Improving the phenomenology of K_{l3} form factors with analyticity and unitarity

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Radiative Corrections for the LHC: 2
Radcor 2011 Symposium
26 - 30 September, 2011

Based on the paper with : B. Ananthanarayan,I. Caprini, I.Sentitemsu Imsong

Phys.Rev.D82:094018,2010.
arXiv:1008.0925

Outline

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Brief Review of Formalism

- Kaon decay into a pion and lepton pair is characterized by the **vector** and **scalar** form factors. The physical region is $m_l^2 \leq t \leq (M_K - M_\pi)^2$ where the form factor is real.
- For a scalar form factor, expansion about $t = 0$

$$f_0(t) = f_+(0)\left(1 + \lambda_0' \frac{t}{M_\pi^2} + \frac{1}{2}\lambda_0'' \frac{t^2}{M_\pi^4} + \cdots\right), \quad (1)$$

  defines slope and curvature parameters where, $f_+(0) = 0.964(5)$. Analogously for vector.
- The method exploits the fact that a bound on an integral involving the modulus squared of the form factors along the unitarity cut is known from the dispersion relation satisfied by a certain QCD correlator.
  Theory of unitarity bounds and low energy form factors
  Gauhar Abbas, B. Ananthanarayan, I. Caprini, I. S. Imsong and S. Ramanan
  Eur. Phys. J. A 45, 389 (2010) arXiv:1004.4257 [hep-ph].

Brief Review of Formalism

- The QCD correlator

$$\chi_0(Q^2) \equiv \frac{\partial}{\partial q^2}\left[q^2\Pi_0\right] = \frac{1}{\pi}\int_{t_+}^{\infty} dt\, \frac{t\mathrm{Im}\Pi_0(t)}{(t+Q^2)^2}\,, \tag{2}$$

$$\mathrm{Im}\Pi_0(t) \geq \frac{3}{2}\frac{t_+t_-}{16\pi}\frac{[(t-t_+)(t-t_-)]^{1/2}}{t^3}|f_0(t)|^2\,, \tag{3}$$

with $t_\pm = (M_K \pm M_\pi)^2$. Positive definite and can be bounded. A different expression for the vector form factor.

- On the other hand, in pQCD when $Q \gg \Lambda_{\mathrm{QCD}}$, m_q, α_S $\overline{MS}$ scheme.

$$\chi_0(Q^2) = \frac{3(m_s-m_u)^2}{8\pi^2 Q^2}\left[1+1.80\alpha_s+4.65\alpha_s^2+15.0\alpha_s^3+57.4\alpha_s^4\ldots\right]. \tag{4}$$

Baikov, Chetyrkin and Kuhn, Phys. Rev. Lett. 96, 012003 (2006)
Baikov, Chetyrkin and Kuhn, Phys. Rev. Lett. 101,012002 (2008)

Brief Review of Formalism

- We can now use the conformal map $t \to z(t)$

$$z(t) = \frac{\sqrt{t_+} - \sqrt{t_+ - t}}{\sqrt{t_+} + \sqrt{t_+ - t}},$$

to transform the dispersion relation to

$$\frac{1}{2\pi}\int_0^{2\pi} d\theta |g(\exp(i\theta))|^2 \leq I \qquad (5)$$

where

$$g(z) = F(t(z))w(z).$$

- Square integrability implies $I = |g_0|^2 + |g_1|^2 + \ldots$ [Parseval theorem]
- Improvement of the bound if f_0 is known at real $z(t)$
$z = x_i,\ i = 1, 2, 3, \ldots$

Brief Review of Formalism

- Further improvement by considering the Omnès function:

$$\mathcal{O}(t) = \exp\left(\frac{t}{\pi}\int_{t_+}^{\infty} dt \frac{\delta(t')}{t'(t'-t)}\right),$$

  where $\delta(t)$ is the $I = 1/2$ elastic S-wave $K\pi$ scattering phase, in the elastic region and arbitrary Lipschitz continuous above t_{in} More mathematical steps required to make it more stringent.

- We make a shift from t_+ to t_{in}. As a result, dispersion contribution from t_+ to t_{in} needs to be removed from pQCD value which is now the input for the bound (first proposed by Caprini) given below -

$$I' = \chi_0(Q^2) - \frac{3}{2}\frac{t_+ t_-}{16\pi^2}\int_{t_+}^{t_{\rm in}} dt \frac{[(t-t_+)(t-t_-)]^{1/2}|f_0(t)|^2}{t^2(t+Q^2)^2}$$

Outline

Experimental and Theoretical Inputs

- $f_+(0) = 1$ in the limit of $m_d = m_u = m_s$ ($SU(3)$ limit where all the eight pseudoscalars are Goldstone particles).
- Corrections to the relation due to $SU(3)$ breaking. Expected to depart by $\sim 20\%$.
- Even smaller due to Ademollo-Gatto theorem.
- Crucial for knowledge of Cabibbo-Kobayashi-Maskawa matrix as the combination $f_+(0)V_{us}$ appears in the expression for rates and Dalitz plot densities.
- Recent determinations from the lattice gives $f_+(0) = 0.964(5)$.
  RBC+UKQCD collaboration [P. A. Boyle et al., Physical Review Letters 100 (2008) 141601]
  They use 2+1 flavour of dynamical wall quarks.

