Alternative subtraction method in QCD using Nagy-Soper scheme

Michael Kubocz

in collaboration with

Giuseppe Bevilacqua, Michał Czakon, Michael Krämer

Institute for Theoretical Particle Physics and Cosmology

RWTH AACHEN

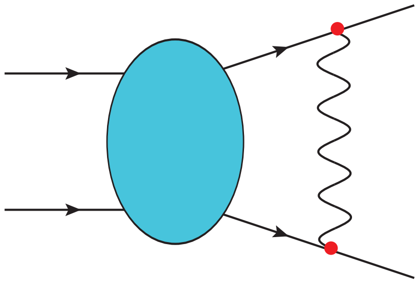
Outline

- NLO calculations, subtraction schemes, ...
- Nagy-Soper subtraction scheme
- Implementation in HELAC
- Summary and Outlook

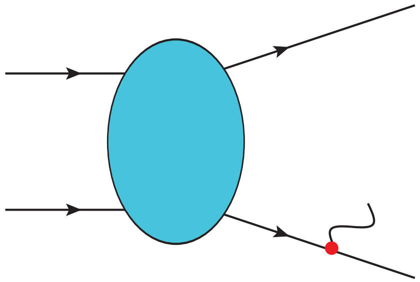
General remarks on NLO calculations

- higher-order contributions are necessary to match experimental accuracies and reduce theoretical uncertainty
- corrections to LHC processes are in general large
- decrease renormalization and factorization scale dependence of the cross section
- source of different methods and schemes to tackle complicated issues
- virtual exchange and real emission of partons lead to divergences

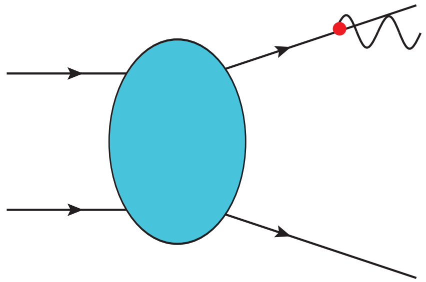
Singularities



virtual-UV:
virtual momentum
arbitrary large



real-IR soft:
gluon energy
arbitrary small



real-IR collinear:
angle between partons
arbitrary small

- pole structure is universal $\longrightarrow$ regularize using dimensional regularization

  $\longrightarrow$ singularities appear as $\frac{1}{\varepsilon^2}$ (soft & coll.) and $\frac{1}{\varepsilon}$ (soft or coll.)

  (after integration over the one parton unresolved phase space)

- but: fully inclusive measurements, which sum over all degenerate states, are free of

  IR-divergences (KNL-theorem)

  $\longrightarrow$ averaging is obtained by integrating σ with a IR-safe jet-function F_j

  $\longrightarrow$ should satisfy $F_j^{m+1} \longrightarrow F_j^m$ in the collinear and soft limits

Subtraction scheme (general remarks)

- construct local counter-terms $\mathrm{d}\sigma^A$, which match the behavior of the real-emission matrix element $\mathcal{M}_{m+1}$ in each soft and collinear limit:

$$\sigma^{\text{NLO}} = \int_m \mathrm{d}\sigma^V + \int_{m+1} \mathrm{d}\sigma^A + \int_{m+1} \left[\mathrm{d}\sigma^R - \mathrm{d}\sigma^A\right]$$
$$= \int_m \left[\mathrm{d}\sigma^V + \sum_i \mathcal{V} \otimes \mathrm{d}\sigma^B\right] + \int_{m+1} \left[\mathrm{d}\sigma^R + \sum_i \mathcal{D} \otimes \mathrm{d}\sigma^B\right]$$

$\rightarrow$ took advantage of the factorization of the real-emission matrix element in the singular limits

$$\mathcal{M}_{m+1}\big(\{p\}_{m+1}\big) \longrightarrow \sum_l v_l\big(\{p\}_{m+1}\big) \otimes \mathcal{M}_m\big(\{p\}_m\big)$$

- mapping $\{p\}_{m+1} \longrightarrow \{p\}_m$ necessary, which satisfies momentum conservation and on-shellness for both configurations

