Status of the theory of saturation of partonic densities

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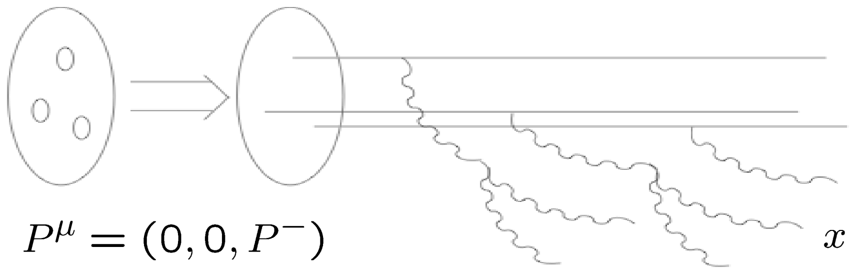
- Introduction to parton saturation in QCD
- Non-linear evolution of color dipoles at leading order
- Higher-order corrections to dipole evolution
- High-p_T corrections to single-hadron production
- Di-hadron production and quadrupole evolution

Map of parton evolution in QCD

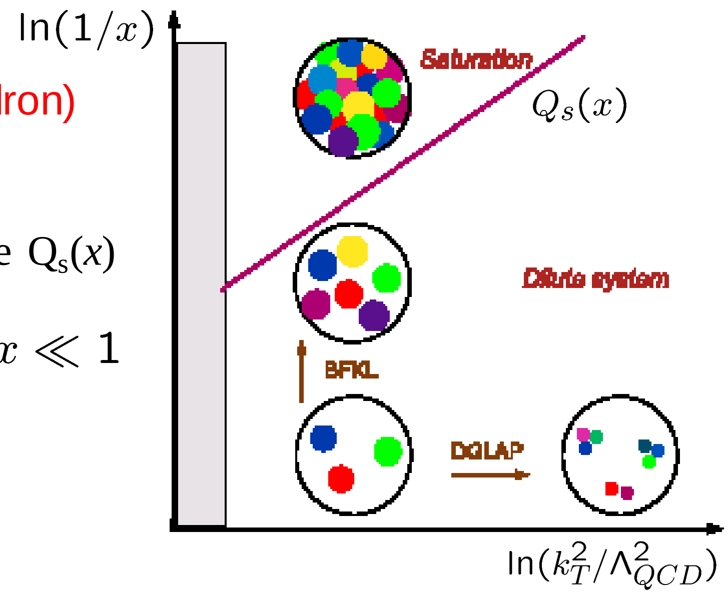


$P^\mu = (0, 0, P^-)$ $\quad x\ P^-, k_T$

x : parton longitudinal momentum fraction

k_T : parton transverse momentum

the distribution of partons as a function of x and k_T :



QCD linear evolutions: $k_T \gg Q_s$

DGLAP evolution to larger k_T (and a more dilute hadron)

BFKL evolution to smaller x (and denser hadron)

dilute/dense separation characterized by the saturation scale $Q_s(x)$

QCD non-linear evolution: $k_T \sim Q_s$ meaning $x \ll 1$

$\rho \sim \frac{xf(x,k_\perp^2)}{\pi R^2}$ gluon density per unit area, it grows with decreasing x

$\sigma_{rec} \sim \alpha_s/k^2$ recombination cross-section

recombinations important when $\rho\ \sigma_{rec} > 1$

the saturation regime: for $k^2 < Q_s^2$ with $Q_s^2 = \frac{\alpha_s x f(x,Q_s^2)}{\pi R^2}$

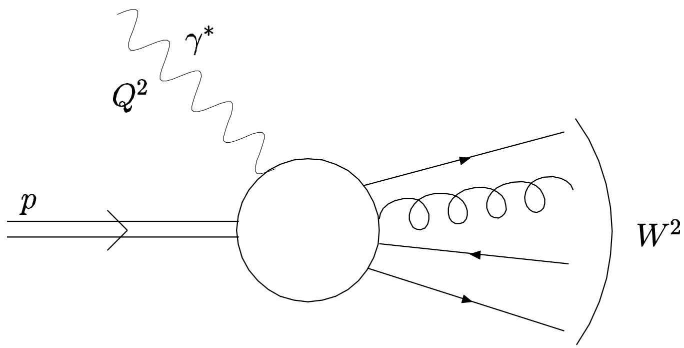
this regime is non-linear yet weakly coupled

