Higgs + jet in the Regge limit

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based on 2504.06184 with **Vittorio Del Duca**

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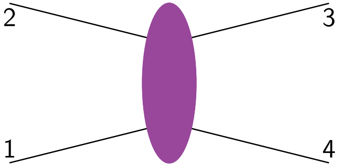
Outline

Outline

The original problem



- Strong ordering in Mandelstam invariants $s_{ij} = (p_i + p_j)^2$ $s_{12} \gg s_{14}$.
  - Signature symmetry $s_{12} \simeq -s_{13}$.
- Large rapidity separation $y_4 \gg y_3$.

$$
\begin{aligned}
p_k^{\perp} &= p_k^x + i\, p_k^y, \\
p_k^{+} &= (p_k^0 + p_k^z) = |p_k^{\perp}|\, e^{y_k}, \\
p_k^{-} &= (p_k^0 - p_k^z) = |p_k^{\perp}|\, e^{-y_k}
\end{aligned}
$$

Extend to **multi-Regge** limit $y_i \gg y_j$, i, j in *final state*.

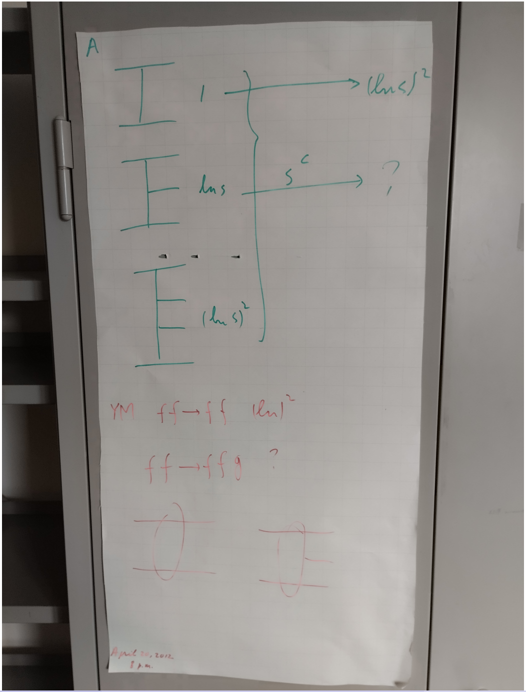
Insight into the all-order structure

- Loop counting $a_s = \frac{g^2}{16\pi^2}$
- High-energy logarithm, here

$$L = \log \frac{s_{12}}{s_{14}} + \log \frac{s_{13}}{s_{14}} \equiv \log \frac{s}{t} + \log \frac{u}{t} = \log \frac{s}{t} - i\frac{\pi}{2}$$

- Double expansion of the amplitude $\hat{\mathcal{M}} = \frac{\mathcal{M}}{\mathcal{M}_0}$

$$\hat{\mathcal{M}} = 1 + \underbrace{a_s L\, \mathcal{M}^{(1,1)} + a_s^2 L^2\, \mathcal{M}^{(2,2)}}_{\text{LL}} + \underbrace{a_s\, \mathcal{M}^{(1,0)} + a_s^2 L\, \mathcal{M}^{(2,1)}}_{\text{NLL}} + \underbrace{a_s^2\, \mathcal{M}^{(2,0)}}_{\text{NNLL}}$$


A
1
(ln s)^2
ln s
s^c
?
(ln s)^2
YM ff→ff (ln)^2
ff→ffg ?
April 20, 2012
8 p.m.

Climbing logarithmic towers

- Diagrammatic analysis in **QCD** $\to$ BFKL amplitudes
  - Leading Logarithm (LL) [Lipatov 1976;Fadin,Kuraev,Lipatov 1975-1977;Balitsky,Lipatov 1978]
  - NLL [Fadin,Lipatov;Ciafaloni,Camici 1998;Fadin,Kozlov,Reznichenko 2015]
  - Towards NNLL [Fadin,Lipatov 2017;Fadin 2016-2025]
- Growing set of tools
  - Wilson lines [Korchemsky 1993;Balitsky 2000;Caron-Huot 2013]
  - Effective Field Theories [Lipatov 1991;Balitsky 2001;Rothstein,Stewart 2016-2025]
- Integrability [Lipatov 1993;Faddeev,Korchemsky 1995] in planar $\mathcal{N} = 4$ sYM [Lipatov 2009]
  - BFLK eigenvalue through N^3LL [Alfimov,Gromov,Sizov 2018;Velizhanin 2021]
  - All-loop, all multiplicity formula [Del Duca *et al.* 2020].

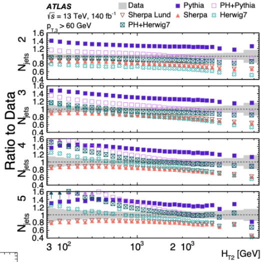
Energy logs at the LHC

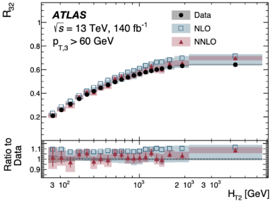
arXiv:2405.20206

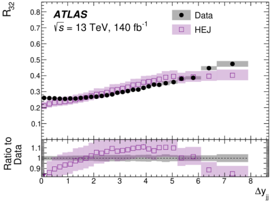
JETS PHYSICS: R3/2 RATIO

$$\frac{d\sigma_{3j}/dx}{d\sigma_{2j}/dx}$$ where x = H_{T2}, $m_{jj,max}$, $|\Delta y_{jj,max}|$, m_{jj}, $|\Delta y_{jj}|$

