Exact Amplitude-Based Resummation in Quantum Field Theory: Recent Results

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Preface

** NEW PHYSICS AT LHC

Must Distinguish from Higher Order SM Processes AND Must Probe Precisely to Specify Uniquely

⇒ Precision QCD for the LHC

** UV LIMIT OF EINSTEIN'S THEORY

Can QFT Handle It?

⇒ Exact Residual Control in Resummation for UV

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The Paradigm:
*An Approach to Precision LHC Physics Theory: Amplitude-Based QED⊗QCD Resummation Realized by MC Methods --

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) d\hat{\sigma}_{res}$$

$$\Rightarrow \Delta\sigma_{th} = \Delta F \oplus \Delta\hat{\sigma}_{res} = \Delta\sigma_{th}(tech) \oplus \Delta\sigma_{th}(phys)$$

$$\Rightarrow$$

$$d\hat{\sigma}_{res} = \sum_n d\hat{\sigma}_n$$

$$= e^{SUM_{IR}(QCED)} \sum_{m,n=0}^{\infty} \frac{1}{m!n!} \int \prod_{j_1=1}^{m} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{n} \frac{d^3 k_{j_2}}{k_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_{j_1} k_{j_1} - \sum_{j_2} k'_{j_2}) + D_{QCED}}$$

$$* \tilde{\bar{\beta}}_{m,n}(k_1,\ldots,k_m;k'_1,\ldots,k'_n) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} \qquad (1)$$

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The Paradigm:

*An Approach to Feynman's Formulation of Einstein's Theory : Amplitude-Based Resummation of the Feynman Propagators in the Theory

$$i\Delta'_F(k) = \frac{i}{k^2 - m^2 - \Sigma_s(k) + i\varepsilon}$$

$$= \frac{ie^{B''_g(k)}}{k^2 - m^2 - \Sigma'_s(k) + i\varepsilon}$$

$$\equiv i\Delta'_F(k)|_{\mathrm{Re}\,summed}$$

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Precision QCD for the LHC

- **Contact with Standard Resummation (Abyat et al.** , Phys. Rev. D 74 (2006) 074004**):**

  2→n process --

$$\mathcal{M}^{[f]}_{\{r_i\}} = \sum_{L=1}^{C} \mathcal{M}^{[f]}_{L}(c_L)_{\{r_i\}}$$
$$= J^{[f]} \sum_{L=1}^{C} S_{LI} H^{[f]}_{I}(c_L)_{\{r_i\}}$$

- **Noting relation of J and SUM$_{IR}$, in our formula**

$$d\hat{\sigma}^m = \frac{e^{2\alpha_s \mathrm{Re}\, B_{QCD}}}{m!} \int \prod_{j=1}^{m} \frac{d^3 k_j}{(k_j^2 + \lambda^2)^{1/2}} \delta(p_1 + q_1 - p_2 - q_2 - \sum_{i=1}^{m} k_i)$$
$$\bar{\rho}^{(m)}(p_1, q_1, p_2, q_2, k_1, \ldots, k_m) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0},$$

we get the identification:

$$\overline{\rho}^{(m)}(p_1, q_1, k_1, \ldots, k_m) = \overline{\sum}_{\text{colors,spin}} |\mathcal{M}^{[f]}_{\{r_i\}}|^2$$

$$\equiv \sum_{\text{spins},\{r_i\},\{r'_i\}} h^{cs}_{\{r_i\}\{r'_i\}} |\overline{\mathbf{J}}^{[f]}|^2 \sum_{L,L'=1}^{C} \mathbf{S}^{[f]}_{LI} \mathbf{H}^{[f]}_{I}(\mathbf{c}_L)_{\{r_i\}} \left(\mathbf{S}^{[f]}_{L'I'} \mathbf{H}^{[f]}_{I'}(\mathbf{c}_{L'})_{\{r'_i\}}\right)^*$$

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- **Shower/ME Matching:**

$$\bar{\tilde{\beta}}_{m,n} \rightarrow \hat{\bar{\tilde{\beta}}}_{m,n}, \text{ shower - subtracted residuals}$$

