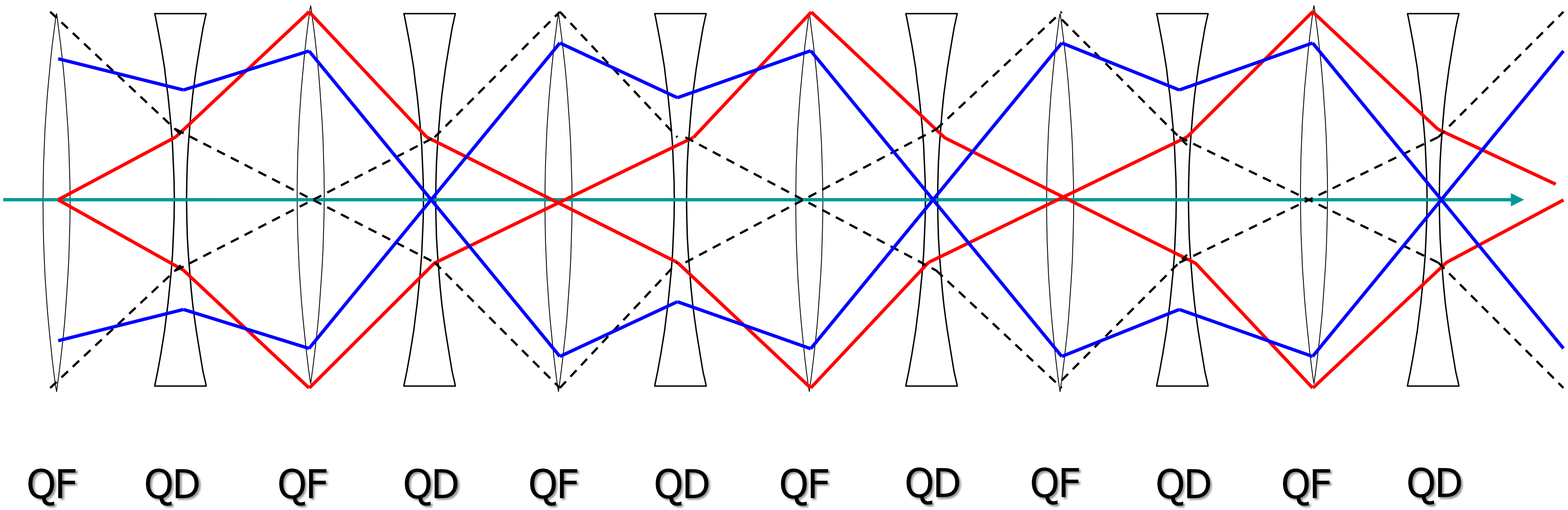
Basic new idea:

Alternate QF and QD



valid for one plane only (H or V) !

Alternating gradient focusing



Alternating gradient focusing:

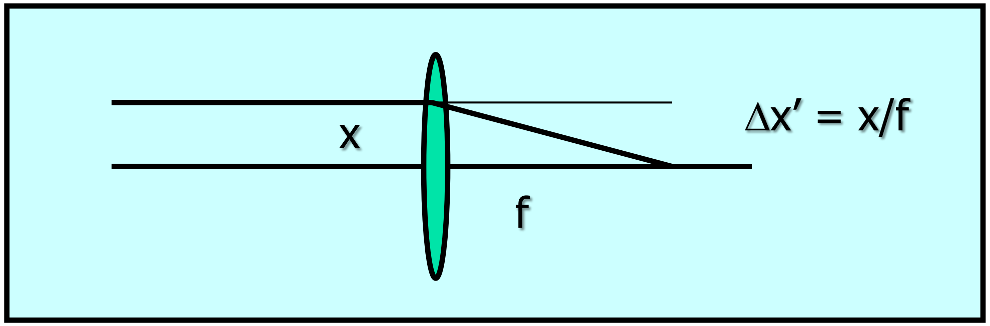
Particles for which $x, x', y, y' \neq 0$ thus oscillate around the ideal particle …

but the trajectories remain inside the vacuum chamber !

Thin lens analogy of AG focusing

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

$$X_{out} = x_{in} + 0.x'_{in}$$
$$x'_{out} = (-1/f).x_{in} + x'_{in}$$

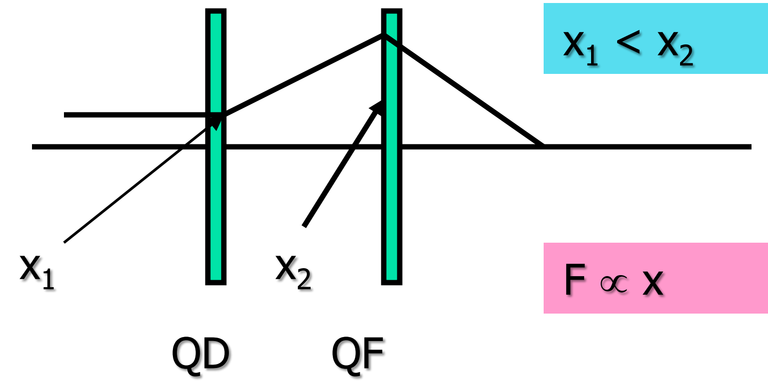


$$\text{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$\text{QF-Drift-QD} = \begin{pmatrix} 1-L/f & L \\ -L/f^2 & 1+L/f \end{pmatrix}$$