$\alpha_s(Q_s^2) \ll 1$

When is saturation relevant ?

in processes that are sensitive to the small-x part of the hadron wavefunction

- deep inelastic scattering at small x_{Bj} :

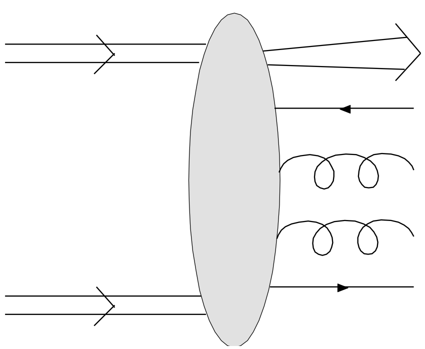


$$x_{Bj} = \frac{Q^2}{W^2 + Q^2}$$

at HERA, $x_{Bj} \sim 10^{-4}$ for $Q^2 = 10$ GeV²

in DIS small x corresponds to high energy

- particle production at forward rapidities y :



$$x_1 \sqrt{s} = p_T \, e^{y}$$

$$x_2 \sqrt{s} = p_T \, e^{-y}$$

at RHIC, $x_2 \sim 10^{-4}$ for $p_T^{\,2} = 10$ GeV²

in particle production, small x corresponds to high energy and forward rapidities

Non-linear evolution of color dipoles at leading order

The dipole scattering amplitude

a fundamental quantity to study high-energy scattering in QCD

- deep inelastic scattering at small *x* :

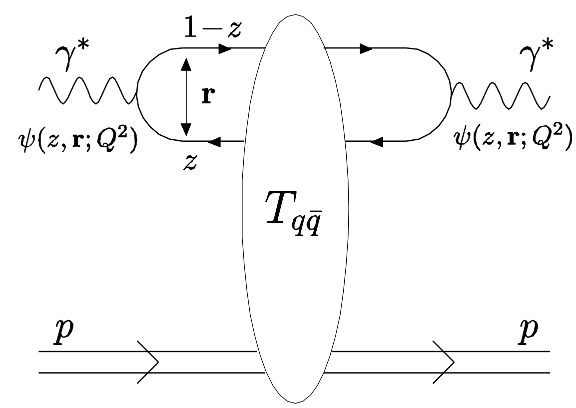
$$\sigma_{T,L}^{\gamma^* p\to X} = 2\int d^2r\, dz\, |\psi_{T,L}(z,\mathbf{r};Q^2)|^2 \underbrace{\int d^2b\, T_{q\bar{q}}(\mathbf{r},\mathbf{b},x_B)}$$

overlap of $\gamma^* \to q\bar{q}$ splitting functions

r = dipole size

dipole-hadron cross-section computed in the CGC

resums powers of $g_S A$ and powers of $\alpha_S \ln(1/x_B)$



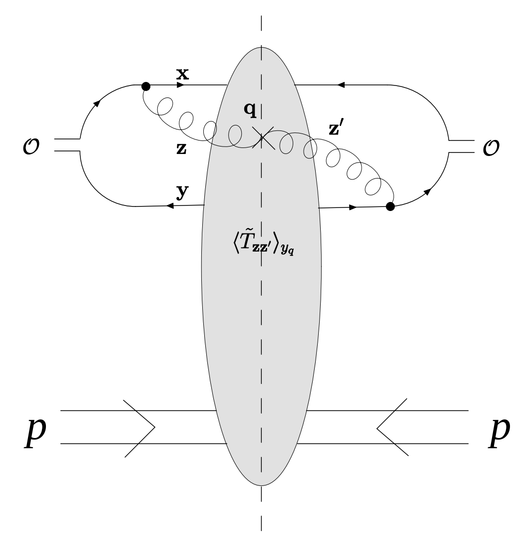
- particle production at forward rapidities:

$$q^2 \frac{d\sigma}{d^2q\, d^2b} \propto \int \frac{d^2r}{(2\pi)^2}\, e^{-i\mathbf{q}\cdot\mathbf{r}}[1 - T_{gg}(\mathbf{r},\mathbf{b},x)]$$

r = z-z'

dipole-hadron scattering amplitude (adjoint or fundamental)