- **IR-Improved DGLAP-CS Theory(PRD81(2010)076008):**

New resummed scheme for P_{AB}, reduced cross section derived from (1) applied to splitting process --

$$\mathrm{F_j}, \hat{\sigma} \rightarrow \mathrm{F'_j}, \hat{\sigma}' \quad \text{for}$$

$$\mathrm{P_{qq}} \rightarrow \mathrm{P_{qq}^{exp}} = \mathrm{C_F} F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+z^2}{1-z}(1-z)^{\gamma_q}, \text{etc.,}$$

giving the same value for σ, with improved MC stability

-- no need for IR cut-off (k_0) parameter

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- **Complete Set:**

$$P_{qq}^{exp}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right],$$

$$P_{Gq}^{exp}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q},$$

$$P_{GG}^{exp}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G} + \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z) \},$$

$$P_{qG}^{exp}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \},$$

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0}, \qquad \delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}),$$

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q + 1} + \frac{1}{\gamma_q + 2},$$

$$\gamma_G = C_G \frac{\alpha_s}{\pi} t = \frac{4C_G}{\beta_0}, \qquad \delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}),$$

$$f_G(\gamma_G) = \frac{n_f}{6C_G F_{YFS}(\gamma_G)} e^{-\frac{1}{2}\delta_G} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} + \frac{1}{(1+\gamma_G)(2+\gamma_G)} + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} + \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)},$$

$$F_{YFS}(\gamma) = \frac{e^{-C\gamma}}{\Gamma(1+\gamma)}, \qquad C = 0.57721566...,$$

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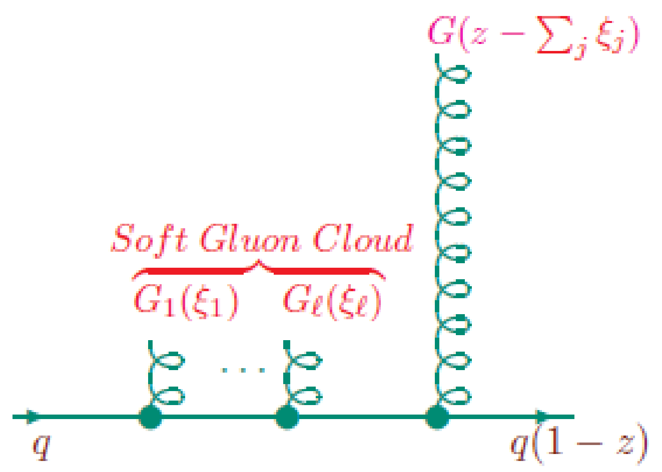
Basic Physical Idea:
Bloch-Nordsiek –

Accelerated Charge ⇒ Coherent State of Soft Gluons (Photons)

⇒ More Physical View of Splitting Process:



$$q \to q(1-z) + G \otimes G_1 \cdots \otimes G_\ell,\ \ell = 0, \cdots, \infty$$

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• Illustration of Calculation

$$\int \frac{\alpha_s(t)}{2\pi} P_{BA}\, dt\, dz = e^{\mathrm{SUM_{IR}(QCD)}(z)} \int \{ \bar{\bar{\beta}}_0 \int \frac{d^4y}{(2\pi)^4} e^{\{iy\cdot(p_1-p_2)+\int^{k<K_{max}} \frac{d^3k}{k} \tilde{S}_{\mathrm{QCD}}(k)\left[e^{-iy\cdot k}-1\right]\}}$$

$$+ \int \frac{d^3k_1}{k_1} \bar{\bar{\beta}}_1(k_1) \int \frac{d^4y}{(2\pi)^4} e^{\{iy\cdot(p_1-p_2-k_1)+\int^{k<K_{max}} \frac{d^3k}{k} \tilde{S}_{\mathrm{QCD}}(k)\left[e^{-iy\cdot k}-1\right]\}}$$

$$+ \cdots \} \frac{d^3p_2}{p_2^0} \frac{d^3q_2}{q_2^0}$$

$$= e^{\mathrm{SUM_{IR}(QCD)}(z)} \int \{ \bar{\bar{\beta}}_0 \int_{-\infty}^{\infty} \frac{dy}{(2\pi)} e^{\{iy\cdot(E_1-E_2)+\int^{k<K_{max}} \frac{d^3k}{k} \tilde{S}_{\mathrm{QCD}}(k)\left[e^{-iyk}-1\right]\}}$$

$$+ \int \frac{d^3k_1}{k_1} \bar{\bar{\beta}}_1(k_1) \int_{-\infty}^{\infty} \frac{dy}{(2\pi)} e^{\{iy\cdot(E_1-E_2-k_1^0)+\int^{k<K_{max}} \frac{d^3k}{k} \tilde{S}_{\mathrm{QCD}}(k)\left[e^{-iy\cdot k}-1\right]\}}$$