Initial: $x = x_0$ and $L < f$
$x' = 0$

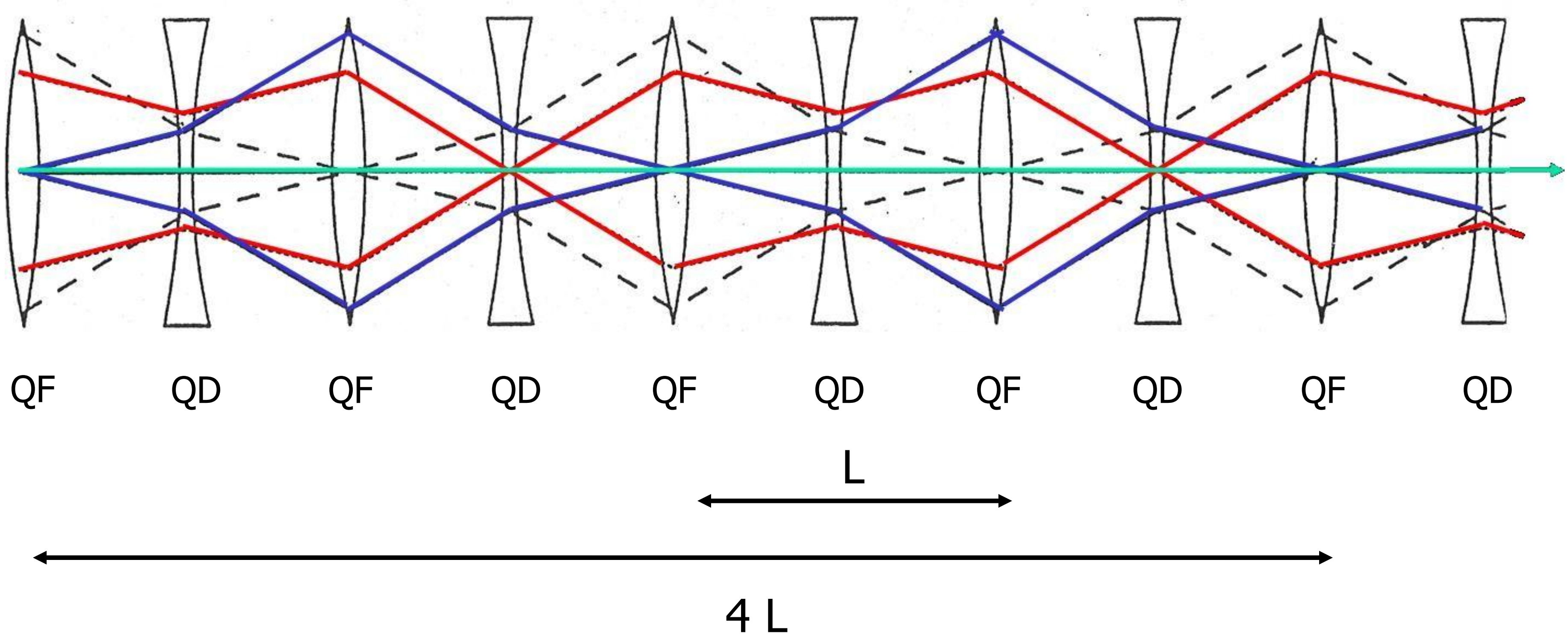
More intuitively:



$x_1 < x_2$

$F \propto x$

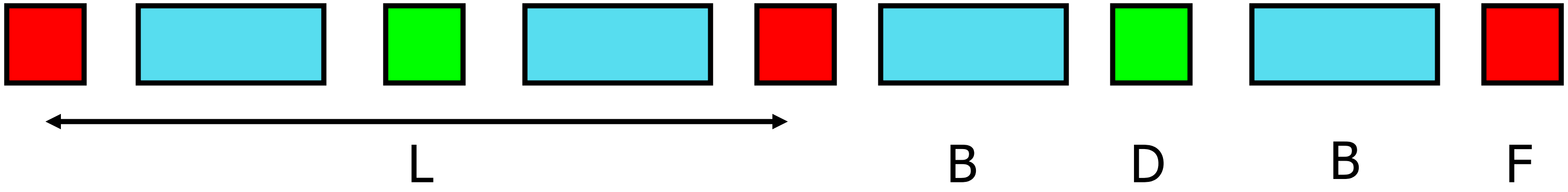
The concept of the « FODO cell »



One complete oscillation in 4 cells ⇒ 90°/ cell ⇒ **μ = 90°**

Circular machines (no errors!)

The accelerator is composed of a periodic repetition of cells:



➢ The phase advance per cell μ can be modified, in each plane, by varying the strength of the quadrupoles.