- **Differential cross sections in different jet multiplicity bins**
  - ➡ **Several p_{T3} thresholds for H_{T2} measurements: explore sensitivity to resummation effects**
  - ➡ **Ratios for better sensitivity to α_s (smaller uncertainties)**
  - ➡ **NNLO needed for a good description of the data.**
  - ➡ **HEJ description is better in regions with large contributions of $\log(p_{Tjet}/\sqrt{s})$**







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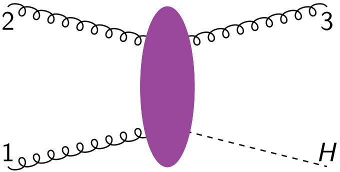
[Ana Rosario Cueto Gòmez, HP2 2024]

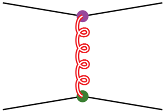
Higgs + jet(s)



- BFKL-like form of the **one loop** amplitudes (NLL) with $m_{\text{top}} \to \infty$ [Xiao,Yuan 2018;Hentschinski,Kutak,van Hameren 2020; Celiberto,Fucilla,Ivanov,Mohammed,Papa 2022].
- Non-trivial pattern of cancellations [Fucilla,Nefedov,Papa 2024]
- Applications to Higgs + jet phenomenology [Celiberto,Ivanov,Mohammed,Papa 2020; Andersen,Hassan,Maier,Paltrinieri,Papaefstathiou,Smillie -HEJ- 2022]

Gluon Reggeization [Lipatov;Kuraev,Fadin,Lipatov 1976]

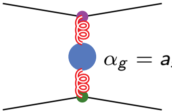
$$\mathcal{M}^{(0)}_{ij\to ij} = g^2 \left(C^{(0)}_i(t)\mathbf{T}^a_i \right) \frac{s^J}{-t} \left(C^{(0)}_j(t)\mathbf{T}^a_j \right) =$$



Tree-level amplitudes $\to$ t-channel exchange of **maximal spin** J.

- $\mathbf{T}^a_i$ group generator in rep. of particle i.
- $C^{(0)}_i$ tree-level impact factor of i.

$$\mathcal{M}^{LL}_{ij\to ij} = \mathcal{M}^{(0)}_{ij\to ij} \left(\frac{s}{-t} \right)^{\alpha_g(t)} =$$



$$\alpha_g = a_s \alpha_g^{(1)} + \ldots$$

$$= \mathcal{M}^{(0)}_{ij\to ij} \left[1 + a_s \alpha_g^{(1)} L + \frac{1}{2} \left(a_s \alpha_g^{(1)} L \right)^2 + \ldots \right.$$

Regge poles

[Caron-Huot,Gardi,Vernazza 2017]

Mellin representation of the amplitude

$$\mathcal{M} = \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{dJ}{\sin(\pi J)} \sin\left(\frac{\pi J}{2}\right) \mathcal{A}_J(t)\, e^{JL}$$

$\mathcal{A}_J(t) \to$ J-th partial wave. Simplest model $\mathcal{A}_J(t) = \frac{1}{J-1-\alpha(t)}$

$$\mathcal{M}_{\text{pole}} \simeq \gamma(t) \left(\frac{s}{-t}\right)^{1+\alpha(t)}$$

Pure QCD amplitudes described by a **Regge pole** exchange to LL.

Regge cuts

[Caron-Huot,Gardi,Vernazza 2017]
The partial wave can have **cuts** in **complex** J-plane e.g.

$$\mathcal{A}_J(t) = \left(\frac{1}{J - 1 - \alpha(t)}\right)^{1+\beta(t)}$$

The Regge cut gives a different expansion in high energy logarithms

$$\mathcal{M}_{\text{cut}} \simeq \tilde{\gamma}(t)\, L^{\beta(t)}\, e^{\alpha(t)\, L}$$

The perturbative expansion doesn't tell the difference.

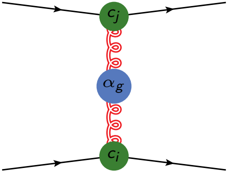
Do Regge cuts arise in QCD amplitudes?

Parton scattering to NLL

The real part of the $2 \to 2$ amplitudes follows the Regge pole ansatz [Fadin,Kozlov,Reznichenko 2015]

$$\mathrm{Re}\left[\mathcal{M}^{\mathrm{NLL}}_{ij\to ij}\right] = c_i(t)\, c_j(t)\, e^{C_A\, \alpha_g(t) L}\, \mathcal{M}^{(0)}_{ij\to ij}$$

Regge trajectory [Fadin,Fiore,Kotsky 1995; Blümlein,Ravindran,van Neerven 1998] and **impact factors** [Fadin,Fiore 1992] expanded to **NLO**

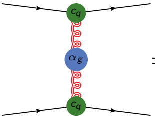


$$\alpha_g(t) = \sum_{n\geq 1} a_s^n\, \alpha_g^{(n)}(t),$$

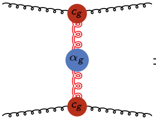
$$c_i(t) = 1 + \sum_{n\geq 1} a_s^n\, c_i^{(n)}(t).$$