$$+ \cdots \} \frac{d^3p_2}{p_2^0 q_2^0}$$

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$$
\begin{aligned}
I_{YFS}(zE,0) &= \int_{-\infty}^{\infty} \frac{dy}{2\pi} e^{[iy(zE)+\int^{k<zE} \frac{d^3k}{k} \tilde{S}_{QCD}(k)(e^{-iyk}-1)]} \\
&= F_{YFS}(\gamma_q) \frac{\gamma_q}{zE} \\
I_{YFS}(zE,k_1) &= \int_{-\infty}^{\infty} \frac{dy}{2\pi} e^{[iy(zE-k_1)+\int^{k<zE} \frac{d^3k}{k} \tilde{S}_{QCD}(k)(e^{-iyk}-1)]} \\
&= (\frac{zE}{zE-k_1})^{1-\gamma_q} I_{YFS}(zE,0)
\end{aligned}
$$

$$
\int \left(\bar{\bar{\beta}}_0 \frac{\gamma_q}{zE} + \int dk_1 k_1 d\Omega_1 \bar{\bar{\beta}}_1(k_1) (\frac{zE}{zE-k_1})^{1-\gamma_q} \frac{\gamma_q}{zE} \right) \frac{d^3 p_2}{E_2 q_2^0} = \int dt \frac{\alpha_s(t)}{2\pi} P_{BA}^0 dz + \mathcal{O}(\alpha_s^2).
$$

differentiation yields

$$
P_{BA} = P_{BA}^0 z^{\gamma_q} F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q}
$$

- Herwiri1.031(PRD81(2010)076008):

w consultation from Bryan Webber, Stefano Frixione, and Mike Seymour, implementation of IR-improved kernels in Herwig 6.5 environment to get Herwiri1.031, MC@NLO/Herwiri1.031.

Observations:

1. SUM_{IR} is an IR effect – It contains as designed only the IR part of the LL, the rest of the LL is in D and the residuals $\hat{\bar{\tilde{\beta}}}_m$, as we show in $\bar{\rho}^m$.
2. Herwiri is just as general as Herwig6.5, as they run the same set of processes

- Some illustrative results follow.

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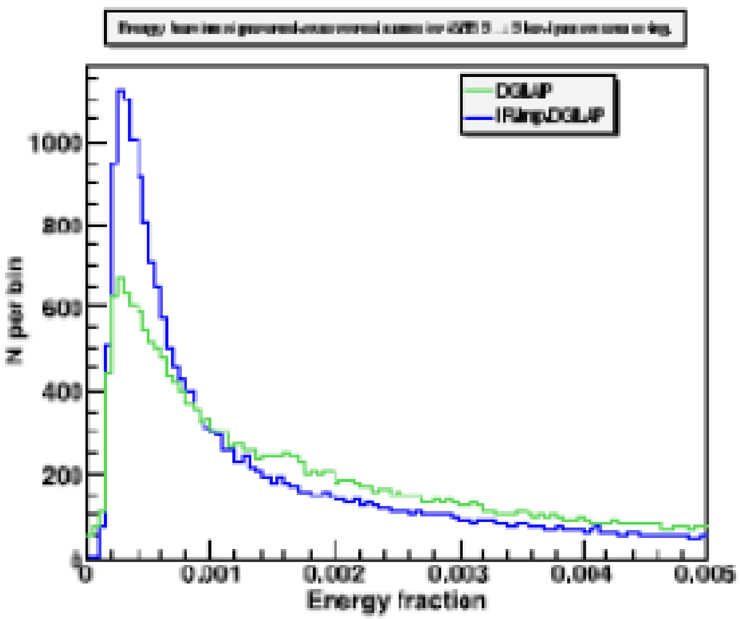

FIG. 1: The z-distribution (ISR parton energy fraction) shower comparison in Herwig 6.5.