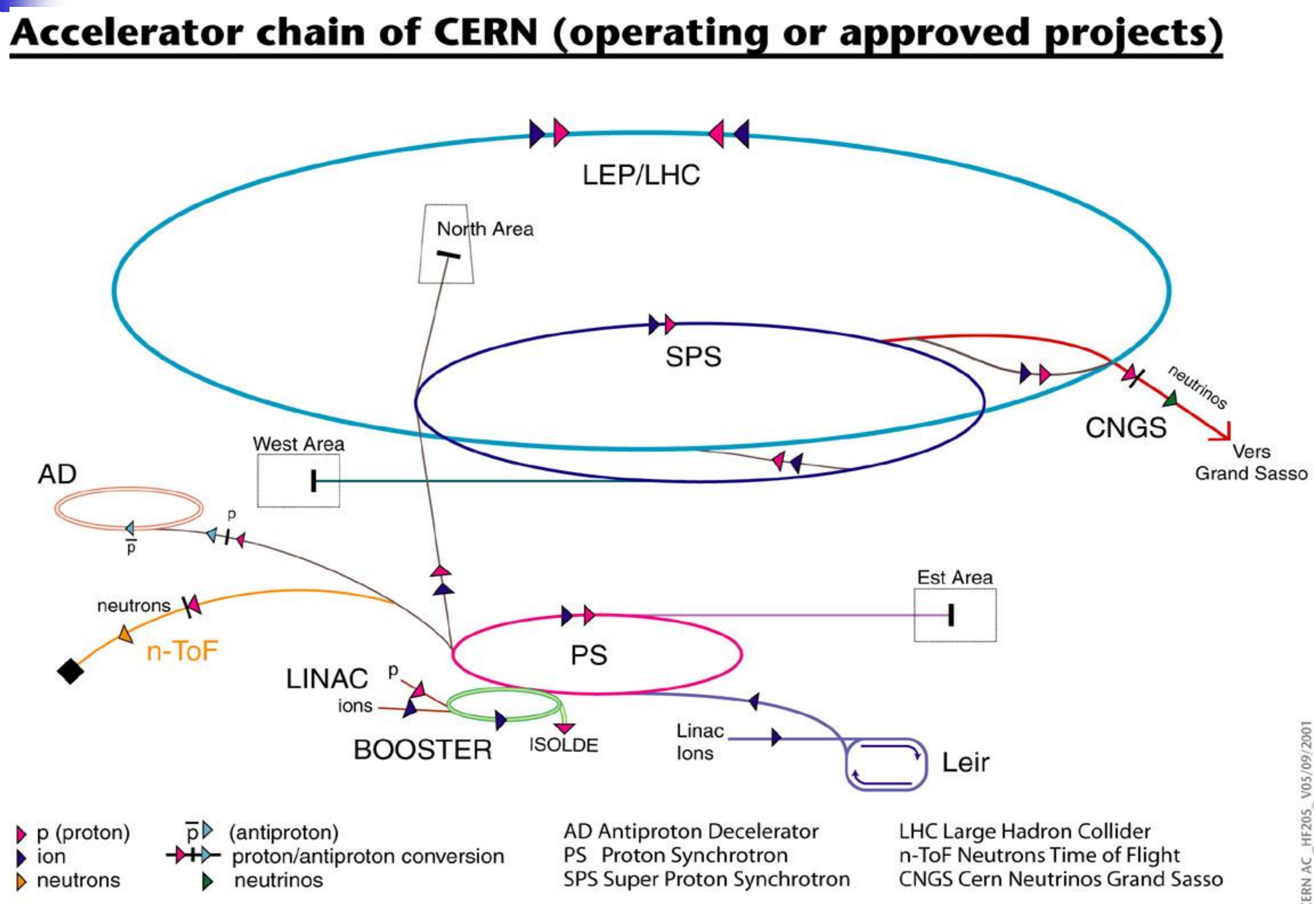
➢ The ideal particle will follow a particular trajectory, which closes on itself after one revolution: the closed orbit.

➢ The real particles will perform oscillations around the closed orbit.

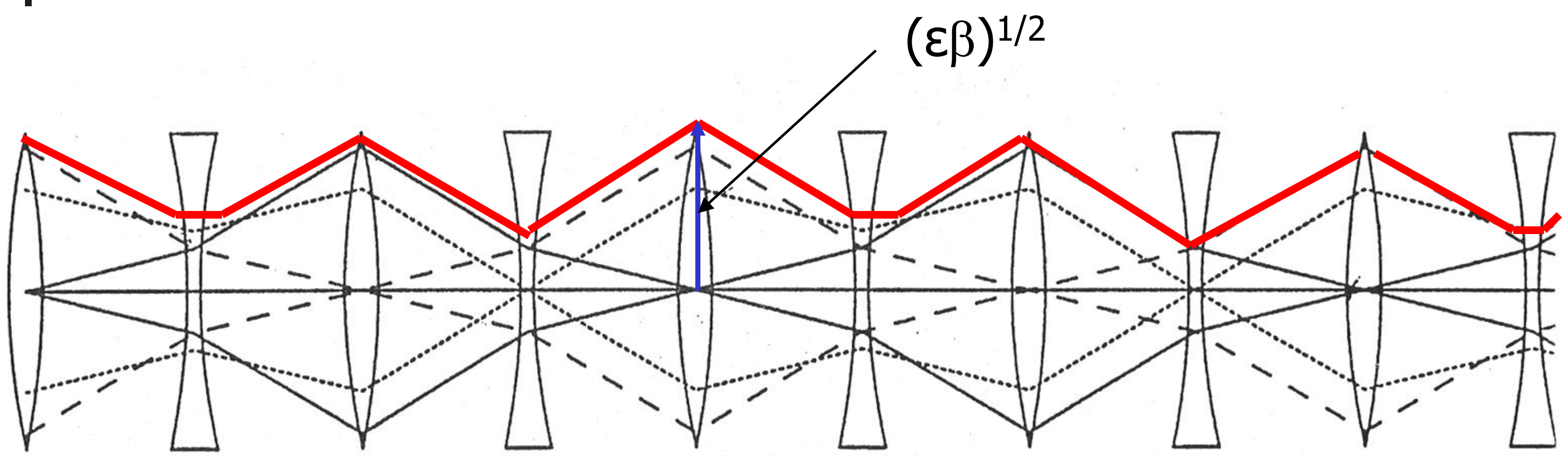
➢ The number of oscillations for a **complete revolution** is called the **Tune Q** of the machine (Qx and Qy).