FT of dipole amplitude ≡ unintegrated gluon distribution



The Balitsky-Kovchegov equation

- for impact-parameter independent solutions $T_{q\bar{q}}(\mathbf{r},\mathbf{b},x) \equiv \mathcal{N}(x,r)$

$$\frac{\partial \mathcal{N}(x,r)}{\partial \ln(x_0/x)} = \bar{\alpha} \int \frac{d^2 r_1}{2\pi} \frac{r^2}{r_1^1 r_2^2} \left[\underbrace{\mathcal{N}(x,r_1) + \mathcal{N}(x,r_2) - \mathcal{N}(x,r)}_{\text{linear evolution : BFKL}} - \mathcal{N}(x,r_1)\mathcal{N}(x,r_2)\right]$$

$r_2 = |\mathbf{r} - \mathbf{r}_1|$

linear evolution : BFKL

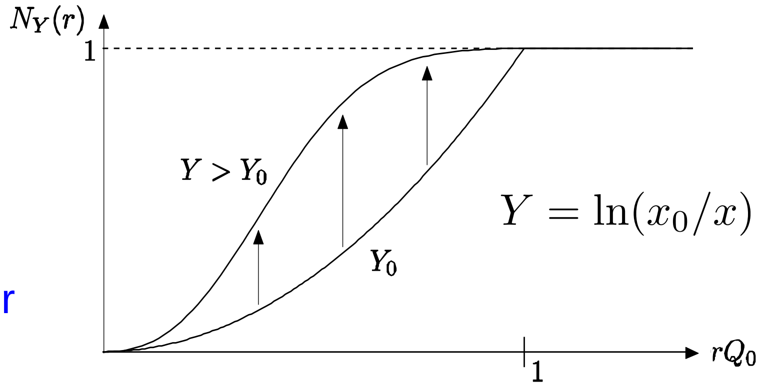
saturation

- solutions: qualitative behavior

at large *x*, $\mathcal{N}$ is small, and the quadratic term can be neglected, the equation reduces then to the linear BFKL equation and $\mathcal{N}$ rises exponentially with decreasing x

as $\mathcal{N}$ gets close to 1 (the stable fixed point of the equation), the non-linear term becomes important, and $d\mathcal{N}/dY \to 0$, $\mathcal{N}$ saturates at 1

with increasing *Y*, the unitarization scale get bigger



The GBW parametrization

- modeling the dipole scattering amplitude

the numerical solution of the BK equation is not useful for phenomenology
(because this is a leading-order calculation)

before

instead, CGC-inspired parameterizations are used for $\mathcal{N}(x,r)$
(with a few parameters adjusted to reproduce the data)

- the original model for the dipole scattering amplitude $\mathcal{N}(x,r)$

Golec-Biernat and Wusthoff (1998)

it features geometric scaling: $\mathcal{N}(x,r) = 1 - \exp[-r^2 Q_s^2(x)/4]$

the saturation scale: $Q_s(x) = (x_0/x)^{\frac{\lambda}{2}}$ GeV

the parameters: $\lambda \simeq 0.3$ and $x_0 \simeq 10^{-4}$ fitted on F_2 data

now

running-coupling corrections to BK evolution have been calculated

Balitsky-Gardi-Kovchegov-Weigert (2007)

one should obtain $\mathcal{N}(x,r)$ from the evolution equation (λ consistent with rcBK)

