NNLL Top-Antitop Production at Threshold

[hep-ph/0611292]
[1102.0269 [hep-ph]]

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Outline

- Heavy quark-antiquark threshold
- The effective theory vNRQCD
- Renormalization
  - Currents
  - Potentials
- Recent Results (QCD)
- Summary/Outlook

Heavy quark-antiquark threshold

Heavy $q\,\bar{q}$ production near threshold:

- perturbative
- nonrelativistic

$t\,\bar{t}$

↓

$e^+e^- \longrightarrow t\,\bar{t}$

threshold scan

↓

$m_t, y_t, \alpha_s, \Gamma_t$

$b\,\bar{b}$

↓

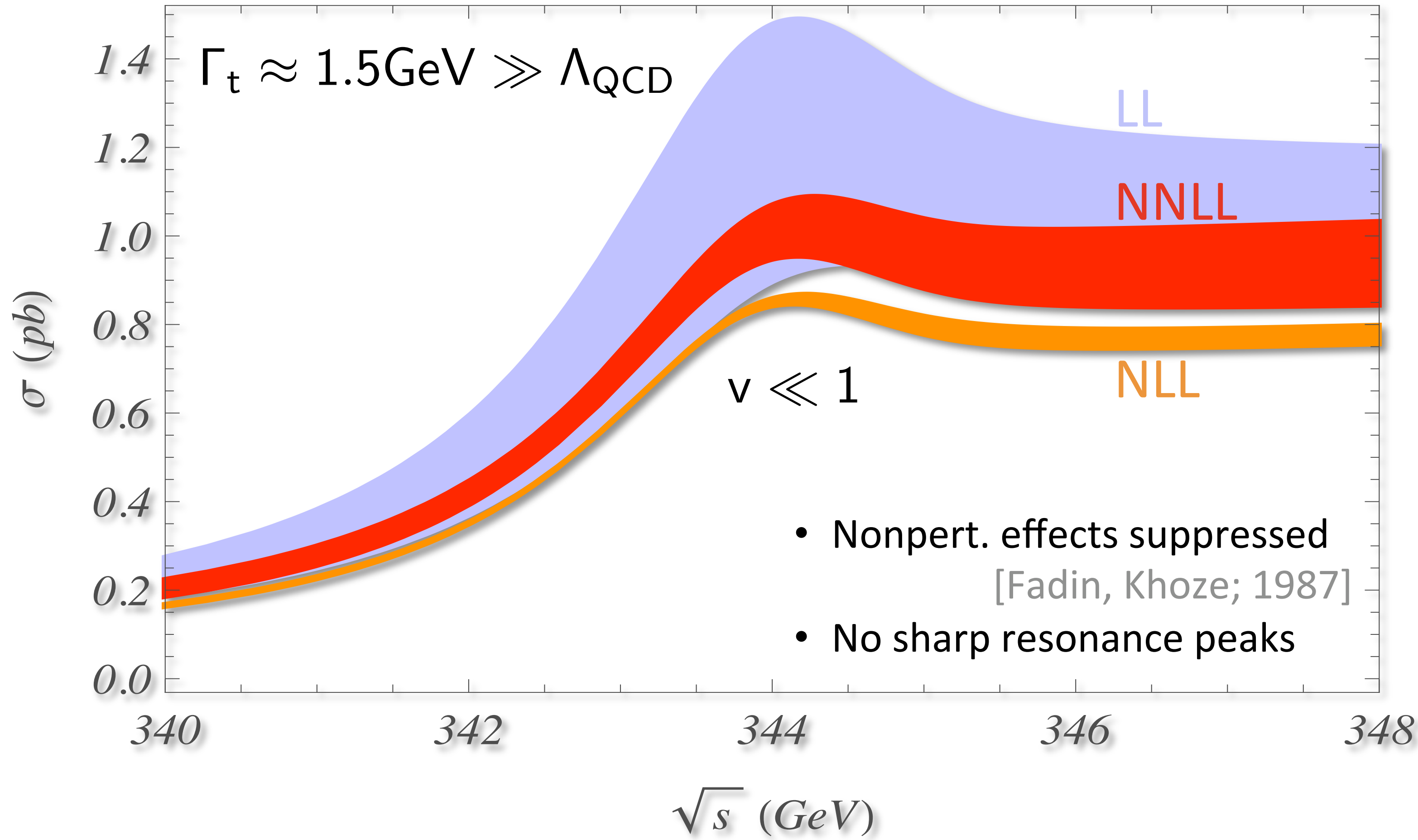
nonrelativistic sumrule analysis

↓

m_b

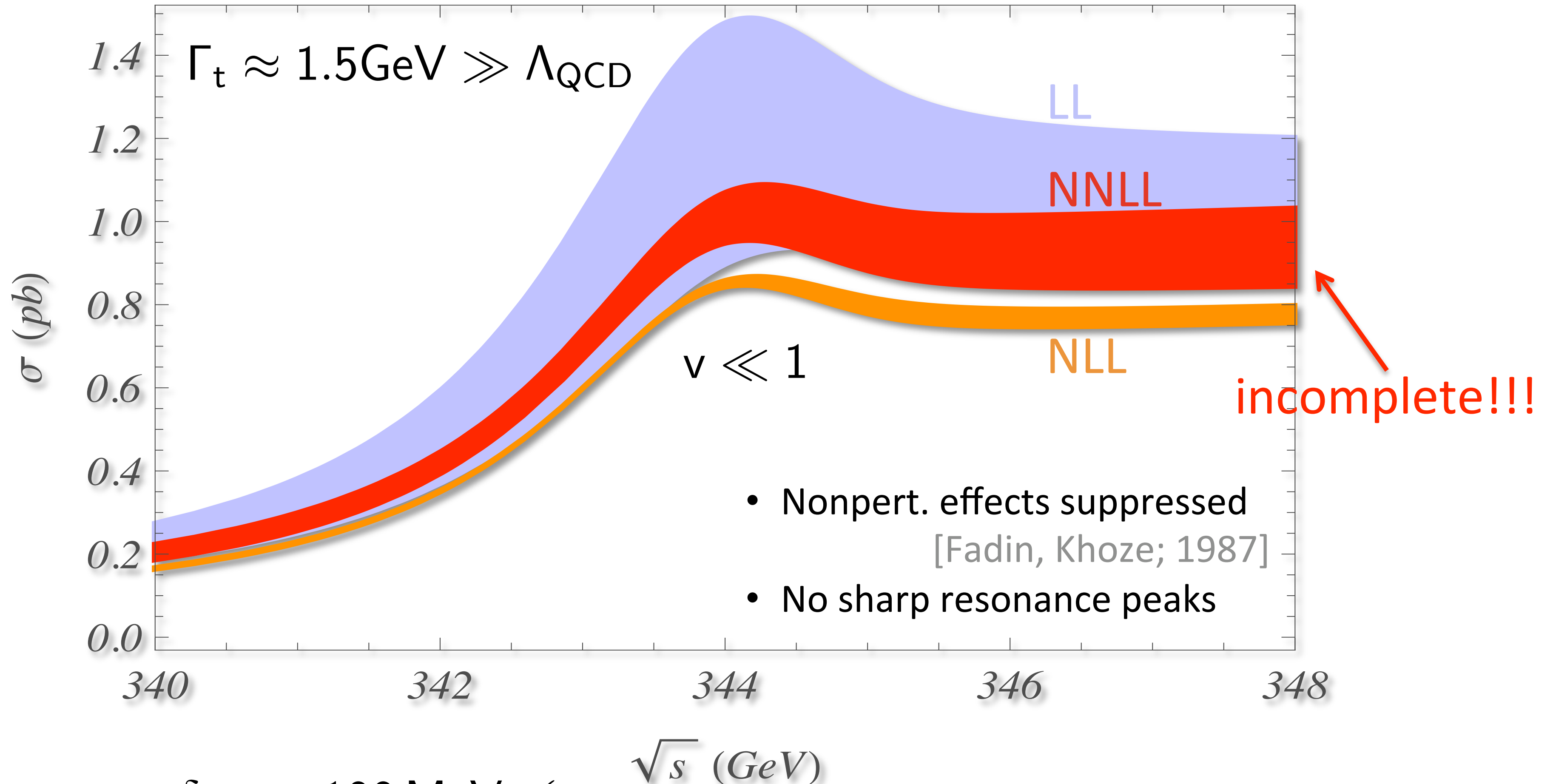
Heavy quark-antiquark threshold

$t\bar{t}$ resonance @ Linear Colider:



Heavy quark-antiquark threshold

$t\bar{t}$ resonance @ Linear Colider:



status: $\delta m_t \sim 100$ MeV ✓
(theory)
$\delta\sigma_{tot}/\sigma_{tot} \sim 6\,\%$, < 3% needed for precise $y_t,\ \alpha_s,\ \Gamma_t$

Heavy quark-antiquark threshold

EW effects:

- LO: Unstable top $\Rightarrow$ $\Gamma_t \approx 1.5\,\mathrm{GeV} \gg \Lambda_{QCD}$ "IR cutoff"

$$v_{eff} \equiv \sqrt{\frac{\sqrt{s}-2m_t}{m_t}} \rightarrow \sqrt{\frac{\sqrt{s}-2m_t+i\Gamma_t}{m_t}} \; ; \qquad |v_{eff}| \gtrsim 0.1$$

[Fadin, Khoze; 1987]

- Higher orders up to NNLL [Hoang, Reisser, Ruiz-Femenia; 2010] [Beneke, Jantzen Ruiz-Femenia; 2010]

QCD near threshold:

$v \sim \alpha_s \ll 1$

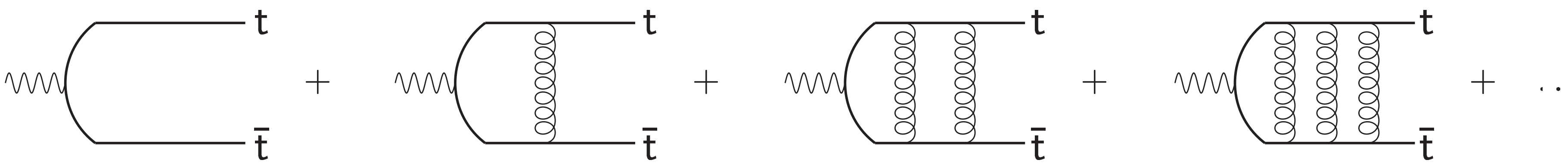
3 scales: $m_t \gg \vec{p} \sim m_t v \gg E_{kin} \sim m_t v^2$ $(\sim \Gamma_t \gg \Lambda_{QCD})$

"hard" "soft" "ultrasoft"

Problems:
- "Coulomb singularities"
- Large logs

Heavy quark-antiquark threshold

Problem of Coulomb singularities:



~ 1 $\sim \alpha_s/v$ $\sim (\alpha_s/v)^2$ $\sim (\alpha_s/v)^3$

Production threshold: $\boxed{\alpha_s \sim v \sim 0.1}$ $\Rightarrow$ breakdown of perturbation theory

Solution:

Nonrelativistic effective field theory → **vNRQCD**

→ summation of $(\alpha_s/v)^n$ terms using Schrödinger Equation !

vNRQCD

Problem of large logarithms:

$$3\text{ scales:}\quad m_t \gg \vec{p} \sim m_t v \gg E_{kin} \sim m_t v^2 \quad (\sim \Gamma_t \gg \Lambda_{QCD})$$

"hard" "soft" "ultrasoft"

→ Logs: $\ln\left(\frac{m^2}{E^2}\right),\ \ln\left(\frac{m^2}{\mathbf{p}^2}\right),\ \ln\left(\frac{\mathbf{p}^2}{E^2}\right)$ e.g. $\alpha_s \ln\left(\frac{m^2}{E^2}\right) \sim -\alpha_s \ln(v^4) \sim 1$

Solution:

Two renormalization scales: $\mu_s = m\nu,\ \mu_u = m\nu^2$ → "v"NRQCD

[Luke, Manohar, Rothstein; 2000]

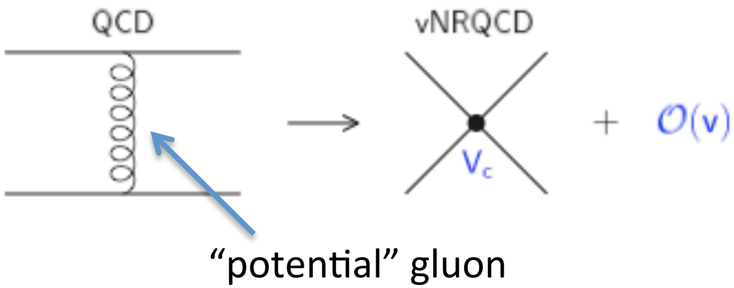
ν "subtraction velocity"

→ RGE's resum $[\alpha_s \ln v]^n$, $\alpha_s [\alpha_s \ln v]^n$, $\alpha_s^2 [\alpha_s \ln v]^n$ ... terms

LL NLL NNLL

vNRQCD

- Nonresonant dof's integrated out, e.g.:



- Resonant dof's $\rightarrow$ fields in the vNRQCD Lagrangian:

nonrel. quark:	$(E, \mathbf{p}) \sim (mv^2, mv)$	$\psi_{\mathbf{p}}(x)$	
soft gluon:	$(q_0, \mathbf{q}) \sim (mv, mv)$	$A_q(x)$	
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

- Systematic expansion in v $\Rightarrow$ consistent power counting in $v \sim \alpha_s$

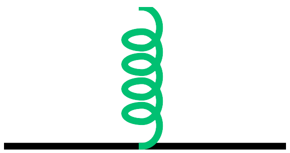
vNRQCD

[Luke, Manohar, Rothstein; 2000]

$$\boxed{\mathcal{L}_{\text{vNRQCD}} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}}$$

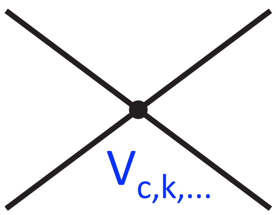
$$D^\mu = \partial^\mu + ig A^\mu(x)$$

$$\mathcal{L}_{\text{usoft}} : \quad \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \ldots \right]\psi_{\mathbf{p}}(x) + \ldots$$

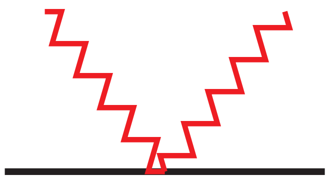


$$\mathcal{L}_{\text{pot}} : \quad - V \;\psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots$$



$\mathcal{L}_{\text{soft}}$:



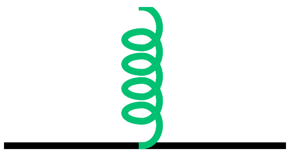
vNRQCD

[Luke, Manohar, Rothstein; 2000]

$$\boxed{\mathcal{L}_{\text{vNRQCD}} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}}$$

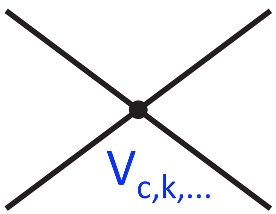
$$D^\mu = \partial^\mu + igA^\mu(x)$$

$$\mathcal{L}_{\text{usoft}}: \quad \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \ldots \right]\psi_{\mathbf{p}}(x) + \ldots$$

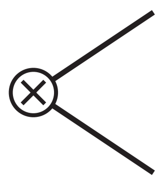


$$\mathcal{L}_{\text{pot}}: \quad -V \; \psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots$$