Regular periodic lattice: The Arc

Synchrotrons …



The beta function $\beta(s)$



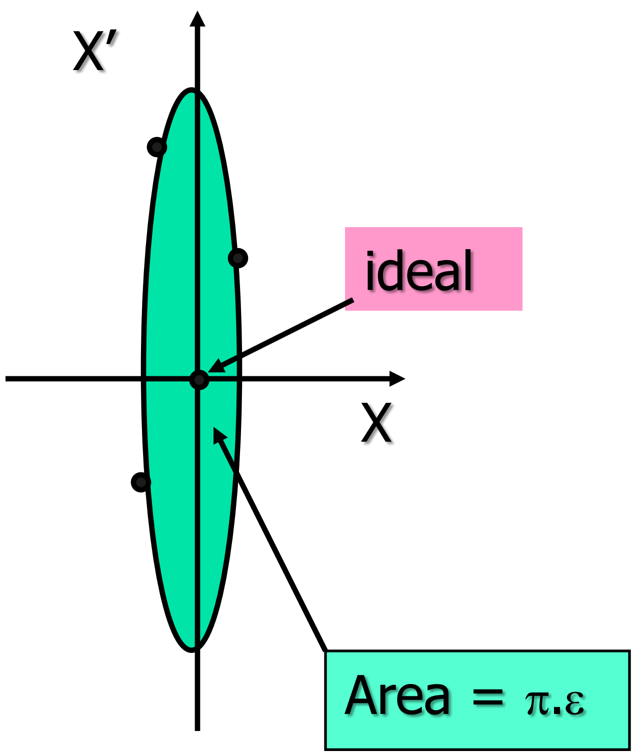
The β-function is the envelope around all the trajectories of the particles circulating in the machine.

The β-function has a minimum at the QD and a maximum at the QF, ensuring the net focusing effect of the lattice.

It is a periodic function (repetition of cells). The oscillations of the particles are called betatron motion or **betatron oscillations**.

