# Higher order QCD corrections for the Drell-Yan process


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# Outline

1. The Drell-Yan (DY) process
2. A fully differential DY NNLO computation
3. $W$ and lepton charge asymmetry at NNLO
4. Associated $W$-Higgs production at NNLO
5. DY transverse-momentum resummation at full NNLL+NLO
6. Conclusions and Perspectives

# Motivations

The Drell-Yan process [Drell,Yan('70)] is the most "classical" hard-scattering process in hadron-hadron collisions. Its study is well motivated:

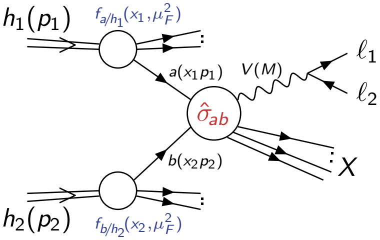
- Large production rates and clean experimental signatures.
- Constraint for fits of PDFs.
- Precise prediction for $M_W$ and Beyond the Standard Model analysis.
- Test of perturbative QCD predictions.

The above reasons and precise experimental data demands for accurate theoretical predictions $\Rightarrow$ computation of higher-order QCD corrections.

# The Drell-Yan (DY) process

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M) + X \;\rightarrow\; \ell_1 + \ell_2 + X$$

where $\quad V = \gamma^*, Z^0, W^\pm \quad$ and $\quad \ell_1\ell_2 = \ell^+\ell^-, \ell\nu_\ell$




According to the QCD factorization theorem:

$$d\sigma(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)\, d\hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; \mu_F^2).$$

$$d\hat{\sigma}_{ab}(\hat{p}_1, \hat{p}_2; \mu_F^2) \;=\; d\hat{\sigma}^{(0)}_{ab}(\hat{p}_1, \hat{p}_2; \mu_F^2) \;+\; \alpha_S(\mu_R^2)\; d\hat{\sigma}^{(1)}_{ab}(\hat{p}_1, \hat{p}_2; \mu_F^2)$$
$$+\; \alpha_S^2(\mu_R^2)\; d\hat{\sigma}^{(2)}_{ab}(\hat{p}_1, \hat{p}_2; \mu_F^2, \mu_R^2) \;+\; \mathcal{O}(\alpha_S^3)\,.$$

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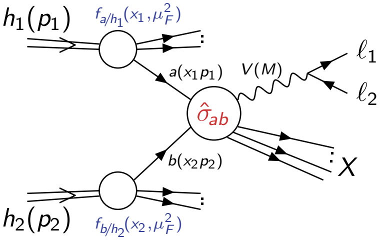


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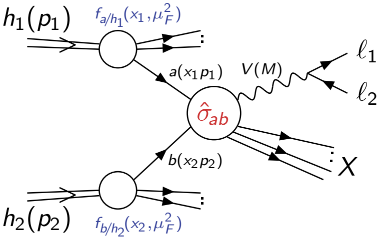


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# State of the art: fixed order perturbative calculations

- QCD corrections:
  - Total cross section known up to NNLO [Hamberg,Van Neerven,Matsuura('91)], [Harlander,Kilgore('02)]
  - Rapidity distribution known up to NNLO [Anastasiou,Dixon,Melnikov,Petriello('03)]
  - Fully exclusive NNLO calculation completed [Melnikov,Petriello('06)], [Catani,Cieri,de Florian,G.F.,Grazzini('09)]
  - Vector boson transverse-momentum distribution known up to NLO [Ellis,Martinelli,Petronzio('83)],[Arnold,Reno('89)], [Gonsalves,Pawlowski,Wai('89)]
- Electroweak corrections are know at $\mathcal{O}(\alpha)$ [Dittmaier,Kramer('02)],[Baur,Wackeroth('02)], [Carloni Calame,Montagna,Nicrosini,Vicini('06)]

# A fully differential Drell-Yan NNLO computation

# DY fully differential NNLO computation

Catani,Cieri,de Florian,G.F.,Grazzini arXiv:0903.2120

- A NNLO extension of the subtraction formalism valid for the production of colourless high-mass system in hadron collisions was proposed and applied for Higgs boson production [Catani,Grazzini('07)].
- This method was used to perform a fully exclusive NNLO calculation for vector boson production. An analogous computation exists [Melnikov,Petriello('06)].
- The calculation is implemented in a parton level Monte Carlo and includes the $\gamma$-$Z$ interference, finite-width effects, the leptonic decay of the vector bosons and the corresponding spin correlations.
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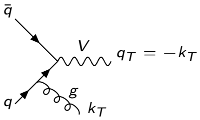
# A NNLO extension of the subtraction method

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M, q_T) + X$$

$V$ is one or more colourless particles (vector bosons, leptons, photons, Higgs bosons,...) [Catani,Grazzini('07)].




- Key point I: at LO the $q_T$ of the $V$ is exactly zero.

$$d\sigma^{V}_{(N)NLO}|_{q_T \neq 0} = d\sigma^{V+\text{jets}}_{(N)LO} \quad ,$$

for $q_T \neq 0$ the NNLO IR divergences cancelled with the NLO subtraction method.

- The only remaining NNLO singularities are associated with the $q_T \to 0$ limit.
- Key point II: treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from $q_T$ resummation formalism [Catani,de Florian,Grazzini('00)].

$$d\sigma^{V}_{N^nLO} \xrightarrow{q_T \to 0} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2 = d\sigma^{V}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} d^2 q_T$$

$$d\sigma^{CT} \xrightarrow{q_T \to 0} d\sigma^{V}_{LO} \otimes \Sigma(q_T/M) dq_T^2$$

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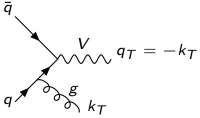


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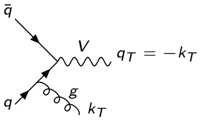


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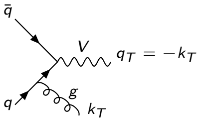


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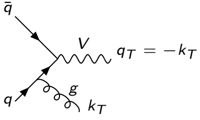


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$$\textit{where} \quad \mathcal{H}^{V}_{NNLO} = \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}^{V(1)} + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{V(2)} \right]$$