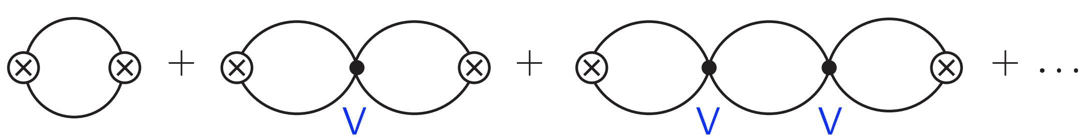
Production/annihilation current ($^3S_1$):



$$\sim \quad c_1(\nu) \cdot \underbrace{\vec{j}_1^{\,\text{eff}}(x)}_{\psi^\dagger_{\mathbf{p}}\vec{\sigma}(i\sigma_2)\chi^*_{-\mathbf{p}}} + \ldots$$

(CMS)

Renormalization

$$\sigma_{\text{tot}} \sim \text{Im}\Big[$$  $$+ \dots \Big]$$

$$\sim |c_1(\nu)|^2 \cdot \text{Im}\left[-i\int d^4x\ e^{i\hat{q}x}\,\langle 0|\,T\,\vec{j}_1^{\,\text{eff}\,*}(x)\,\vec{j}_1^{\,\text{eff}}(0)\,|0\rangle\right] + \dots$$

$$\sim |c_1(\nu)|^2 \cdot \text{Im}\left[G(0,0,E,\nu)\right] + \dots$$

LO SG: $\left[-\frac{\nabla^2_{\vec{r}}}{m} + V_c(r) - E\right] G(\vec{r},\vec{r}\,',E) = \delta^{(3)}(\vec{r}-\vec{r}\,')$

G^{NNLL} known ✓ [Hoang, Manohar, Stewart, Teubner; 2002] [Pineda, Signer; 2006]

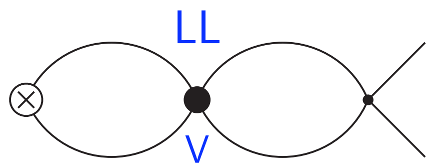
G^{NNNLO} known ✓ [Beneke, Kiyo, Schuller; 2007]

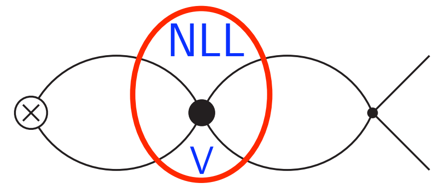
Renormalization

$$\sigma_{\mathrm{tot}} \sim |c_1(\nu)|^2 \cdot \mathrm{Im}\left[G(0,0,E,\nu)\right] + \ldots$$

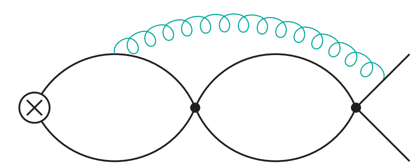
G^{NNLL} known ✓

current renormalization





$$\ln\left[\frac{c_1(\nu)}{c_1(1)}\right] = \underbrace{\xi^{\mathrm{LL}}}_{0} + \xi^{\mathrm{NLL}} + \xi^{\mathrm{NNLL}}_{\mathrm{mix}} + \xi^{\mathrm{NNLL}}_{\mathrm{nonmix}}$$



[Luke, Manohar, Rothstein; 2000]
[Pineda; 2002] [Hoang, Stewart; 2003]

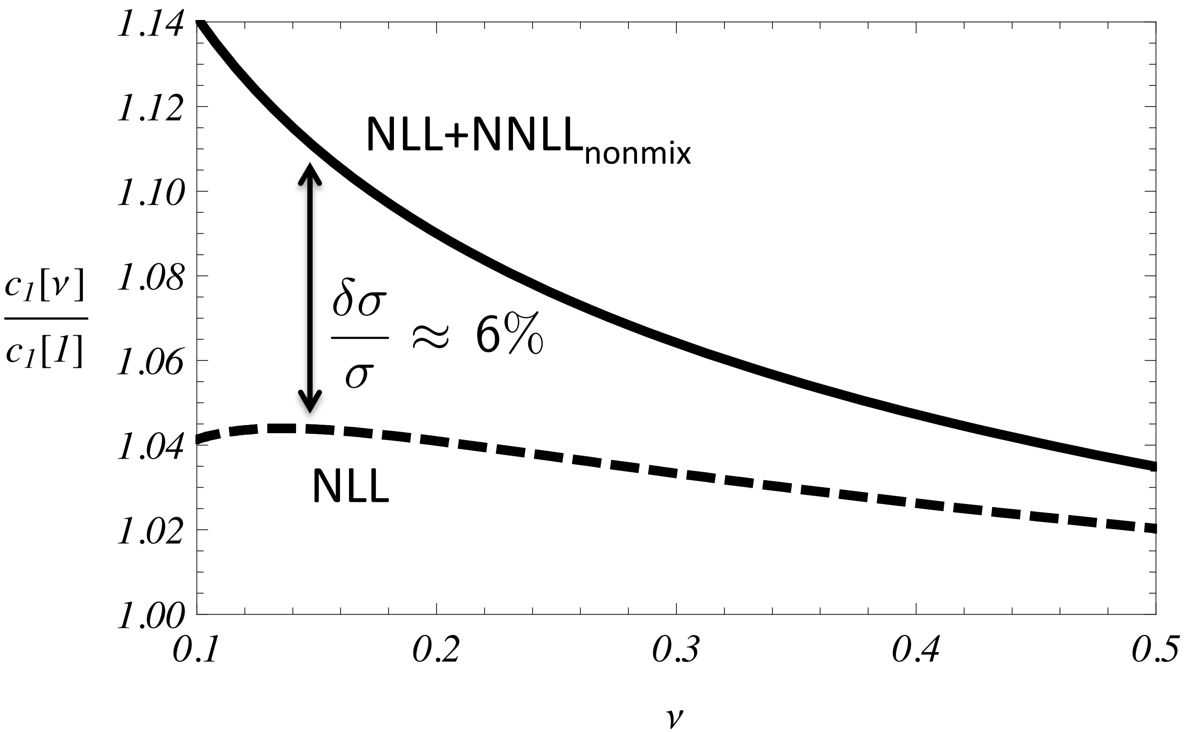
[Hoang; 2003]

missing

Renormalization

$$\sigma_{\text{tot}} \sim |c_1(\nu)|^2 \cdot \text{Im}\left[G(0,0,E,\nu)\right] + \ldots$$

G^{NNLL} known ✓



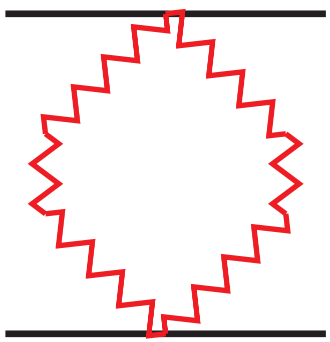
Missing NNLL_{mix} contribution may reduce th. error of σ_{tot} ?

$\Rightarrow$ $V^{\text{NLL}}(\nu)$ needed !