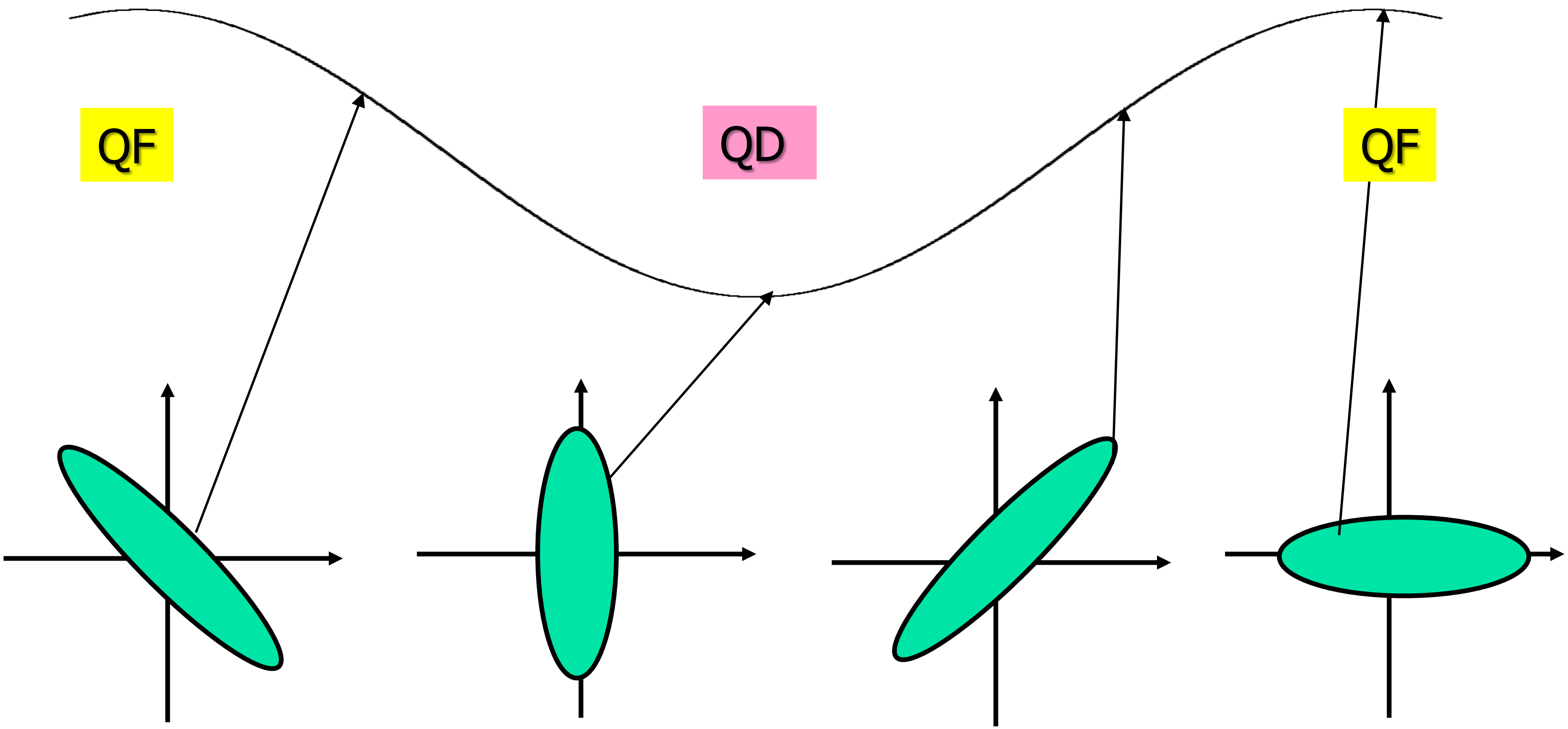
Phase space

- Select a particle in the beam being at 1 sigma (68%) of the distribution and plot its position vs. its phase (x vs. x') at some location in the machine for many turns.



- ε Is the emittance of the beam [mm mrad]
- ε describes the quality of the beam
- Measure of how much particle depart from ideal trajectory.
- β is a property of the machine (quadrupoles).

Emittance conservation



The shape of the ellipse varies along the machine, but its area (the emittance ε) remains constant at a given energy.

Why introducing these functions?

The β function and the emittance are fundamental parameters, because they are directly related to the beam size (measurable quantity !):

Beam size [m]

$$\sigma_{x,y}(s) = (\varepsilon.\beta_{x,y}(s))^{1/2}$$

σ (IP) = 17 μm

at 7 TeV (β=0.55 m)

The emittance ε characterises the quality of the injected beam (kind of measure how the particules depart from ideal ones). It is an invariant at a given energy.

ε = beam property

β = machine property (quads)

Recapitulation 1

- The **fraction** of the oscillation performed in a periodic cell is called the **phase advance μ per cell** (x or y).

- The total number of oscillations over one full turn of the machine is called the **betatron tune Q** (x or y).

- The envelope of the betatron oscillations is characterised by the **beta function $\beta(s)$.** This is a property of the quadrupole settings.

- The quality of the (injected) beam is characterised by the **emittance ε**. This is a property of the beam and is invariant around the machine for a given energy.

- The r.m.s. beam size (measurable quantity) is $\sigma = (\beta.\varepsilon)^{1/2}$.

Off momentum particles:

- These are “non-ideal” particles, in the sense that they do not have the right energy, i.e. all particles with $\Delta p/p \neq 0$

What happens to these particles when traversing the magnets ?

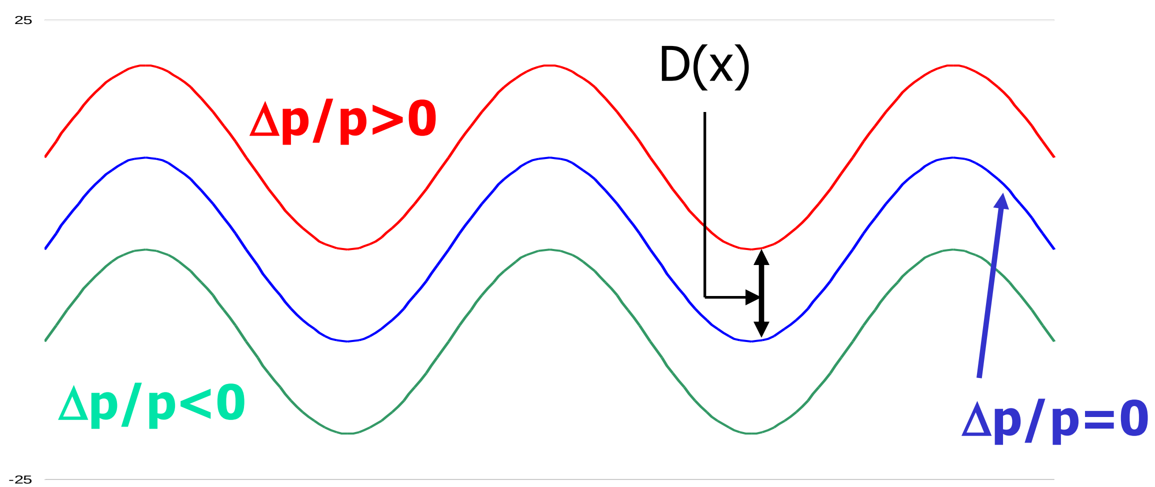
Off momentum particles ($\Delta p/p \neq 0$)

Effect from Dipoles

- If $\Delta p/p > 0$, particles are less bent in the dipoles ➔ should spiral out !
- If $\Delta p/p < 0$, particles are more bent in the dipoles ➔ should spiral in !