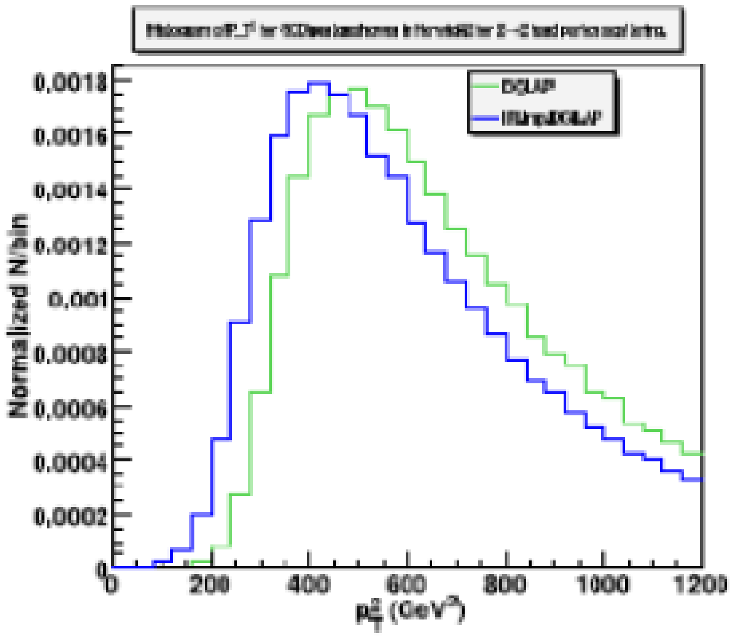

FIG. 2: The p_T^2-distribution (ISR parton) shower comparison in Herwig 6.5.

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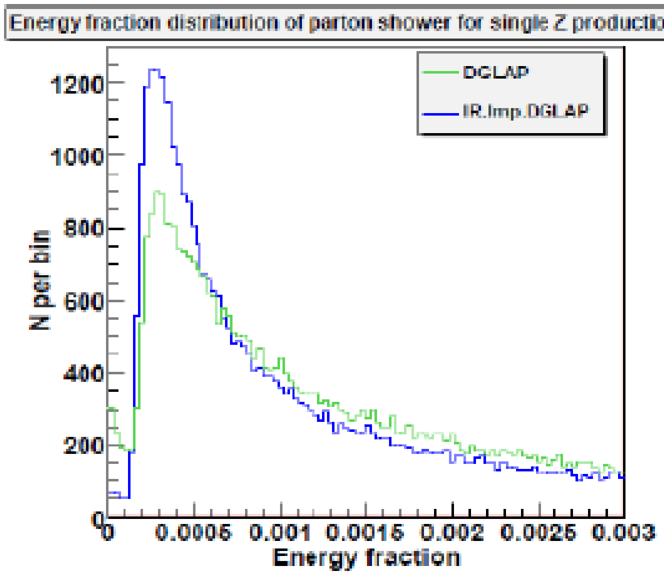

FIG.3: The z-distribution(ISR parton energy fraction) shower comparison

in HERWIG6.5.

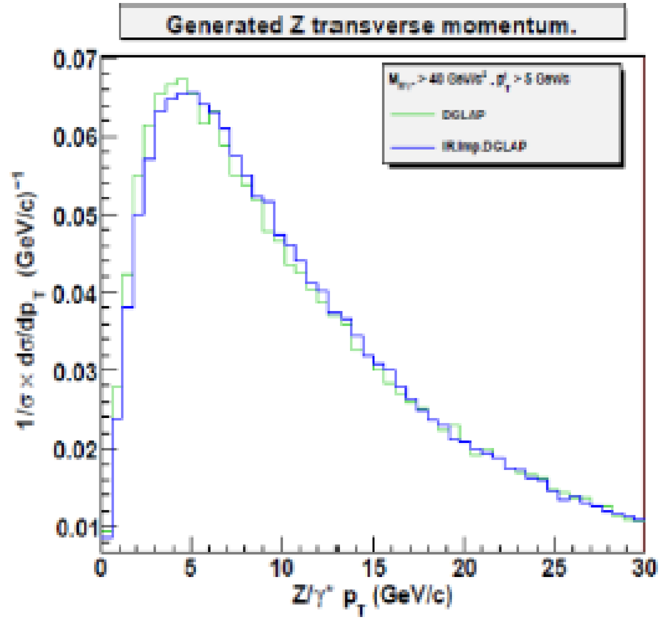

FIG.4 : The Z p_T –distribution(ISR parton shower effect) comparison

in HERWIG6.5.