Experimental and Theoretical Inputs

- A soft-pion theorem due to Callan and Treiman

$$f_0(M_K^2 - M_\pi^2) = F_K/F_\pi + \Delta_{CT}$$

$\Delta_{CT} \simeq 0$ to two-loops in chiral perturbation theory.
J. Bijnens and P. Talavera, Nuclear Physics B 669 (2003) 341.

- Knowledge of F_K/F_π at high precision is therefore crucial.
- A soft-kaon theorem due to Oehme

$$f_0(M_\pi^2 - M_K^2) = F_\pi/F_K + \overline{\Delta}_{CT}$$

$\overline{\Delta}_{CT} = 0.03$ is one-loop in chiral perturbation theory.
J. Gasser and H. Leutwyler, Nuclear Physics B250 (1985) 517.
Not known at two-loops.

- Our work predicts $-0.046 \leq \bar{\Delta}_{CT} \leq 0.014$ for higher order corrections.

- $F_K/F_\pi = 1.193 \pm 0.006$ according to recent lattice evaluations, see e.g., L. Lellouch, arXiv:0902.4545;
  see also A. Bazavov et al. [MILC collaboration],PoS CD09 (2009) 007,
  which uses 2+1 flavor with improved staggered quark action.
  Confirmed by S. Dürr et al. [BMW collaboration], Physical Review D81 (2010) 054507.
- We used the phase of the S-wave of $I = 1/2$ of the elastic$K\pi$ scattering for the scalar form factor, and the phase of the P-wave of $I = 1/2$ for the vector form factor.

- To extimate the low-energy integral, we used the Breit-Wigner parameterizations of $|f_+(t)|$ and $|f_0(t)|$ in terms of the resanances given by the Belle Collaboration for fitting the rate of $\tau \to K\pi\nu_\tau$ decay.
- D. Epifanov et al., Physics Letters B 654, (2007), 65
  reports measurement of modulus and phase of the $K\pi$ form factors in terms of resonances, based on about 53,000 lepton tagged events.
- Mushkelishvili-Omnès study of πK, πK^*, $K\rho$ and use of high statistics LASS experiment phase shifts used to produce the πK vector form factor and compared with BELLE
  B. Moussallam, European Physical Journal C 53 (2008) 401
  M. Jamin, J. A. Oller and A. Pich, JHEP 0402, 047 (2004)

Outline

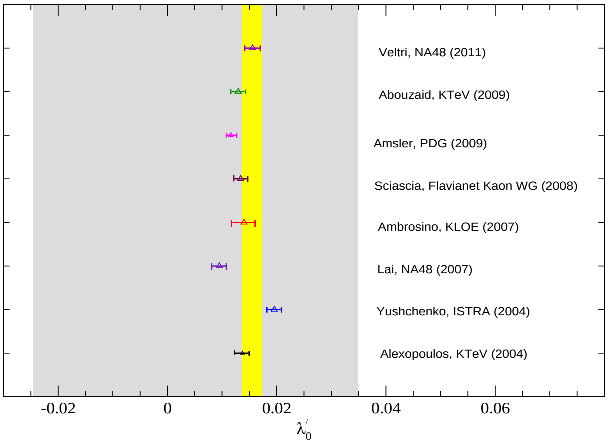

Figure: Allowed bands for the slope of the scalar form factor, narrow band for when we include Callan-Treiman constraint.

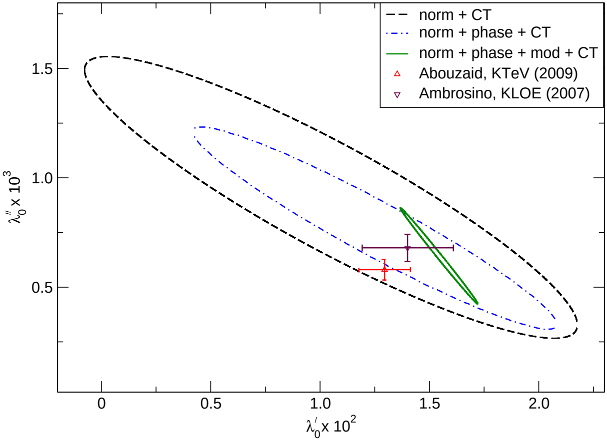

Figure: The slope and curvature of the scalar form factor, when we include phase, modulus and Callan-Treiman constraint.