Motivation for a new scheme

- one can use splitting functions (proposed in the context of a parton shower with quantum interference) as dipole subtraction terms

  Z. Nagy, D. Soper, arXiv:0706.0017v2; 0801.1917v1; 0805.0216v1 [hep-ph]

  $\longrightarrow$ have same behavior in singular limits (same pole structure, but different finite parts)

  $\longrightarrow$ when combining NLO-calculations with parton shower, less counter-terms (avoid double counting) have to be added

- new mapping between $\{p\}_m$ and $\{\hat{p}\}_{m+1}$ phase spaces

  $\longrightarrow$ leads to smaller number of subtractions terms

  $\Longrightarrow$ for a $2 \longrightarrow N$ process:

  - CS: momentum fraction of a single designated spectator (final state) $\curvearrowright N^3_{\text{Dipoles}}$
  - NS: momentum fraction of all non-splitting final state partons $\curvearrowright N^2_{\text{Dipoles}}$

- it's fun

Nagy-Soper subtraction scheme - example: final state mapping

- $\{\hat{p}\}_{m+1}$ configuration (starting point) : $Q = \hat{p}_a + \hat{p}_b = \sum_{j=1}^{m+1} \hat{p}_j$
- $\{\hat{p}\}_m$ configuration via $P_i = \hat{p}_i + \hat{p}_j \to p_i$: $p_i = \frac{1}{\lambda} P_i - \frac{1-\lambda+y}{2\lambda a_i} Q$

  with $y = \frac{P_i^2 - m^2(f_i)}{2p_i \cdot Q}; \quad a_i = \frac{Q^2}{2p_i \cdot Q}; \quad \lambda = \lambda(y, a_i, \ldots)$
- total Momentum of spectators : before splitting $K = Q - p_i$ after splitting $\hat{K} = Q - P_i$
- transfer of momentum via LT : $\Lambda^\mu{}_\nu(\hat{K}, K) = g^\mu{}_\nu - \frac{2(\hat{K}+K)^\mu(\hat{K}+K)_\nu}{(\hat{K}+K)^2} + \frac{2\hat{K}^\mu K_\nu}{K^2}$

$\Longrightarrow \quad K^\mu = \Lambda^\mu{}_\nu(K, \hat{K})\hat{K}^\nu$ or rather $p_n^\mu = \Lambda^\mu{}_\nu(K, \hat{K})\hat{p}_n^\nu \quad n \notin \{l, j\}$

Nagy-Soper subtraction scheme - subtraction function

- subtraction function : $\mathcal{D}_{ij}(\{\hat{p},\hat{f}\}_{m+1}) = \Big\langle M(\{p,f\}_m^{ij})\Big|\boldsymbol{P}_{ij}(\{Q_{ij},\hat{p},\hat{f}\}_{m+1})\Big|M(\{p,f\}_m^{ij})\Big\rangle$
  ($f \mathrel{\widehat{=}}$ flavor, $Q_{ij} \mathrel{\widehat{=}}$ momentum mapping)
- the operator $\boldsymbol{P}_{ij}$ acts on the color $\otimes$ spin space :

$$\begin{aligned}\boldsymbol{P}_{ij}(\{Q_{ij},\hat{p},\hat{f}\}_{m+1}) &= \underbrace{C(\hat{f}_i,\hat{f}_j)V_{ij}(\{Q_{ij},\hat{p},\hat{f}\}_{m+1})V_{ij}^{\dagger}(\{Q_{ij},\hat{p},\hat{f}\}_{m+1})}_{\text{direct terms}} \\ &\underbrace{\begin{aligned}&+\sum_{\tilde{k}\neq i,j} T_{\tilde{i}}\cdot T_{\tilde{k}}\Big\{V_{kj}^{\text{soft}}(\{Q_{ij},\hat{p},\hat{f}\}_{m+1})V_{ij}^{\dagger,\text{soft}}(\{Q_{ij},\hat{p},\hat{f}\}_{m+1}) \\ &\qquad +\theta(i\geq 1)V_{ki}^{\text{soft}}(\{Q_{ij},\hat{p},\hat{f}\}_{m+1})V_{ji}^{\dagger,\text{soft}}(\{Q_{ij},\hat{p},\hat{f}\}_{m+1})\Big\}\end{aligned}}_{\text{interference terms}}\end{aligned}$$