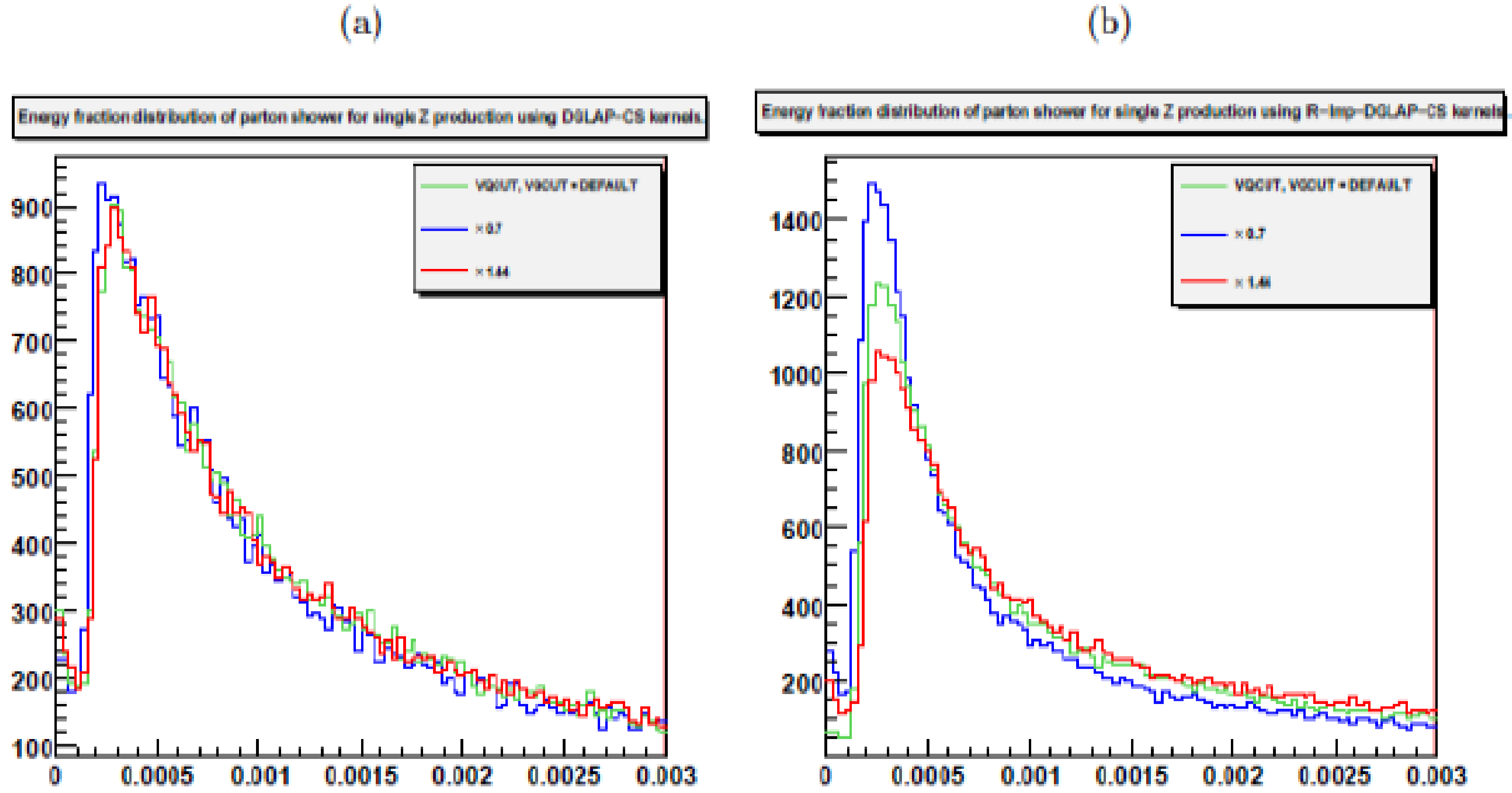

FIG.5: IR-cut-off sensitivity in z-distributions of the ISR parton energy fraction: (a), DGLAP-CS; (b), IR-I DGLAP-CS.