Higher-order corrections to dipole evolution

Running-coupling BK evolution

- running-coupling (RC) corrections to the BK equation

$$\alpha_s(\mathbf{r}^2) = \left[-\frac{11N_c - 2N_f}{12\pi}\ln\left(\mathbf{r}^2\Lambda_{QCD}^2\right)\right]^{-1}$$

taken into account by the substitution

$$\frac{\bar{\alpha}}{2\pi}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \xrightarrow[\text{Weigert}]{\text{Kovchegov}} \frac{N_c}{2\pi^2}\left[\frac{\alpha_s((\mathbf{x}-\mathbf{z})^2)}{(\mathbf{x}-\mathbf{z})^2} - 2\frac{\alpha_s((\mathbf{x}-\mathbf{z})^2)\alpha_s((\mathbf{z}-\mathbf{y})^2)}{\alpha_s((\mathbf{x}-\mathbf{y})^2)} + \frac{\alpha_s((\mathbf{z}-\mathbf{y})^2)}{(\mathbf{z}-\mathbf{y})^2}\right]$$

Balitsky ↓ (2007)

$$\frac{N_c\alpha_s((\mathbf{x}-\mathbf{y})^2)}{2\pi^2}\left[\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} + \frac{1}{(\mathbf{x}-\mathbf{z})^2}\left(\frac{\alpha_s((\mathbf{x}-\mathbf{z})^2)}{\alpha_s((\mathbf{z}-\mathbf{y})^2)} - 1\right) + \frac{1}{(\mathbf{z}-\mathbf{y})^2}\left(\frac{\alpha_s((\mathbf{z}-\mathbf{y})^2)}{\alpha_s((\mathbf{x}-\mathbf{z})^2)} - 1\right)\right]$$

RC corrections represent most of the NLO contribution

- the begining of the NLO-CGC era

first numerical solution Albacete and Kovchegov (2007)

first phenomenological implementation Albacete, Armesto, Milhano and Salgado (2009)
to successfully describe the proton structure function F_2 at small x

more confrontation to data now, both for DIS and forward particle production

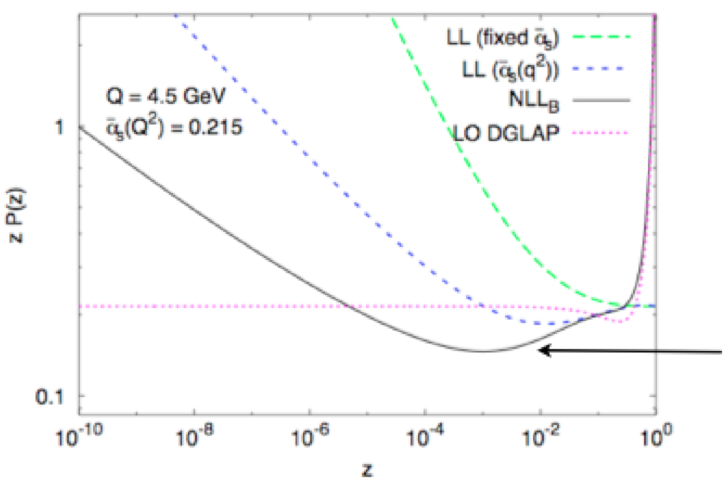
Towards full NLO-CGC calculations

- the full NLO evolution equation is known Balitsky and Chirilli (2008)

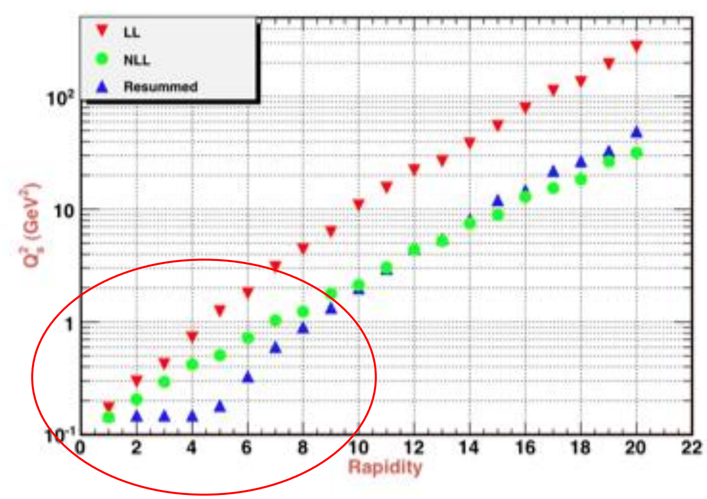
but (linear) BFKL evolution suffers from spurious singularities
collinear resummations are needed to get meaningful results