Regge cuts in $\mathrm{Im}[\mathcal{M}]$ [Caron-Huot 2013;+Gardi,Reichel,Vernazza 2017-2020]

Regge pole universality



$$= 2\,a_s\,c_q^{(1)} + a_s^2 L\left(\alpha_g^{(2)} + 2\,\alpha_g^{(1)}\,c_q^{(1)}\right) + \ldots$$



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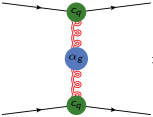

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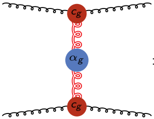
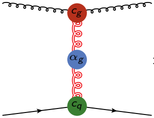


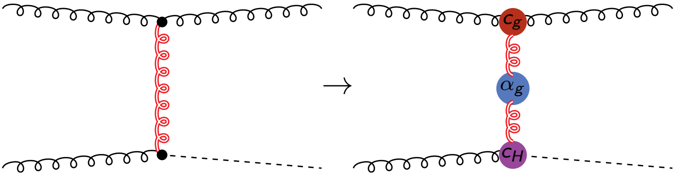
$$= 2\, a_s\, c_g^{(1)} + a_s^2 L \left(\alpha_g^{(2)} + 2\, \alpha_g^{(1)}\, c_g^{(1)} \right) + \ldots$$



$$= a_s \left(c_q^{(1)} + c_g^{(1)} \right) + a_s^2 L \left(\alpha_g^{(2)} + \alpha_g^{(1)} \left(c_q^{(1)} + c_g^{(1)} \right) \right) + \ldots$$

Higgs + jet to NLL

Effective theory in the infinite top mass limit $\mathcal{L}_{\text{eff.}} \sim \lambda H F^{\mu\nu;a} F^a_{\mu\nu}$



Tree-level amplitudes $\propto \mathcal{M}^{(0)}_{ig\to iH} = g\lambda\, C^{(0)}_i(t) \left(\frac{s}{t}\right)\; C^{(0)}_H(t)\, (\mathbf{T}^x_i)_{a_3 a_2} \delta^x_{a_1}$

$$\mathcal{M}^{LL+NLL}_{ig\to iH} = \frac{1}{2}\left[\left(\frac{s}{\tau}\right)^{\alpha_g(t)} + \left(\frac{-s}{\tau}\right)^{\alpha_g(t)}\right] \mathcal{M}^{(0)}_{ig\to iH}\, c_i(t,\tau)\, c_H(t, m_H^2, \tau),$$

- Universal behaviour in $gg \to gH$ and $qg \to qH$.
- Verified at one loop [Celiberto,Fucilla,Ivanov,Mohammed,Papa 2022].

Breaking Regge pole factorisation in pure QCD

- Colour structures that are absent at tree level
- Universality breaking [Del Duca,Glover 2001]

$$\mathrm{Re}\mathcal{M}^{\mathrm{NNLO}}_{qg\to qg} \neq \underbrace{c_q^{(2)} + c_g^{(2)} + c_q^{(1)} c_g^{(1)} - \frac{\pi^2}{N_c^2}(\alpha_g^{(1)})^2}_{\text{from pole ansatz}} + \mathcal{O}(a_s^3)$$

There is a mismatch arising from **Regge cuts** starting at NNLL

$$\delta_{\mathrm{fact}} = \mathrm{Re}\mathcal{M}^{\mathrm{NNLO}}_{qg\to qg} - \text{pole ansatz} = \frac{a_s^2 \pi^2}{\epsilon^2} \frac{3}{16} \left(\frac{N_c^2+1}{N_c^2} \right) (1 - \epsilon^2 \zeta_2).$$

- IR divergent $\sim \epsilon^{-2}$
- Subleading in N_c.

Questions

- What is the structure of the amplitudes for Higgs + jet at **NNLL** in the Regge limit?
  - Is it purely a Regge pole or do Regge cuts arise?

- How to proceed?
  - Find methods to compute the contribution of the Regge cuts.
  - Determine the missing parameter in the Regge pole ansatz, i.e. the **two-loop Higgs impact factor**.

Outline

Long distance origin

Regge cut in the QCD amplitudes start as a $\frac{1}{\epsilon^2}$ effect $\rightarrow$ IR origin!

- In the EFT language, Regge cuts from the interplay of **collinear** and **Glauber** modes [Gao,Moult,Raman,Ridgway,Stewart 2024].

Use Infrared Factorisation [Del Duca,Duhr,Gardi,Magnea,White 2011]

$$\mathcal{M} = \underbrace{\mathbf{Z}\left(\log\frac{s}{-t}, \frac{\mu^2}{-t}, \frac{1}{\epsilon}\right)}_{\text{Predicted by IR fact.}} \mathcal{H}\left(\frac{\mu^2}{s}, \frac{\mu^2}{-t}, \epsilon\right)$$

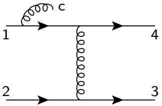
$\mathcal{H}$ is **finite** as $\epsilon \rightarrow 0$, $\mathbf{Z}$ is a universal operator

$$\mathbf{Z} = \mathcal{P}\mathrm{exp}\left[-\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\underbrace{\mathbf{\Gamma}(\lambda^2, a_s(\lambda^2))}_{\text{Soft anomalous dim.}}\right]$$

Operators in colour space

$(\mathbf{T_i})^b_{a_1 a_2}$ is the **colour generator** in the representation i.