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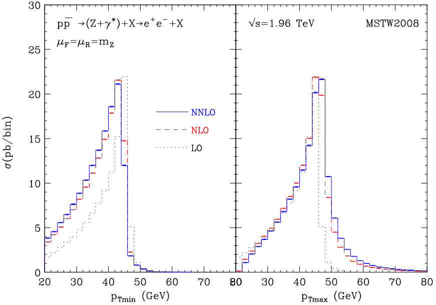
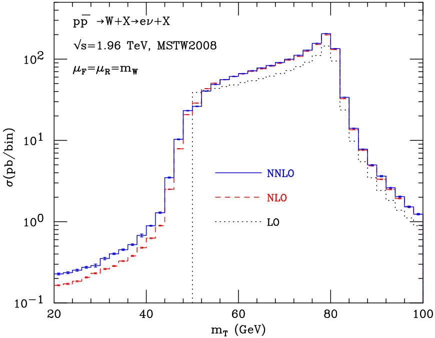
- NLO calculation requires $d\sigma^{V+\mathrm{jets}}_{LO}$ and $\mathcal{H}^{V(1)}$ [de Florian, Grazzini('01)].
- At NNLO we need also $d\sigma^{V+\mathrm{jets}}_{NLO}$ [Giele et al.('93), MCFM] and $\mathcal{H}^{V(2)}$ [Catani,Cieri,de Florian,Grazzini,G.F.: [arXiv:0903.2120]].




Minimum (left) and maximum (right) lepton $p_T$ distribution for $Z$ production at the Tevatron.
The error bars in the histograms refer to the Monte Carlo numerical errors.

Cuts: $p_{T\,min} \geq 20\ GeV$ ; $|\eta| < 2$ ;
$70\ GeV \leq m_{e^+e^-} \leq 110\ GeV$




Transverse mass distribution for $W$ production at the Tevatron:

$$m_T = \sqrt{2p_T^l p_T^{miss}(1 - \cos\phi_{l\nu})}$$

Cuts: $p_T{}^{miss} \geq 25\ GeV$ ; $|\eta| < 2$ ;
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# $W$ and lepton charge asymmetry at NNLO

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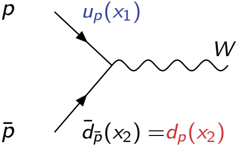
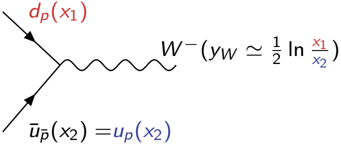
Catani,G.F.,Grazzini arXiv:1002.3115

Consider $p\bar{p}$ collisions: the LO cross section is controlled by the partonic subprocesses

$$U + \overline{D} \to W^+ \to l^+ \nu_l \ , \qquad D + \overline{U} \to W^- \to l^- \bar{\nu}_l \ ,$$




$$\Rightarrow \sigma_{W^+}(y_W \simeq \tfrac{1}{2} \ln \tfrac{x_1}{x_2}) \sim u_p(x_1) d_p(x_2) \qquad \sigma_{W^-}(y_W \simeq \tfrac{1}{2} \ln \tfrac{x_1}{x_2}) \sim u_p(x_2) d_p(x_1)$$

IF $u_p$ in proton moves (on average) faster than $d_p$
( i.e. $u(x) > d(x)$ for $x \sim \frac{M_W}{\sqrt{s}} \simeq 0.04$)
THEN $W^+(W^-)$ produced mainly in proton (antiproton) direction.

Mainly sensitivity to $u(x) - d(x)$ proton density

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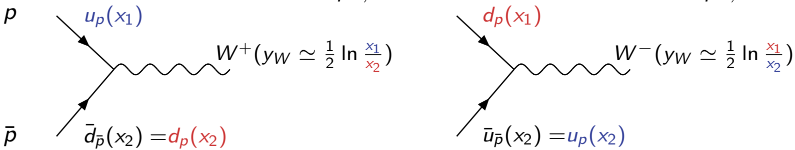


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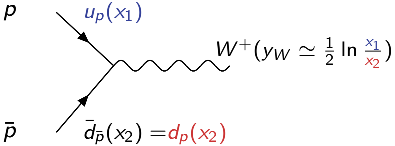


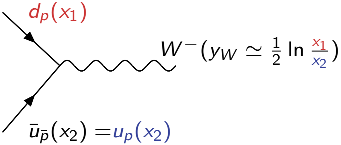


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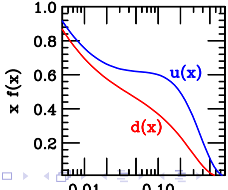
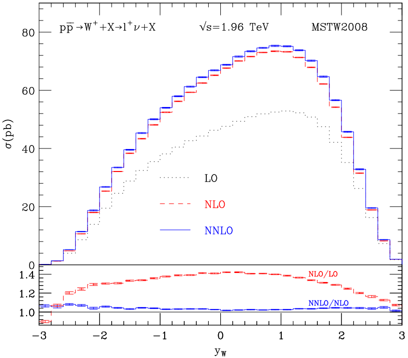


# W rapidity distribution




Rapidity distribution of an on-shell $W^+$ boson at the Tevatron Run II in LO (PDFs at LO, 1-loop $\alpha_S$), NLO (PDFs at NLO, 2-loops $\alpha_S$) and NNLO (PDFs at NNLO, 3-loop $\alpha_S$) QCD.