($C \mathrel{\widehat{=}}$ Casimir, $T_i \mathrel{\widehat{=}}$ color matrix, $V_{ij} \mathrel{\widehat{=}}$ splitting operator on the spin space)

$\rightarrow$ direct terms contribute leading singularities: $\hat{p}_i, \hat{p}_j$ coll., $\hat{p}_j$ soft but not coll. to $\hat{p}_i$

$\rightarrow$ interference terms have no leading singularity when $\hat{p}_j$ is coll. with $\hat{p}_i$ or $\hat{p}_k$

$\hookrightarrow$ use eikonal approximation

Nagy-Soper subtraction scheme - splitting functions, direct terms

- define the splitting functions v_{ij} from the QCD vertices, spinors and polarization vectors for on-shell partons :

$$\left\langle \{\hat{s}\}_{m+1} \middle| V_{ij}^{\dagger}\left(\{Q_{ij}, \hat{p}, \hat{f}\}_{m+1}\right) \middle| \{\hat{s}\}_m \right\rangle = \left(\prod_{k \notin \{i,j\}} \delta_{\hat{s}_k s_{\tilde{k}}} \right) v_{ij}\left(\{\hat{p}, \hat{f}\}_{m+1}, \hat{s}_j, \hat{s}_i, s_{\tilde{i}}\right)$$

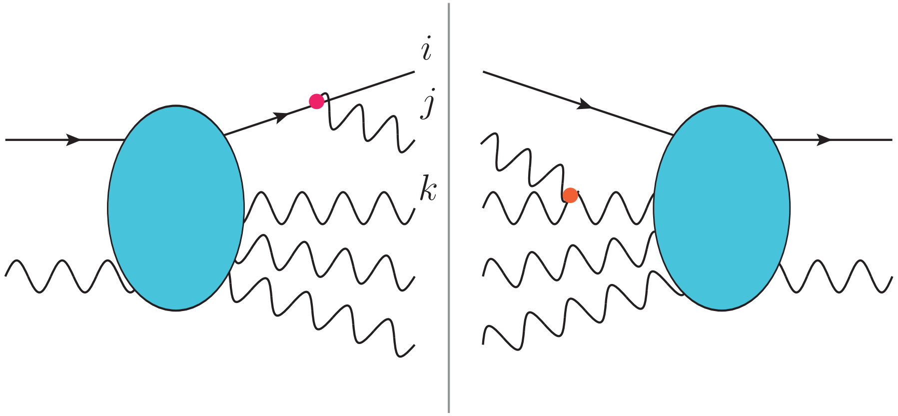
- we use the axial gauge: $n_{\tilde{i}} \cdot A = 0$, with $D^{\mu\nu}(P;n) = -g^{\mu\nu} + \dfrac{P^\mu n^\nu + n^\mu P^\nu}{(P \cdot n)}$

- splitting functions examples:

  ◦ final state, $q \to qg$: $\quad v_{ij} = \varepsilon_\mu\left(\hat{p}_j, \hat{s}_j; \hat{Q}\right)^* \dfrac{\overline{U}(\hat{p}_i \hat{s}_i)\gamma^\mu \left[\hat{\not{p}}_i + \hat{\not{p}}_j + m(f_{\tilde{i}})\right] \not{n}_{\tilde{i}} U(\hat{p}_{\tilde{i}} \hat{s}_{\tilde{i}})}{2 p_{\tilde{i}} \cdot n_{\tilde{i}} \left[(\hat{p}_i + \hat{p}_j)^2 - m^2(f_{\tilde{i}})\right]} t^a$