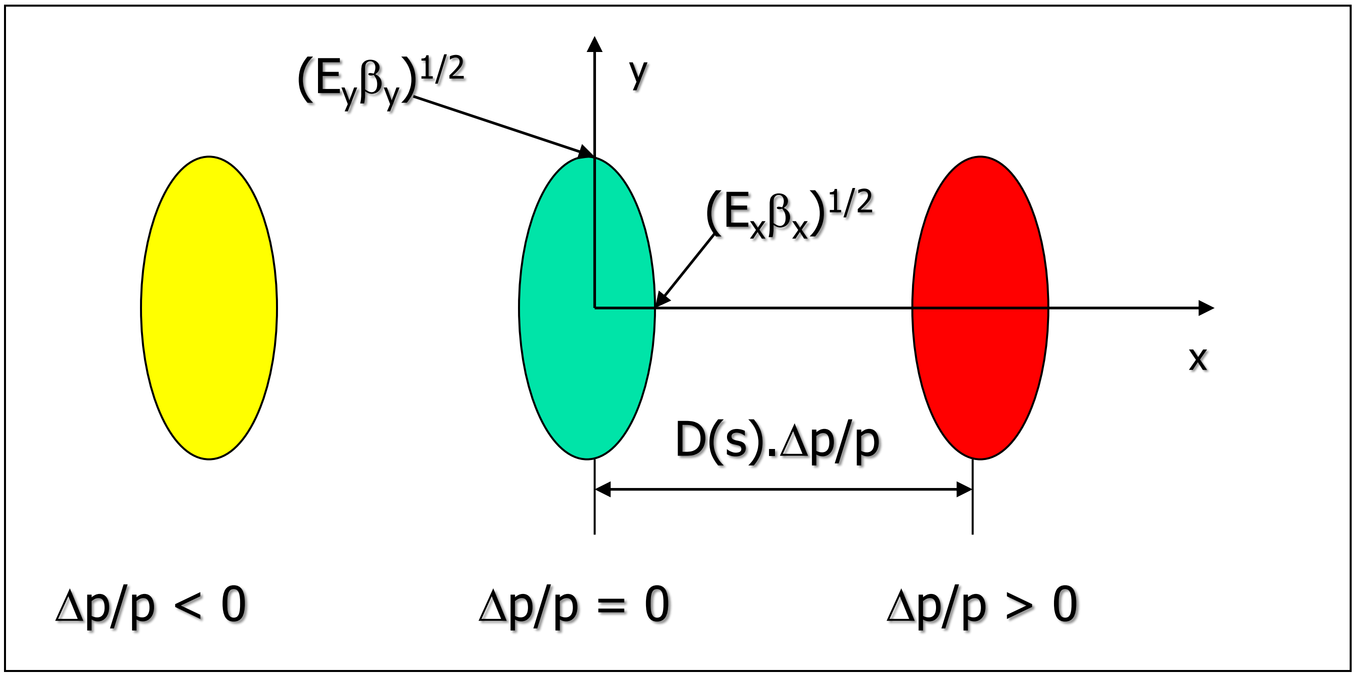
No!

There is an equilibrium with the restoring force of the quadrupoles



Dispersion

In general:



Only extreme values of Δp/p are shown.

The vacuum chamber must accomodate the full width.

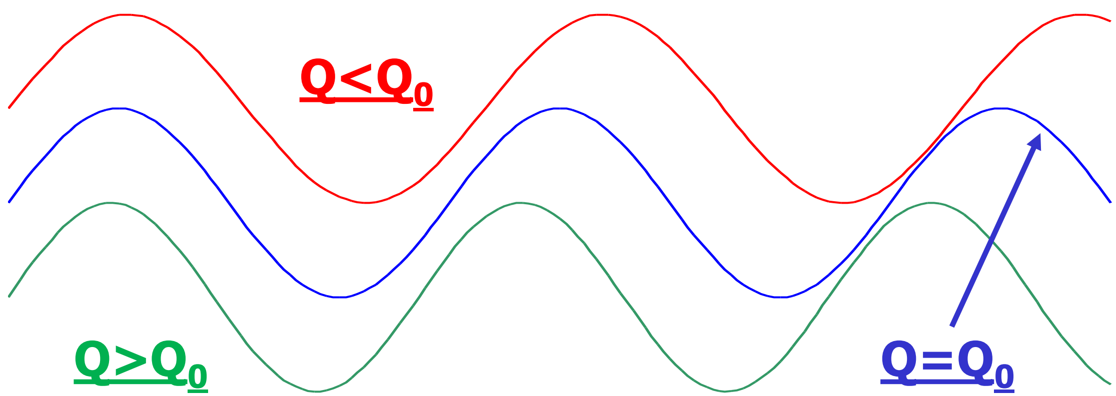
VH: $A_y(s) = (E_y\beta_y(s))^{1/2}$ and HW: $A_x(s) = (E_x\beta_x(s))^{1/2} + D(s).\Delta p/p$

Off momentum particles ($\Delta p/p \neq 0$)

Effect from Quadrupoles

- If $\Delta p/p > 0$, particles are less focused in the quadrupoles ➔ **lower Q !**
- If $\Delta p/p < 0$, particles are more focused in the quadrupoles ➔ **higher Q !**

Particles with different momenta would have a different betatron tune $Q=f(\Delta p/p)$!



The chromaticity Q'

Particles with different momenta ($\Delta p/p$) would thus have different tunes Q.
So what ?