- Owing to CP invariance $W$ rapidity distribution in $p\bar{p}$ collisions fulfills $d\sigma(W^+)/dy_W = d\sigma(W^-)/d(-y_W)$
- No cuts are applied on final states.
- In the lower panel we show the K-factors $K_{(N)NLO}(y) = \left[d\sigma/dy\right]_{(N)NLO} \Big/ \left[d\sigma/dy\right]_{(N)LO}$.
- In the rapidity region $|y_W| \lesssim 2$:
  $K_{NLO}(y_W) \sim\ 1.3 - 1.4$
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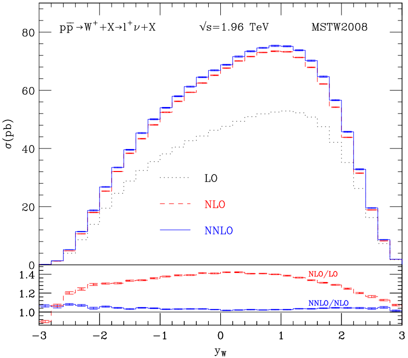


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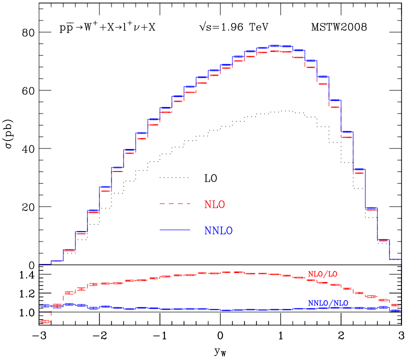


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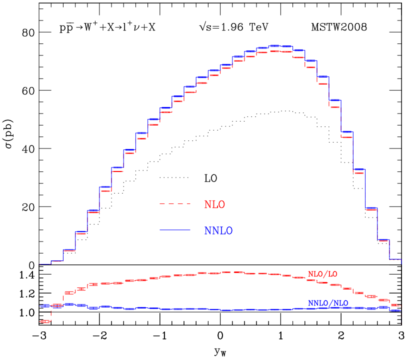


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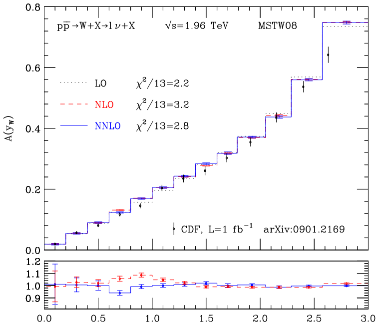


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Lower panel: NLO and NNLO K-factors.

- CDF data on W asymmetry [arXiv:0901.2169]: no selection cuts on final states.
- Very stable perturbative predictions:
  $K_{NLO}(y_W) \sim 0.98 - 1.08$
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- Good agreement between data and theory:

$$\frac{\chi^2}{N_{\rm pts.}} = \frac{1}{N_{\rm pts.}} \sum_{i=1} \frac{(\mathrm{th}_i - \exp_i)^2}{\Delta^2_{i,\exp}} .$$

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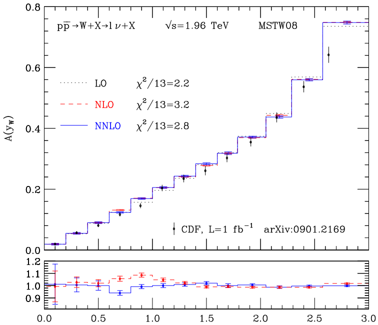


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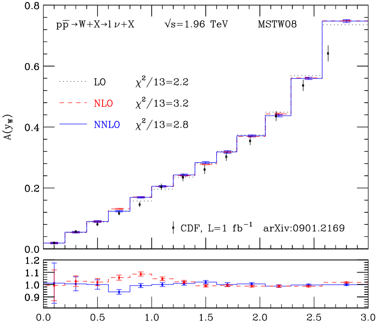


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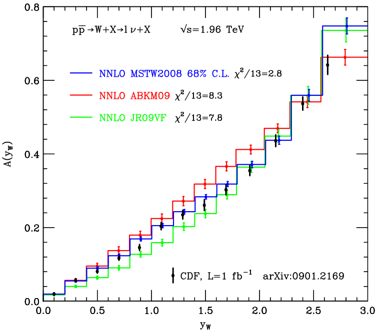


The NNLO $W$ charge asymmetry at the Tevatron with MSTW08 (blue), ABKM09 (red) and JR09VF (green) PDFs with errors compared with CDF data. Now also NNPDF partons available at NNLO.

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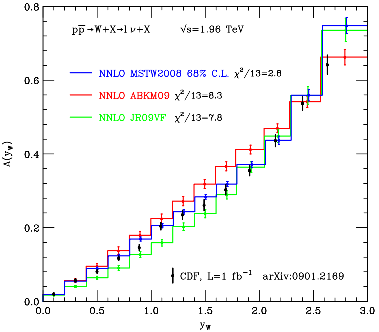


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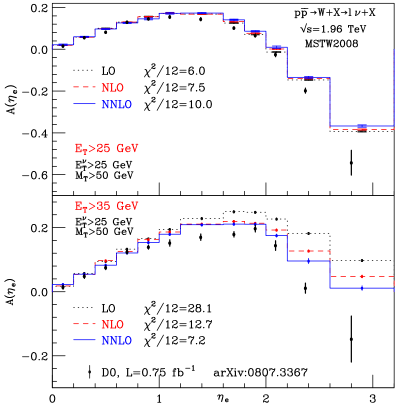


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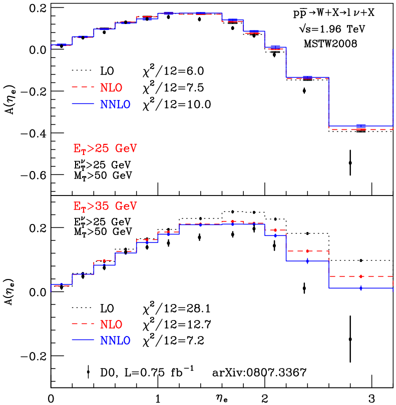


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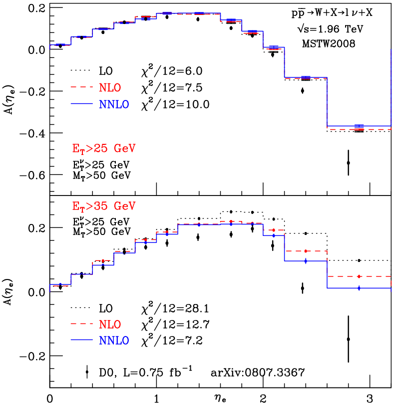