belief/hope: saturation cures the BFKL instabilities, no need for collinear resummations when non-linear effects are included

this is wrong, resummations are needed and may have sizable effects



the dip of the resummed splitting function delays the onset of saturation



Avsar, Stasto, Triantafyllopoulos and Zaslavsky (2011)

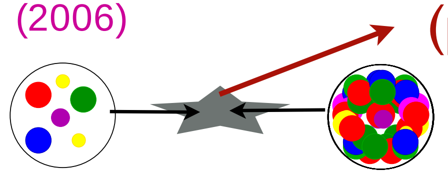
High-p_T corrections to single-hadron production

Forward particle production

- forward rapidities probe small values of *x*

Dumitru, Hayashigaki and Jalilian-Marian (2006)

$$x_{1(2)} \sim \frac{m_t}{\sqrt{s}} \exp(\pm y_h)$$



(p_t, y_h>>0)

large-x parton from proj. (pdf)

small-x glue from target (CGC)

$$\frac{dN_h}{dy_h\, d^2p_t} = \frac{K}{(2\pi)^2} \sum_q \int_{x_F}^1 \frac{dz}{z^2} \left[x_1 f_{q/p}(x_1, p_t^2)\, \tilde{N}_F\left(x_2, \frac{p_t}{z}\right) D_{h/q}(z, p_t^2) + x_1 f_{g/p}(x_1, p_t^2)\, \tilde{N}_A\left(x_2, \frac{p_t}{z}\right) D_{h/g}(z, p_t^2) \right]$$

fragmentation

the coupling α_s does not appear in this formula because it is compensated by the strong color field of the nucleus $A\sim 1/g_s$

- merging to the high-p_T leading-twist regime ?

Altinoluk and Kovner (2011)

at high-pt the color field becomes *O(1)* and this cross section $\mathcal{O}(\alpha_s)$

then another $\mathcal{O}(\alpha_s)$ contribution (which is an NLO contribution when $A\sim 1/g_s$) is needed to fully recover to correct high-p_T limit

RHIC vs LHC kinematics

- typical values of x being probed at forward rapidities

RHIC $x_d \simeq 0.5 \quad x_A \simeq 5.10^{-3}$

y~3

deuteron dominated by valence quarks

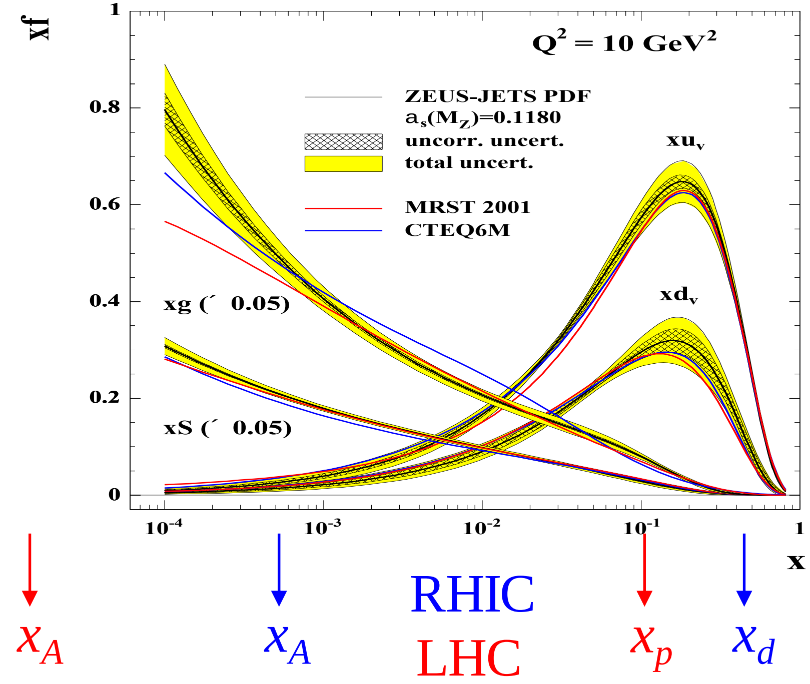
nucleus dominated by early CGC evolution