$$(\mathbf{T_1})^c_{a_1 a_1'}\,(t^b_{a_4 a_1'}\,t^b_{a_3 a_2}) =$$



$$= t^b_{a_4 a_1'}\,t^c_{a_1' a_1}\,t^b_{a_3 a_2}$$

Colour channel operators

$$\mathbf{T}_s^2 = (\mathbf{T_1} + \mathbf{T_2})^2,\ \mathbf{T}_t^2 = (\mathbf{T_1} + \mathbf{T_4})^2,\ \mathbf{T}_u^2 = (\mathbf{T_1} + \mathbf{T_3})^2.$$

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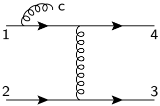
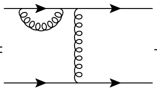


$$= t^b_{a_4a'_1}t^c_{a'_1a_1}t^b_{a_3a_2}$$

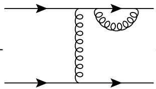
Colour channel operators

$$\mathbf{T}_s^2 = (\mathbf{T_1}+\mathbf{T_2})^2,\ \mathbf{T}_t^2 = (\mathbf{T_1}+\mathbf{T_4})^2,\ \mathbf{T}_u^2 = (\mathbf{T_1}+\mathbf{T_3})^2.$$

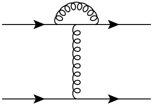
$$\mathbf{T}_t^2\,(t^b_{a_4a_1}t^b_{a_3a_2}) =$$



$$+$$



$$-2$$



$$= C_A\,(t^b_{a_4a_1}t^b_{a_3a_2})$$

Structure of 2 → 2 scattering

$$\mathbf{\Gamma}_{ij\to ij} = \boxed{\frac{\gamma_K(a_s)}{2}\left[L\mathbf{T}_t^2 + i\pi\mathbf{T}_{s-u}^2\right] + (\Gamma_i + \Gamma_j)\mathbf{1}} + \underbrace{\mathbf{\Delta}}_{O(a_s^3)}$$

Dipole formula [Becher,Neubert;Gardi,Magnea 2009]

- $L\mathbf{T}_t^2 \to$ Reggeization at LL [Del Duca,Duhr,Gardi,Magnea,White 2011]

$$\frac{\gamma_K(a_s)}{2} L\,\mathbf{T}_t^2 \mathcal{M}^{(0)}_{ij\to ij} = \frac{\gamma_K(a_s)}{2} L\,C_A \mathcal{M}^{(0)}_{ij\to ij}$$

- $\mathbf{T}_{s-u}^2 \equiv \frac{\mathbf{T}_s^2 - \mathbf{T}_u^2}{2}$ is **not** diagonal → **breaks factorisation**
  [Del Duca,GF,Magnea,Vernazza 2013-2015]
- $\Gamma_i \to$ Collinear singularities for parton $i = q, g$.

IR structure for Higgs + 3 partons

Tight constraints from **colour conservation** with 3 partons

$$\mathbf{T}_t^2 = C_A\,\mathbf{1}, \qquad\qquad \mathbf{T}_{s-u}^2 = 0.$$

- The source of Reggeization breaking, $\mathbf{T}_{s-u}^2$, **disappeared**.
- From operator to scalar anom. dim. [Del Duca,GF 2025]

$$\Gamma_{ig\to iH} = \frac{\gamma_K(a_s)}{2} C_A \left[\log \frac{s_{12}}{\sqrt{-s_{13}}\,m_{H\perp}} - i\frac{\pi}{2}\right] + (\Gamma_i + \Gamma_{gH}) + \mathcal{O}(a_s^3)$$

where $m_{H\perp}^2 = m_H^2 + |p_H^{\perp}|^2$, $\Gamma_{gH} \to$ collinear sing.

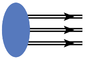
- $\mathcal{O}(a_s^3)$ do not depend on kinematics [Almelid,Duhr,Gardi 2015]
  $\mathcal{O}(a_s^4) \sim \log(s)$ at most [GF,Gardi,Maher,Milloy,Vernazza 2022]

$\Gamma_{ig\to iH}$ conjecturally exact to **NNLL without** Regge cuts!

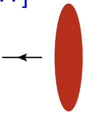
OPE for high-energy scattering

Scattering of Wilson lines

[Mueller,Patel,Balitsky,Korchemskaya,Korchemsky,Kovchegov,Jalilian-Marian,Kovner,McLerran,Weigert,Iancu,Leonidov,...]



$$\Psi_{\text{proj}} \sim \mathrm{T}\left[U(z_1)U(z_2)\ldots\right]$$



$$U(z) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dt\, A_+(x^+ = t, x^- = 0, x^\perp = z)\right] \equiv \mathcal{P}\exp\left[ig\, W(z)\right]$$

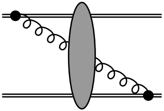
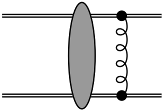
- **Rapidity divergence**: infinite Wilson lines on the lightcone
- Regulator: tilt **off** the lightcone $x^+ = e^\eta/\gamma\, t$, $x^- = e^{-\eta}/\gamma\, t$

$$\eta = \frac{1}{2}\log\frac{dx^+}{dx^-} = \frac{1}{2}\log\frac{s}{-t} \longrightarrow U_\eta(z)$$

Evolution in rapidity

Evolution of the projectile in the background of the target
[Balitsky,Jalilian-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner: B-JIMWLK, 1995-2000; Caron-Huot 2013]

$$\frac{d}{d\eta}\mathrm{T}\left[U(z_1)\dots U(z_n)\right] = -\sum_{i,j}\int d^2z\, H_{ij}[U_{\mathrm{adj}}(z)]\otimes\mathrm{T}\left[U(z_1)\dots U(z_n)\right]$$

$H_{ij}^{\mathrm{LO}} \sim$  $+$  $= \mathcal{O}(a_s)$

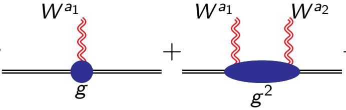
Two terms in the L.O. Hamiltonian

- **Add** one Wilson line in the adjoint rep. (gluon emission)
- **Colour rotation** of the dipole.