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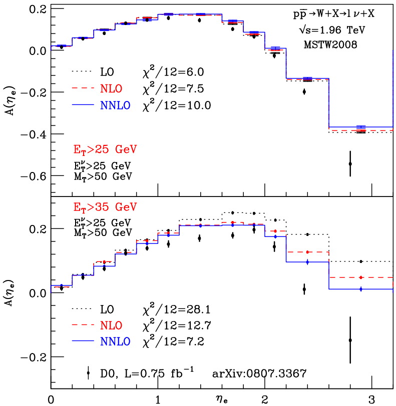


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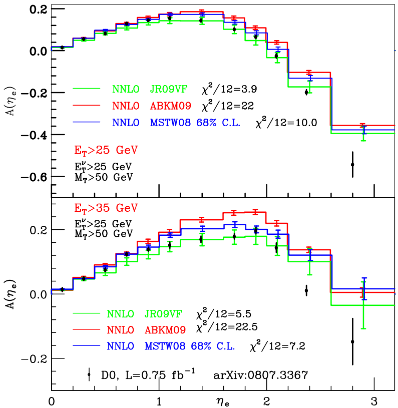


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# Associated $W$-Higgs production at NNLO

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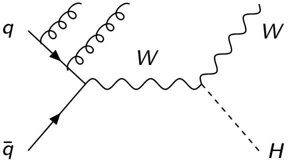
G.F.,Grazzini,Tramontano arXiv:1107.1164




It's a DY-like process

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- At the LHC: promising search mode through boosted analysis with jet reconstruction and decomposition techniques [Butterworth et al.('08)].
- Already known: NLO QCD corrections [Han et al.('90)], EW corr. [Ciccolini et al.('03)] and (for total cross section only) NNLO QCD corr. [Brein et al.('03)].
- We include in a fully exclusive parton level MC code the DY-like NNLO corrections (additional heavy-quark loop diagrams eximated to give a contribution $< 1\%$ ).
- We include $H \to b\bar{b}$ and $W \to l\nu$ decays with spin correlations.

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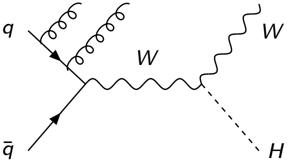


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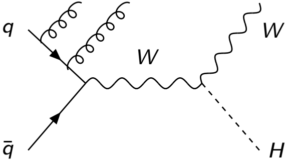
G.F.,Grazzini,Tramontano arXiv:1107.1164

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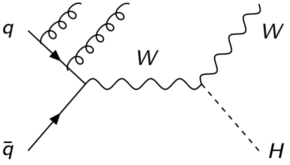


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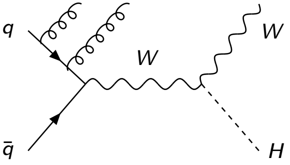


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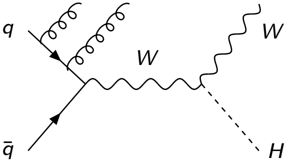


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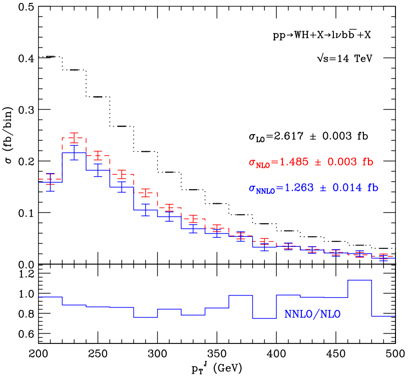


$p_T$ spectra of the fat jet at the LHC for $m_H = 120 GeV$ at LO (dots), NLO (dashes) and NNLO (solid).

- Selection strategy of [Butterworth et al.('08)]: search a large-$p_T$ Higgs boson thorough a collimated $b\bar{b}$ pair decay.
  Cuts:
  Leptons: $p_T^l > 30 GeV$, $|\eta^l| < 2.5$, $p_T^{miss} > 30 GeV$, $p_T^W > 200 GeV$.
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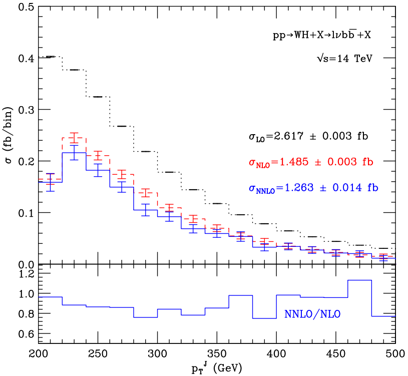


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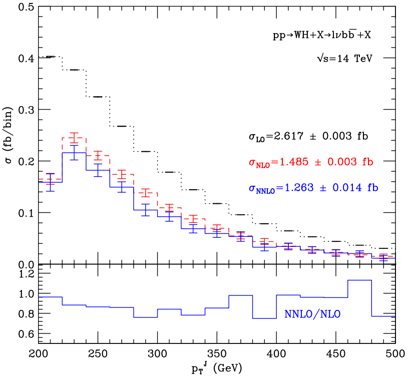


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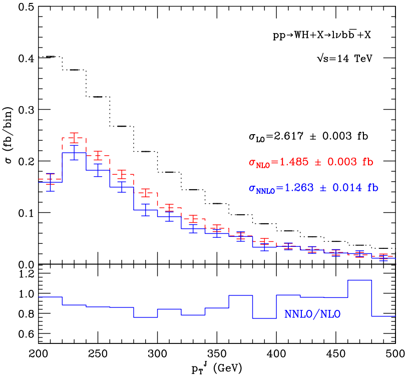


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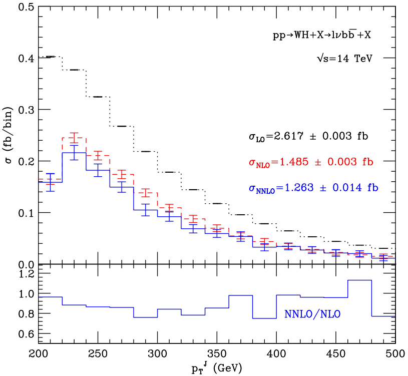