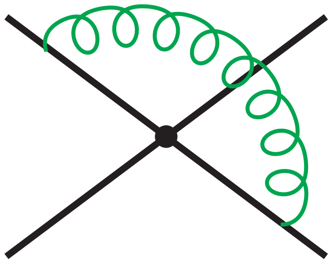
Renormalization

NLL: $$\nu \frac{\partial}{\partial \nu} \ln[\mathsf{c}_1(\nu)] = -\frac{\mathcal{V}_\mathsf{c}(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_\mathsf{c}(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_\mathsf{r}(\nu) + \mathbf{S}^2\,\mathcal{V}_\mathsf{s}(\nu)\right] + \frac{1}{2}\mathcal{V}_\mathsf{k}(\nu)$$

soft



ultrasoft

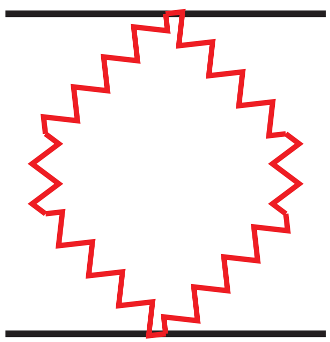


Renormalization

NLL: $$\nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

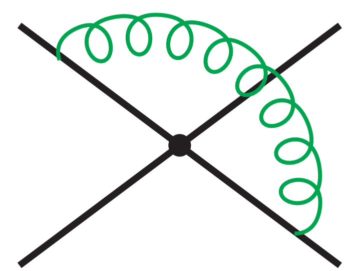
NLL corrections

soft

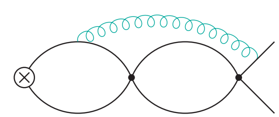


- known soft contributions small

ultrasoft



- dominant: $\alpha_s(\mathrm{mv}^2) > \alpha_s(\mathrm{mv})$
- large contribution to $\xi^{\mathrm{NNLL}}_{\mathrm{nonmix}}$



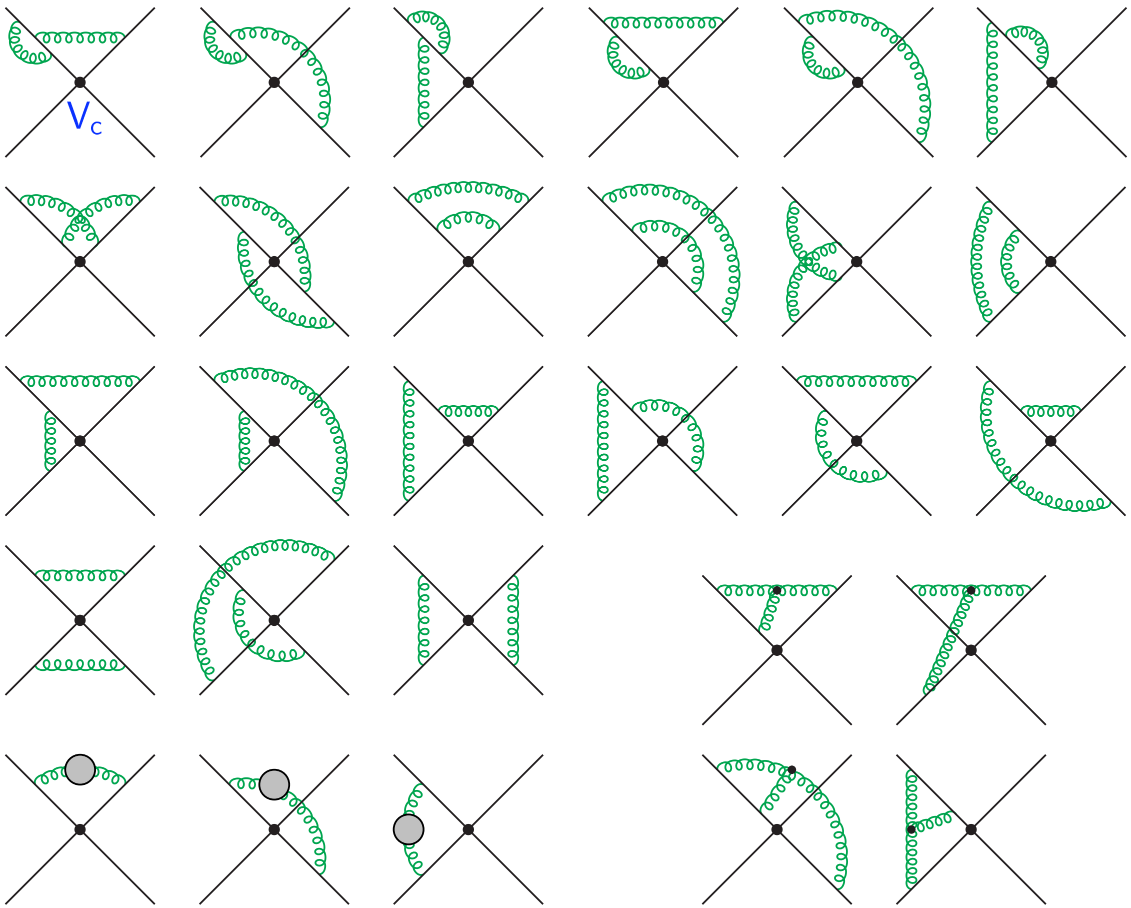
[Hoang; 2003]

Renormalization

NLL: $$\nu\frac{\partial}{\partial\nu}\ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

renormalize

Two usoft loops:



- Feynman gauge
- $\overline{\mathrm{MS}}$, dim. reg.
- $O(10^3)$ diagrams

$$\Rightarrow \quad \delta\mathcal{V}_{r,2}^{2\,\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_{r,2}^{\text{NLL}}(\nu)$$

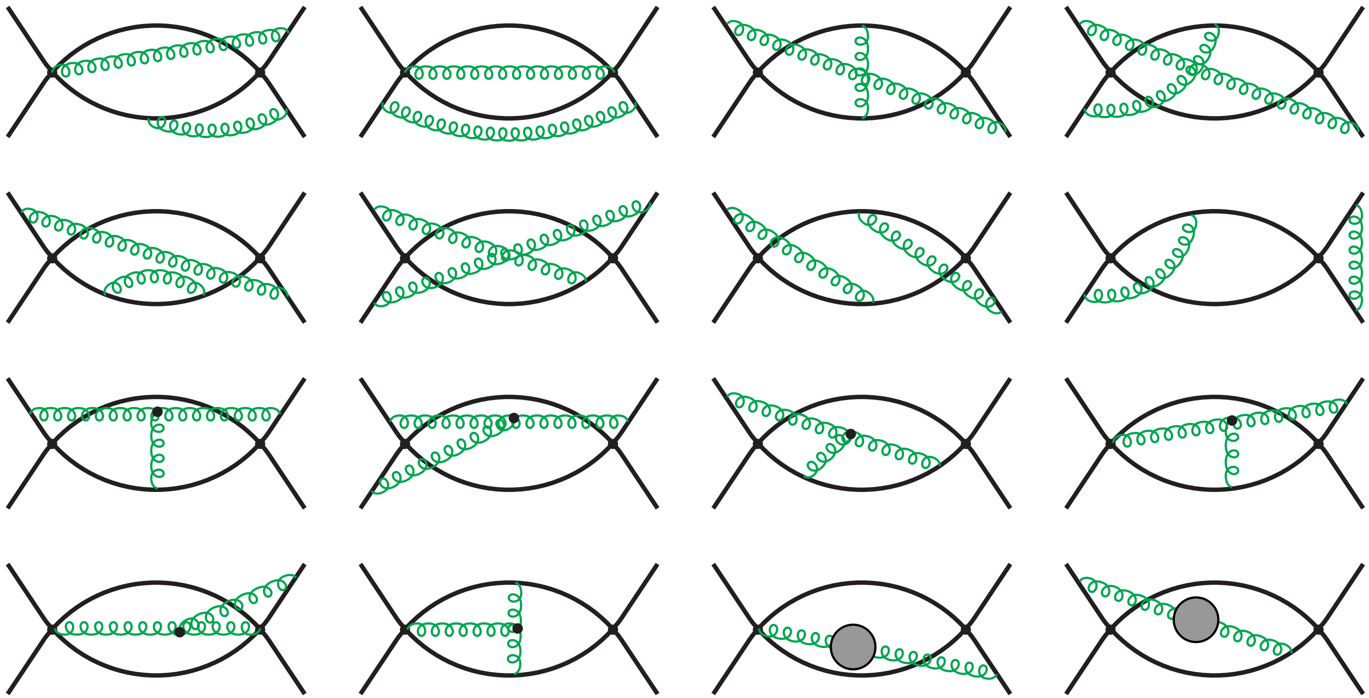
[MS, Hoang; hep-ph/0611292]

Renormalization

NLL: $$\nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2} \left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\, \mathcal{V}_s(\nu) \right] + \frac{1}{2} \mathcal{V}_k(\nu)$$

renormalize

Two usoft loops:



- 3 loops:
  2 x usoft
  1 x potential (finite)
- Feynman gauge
- $\overline{\mathrm{MS}}$, dim. reg.
- O(10^4) diagrams
- Generation:
  own **Mathematica** code
- Integrals:
  IBP & partial frac.