Partonic Amplitudes [Caron-Huot 2013;+Gardi,Vernazza 2017]

$$\mathcal{M} \sim \underbrace{\langle \Psi_j |}_{\text{target}} e^{-H\eta} \underbrace{| \Psi_i \rangle}_{\text{projectile}}$$

- $|\Psi_i\rangle$ **dilute**: $U(z) = exp\left[ig\, W^a(z)\, \mathbf{T^a}\right] \sim 1$, $W^a(z)$ **Reggeon**

$|\psi_i\rangle \sim$ 

$+\cdots \sim \begin{pmatrix} W^{a_1} \\ W^{a_1} W^{a_2} \\ \cdots \end{pmatrix}$

- Iteratively act with H and generate powers of $\eta = \frac{1}{2}\log\frac{s}{-t}$.

$$H = \begin{pmatrix} H_{1\to1} & 0 & H_{3\to1} & \cdots \\ 0 & H_{2\to2} & 0 & \cdots \\ H_{1\to3} & 0 & H_{3\to3} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}, \qquad \begin{array}{l} H_{k\to k} = \mathcal{O}(a_s) \\ \\ H_{1\to3}, H_{3\to1} = \mathcal{O}(a_s^2) \end{array}$$

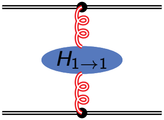
- Reggeons as free fields $\langle W^a(z_1)|W^b(z_2)\rangle \sim \delta(z_1 - z_2)\delta^{ab}$

Single Reggeon vs multiple Reggeons

$$\mathcal{M}_{ij\to ij} = \mathcal{M}^{\text{SR}}_{ij\to ij} + \mathcal{M}^{\text{MR}}_{ij\to ij}$$

The contribution of a single Reggeon has a **pole** structure

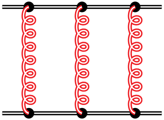
$$\mathcal{M}^{\text{SR}}_{ij\to ij} =$$



$$\simeq \frac{1}{t}\, e^{H_{1\to 1}\, L}$$

Multi-Reggeon exchanges are related to **Regge cuts**. E.g. $\mathcal{O}(a_s^2 L^0)$

$$\mathcal{M}^{\text{MR},(2)}_{ij\to ij} = \langle W^{a_1} W^{a_2} W^{a_3}(z) | W^{b_1} W^{b_2} W^{b_3}(x)\rangle =$$



Disentangle the pole from the cut

$$\mathcal{M}^{\mathrm{MR},(2)}_{ij\to ij} = \pi^2\, S(\epsilon)\Big[\frac{N_c^2}{6} + \underbrace{\Big((\mathbf{T}^2_{s-u})^2 - \frac{C_A^2}{4}\Big)}_{N_c-\text{sublead}}\Big]\mathcal{M}^{(0)}_{ij\to ij}$$

- **Leading colour** $\to \propto \mathcal{M}^{(0)}$ and **universal** $\to$ **POLE**
- **Subleading colour** $\to$ factorisation breaking $\mathbf{T}^2_{s-u}$ $\to$ **CUT**

Define a **scheme** through NNLL [GF,Gardi,Maher,Milloy,Vernazza 2021]

$$\mathcal{M}_{ij\to ij} = \frac{c_i(t,\tau)}{2}\left[\left(\frac{s}{\tau}\right)^{\alpha_g(t)} + \left(\frac{-s}{\tau}\right)^{\alpha_g(t)}\right] c_j(t,\tau)\mathcal{M}^{(0)}_{ij\to ij} + \mathcal{M}^{\mathrm{MR}}_{ij\to ij}\Big|_{N_c\text{-sublead.}}$$

- Self-consistent through 4 loops [GF,Gardi,Maher,Milloy,Vernazza 2021]
- Extended to $2\to 3$ amplitudes at 2 loops [Abreu,De Laurentis,GF,Gardi,Milloy,Vernazza; Buccioni,Caola,Devoto,Gambuti 2024]

Outline

Higgs + jet at NNLL

$$\mathcal{M}_{ig\to iH} = \frac{c_i(t,\tau)}{2}\left[\left(\frac{s}{\tau}\right)^{\alpha_g(t)} + \left(\frac{-s}{\tau}\right)^{\alpha_g(t)}\right] c_H(t,\tau)\, \mathcal{M}^{(0)}_{ig\to iH} + \mathcal{M}^{\text{cut}}_{ig\to iH}$$

- Already known $c_i^{(L)}$, $i = q, g$ and $L \leq 2$ and $\alpha_g^{(L)}$ for $L \leq 3$ [GF,Gardi,Maher,Milloy,Vernazza 2021; Caola,Chakraborty,Gambuti,von Manteuffel,Tancredi 2021]
- $c_H^{(2)}$ unknown term in the **Regge pole** component
- $\mathcal{M}^{\text{cut}}_{ig\to iH}$ must be disentangled from the rest.