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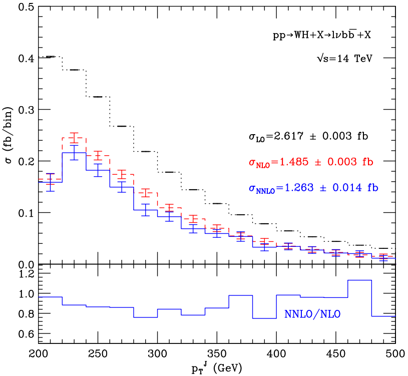


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# Drell-Yan transverse-momentum resummation at full NNLL+NLO

# State of the art: transverse-momentum ($q_T$) resummation

- The method to perform the resummation of the large logarithms of $q_T$ is known
  [Dokshitzer,Diakonov,Troian ('78)], [Parisi,Petronzio('79)], [Kodaira,Trentadue('82)],[Altarelli et al.('84)], [Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)] [Catani,Grazzini('10)]
- Various phenomenological studies of the vector boson transverse momentum distribution exist
  [Balasz,Qiu,Yuan('95)],[Balasz,Yuan('97)],[Ellis et al.('97)], [Kulesza et al.('02)]
- Recently various results for transverse momentum resummation in the framework of Effective Theories appeared [Gao,Li,Liu('05), Idilbi,Ji,Yuan('05), Mantry,Petriello('10), Becher,Neubert('10)].

# DY $q_T$ resummation at NNLL+NLO:

Bozzi,Catani,de Florian,G.F., Grazzini arXiv:1007.2351

- We have applied for Drell-Yan transverse-momentum distribution the resummation formalism developed by [Catani,de Florian, Grazzini('01)] already applied for the case of Higgs boson production [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)].
- We have performed the resummation up to NNLL+NLO. It means that our complete formula includes:
  - NNLL logarithmic contributions to all orders;
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# Transverse momentum resummation

$$\frac{d\hat{\sigma}_{ab}}{dq_T^2} = \frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2}; \qquad \int_0^{Q_T^2} dq_T^2 \Big[\frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2}\Big]_{f.o.} \overset{Q_T\to 0}{=} 0\,;$$

$$\int_0^{Q_T^2} dq_T^2 \Big[\frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2}\Big]_{f.o.} \overset{Q_T\to 0}{\sim} 1 + \sum_n \sum_{m=1}^{2n} c_{nm}\, \alpha_S^n \log^m\!\Big(\frac{M^2}{Q_T^2}\Big).$$

Resummation holds in impact parameter space:

$$\frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int_0^\infty db\, \frac{b}{2} J_0(bq_T)\, \mathcal{W}_{ab}(b, M), \qquad q_T \ll M \Leftrightarrow Mb \gg 1, \quad \log M^2/q_T^2 \gg 1 \Leftrightarrow \log Mb \gg 1$$

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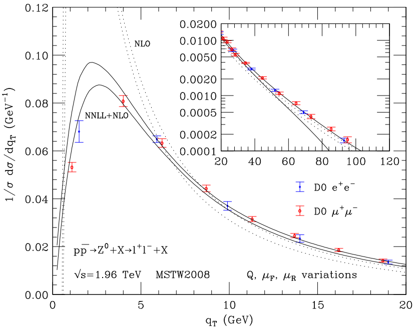


- D0 data compared with our NNLL+NLO result.
- The NNLL+NLO band obtained varying $\mu_R$, $\mu_F$, $Q$ independently: $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$ to avoid large logarithmic contributions ($\sim \ln(\mu_F^2/\mu_R^2)$, $\ln(Q^2/\mu_R^2)$) in the evolution of the parton densities and in the the resummed form factor.
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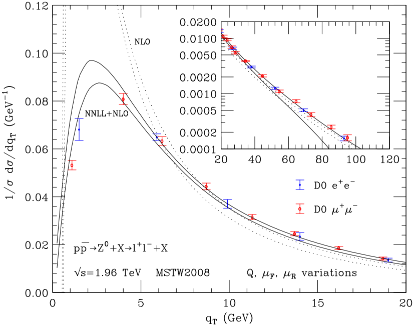


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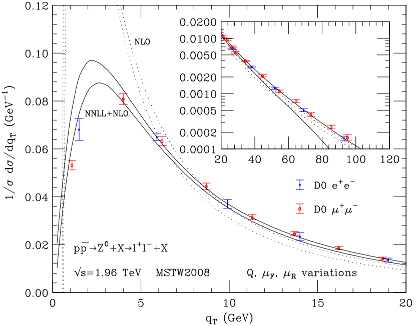


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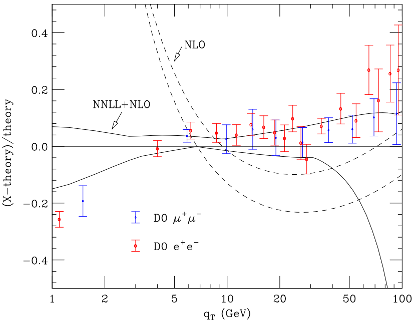


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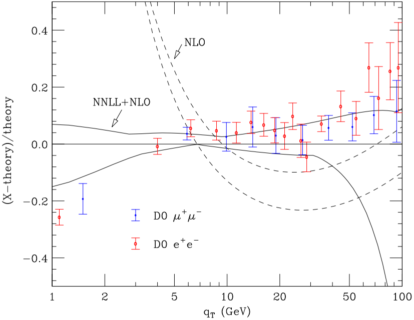


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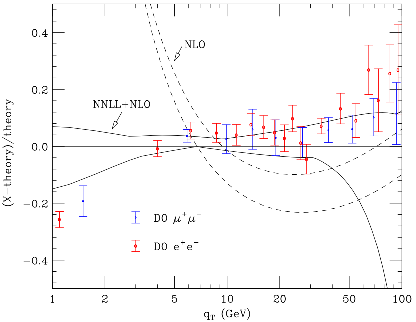


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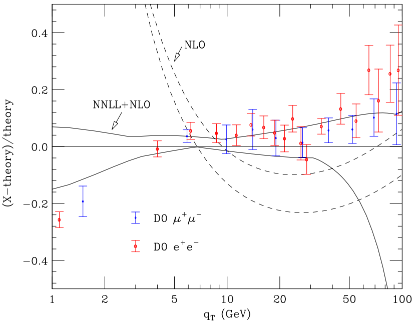