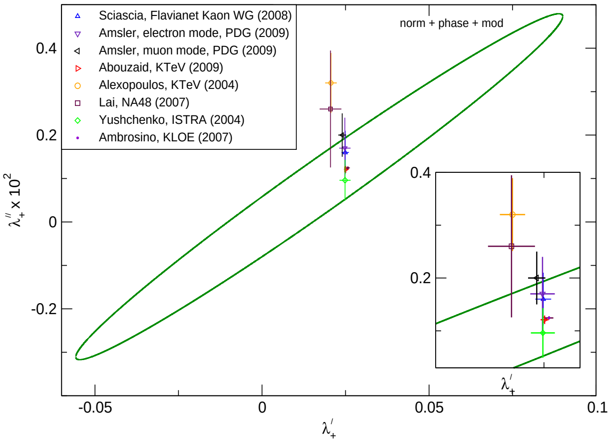

Figure: The best constraints for the slope and curvature of the vector form factor.

Zeros

- The question of the zeros : important from theoretical and phenomenological point of view.
- Example Adler zeros in scattering amlitude. No statement for form factors.
- The absence of zeros is assumed in the recent analysis of KTeV data.
  E. Abouzaid *et al.* [KTeV collaboration]
  Phys. Rev. D 81 , 052001 (2010) [arXiv:0912.1291 [hep-ex]].

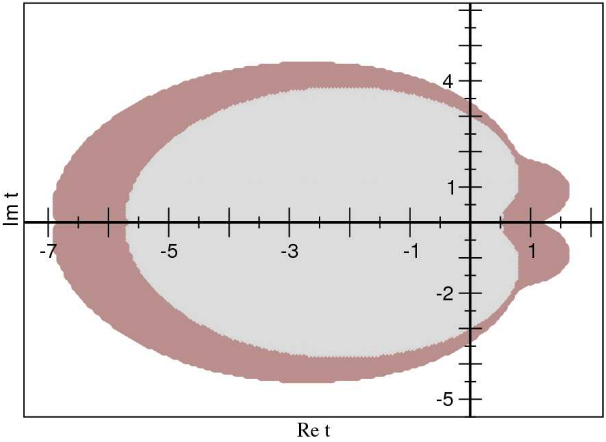

Figure: Domain without zeros for the scalar form factor, the small domain is obtained without including phase and modulus in the elastic region, bigger one using phase, modulus and Callan-Treiman constraint

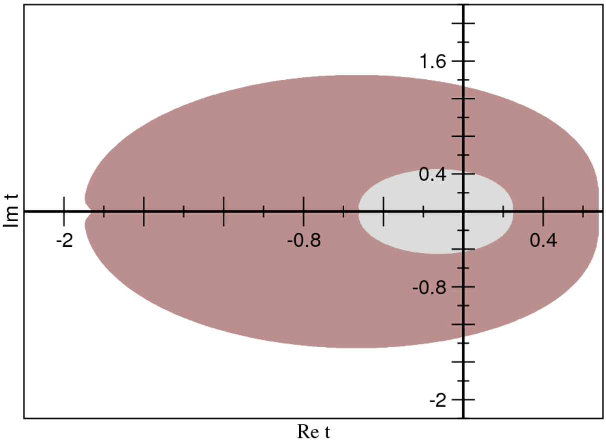

Figure: Domain without zeros for the vector form factor, the small domain is obtained without including phase and modulus in the elastic region.

Results on zeros

- Our results show that the zeros are excluded in a rather large domain at low energies. This provides confidence in the semiphenomenological analyses based on Omnès representations.
- In the case of complex zeros as well, the allowed zeros are rather remote to produce visible effects.

Outline

Conclusion

- We have reviewed the status of the vector and scalar form factors which are of fundamental importance to the standard model.
- The results are very stringent in the scalar form factor case.
- Note the most recent results from NA48 (Veltri et.al) respect our prediction for the slope of scalar form factor.
- Restricts the range of the slope to $\sim 0.01 - 0.02$, gives a near linear correlation with the curvature.
- Eliminates zeros in significant part of the real energy line and complex energy plane
- We do not need assumptions on the zeros or the phase above t_{in} (like in the recent Omnes representations). The model independence has a price: we only are able to predict bounds. But the precision of the input is already so large, that the bounds are stringent and improve our knowledge on the form factors.