Regge cuts in Higgs + jet?

IR poles consistent with **no cuts** through 4 loops [Del Duca,GF 2025]
In the prescription of [GF,Gardi,Maher,Milloy,Vernazza 2021]

$$\mathcal{M}^{\text{cut}}_{ig\to iH} = \mathcal{M}^{\text{MR}}_{ig\to iH}\Big|_{N_c\text{-sublead.}} \quad \text{with} \quad \mathcal{M}^{\text{MR}}_{ig\to iH} = P(\mathbf{T}_t^2, \mathbf{T}_{s-u}^2)\,\mathcal{M}^{(0)}_{ig\to iH}$$

P polynomial has Casimir in **adjoint** rep. $\mathcal{M}^{\text{MR}}_{ig\to iH} = \sum_L \mathcal{M}^{\text{MR}\,(L)}_{ig\to iH}$

$$\mathcal{M}^{\text{MR}\,(L)}_{ig\to iH} = a_s^L\, f_L(\epsilon)\, C_A^L\, \mathcal{M}^{(0)}_{ig\to iH}, \quad L \leq 3 \text{ [Del Duca,GF 2025]}$$

since $\mathbf{T}_t^2 = C_A \mathbf{1}$, $\mathbf{T}_{s-u}^2 = 0$. Therefore

$$\mathcal{M}^{\text{cut}}_{ig\to iH} = \mathcal{O}(a_s^4)$$

Input from fixed order

$$\frac{\mathcal{M}^{(2)}_{ig\to iH}}{\mathcal{M}^{(0)}_{ig\to iH}} = c_H^{(2)} + c_i^{(2)} + c_i^{(1)} c_H^{(1)} - \frac{\pi^2}{8}\left(\alpha^{(1)}\right)^2 + L\left(\alpha^{(2)} + \alpha^{(1)}(c_i^{(1)} + c_H^{(1)})\right)$$

$$+ \frac{\left(\alpha^{(1)}\right)^2}{2} L^2, \text{ where } L = \log\frac{s}{\tau} - i\frac{\pi}{2}$$

- Two-loop amplitudes known in general kinematics through $\mathcal{O}(\epsilon^2)$ [Gehrmann,Jakubčík,Mella,Syrrakos,Tancredi 2023]
- Results can be written in terms of HPLs in $v = \frac{m_H^2}{s}$ and 2dHPLs with letters $\{0, 1, -v, 1-v\}$ and argument $u = u\left(\frac{-t}{m_H^2}, v\right)$.
- Asymptotic expansion as $v \to 0$ from the differential eqs. for 2dHPLs [Del Duca,Falcioni 2025]

The two-loop Higgs impact factor

Factorisation of the IR poles

$$c_H(t, m_H^2, \tau) = \frac{Z_{\mathrm{col}\, gH}\left(\frac{m_{H\perp}^2}{\mu^2}, a_s\right)}{\cos\left(\frac{\pi\alpha_g(t)}{2}\right)} \left(\frac{\tau}{m_{H\perp}^2}\right)^{\frac{\alpha_g(t)}{2}} \bar{D}_H(a_s, x, \mu^2),$$

where $x = \frac{-t}{m_H^2}$, $Z_{\mathrm{col}\, gH} = \exp[-\int \frac{d\lambda^2}{2}\Gamma_{gH}(\lambda^2)]$, $\bar{D}$ **finite** as $\epsilon \to 0$.

The result is expanded as $\bar{D} = \sum a_s^L \, \bar{D}^{(L)}$ and organised in weight

$$\bar{D}_H^{(2)}(x, \mu^2 = -t) = \bar{D}_{H,w=4}^{(2)}(x) + \underbrace{\bar{D}_{H,\beta_0}^{(2)}(x)}_{\text{weight 3}} + \sum_{i \le 2} \bar{D}_{H,w=i}^{(2)}(x)$$

Result continued

Leading transcendental weight

$$
\begin{aligned}
\bar{D}^{(2)}_{H,w=4}(x) = 8N_c^2 \Bigg\{ & \mathrm{Li}_4\left(\frac{x}{1+x}\right) - \frac{1}{2}\mathrm{Li}_4(-x) + \frac{1}{2}\mathrm{Li}_3(-x)\log(x) - \frac{1}{4}\mathrm{Li}_2(-x)\log^2(x) \\
& + \frac{\log^4(x)}{16} + \frac{\log^4(1+x)}{24} + \frac{1}{4}\log^2(x)\log^2(1+x) - \frac{1}{4}\log^3(x)\log(1+x) \\
& - \frac{1}{6}\log(x)\log^3(1+x) + \zeta_2\Bigg(\frac{15}{8}\mathrm{Li}_2(-x) - \frac{31}{16}\log^2(x) + \frac{31}{8}\log(x)\log(1+x) \\
& - \log^2(1+x)\Bigg) + \frac{\zeta_3}{8}\log\left(\frac{x}{1+x}\right) + \frac{277}{128}\zeta_4 + i\pi\Bigg[\frac{1}{2}\mathrm{Li}_3(-x) - \frac{1}{2}\mathrm{Li}_2(-x)\log(x) \\
& + \frac{1}{4}\log^3(x) - \frac{3}{4}\log^2(x)\log(1+x) + \frac{1}{2}\log(x)\log^2(1+x) - \frac{1}{6}\log^3(1+x) \\
& - \frac{7}{8}\zeta_2\log\left(\frac{x}{1+x}\right)\Bigg]\Bigg\}.
\end{aligned} \tag{1}
$$

Weight 4 terms written as classical polylogarithms [Duhr 2012].