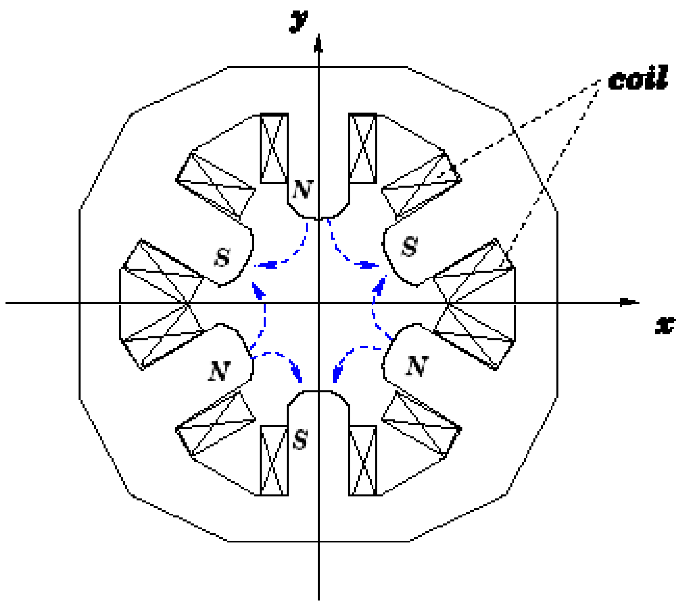
unfortunately

- The tune dependence on momentum is of fundamental importance for the stability of the machine. It is described by the chromaticity of the machine Q':

$$Q' = \Delta Q / (\Delta p/p)$$

The chromaticity has to be carefully controlled and corrected for stability reasons.

The sextupoles (SF and SD)

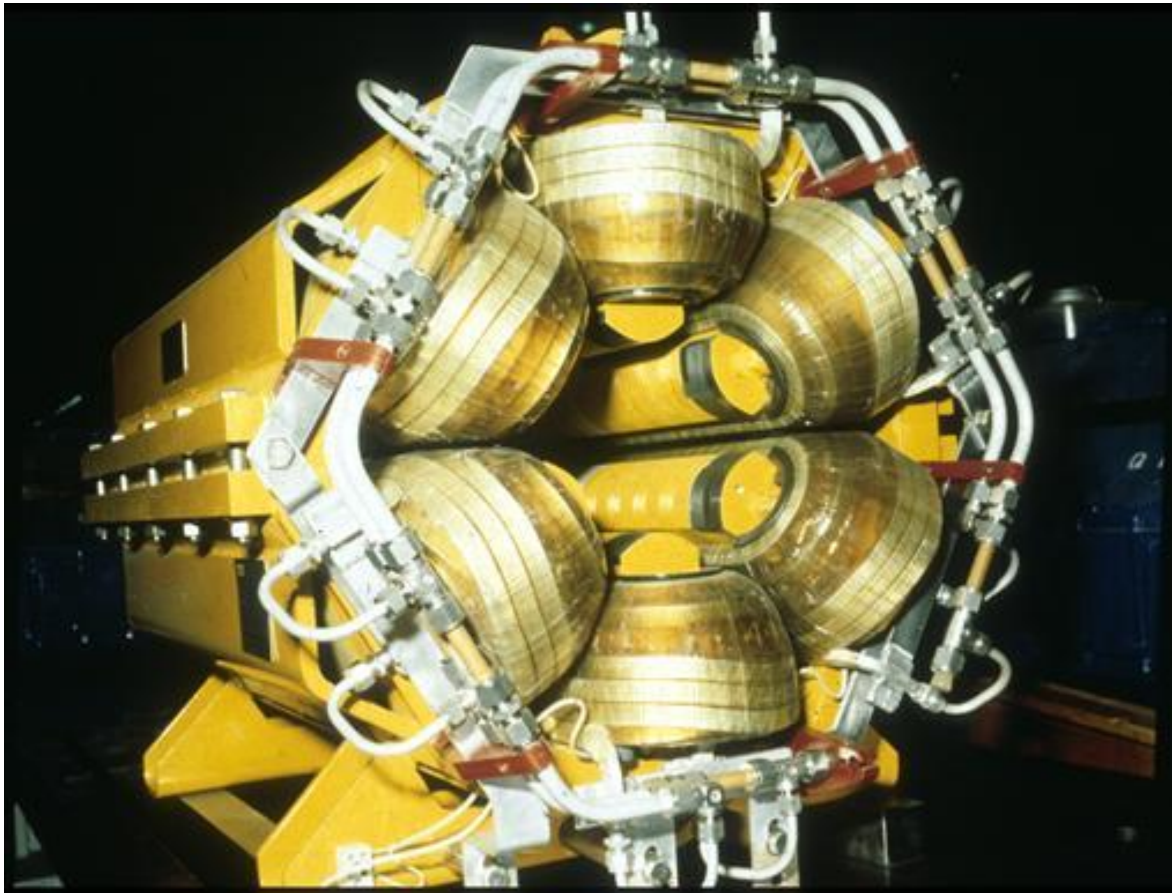


- A SF sextupole basically « adds » focusing for the particles with $\Delta p/p > 0$, and « reduces » it for $\Delta p/p < 0$.
- The chromaticity is corrected by adding a sextupole after each quadrupole of the FODO lattice.