LHC $x_p \simeq 0.1 \quad x_A \simeq 10^{-5}$

y~5

the proton description should include both quarks and gluons

on the nucleus side, the non-linear evolution would be better tested



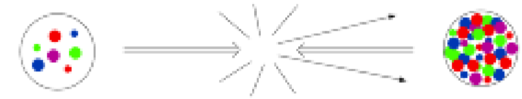
- smaller x_p: suppression of large-x effects who might play a role at RHIC
- larger p_T: the transition to leading-twist regime can really be tested, also at forward rapidities

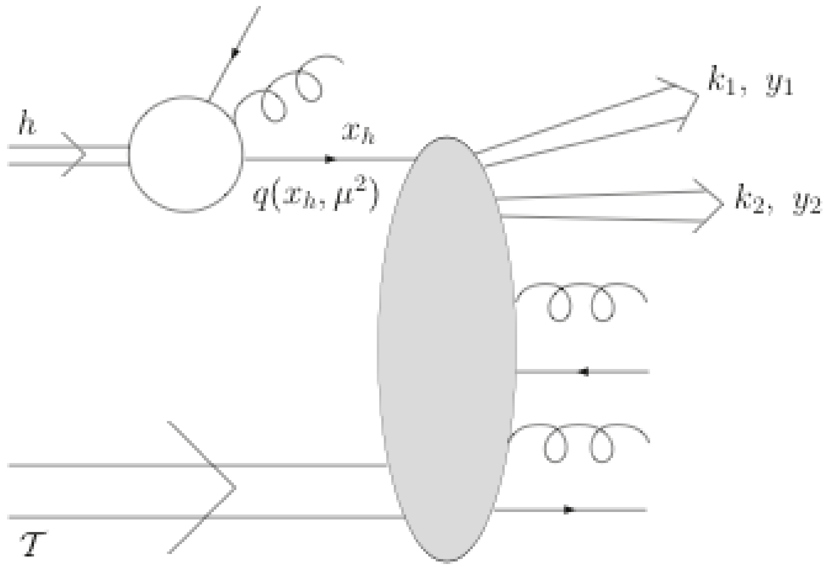
Jalilian-Marian and Rezaeian (2011)

Di-hadron production and quadrupole evolution

Forward di-hadron production

in p+A type collisions $x_A = \frac{k_1 e^{-y_1} + k_2 e^{-y_2}}{\sqrt{s}} \ll 1$





CM (2007)

the saturation regime is better probed compared to single particle production

$$\frac{d\sigma^{dAu\to h_1 h_2 X}}{d^2k_1 dy_1 d^2k_2 dy_2}$$

is sensitive to multi-parton distributions, and not only to the gluon distribution

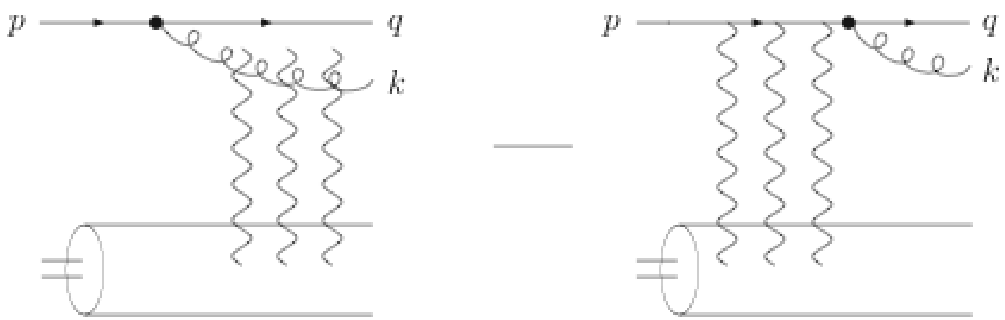
the CGC cannot be described by a single gluon distribution