Checks

- Universality: $\mathcal{M}_{gg\to gH}$ and $\mathcal{M}_{qg\to qH}$ give the **same** Higgs impact factor.
- Three-loop amplitude at NNLL

$$\frac{\mathcal{M}^{(3)}_{ig\to iH}}{\mathcal{M}^{(0)}_{ig\to iH}} = \frac{\left(\alpha^{(1)}L\right)^3}{6} + \alpha^{(1)}L^2\left[\frac{\alpha^{(1)}}{2}\left(c_i^{(1)}+c_H^{(1)}\right)+\alpha^{(2)}\right] + L\left[\alpha^{(3)}\right.$$
$$\left. +\alpha^{(2)}\left(c_i^{(1)}+c_H^{(1)}\right)+\alpha^{(1)}\left(c_H^{(2)}+c_i^{(2)}+c_i^{(1)}c_H^{(1)}-\frac{\pi^2}{8}\left(\alpha^{(1)}\right)^2\right)\right]$$

All the ϵ-poles match the IR factorisation formula

$$\mathcal{M}^{(3)}_{ig\to iH} = Z^{(3)}\,\mathcal{M}^{(0)}_{ig\to iH} + Z^{(2)}\,\mathcal{H}^{(1)}_{ig\to iH} + Z^{(1)}\,\mathcal{H}^{(2)}_{ig\to iH} + \mathcal{O}(\epsilon^0)$$

Outline

Conclusion

- Simple structure of Higgs + jet at **NNLL** in the Regge limit
  - No Regge cuts up to (and including) three loops.
  - The amplitudes are given in terms of the 3-loop gluon Regge trajectory, 2-loop partonic and **Higgs impact factor**, $c_H^{(2)}$.
- The impact factor $c_H^{(2)}$ from the high-energy limit of the two-loop amplitudes in [Gehrmann,Jakubčík,Mella,Syrrakos,Tancredi 2023]
- Verified **universality** of Regge pole factorisation in

$$gg \to gH, \qquad qg \to qH$$

.

- Pole structure at 3 loops agrees with IR factorisation.

Outlook

- Three-loop amplitudes complete [Chen,Guan,Mistlberger 2025] $\rightarrow$ a look beyond NNLL.
- Reggeization in **full QCD** $\rightarrow$ include top mass
  One-loop corrections for $g \rightarrow gH$ impact factor recently computed [Celiberto,Delle Rose,Fucilla,Gatto,Papa 2024]
- Extend to the multi-Regge limits with **more** than 1 jet
  See [Abreu,De Laurentis,GF,Gardi,Milloy,Vernazza; Buccioni,Caola,Devoto,Gambuti 2024]
- Correspondence between amplitudes for H $+$ 3 g and stress-tensor form factor known through 8 loops in $\mathcal{N} = 4$ sYM [Dixon,McLeod,Wilhelm 2020;Dixon,Gurdogan,McLeod,Wilhelm 2022].

Thank you!

Expansion of the two-loop amplitudes (I)

Introduce the 2dHPL [Gehrmann,Remiddi 2000-2001] as

$$G(a_1,\ldots,a_n;u) = \int_0^u \frac{dt}{t-a_1}\, G(a_2,\ldots,a_n;t), \qquad a_i \in \{0,1,-v,1-v\}$$

Pick independent basis under shuffle relations,
$\vec{f}(x,v) = \{\varepsilon^6 G(0,0,0,0,0,1-v,u),\ldots,\varepsilon G(0,u),1\}$, and write

$$\frac{\partial}{\partial v}\vec{f}(v,x) = \varepsilon\,\mathbf{M_v}(v,x)\,\vec{f}(v,x),$$
$$\frac{\partial}{\partial x}\vec{f}(v,x) = \varepsilon\,\mathbf{M_x}(v,x)\,\vec{f}(v,x),$$

with $v = \frac{m_H^2}{s}$ and $x = \frac{-t}{m_H^2}$. Singular behaviour as $v \to 0$

$$\mathbf{M}_v = \frac{1}{v}\mathbf{M_v^{(0)}} + \sum_{k\geq 0} v^k\,\mathbf{M_v^{(1+k)}}(x),\,.$$

Expansion of the two-loop amplitudes (II)