$\Rightarrow$ $\delta\mathcal{V}_k^{2\,\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_k^{\text{NLL}}(\nu)$ [MS, Hoang, 1102.0269 [hep-ph]]

Results

RGE's + matching at hard scale $(\nu = 1)$ give:

LL | NLL

$$\left[\mathcal{V}_2(\nu)\right]^{\rm NLL}_{\rm usoft} = 4\pi\alpha_s(m\nu)\left[-\frac{4\pi}{\beta_0}A_2 \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_2 - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_2\right)\right]$$

$$\left[\mathcal{V}_r(\nu)\right]^{\rm NLL}_{\rm usoft} = 8\pi\alpha_s(m\nu)\left[-\frac{4\pi}{\beta_0}A_r \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_r - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_r\right)\right]$$

$$\left[\mathcal{V}_k(\nu)\right]^{\rm NLL}_{\rm usoft} = 2\alpha_s^2(m\nu)\left[-\frac{4\pi}{\beta_0}A_k \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_k - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_k\right)\right]$$

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = C_F(C_A - 2C_F)\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ B_r \end{bmatrix} = -C_A C_F\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A_k \\ B_k \end{bmatrix} = -C_A C_F(C_A - 2C_F)\begin{bmatrix} A \\ B \end{bmatrix}$$

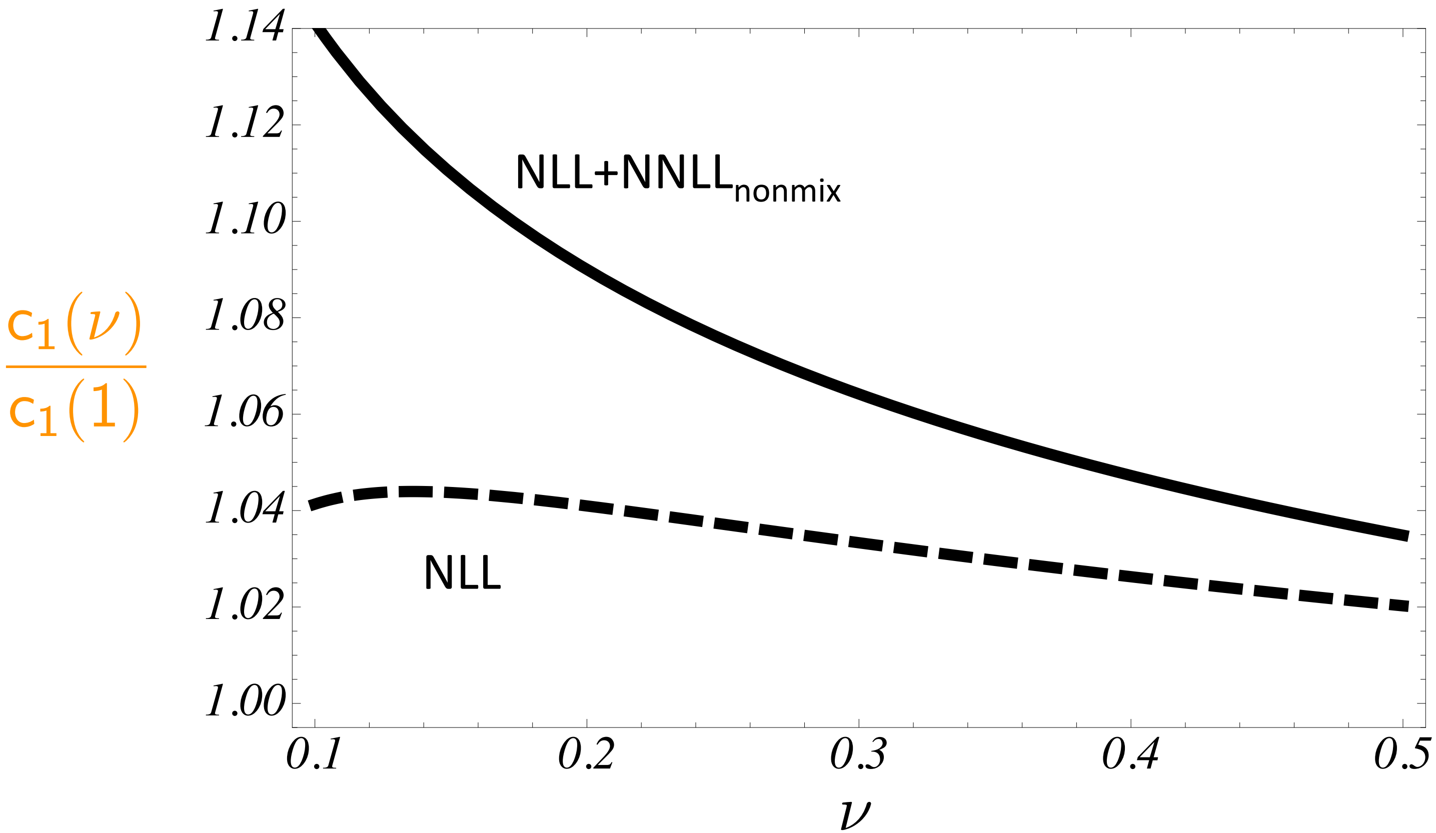
$$A = \frac{1}{3\pi}$$

$$B = \frac{C_A(47 + 6\pi^2) - 10 n_f T}{108\pi^2}$$

[MS, Hoang; 2011]

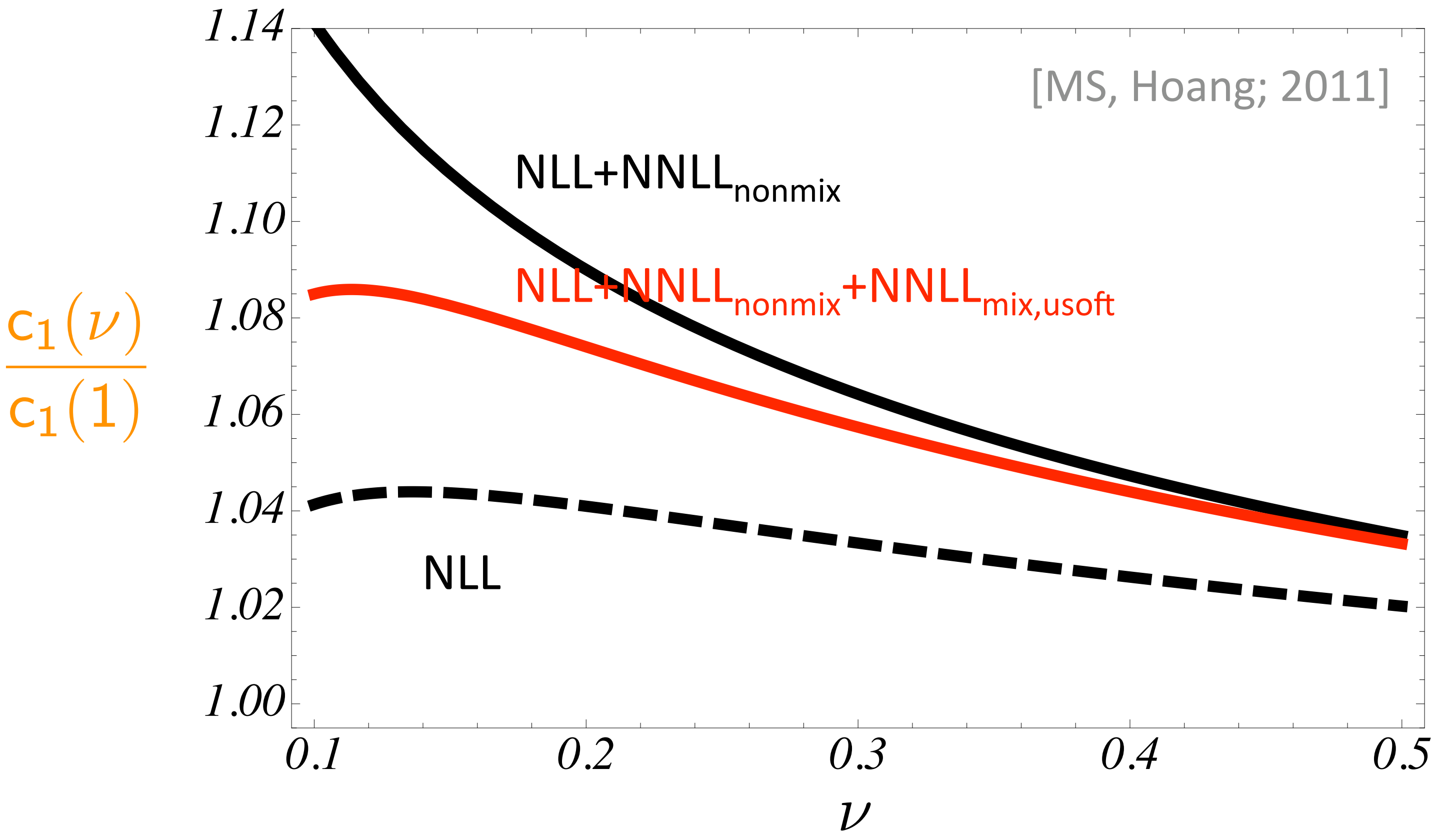
[Pineda; 2011]