$$\frac{d\sigma^{dAu\to h_1 h_2 X}}{d^2k_1 dy_1 d^2k_2 dy_2}$$

involves 2-, 4- and 6- point functions

no k_T factorization

The two-particle spectrum



b: quark in the amplitude
x: gluon in the amplitude
b': quark in the conj. amplitude
x': gluon in the conj. amplitude

collinear factorization of quark density in deuteron

Fourier transform $k_\perp$ and $q_\perp$ into transverse coordinates

$$\frac{d\sigma^{dAu\to qgX}}{d^2k_\perp dy_k d^2q_\perp dy_q} = \alpha_S C_F N_c\, x_d q(x_d,\mu^2) \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2b}{(2\pi)^2}\frac{d^2b'}{(2\pi)^2}\, e^{ik_\perp.(\mathbf{x}'-\mathbf{x})} e^{iq_\perp.(\mathbf{b}'-\mathbf{b})}$$

$$\left|\Phi^{q\to qg}(z,\mathbf{x}-\mathbf{b},\mathbf{x}'-\mathbf{b}')\right|^2 \Big\{ S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_A] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_A]$$

$$-S^{(3)}_{\bar{q}gq}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_A] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_A]\Big\}$$

pQCD $q \to qg$ wavefunction

interaction with target nucleus

$$z = \frac{|k_\perp|e^{y_k}}{|k_\perp|e^{y_k} + |q_\perp|e^{y_q}}$$

n-point functions that resums the powers of $g_S A$ and the powers of $\alpha_S \ln(1/x_A)$

Dealing with the 4-point function

- in the large-Nc limit, the cross section is obtained from

$$S^{(4)} = \frac{1}{N_c}\left\langle \mathrm{Tr}(W_{\mathbf{x}}W_{\mathbf{y}}^{\dagger}W_{\mathbf{u}}W_{\mathbf{v}}^{\dagger})\right\rangle_{x_A} \text{ and } S^{(2)} = \frac{1}{N_c}\left\langle \mathrm{Tr}(W_{\mathbf{x}}W_{\mathbf{y}}^{\dagger})\right\rangle_{x_A}$$

the 2-point function is fully constrained
by e+A DIS and d+Au single hadron data

- in principle the 4-point function should be obtained from an evolution equation (equivalent to JIMWLK + large Nc)

Jalilian-Marian and Kovchegov (2005)

- in practice one uses an approximation that allows to express $S^{(4)}$ as a (non linear) function of $S^{(2)}$ C.M. (2007)

this approximation misses some leading-Nc terms Dumitru and Jalilian-Marian (2010)
they may become dominant for $|k_\perp + q_\perp| \ll |k_\perp|,\ |q_\perp|$ Dominguez, Xiao and Yuan (2010)

- very recent results: 4-point function obtained from a numerical solution of the JIMWLK equation Schenke and Venugopalan (in progress)

the so-called dipole approximation used in the calculation show ~10% deviations

Conclusions

- Theory of parton saturation well established at leading order
  - cornerstone: the Balitsky-Kovchegov equation (or JIMWLK hierarchy)
  - but not sufficient for successful phenomenology

- Most important recent progress: running-coupling corrections
  - needed for the compatibility of the non-linear QCD evolution with data
  - already successfully tested but the LHC p+A run will contribute

- Theoretical developments that will benefit from a p+A run at the LHC
  - quantifying the transition from the saturation regime to the leading-twist regime (i.e. how R_{pA} goes back towards unity at high-p_T)
  - testing the quadrupole evolution and constraining the initial condition

- Other important theoretical developments whose relevance for the LHC is not clearly established
  - including Pomeron loops in the evolution
  - going beyond the large-Nc limit