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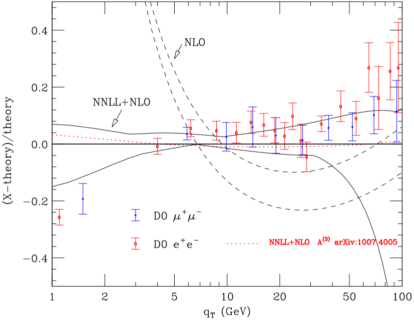


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  Perspectives: extend to the $Z$-Higgs case.
- NNLL+NLO DY $q_T$-resummation [Bozzi,Catani,de Florian,G.F., Grazzini [arXiv:1007.2351]].
  Reduction of scale uncertainties from NLL+LO to NNLL+NLO accuracy. The NNLL+NLO results (without any non-perturbative effects) consistent with the experimental data in a wide region of $q_T$.
  Perspectives: add the dependence on the vector boson rapidity and on the decay leptons variables, compare with LHC data.

# Conclusions and Perspectives

- DY fully exclusive NNLO QCD calculation [Catani,Cieri,de Florian,G.F.,Grazzini: [arXiv:0903.2120]]: implemented in a parton level Monte Carlo code, DYNNLO, which allows to apply arbitrary kinematical cuts and to compute distributions in histograms.
- $W$ and lepton charge asymmetry up to NNLO QCD, [Catani,G.F.,Grazzini [arXiv:1002.3115]] with different PDFs and compared with Tevatron data: lepton asymmetry is more sensitive to the partonic dynamics and kinematics. Perspectives: compare with LHC data.
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Back up slides

- The general relation between $\mathcal{H}^{V(2)}$ and the IR finite part of the two-loops correction to a generic process is unknown. We explicit computed it for the DY process with the following method.

$$\sigma^{V,tot}_{NNLO} = \int_0^\infty dq_T^2 \frac{d\sigma^V_{NLO}}{dq_T^2}.$$

- We decompose the $q_T$ distribution as following:

$$\frac{d\sigma^V_{NLO}}{dq_T^2} = \frac{d\sigma^{V,(res.)}_{NLO}}{dq_T^2} + \frac{d\sigma^{V,(fin.)}_{NLO}}{dq_T^2},$$

where the first term on the r.h.s. contains all the the logarithmically-enhanced contributions at small $q_T$ while the second term is free of such contributions.

- Following the [Bozzi, Catani, de Florian, Grazzini('06)] formalism we can then write

$$\sigma^{V,tot}_{NNLO} = \sigma^V_{LO} \mathcal{H}^V_{NNLO} + \int_0^\infty dq_T^2 \frac{d\sigma^{V,(fin.)}_{NLO}}{dq_T^2}.$$

- This formula allows us to analytically compute $\mathcal{H}^V_{NNLO}$ from the knowledge of the NNLO total cross section and the NLO $q_T$ distribution.

# The $q_T$ resummation formalism

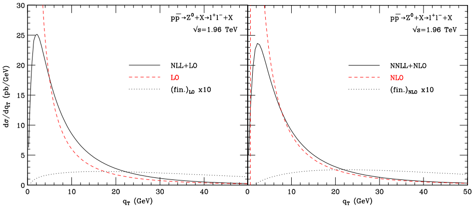
The main distinctive features of the formalism we use: [Catani,de Florian, Grazzini('01)], [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)]:

- Resummation performed at partonic cross section level: PDF evaluated at $\mu_F \sim M$: no PDF extrapolation in the non perturbative region, study of $\mu_R$ and $\mu_F$ dependence as in fixed-order calculations.
- Possible to make prediction without introducing non perturbative effects: Landau singularity of the QCD coupling regularized using a *Minimal Prescription* [Laenen,Sterman,Vogelsang('00)],[Catani et al.('96)].
- Resummed effects exponentiated in a universal Sudakov form factor $\mathcal{G}_N(\alpha_S, L)$; process-dependence factorized in the hard scattering coefficient $\mathcal{H}_N(\alpha_S)$.
- Perturbative unitarity constrain and resummation scale $Q$:

$$\ln\left(\frac{M^2 b^2}{b_0^2}\right) \to \widetilde{L} \equiv \ln\left(\frac{Q^2 b^2}{b_0^2}+1\right) \;\Rightarrow\; \exp\left\{\mathcal{G}_N(\alpha_S, \widetilde{L})\right\}\big|_{b=0} = 1 \;\Rightarrow\; \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right)_{NLL+LO} = \hat{\sigma}^{(tot)}_{NLO};$$

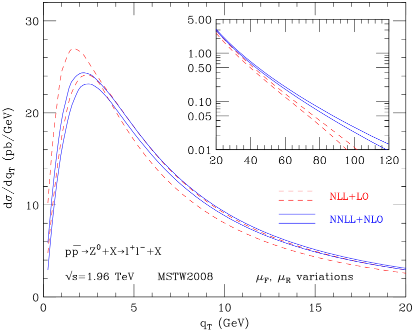
  - avoids unjustified higher-order contributions in the small-$b$ region: no need for unphysical switching from resummed to fixed-order results.
  - allows to recover *exactly* the total cross-section upon integration on $q_T$
  - variations of the resummation scale $Q \sim M$ allows to estimate the uncertainty from higher orders uncalculated logarithmic corrections.

# Resummed results: $q_T$ spectrum of $Z$ boson at the Tevatron $\sqrt{s} = 1.96\ TeV$




- Left side: NLL+LO result compared with fixed LO result.
  Resummation cure the fixed order divergence at $q_T \to 0$.
- Right side: NNLL+NLO result compared with fixed NLO result.
- The $q_T$ spectrum is slightly harder at NNLL+NLO accuracy than at NLL+LO accuracy.
- Integral of the NLL+LO (NNLL+NLO) curve reproduce the total NLO (NNLO) cross section to better 1% (check of the code).