Results



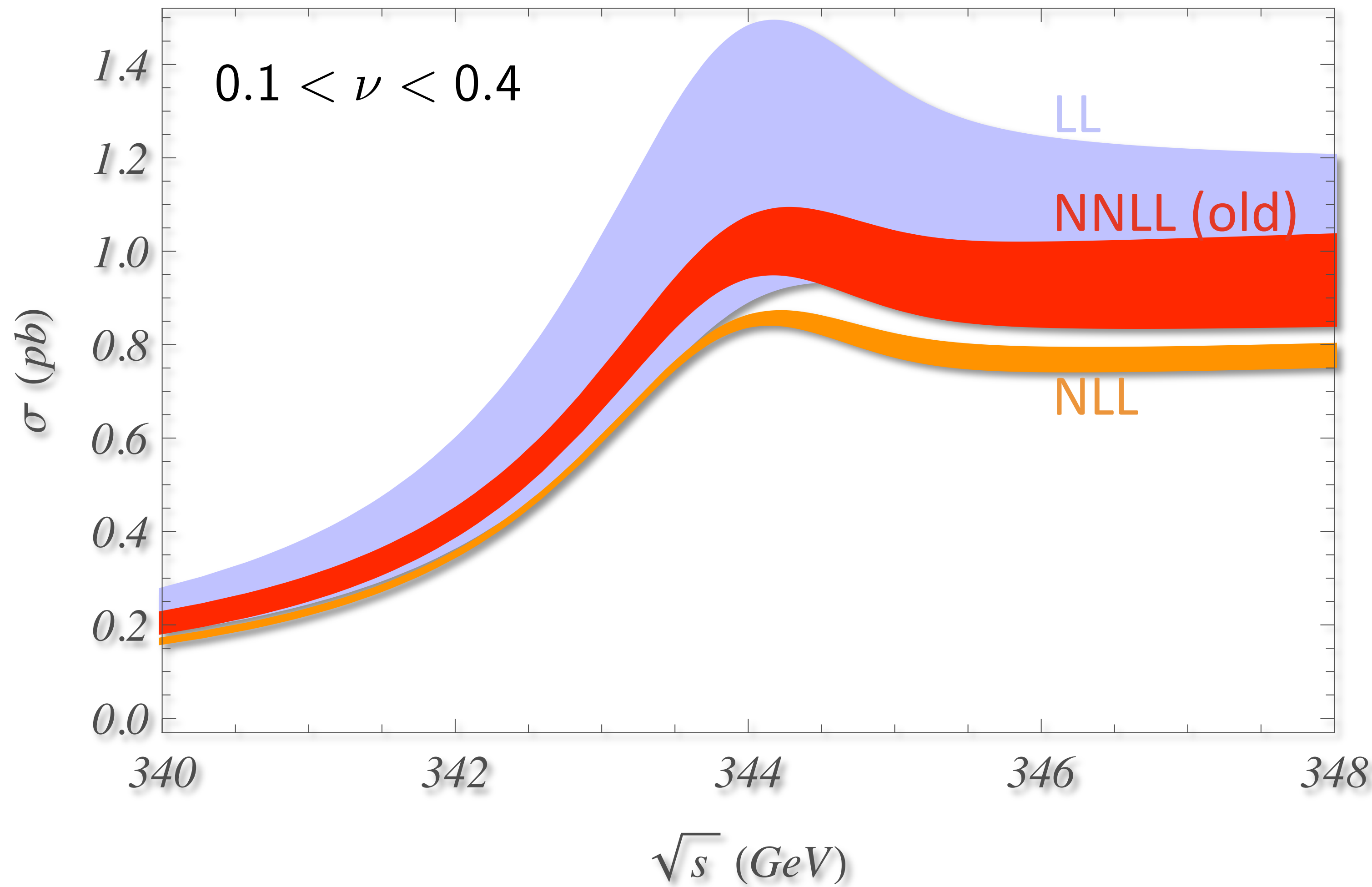
1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
$\frac{c_1(\nu)}{c_1(1)}$
NLL+NNLL$_{nonmix}$
NLL
0.1
0.2
0.3
0.4
0.5
ν