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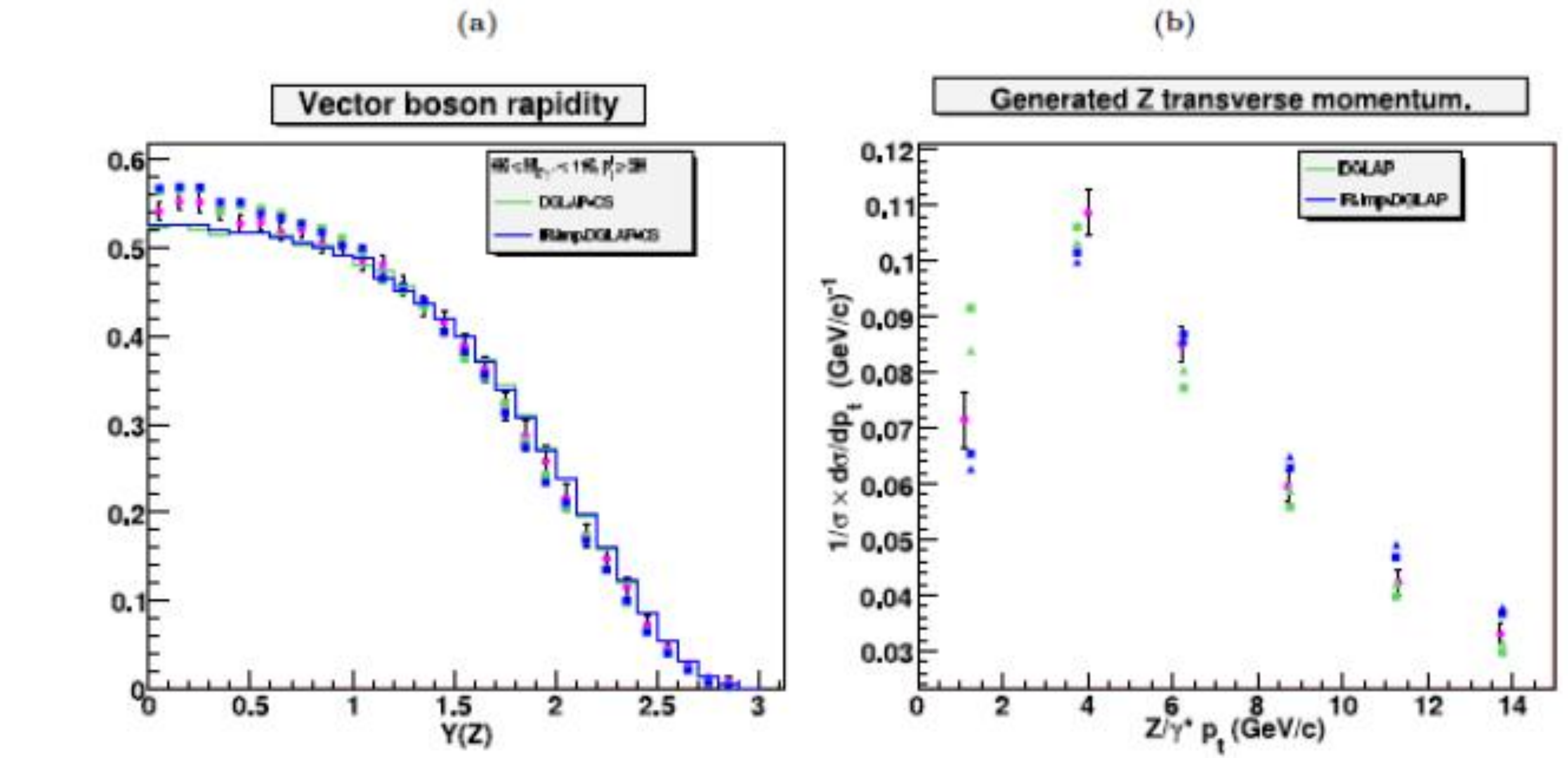

FIG. 6: Comparison with FNAL data: (a), CDF rapidity data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data, the green(blue) lines are HERWIG6.510(HERWIRI1.031); (b), D0 p_T spectrum data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data, the blue triangles are HERWIRI1.031, the green triangles are HERWIG6.510 – in both (a) and (b) the blue squares are MC@NLO/HERWIRI1.031, and the green squares are MC@NLO/HERWIG6.510. These are untuned theoretical results.

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- **HERWIRI1.031 -- PRD81 (2010) 076008:**

  – For the CDF rapidity data, HERWIRI1.031 is closer to the data than is HERWIG6.510 (1.54 vs 1.77 for χ^2/d.o.f. resp.);
  for MC@NLO/HERWIRI1.031 and MC@NLO/HERWIG6.510
  the χ^2/d.o.f are 1.42 and 1.40 resp., both are within 10% of the data
  $\Rightarrow$ Need NNLO level, in progress.

  – For the D0 p_T data, HERWIRI1.031 gives a better fit
  to the data compared to HERWIG6.5 for low p_T,
  for $p_T < 12.5$GeV, the χ^2/d.o.f. are $\sim$ 2.5 and 3.3 respectively
  - we add the statistical and systematic errors,
  showing that the IR-improvement makes a better representation
  of QCD in the soft regime for a given fixed order in perturbation theory.

