

# AD Mission Playground: An Interactive Tool for Elimination Techniques

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$$\begin{aligned} \mathbf{y} &= F(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^m \\ &= F_q \circ F_{q-1} \circ \dots \circ F_2 \circ F_1 \end{aligned}$$

$$\mathbf{z}_i = F_i(\mathbf{z}_{i-1}) : \mathbb{R}^{n_i} \rightarrow \mathbb{R}^{m_i} \quad i = 1, \dots, q \quad (\mathbf{z}_0 = \mathbf{x}, \mathbf{z}_q = \mathbf{y})$$

$$\Rightarrow F' \equiv \frac{dF}{d\mathbf{x}} = F'_q \cdot F'_{q-1} \cdot \dots \cdot F'_1 \in \mathbb{R}^{m \times n}.$$

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### Dense Jacobian Chain Product Bracketing

$$\mathbf{fma}_{j,i} = \begin{cases} |E_j| \cdot \min\{n_j, m_j\} & j = i \\ \min_{i \leq k < j} (\mathbf{fma}_{j,k+1} + \mathbf{fma}_{k,i} + m_j \cdot m_k \cdot n_i) & j > i. \end{cases}$$

## Matrix-Free Dense Jacobian Chain Product Bracketing

$$\text{fma}_{j,i} = \left\{ \begin{array}{l} |E_j| \cdot \min\{n_j, m_j\} \\ \min_{i \leq k < j} \left\{ \min \left\{ \begin{array}{l} \text{fma}_{j,k+1} + \text{fma}_{k,i} + m_j \cdot m_k \cdot n_i, \\ \text{fma}_{j,k+1} + m_j \cdot \sum_{\nu=i}^k |E_\nu|, \\ \text{fma}_{k,i} + n_i \cdot \sum_{\nu=k+1}^j |E_\nu| \end{array} \right\} \right\} \end{array} \right\} \begin{array}{l} j = i \\ \\ j > i. \end{array}$$

## Scheduled Limited-Memory Matrix-Free Dense Jacobian Chain Product Bracketing (Limited)

$$\text{fma}_{j,i}^{(t)} = \left\{ \begin{array}{l} |E_j| \cdot \begin{cases} n_j & |E_j| > \bar{M} \\ \min(n_j, m_j) & \text{otherwise} \end{cases} & j = i \\ \min_{i \leq k < j} \left\{ \min \left\{ \begin{array}{l} m_j \cdot m_k \cdot n_i + \min \left\{ \begin{array}{l} \text{fma}_{j,k+1}^{(t)} + \text{fma}_{k,i}^{(t)} \\ \min_{1 \leq t^* < t} \left( \max \left( \text{fma}_{j,k+1}^{(t^*)}, \text{fma}_{k,i}^{(t-t^*)} \right) \right) \end{array} \right\} \\ \text{fma}_{j,k+1}^{(t)} + m_j \cdot \sum_{\nu=i}^k |E_\nu| \quad \text{if } \sum_{\nu=i}^k |E_\nu| \leq \bar{M} \\ \text{fma}_{k,i}^{(t)} + n_i \cdot \sum_{\nu=k+1}^j |E_\nu| \end{array} \right\} \right\} & j > i. \end{array} \right.$$

### ▶ VERTEX ELIMINATION

A. Griewank, S. Reese: *On the Calculation of Jacobian Matrices by the Markowitz Rule*. AD @ Breckenridge, 1991.

### ▶ EDGE ELIMINATION

U. Naumann: *Elimination Techniques for Cheap Jacobians*. AD @ Nice, 2000.

### ▶ FACE ELIMINATION

U. Naumann: *Optimal Accumulation of Jacobian Matrices by Elimination Methods on the Dual Computational Graph*. Math. Prog., 2004.

### ▶ GENERALIZED FACE ELIMINATION

U. Naumann, E. Schneiderei, S. Märtens, and M. Towara: *Elimination Techniques for Algorithmic Differentiation Revisited*. SIAM ACDA, 2023.

### ▶ SCHEDULED JACOBIAN CHAINING

S. Märtens, U. Naumann: *Scheduled Jacobian Chaining*. SIAM ACDA, 2025.

## Design goals:

- ▶ A drawable canvas to manually create a DAG.
- ▶ Manual application of all elimination techniques.
- ▶ Integration of our existing solvers (<https://github.com/STCE-at-RWTH/Jacobian-Chaining>)
- ▶ Static website without server backend.



<https://ad-elimination-playground-accadf.pages.stce.rwth-aachen.de/>