Results



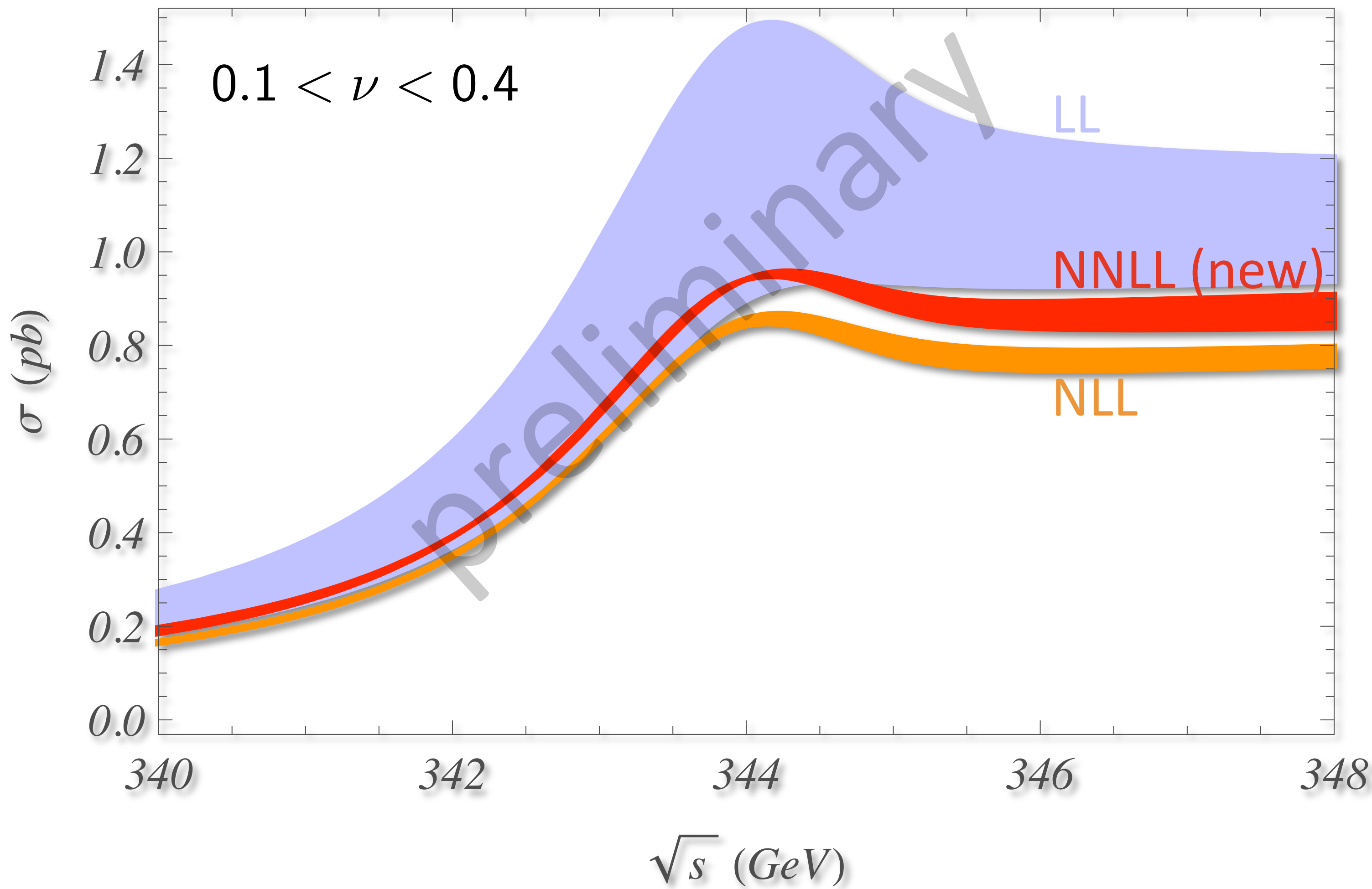
- large usoft NNLL contributions compensate each other!
- good convergence: $[c_1(\nu = 0.15)]^2 = 1 - 0.096|_{\text{NLL}} - 0.029|_{\text{NNLL}}$

Results



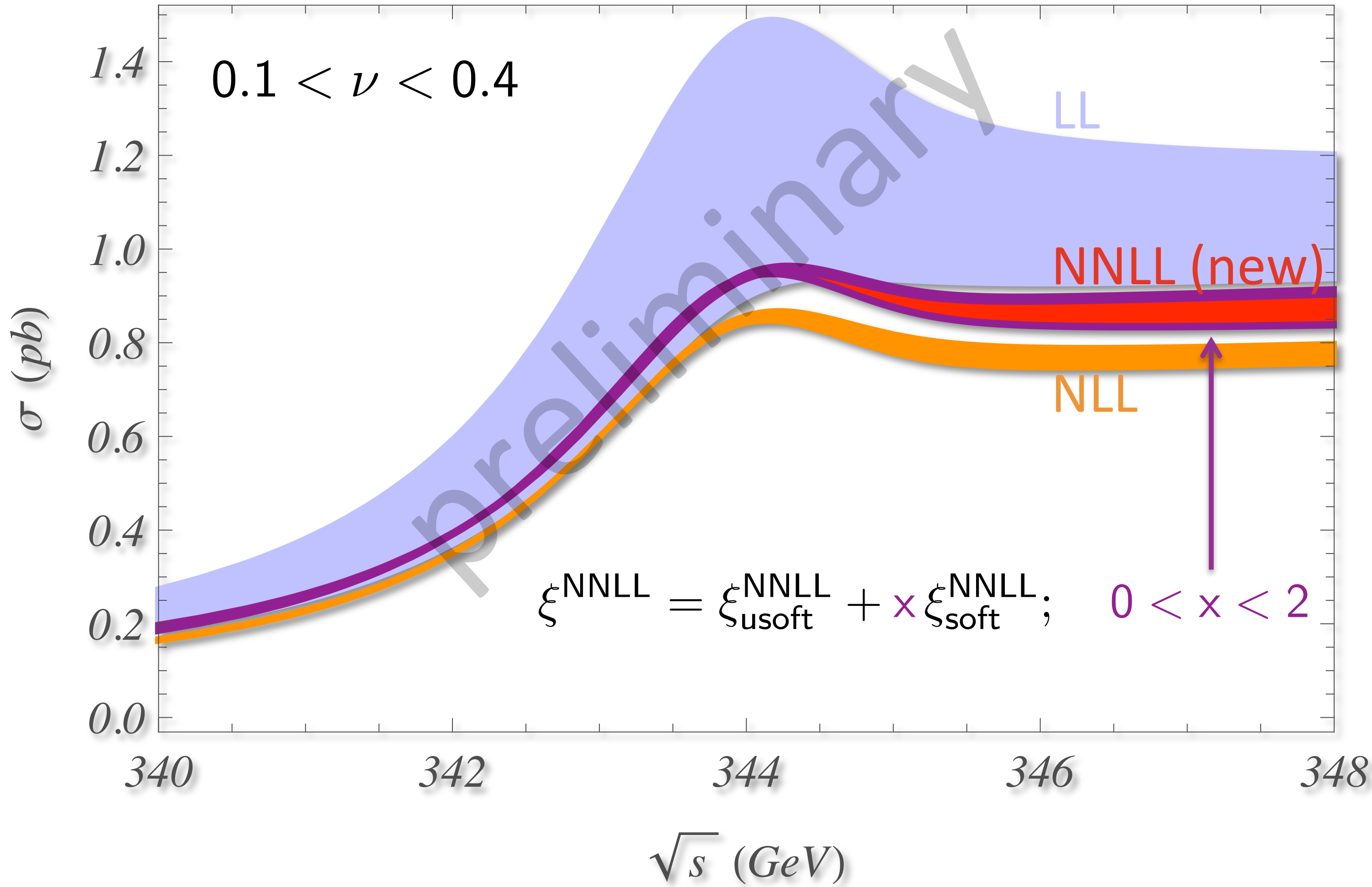
$0.1 < \nu < 0.4$
LL
NNLL (old)
NLL
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
σ (pb)
340
342
344
346
348
$\sqrt{s}$ (GeV)

Results



0.1 < ν < 0.4
LL
NNLL (new)
NLL
preliminary
σ (pb)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
340
342
344
346
348
√s (GeV)

Results



- known soft NNLL contributions small
- detailed error analysis (incl. EW effects, variation of matching scale, ...) → WIP