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- **HERWIRI1.031:** PRD**81 (2010) 076008 –**
HERWIG++(SOON, thanks to Mike and Bryan)

Compare CMS(Mans) (soon) ⇒

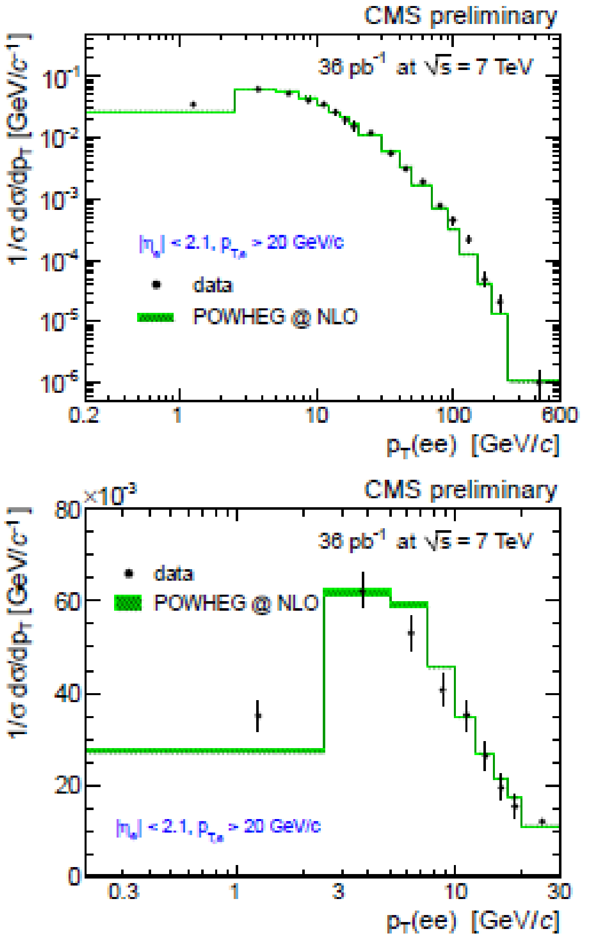
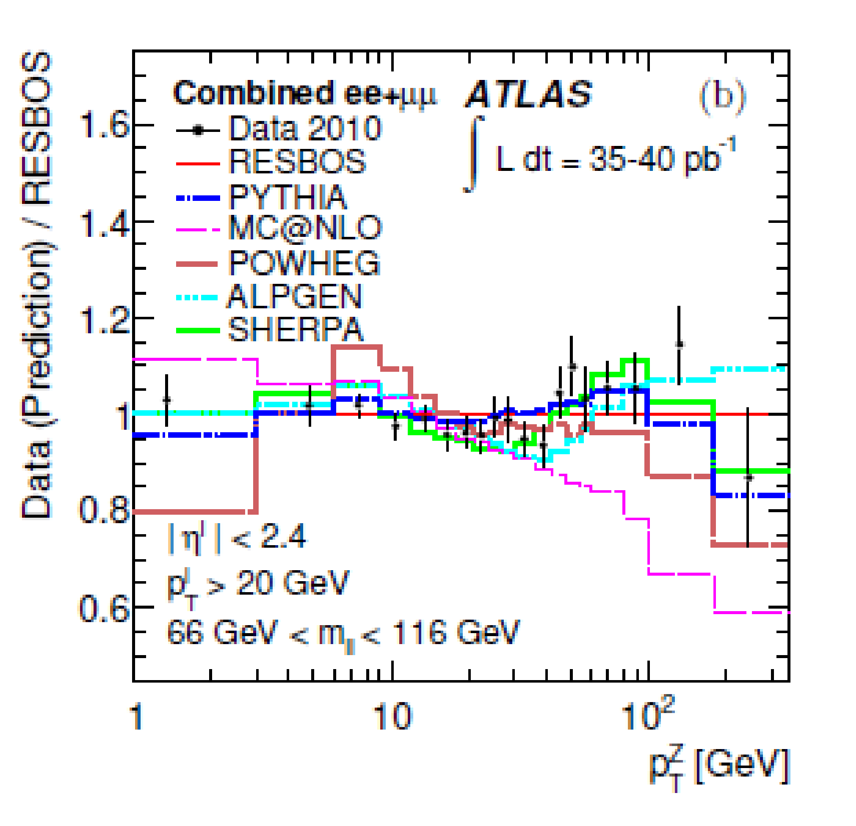


Figure 10: Comparison between the $Z \to e^+e^-$ differential cross section shape to the POWHEG prediction for $|\eta_e| < 2.1$ and $p_{T,e} > 20$ GeV/c. The same comparison is presented for the whole p_T range and zoomed in for the low p_T range (< 30 GeV/c).