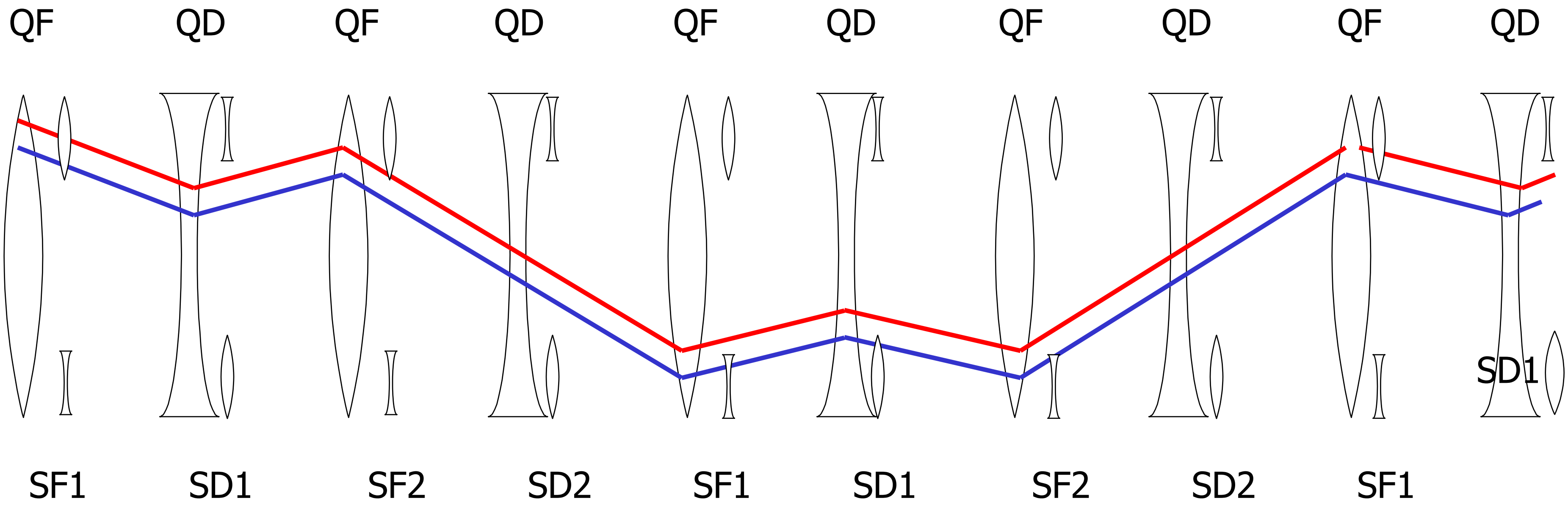
➢ $\Delta x' \propto x^2$

Sextupoles:

SPS



Effect of sextupoles



Recapitulation 2

- For off momentum particles ($\Delta p/p \neq 0$), the magnets induce other important effects, namely:

➢ The dispersion (dipoles)

➢ The chromaticity (quadrupoles)

The concept of luminosity

$$L = \frac{n_b I_b^{\ 2}}{4\pi e^2 f_0 \sigma_x \sigma_y}$$

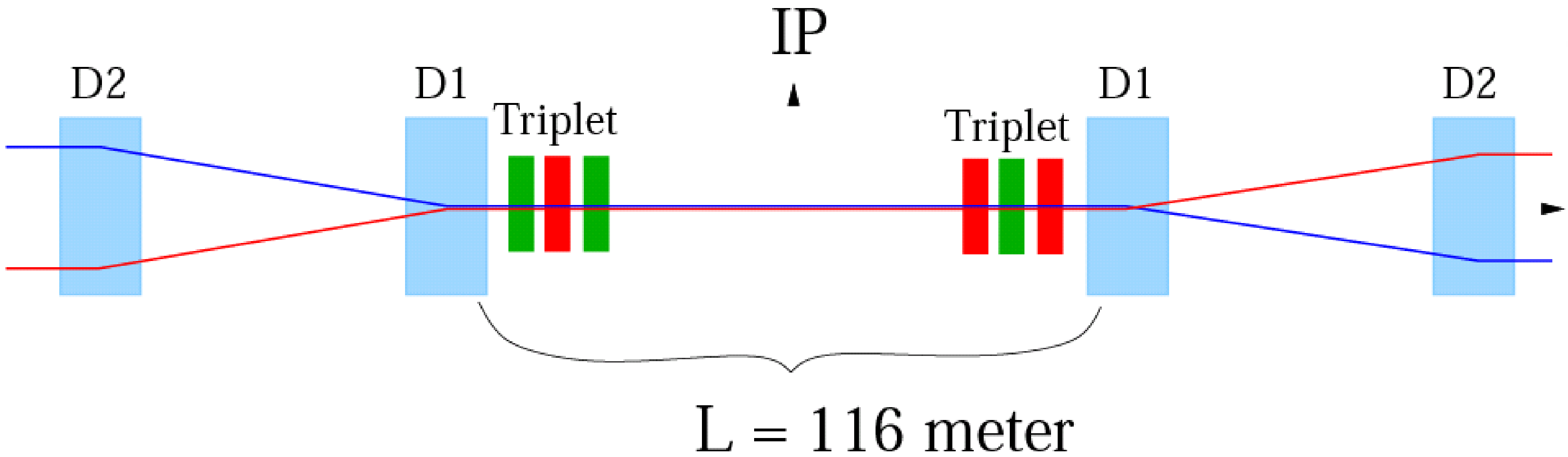
L is proportional to the square of the beam intensity

L is inversely proportional to the overlap size of the two beams

The event rate of physics is the product of the cross section (CS) and Luminosity (L).

Ex: L= 10^34 cm-2 s-1, CS = 1 nb (1 barn = 10^-24 cm2)
→ event rate = 10 events per second

Insertion



$$L = \frac{n_b I_b^{\,2}}{4\pi e^2 f_0 \sigma_x \sigma_y}$$

Clearly we have to collide the beams
And then get the beam sizes at the interaction point as small as possible

- β^* = 0.5 m
- $\varepsilon = 5 \times 10^{-10}$ m $\rightarrow \sigma^* = 16\ \mu$m

$$\varepsilon = \frac{\sigma^2}{\beta}$$