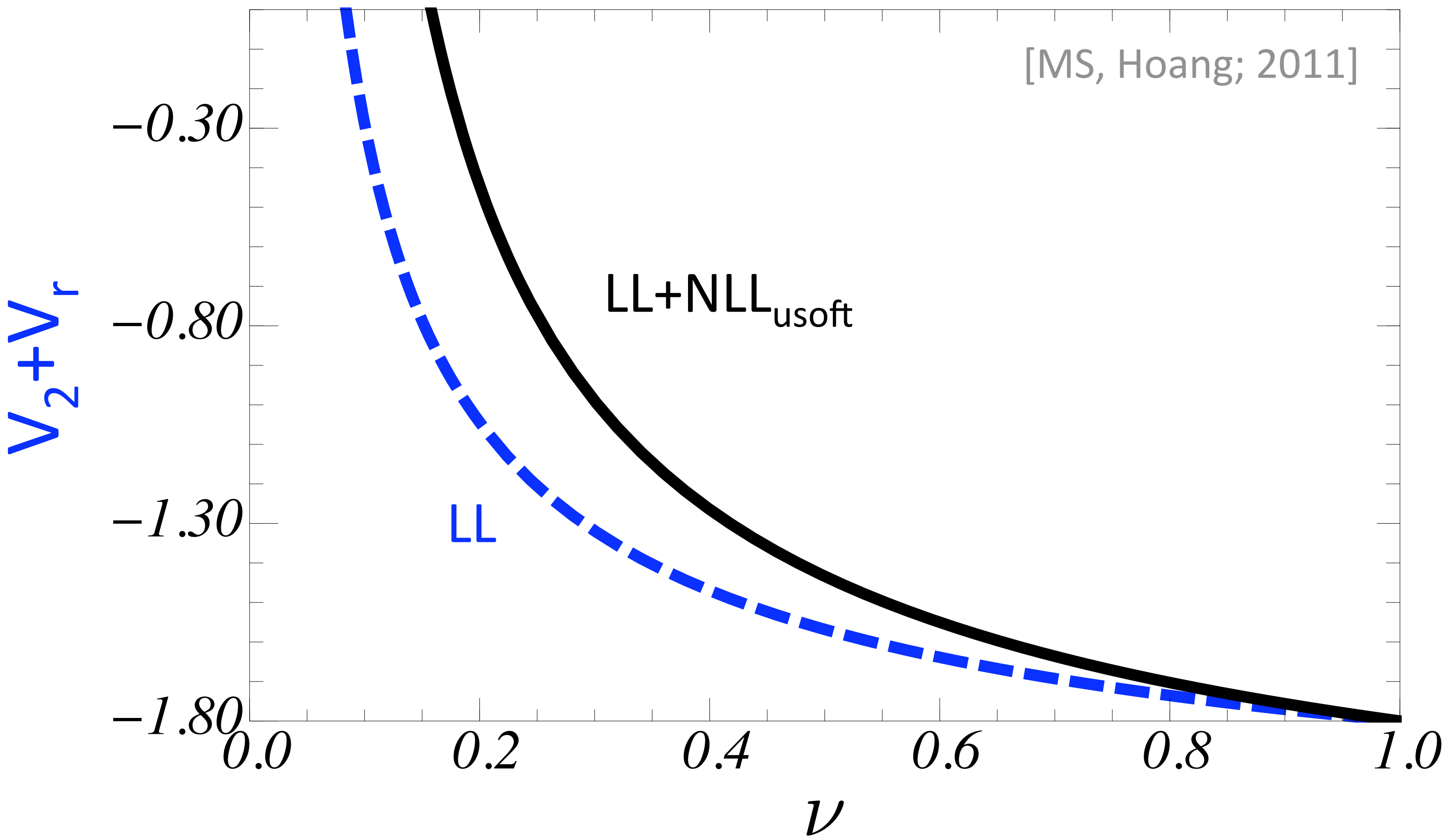
Summary/Outlook

- precise $m_t, y_t, \alpha_s, \Gamma_t$ from $\sigma_{tot}(e^+e^- \to t\bar{t})$ at threshold
- $\sigma_{tot} \sim |c_1(\nu)|^2 \cdot \mathrm{Im}\left[G(0,0,E,\nu)\right] + \ldots$
- $G(0,0,E,\nu)$ known up to NNLL ✓ EW conributions up to NNLL ✓
- New $\mathrm{NNLL}_{\mathrm{mix,usoft}}$ compensates for large $\mathrm{NNLL}_{\mathrm{nonmix}}$ contribution to $c_1(\nu)$

→ $\frac{\delta\sigma_{tot}^{th}}{\sigma_{tot}}$ decreases substantially! → RG Improvement important!

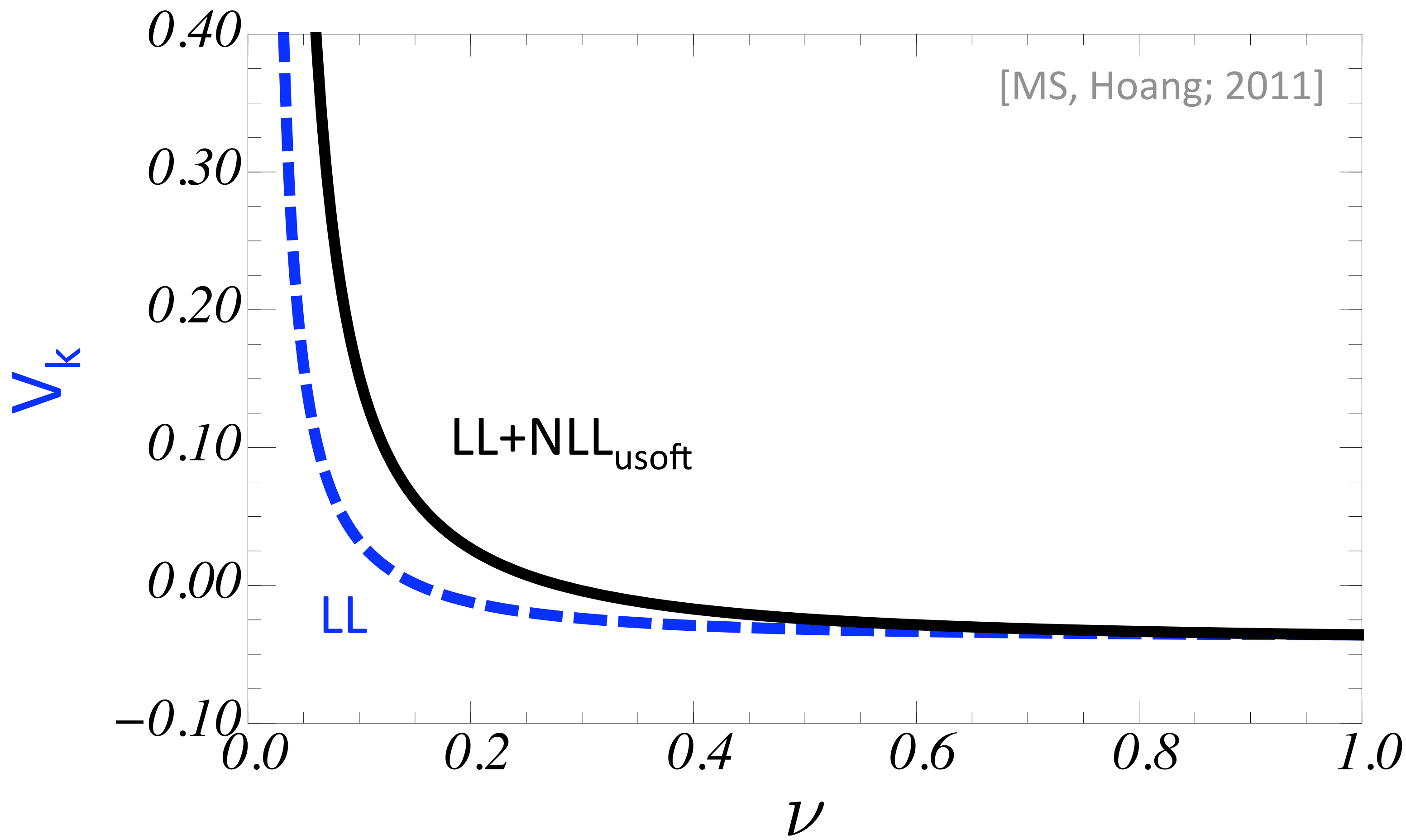
- Outlook:
  - Detailed error analysis for $\sigma_{tot}(e^+e^- \to t\bar{t})$ at threshold
  - Determination of bottom mass from nonrel. Υ sum rules

Backup



[MS, Hoang; 2011]
−0.30
−0.80
−1.30
−1.80
0.0
0.2
0.4
0.6
0.8
1.0
ν
V2+Vr
LL+NLLusoft
LL

Backup



0.40
0.30
0.20
0.10
0.00
−0.10
0.0
0.2
0.4
0.6
0.8
1.0
ν
V_k
[MS, Hoang; 2011]
LL+NLL$_{usoft}$
LL