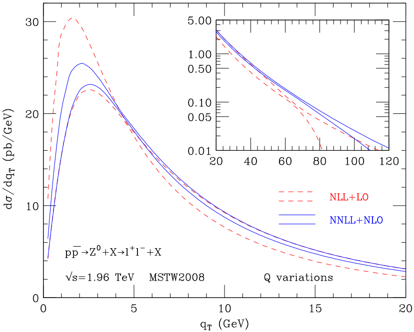
# Resummed results: $q_T$ spectrum of $Z$ boson at the Tevatron $\sqrt{s} = 1.96\ TeV$




NLL+LO: pdf=MSTW08 NLO, 2-loops $\alpha_S$
NNLL+NLO: pdf=MSTW08 NNLO, 3-loops $\alpha_S$

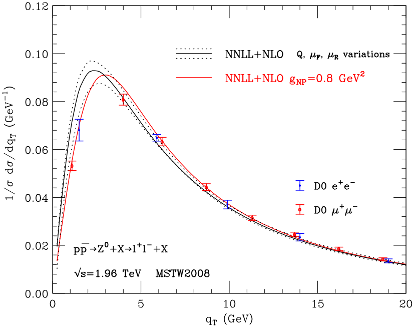
- Our calculation implements $\gamma^* Z$ interference and finite-width effects. Here we use the narrow width approximation (differences within 1% level).
- Uncertainty bands obtained by performing renormalization and factorization scale variations: $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R\} \leq 2$, with $Q = m_Z/2$.
  In the region $q_T \lesssim 30$ the NNLL+NLO and NLL+LO bands overlap (contrary to the fixed-order case).
- We observe a significative reduction of scale dependence going from NLL+LO to NNLL+NLO accuracy.
- Suppression of NLL+LO result in the large-$q_T$ region ($q_T \gtrsim 60\ GeV$) (strong dependence from the resummation scale, see next plot).

# Resummed results: $q_T$ spectrum of $Z$ boson at the Tevatron $\sqrt{s} = 1.96\ TeV$




- Uncertainty bands obtained by performing resummation scale variations (estimate of higher-order logarithmic contributions): $m_Z/4 \leq Q \leq m_Z$ with $\mu_F = \mu_R = m_Z$.
- The resummation scale dependence at NNLL+NLO (NLL+LO) is about $\pm 5\%$ ($\pm 12\%$) around the peak and $\pm 5\%$ ($\pm 16\%$) in the $q_T \gtrsim 20\ GeV$ region and it is larger than the renormalization and factorization scale dependence.
- Going from the NLL+LO to the NNLL+NLO calculation the resummation scale dependence is reduce by roughly a factor 2 in the wide region $5\ GeV \lesssim q_T \lesssim 50\ GeV$.

# Non perturbative effects: $q_T$ spectrum of $Z$ boson at the Tevatron $\sqrt{s}=1.96$ TeV



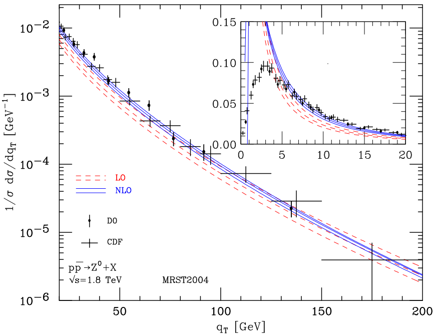

- Up to now result in a complete perturbative framework.
- Non perturbative effects parametrized by a NP form factor $S_{NP}=\exp\{-g_{NP}b^2\}$:

$$\exp\{\mathcal{G}_N(\alpha_S,\widetilde{L})\} \quad\rightarrow\quad \exp\{\mathcal{G}_N(\alpha_S,\widetilde{L})\}\, S_{NP}$$

$$g_{NP}=0.8\,GeV^2$$ [Kulesza et al.('02)]

- With NP effects the $q_T$ spectrum is harder.
- Quantitative impact of such NP effects is comparable with perturbative uncertainties.

# Fixed order results: $q_T$ spectrum of $Z$ boson at the Tevatron $\sqrt{s} = 1.8\ TeV$

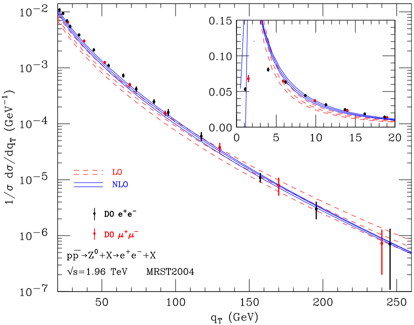



LO: pdf=MRST02 LO, 1-loop $\alpha_S$
NLO: pdf=MRST04 NLO, 2-loops $\alpha_S$

- CDF data: $66\ GeV < M^2 < 116\ GeV$, $\sigma_{tot} = 248 \pm 11\ pb$ [CDF Coll. ('00)]
  D0 data: $75\ GeV < M^2 < 105\ GeV$, $\sigma_{tot} = 221 \pm 11\ pb$ [D0 Coll. ('00)]
- Factorization and renormalization scale variations: $\mu_F = \mu_R = m_Z$, $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R\} \leq 2$,
  $q_T \sim m_Z$ : $LO \pm 25\%$, $NLO \pm 8\%$
  $q_T \sim 20\ GeV$ : $LO \pm 20\%$, $NLO \pm 7\%$
- Good agreement between NLO results and data up to $q_T \sim 20\ GeV$.
- In the small $q_T$ region ($q_T \lesssim 20\ GeV$) LO and NLO result diverges to $+\infty$ and $-\infty$ (accidental partial agreement at $q_T \sim 5-7\ GeV$): need for resummation.