  ◦ final state, $g \to q\overline{q}$: $\quad v_{ij} = -\varepsilon_\mu\left(\hat{p}_{\tilde{i}}, \hat{s}_{\tilde{i}}; \hat{Q}\right) D_{\mu\nu}(\hat{p}_i + \hat{p}_j, n_{\tilde{i}}) \dfrac{\overline{U}(\hat{p}_i, \hat{s}_i)\gamma^\nu V(\hat{p}_j, \hat{s}_j)}{(\hat{p}_i + \hat{p}_j)^2} t^a$

Nagy-Soper subtraction scheme - splitting functions, interference terms



- soft splitting function:

$$v_i^{\text{soft}}\left(\{\hat{p},\hat{f}\}_{m+1},\hat{s}_j,\hat{s}_i,s_{\tilde{i}}\right)$$

$$=\delta_{\hat{s}_i,s_{\tilde{i}}}\frac{\varepsilon\left(\hat{p}_j,\hat{s}_j;\hat{Q}\right)^*\cdot\hat{p}_i}{\hat{p}_i\cdot\hat{p}_j}$$

- ambiguity with momentum mapping in squared expression: $W_{ik}=v_i^{\text{soft}}\left(v_k^{\text{soft}}\right)^*\delta_{\hat{s}'_i,s'_{\tilde{i}}}\delta_{\hat{s}_k,s_{\tilde{k}}}$

  $\hookrightarrow$ mapping from $p_{\tilde{i}}\to\hat{p}_i+\hat{p}_j$ or $p_{\tilde{k}}\to\hat{p}_k+\hat{p}_j$

  $\to$ define $W_{ik}^{(i)}$ with weight A_{ik} and $W_{ik}^{(k)}$ with weight A_{ki}

  $\to$ default value for weight functions: $A_{ik}=A_{ki}=1/2$, better: $A\left(\{p\}_{m+1}\right)$

- example: spin averaged i-k interference function: $\overline{W}_{ik}=A_{ik}\dfrac{\hat{p}_i\cdot D(\hat{p}_j,\hat{Q})\cdot\hat{p}_k}{\hat{p}_j\cdot\hat{p}_i\,\hat{p}_j\cdot\hat{p}_k}$

with $$A_{ik}=\frac{\left(\hat{p}_j\cdot\hat{p}_k\right)^2\hat{p}_i\cdot D(\hat{p}_j,\hat{Q})\cdot\hat{p}_i}{\left(\hat{p}_j\cdot\hat{p}_k\right)^2\hat{p}_i\cdot D(\hat{p}_j,\hat{Q})\cdot\hat{p}_i+\left(\hat{p}_j\cdot\hat{p}_i\right)^2\hat{p}_k\cdot D(\hat{p}_j,\hat{Q})\cdot\hat{p}_k}$$

Nagy-Soper subtraction scheme - integrated dipoles

- main work: calculation of the $\boldsymbol{I}$ operator ($+$ finite parts $\rightarrow$ $\boldsymbol{K}\dots$)

$$\int_{m+1} \mathrm{d}\sigma^A = \sum_{\text{dipoles}} \int_m \mathrm{d}\sigma^B \otimes \int_1 \mathrm{d}V_{\text{dipole}} = \int_m \left[\mathrm{d}\sigma^B \otimes \boldsymbol{I}\right]$$

$\rightarrow$ contains the singularity structure that cancels all ε poles in the virtual contribution $\mathrm{d}\sigma^V$

$$\dots + \int_0^1 \mathrm{d}x \int_m \mathrm{d}\sigma^B(xp) \otimes \left[\boldsymbol{K} + \dots\right]$$

$\rightarrow$ contains finite parts as well as collinear singularities (of initial state partons ...)