Solution in generalised series [Caron-Huot,Chicherin,Henn,Zhang,Zoia 2020]

$$\vec{f}(v,x) = T(\varepsilon,v,x)\exp\left[\varepsilon\,\log(v)\,\mathbf{M}_{\mathbf{v}}^{(\mathbf{0})}\right]\,\mathbb{P}\mathrm{exp}\left[\varepsilon\int_1^x \mathbf{M}_{\mathbf{x}}(v=0,t),dt\right]\,\vec{g}_0,$$

where $T(\varepsilon,v,x) = \mathbb{I} + \sum_{k,j\geq 1} v^k\,\varepsilon^j\,T^{(k,j)}(x)$

$$T^{(k,1)}(x) = \frac{\mathbf{M}_{\mathbf{v}}^{(\mathbf{k})}(x)}{k},$$

$$T^{(k,j)}(x) = \frac{1}{k}\left\{\left[\mathbf{M}_{\mathbf{v}}^{(\mathbf{0})},\,T^{(k,j-1)}(x)\right] + \sum_{q=1}^{k-1}\mathbf{M}_{\mathbf{v}}^{(\mathbf{k-q})}(x)\,T^{(q,j-1)}(x)\right\}.$$

and the boundary conditions [Hidding 2020] is

$$\lim_{v\to 0}\vec{f}(v,1) = \exp\left[\varepsilon\,\log(v)\,\mathbf{M}_{\mathbf{v}}^{(\mathbf{0})}\right]\,\vec{g}_0(\varepsilon),$$

The soft anomalous dimension

3-loop $\mathbf{\Gamma}$ [Almelid,Duhr,Gardi 2015] + 4-loop ansatz [Becher,Neubert 2019]

$$\mathbf{\Gamma}_n\left(\{s_{ij}\},\lambda,a_s\right) = \mathbf{\Gamma}_n^{\text{dip.}}\left(\{s_{ij}\},\lambda,a_s\right) + \mathbf{\Gamma}_{n,\text{4T-3L}}(a_s) + \mathbf{\Gamma}_{n,\text{Q4T-2,3L}}(a_s) + \mathbf{\Delta}_4\left(\{s_{ij}\},\lambda,a_s\right)$$

$\mathbf{\Delta}_4 = \mathcal{O}(a_s^3)$ **vanishes** with 3 partons

$$\mathbf{\Gamma}_n^{\text{dip.}} = -\frac{\gamma_K(a_s)}{4}\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\log\frac{-s_{ij}}{\lambda^2} + \sum_i \gamma_i(a_s),$$

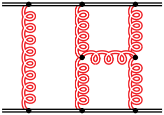
$$\mathbf{\Gamma}_{n,\text{4T-3L}}(a_s) = \underbrace{f(a_s)}_{\mathcal{O}(a_s^3)}\sum_{(i,j,k)} f^{ade}f^{bce}\{\mathbf{T}_i^a,\mathbf{T}_i^b,\mathbf{T}_j^c,\mathbf{T}_k^d\}_+,$$

$$\mathbf{T}_{n,\text{Q4T-2,3L}} = -\frac{1}{2}\sum_R \underbrace{g_R(a_s)}_{\mathcal{O}(a_s^4)}\left[\sum_{(i,j)}\left(\mathcal{D}^R_{iijj} + 2\mathcal{D}^R_{iiij}\right)\log\frac{-s_{ij}}{\lambda^2} + \sum_{(i,j,k)}\mathcal{D}^R_{ijkk}\log\frac{-s_{ij}}{\lambda^2}\right]$$

with $\mathcal{D}^R_{ijkl} = \frac{1}{4!}\sum_{\sigma\in S_4}\text{Tr}\left[T_R^{\sigma(a)}T_R^{\sigma(b)}T_R^{\sigma(c)}T_R^{\sigma(d)}\right]\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_k^c\mathbf{T}_l^d$.

Multi-Reggeon exchanges at higher order (I)

[Caron-Huot,Gardi,Vernazza 2017;GF,Gardi,Milloy,Vernazza 2020;+Maher 2021]

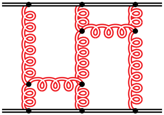


$$+ \ldots = a_s^3 L(i\pi)^2 \Big[N_c^3\, S_C(\epsilon) + \mathbf{T}_{s-u}^2[\mathbf{T}_{s-u}^2, \mathbf{T}_t^2] S_A(\epsilon) + [\mathbf{T}_{s-u}^2, \mathbf{T}_t^2]\mathbf{T}_{s-u}^2 S_B(\epsilon) \Big] \mathcal{M}^{(0)}_{ij\to ij}$$

- The operator $[\mathbf{T}_{s-u}^2, \mathbf{T}_t^2]$ is subleading at large N_c
- The leading term at large N_c is **universal**
  - Absorbed in the definition of the 3-loop Regge trajectory [GF,Gardi,Maher,Milloy,Vernazza; Caola,Chakraborty,Gambuti,von Manteuffel,Tancredi 2021]

Multi-Reggeon exchanges at higher order (II)

[GF,Gardi,Milloy,Vernazza 2020;+Maher 2021]

 $+\ldots = a_s^4 L^2 (i\pi)^2 \left[\frac{\mathbf{K}^{(4)}}{\epsilon^4} + \frac{\mathbf{K}^{(1)}}{\epsilon} \right] \mathcal{M}^{(0)}_{ij\to ij}$

- $\mathbf{K}^{(4)}$ and $\mathbf{K}^{(1)}$ involve either the operator $[\mathbf{T}^2_{s-u}, \mathbf{T}^2_t]$ or

$$\frac{d_{AA}}{N_A} - \frac{C_A^4}{24} = \mathcal{O}(N_c^2),$$

since $\frac{d_{AA}}{N_A} = \frac{N_c^4 + 36 N_c^2}{24}$.

- Multi-Regge exchanges at 4 loops are subleading in N_c.
- The Regge pole can NOT absorb a would-be leading term in N_c!