LO and NLO scale variations bands overlap only for $q_T > 70\ GeV$

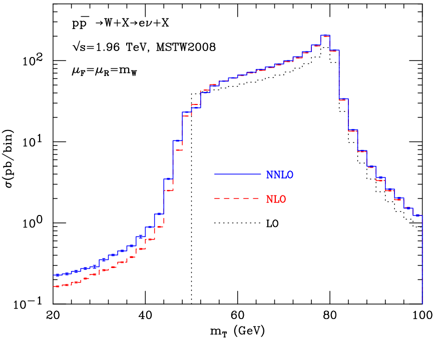
# Fixed order results: $q_T$ spectrum of $Z$ boson at the Tevatron $\sqrt{s} = 1.96\ TeV$




- D0 data [D0 Coll.('08,'10)].
- Scale variations as before: $\mu_F = \mu_R = m_Z$, $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R\} \leq 2$,
- Experimental errors very small but bins are larger.
- Qualitatively same situation of Tevatron Run I data.
- LO and NLO scale variations bands overlap only for $q_T > 60\ GeV$
- Good agreement between NLO results and data up to $q_T \sim 20\ GeV$.

In the small $q_T$ region ($q_T \lesssim 20\ GeV$) effects of soft-gluon resummation are essential

At Tevatron 90% of the $W^\pm$ and $Z^0$ are produced with $q_T \lesssim 20\ GeV$

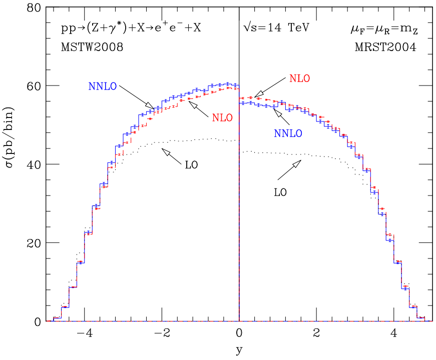



Transverse mass distribution for $W$ production at the Tevatron:

$$m_T = \sqrt{2p_T^l p_T^{miss}(1-\cos\phi_{l\nu})}$$

Cuts: ${p_T}^{miss} \geq 25\ GeV$ ; $|\eta| < 2$ ; ${p_T}^l \geq 20\ GeV$

- The LO distribution is bounded at $m_T = 50$ GeV. At LO the $W$ is produced with $q_T = 0$ therefore, the requirement $p_T^{\rm miss} > 25$ GeV sets $m_T \geq 50$ GeV.
- Around this region there are perturbative instabilities in going from LO to NLO and to NNLO.
- The origin of such instabilities is due to (integrable) logarithmic singularities in the vicinity of the boundary (Sudakov shoulder [Catani,Webber ('97)]).
- Below the boundary, the $\mathcal{O}(\alpha_S^2)$ corrections are large (for istance +40% at $m_T \sim 30$ GeV). This is not unexpected, since in this region the $\mathcal{O}(\alpha_S^2)$ result is actually only a NLO calculation.
- Accepted cross sections (errors refer to Monte Carlo numerical errors ):
  $\sigma_{LO} = 1.61 \pm 0.001\ nb$
  $\sigma_{NLO} = 1.550 \pm 0.001\ nb$
  $\sigma_{NNLO} = 1.586 \pm 0.002\ nb$

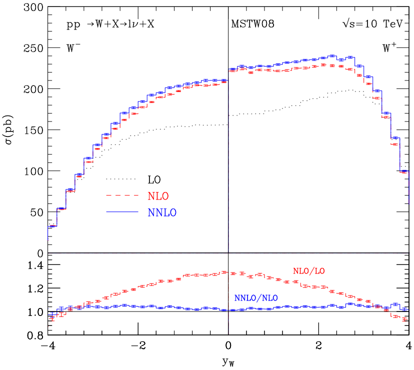



Rapidity distribution for $Z$ production at the LHC (no cuts).

- Left panel: MSTW 2008 pdf. Going from NLO to NNLO the total cross section increase by about 3%: $\sigma_{NLO} = 2.030 \pm 0.001$ *nb* and $\sigma_{NNLO} = 2.089 \pm 0.003$ *nb* (errors refer to Monte Carlo numerical errors).

- Right panel: MRST 2004 pdf. Going from NLO to NNLO the total cross section decrease by about 2%: $\sigma_{NLO} = 1.992 \pm 0.001$ *nb* and $\sigma_{NNLO} = 1.954 \pm 0.003$ *nb*.

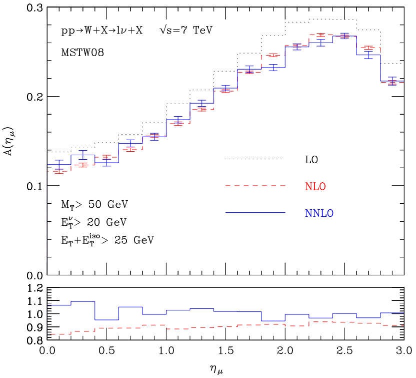
# Lepton charge asymmetry in *pp* collisions




The $W$ charge rapidity distribution in LO, NLO and NNLO QCD with MSTW08 PDFs in $pp$ collision at $\sqrt{s} = 10\ TeV$.

- Owing to CP invariance $W^{\pm}$ rapidity distribution in $pp$ collision is forward-backward symmetric: $d\sigma(W^{\pm})/dy_W = d\sigma(W^{\pm})/d(-y_W)$
- $W^+$ is mainly produced by $u\bar{d}$ collisions while $W^-$ is mainly produced by $d\bar{u}$. Since $\bar{u}(x) \sim \bar{d}(x)$ and $u(x) > d(x)$, $W^+$ production is larger and $W^+$ is produced at larger rapidities than $W^-$.
- Lepton charge asymmetry at the LHC is sensitive to PDFs with typical momentum fractions smaller (up about a factor 7) than those proved at the Tevatron
- We show the muon asymmetry at the LHC with typical selection cuts (CMS Coll. ).
- Good convergence of the perturbative expansion.

# Lepton charge asymmetry in *pp* collisions




The muon asymmetry in LO, NLO and NNLO QCD with MSTW08 PDFs in *pp* collision at $\sqrt{s} = 7\ TeV$.
Lower panel: NLO and NNLO K-factors.

- Owing to CP invariance $W^{\pm}$ rapidity distribution in *pp* collision is forward-backward symmetric: $d\sigma(W^{\pm})/dy_W = d\sigma(W^{\pm})/d(-y_W)$
- $W^+$ is mainly produced by $u\bar{d}$ collisions while $W^-$ is mainly produced by $d\bar{u}$.
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