- due to different mapping, integrals are more complicated in comparison to CS
- one particle phase-space measure:

$$\mathrm{d}\xi_p \int \mathrm{d}P_i^2 \frac{\mathrm{d}^d\hat{p}_i}{(2\pi)^{d-1}} \delta_+\left(\hat{p}_i^2 - m_i^2\right) \frac{\mathrm{d}^d\hat{p}_j}{(2\pi)^{d-1}} \delta_+\left(\hat{p}_j^2 - m_i^2\right) \left[\frac{\lambda\left(Q^2, P_i^2, M^2\right)}{\lambda\left(Q^2, m_{\tilde{i}}, M^2\right)}\right]^{\frac{d-3}{2}} \times$$
$$\times\ (2\pi)^d \delta^{(d)}\left(\hat{p}_i + \hat{p}_j - P_i\right); \quad \boldsymbol{P}_i \neq 0$$

$\rightarrow$ parametrization is a little bit more complicated, some parts have to be evaluated numerically

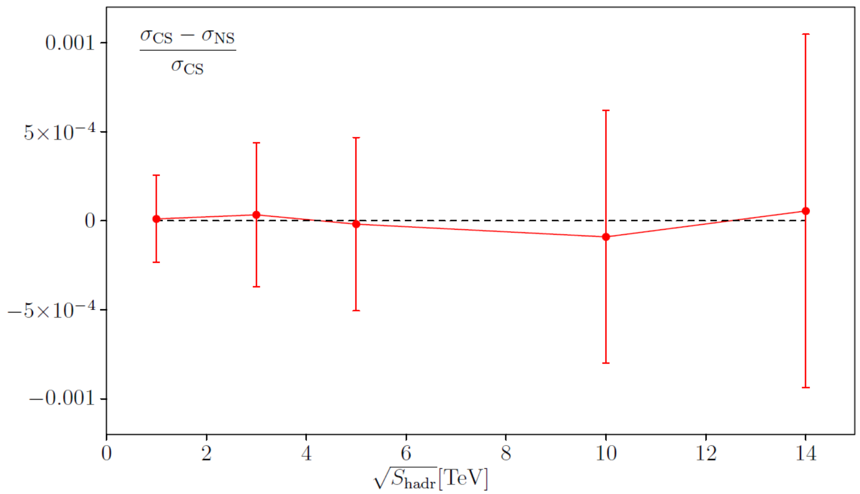
Implementation

- first numerical test and comparisons with Catani-Seymour scheme on simple collider processes with up to two massless particles in the final state can be looked up here:

  C. H. Chung, T. Robens, arXiv:1001.2704 [hep-ph], C. H. Chung, M. Krämer, T. Robens, arXiv:1012.4948 [hep-ph]

  $\longrightarrow$ reproduced results from the literature, implementation agrees with results obtained using Catani-Seymour scheme (private code)

- example: results for single W, plot: difference between CS and NS: $\dfrac{\sigma_{\text{CS}} - \sigma_{\text{NS}}}{\sigma_{\text{CS}}}$



$\Longrightarrow$ generalization to the massive case with arbitrary number of partons in final state

- extension of the HELAC-DIPOLES package

  M. Czakon, C. G. Papadopoulos, M. Worek, JHEP 0908 (2009) 85

HELAC-DIPOLES package implementation (current status)

- new NS-mapping subroutine ✓
- NS-splitting functions for real-emission (included with minimal modification of the original CS-based code) ✓

  $\rightarrow$ in singular limits same or better reproduction of σ^R
- integrated NS-Dipoles for final states (massless/massive) ✓ (ongoing work: initial states) ○
- ready for arbitrary processes ✓

  $\rightarrow$ with massive and massless external states ✓
- uses further features of the HELAC-DIPOLES package

  $\rightarrow$ helicity sampling for partons ✓

  $\rightarrow$ random polarizations for non-partons (soon) ○

  $\rightarrow$ . . .

Final remarks and conclusion

- new NLO subtraction scheme, based on context of an improved shower formalism
- global mapping leads to less dipole configurations
- interesting for multi-parton processes (with additional parton shower)
- implementation has (will have) access to all features of HELAC-DIPOLES package
- Open questions which increase curiosity and motivation:

  $\rightarrow$ How is the numerical behavior in comparison to CS (different $A_{ik}(\{p\}_{m+1})$ weight functions, . . .) ?

  $\rightarrow$ How big is the gain of speed due to the new mapping after optimization of the code?

  $\rightarrow$ How will look the outcome if we attach the NS parton shower

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Work in progress . . .