- **HERWIRI1.031:** PRD**81 (2010) 076008 – HERWIG++(SOON, thanks to Mike and Bryan)**

Compare ATLAS(1107.2381) (soon) ⇒



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Near Future

* Herwig++(soon, running , under cross checks)
* Pyhtia 8,6 (w consultation from Peter Skands and Torbjorn Sjostrand
* Sherpa (w consultation from Jan Winter)

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- Recent Progress: Cosmological Constant Λ

In arXiv.org:1008.1048, using

$$B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right)$$

we show that we get the UV limit

$k^2 G_N(k) \rightarrow .0442$

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and the scalar contribution to Λ as

$$\Lambda_s = -8\pi G_N \frac{\int d^4k}{2(2\pi)^4} \frac{(2k_0^2)e^{-\lambda_c(k^2/(2m^2))\ln(k^2/m^2+1)}}{k^2+m^2}$$
$$\cong -8\pi G_N [\frac{1}{G_N^2 64\rho^2}], \quad \rho = \ln\frac{2}{\lambda_c}$$

for $\lambda_c = \frac{2m^2}{M_{Pl}^2}$.

A Dirac fermion gives -4 times Λ_s.

⇒ UV limit

$$\Lambda(k) \xrightarrow[k^2\to\infty]{} k^2\lambda_*,$$
$$\lambda_* = -\frac{c_{2,eff}}{2880}\sum_j(-1)^{F_j} n_j/\rho_j^2$$
$$\cong 0.0817$$

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Comparison with EFRG(Reuter et al., Percacci et al, Litim ,….):

Illustration(Laucsher&Reuter(PRD65(2002)025013))--

UV Fixed Point:

$$\beta_\lambda(\lambda_k, g_k; \alpha, d) = -2\lambda_k + \nu_d\, d\, g_k + \left[2d(d-1+2\alpha)(4\pi\ )^{1-\frac{d}{2}}\,\Phi^2_{d/2}(0) - (d-2)\,\omega_d\right]\lambda_k\, g_k + \frac{1}{2}d(d+1)(d-2)(4\pi)^{1-\frac{d}{2}}\omega_d\,\Phi^1_{d/2}(0)\,g_k^2 + \mathcal{O}\left(\mathrm{g}^3\right)\ ,$$

$$\beta_g(\lambda_k, g_k; \alpha, d) = (d-2)\,g_k - (d-2)\omega_d\,g_k^2 + \mathcal{O}\left(\mathrm{g}^3\right)$$

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For d=4, cut-off profile

$R^{(0)}(y) = y/(e^y - 1)$,

$g_* \cong \pi/(13\pi^2/144+55/24+\alpha)$

$\lambda_* \cong 3\zeta(3)/(13\pi^2/144+19/24)$

Evidently, for appropriate α and $R^{(0)}(y)$ we can have qualitative agreement with our pure gravity results

$g_* \cong 0.0533$

$\lambda_* \cong -0.000189$

An Estimate of Λ:

Planck Scale Cosmology --

(Bonanno&Reuter(J.Phys.Conf.Series140(2008)012008))

Transition between Planck regime and classical FRW regime at

$$t_{tr} \cong 25t_{Pl}$$

$\Rightarrow$

$$\rho_\Lambda(t_{tr}) \equiv \frac{\Lambda(t_{tr})}{8\pi G_N(t_{tr})} = \frac{-M_{Pl}^4(k_{tr})}{64}\sum_j \frac{(-1)^F n_j}{\rho_j^2}$$

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For t_{eq} = time of radiation matter equality

we get (see Branchina&Zappala (G.R.Grav.42(2010)141))

$$\rho_\Lambda(t_0) \cong \frac{-M_{Pl}^4(1+c_{2,eff}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64}\sum_j \frac{(-1)^F n_j}{\rho_j^2} \times \frac{t_{tr}^2}{t_{eq}^2} \times \left(\frac{t_{eq}^{2/3}}{t_0^{2/3}}\right)^3$$

$$\cong \frac{-M_{Pl}^2(1.0362)^2(-9.197\times 10^{-3})}{64}\frac{(25)^2}{t_0^2}$$

$$\cong (2.400\times 10^{-3}eV)^4.$$

Compare:

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$$\rho_\Lambda(t_0)|_{\text{expt}} \cong (2.368\times 10^{-3}eV(1\pm 0.023))^4$$

CONCLUSIONS

* Herwiri1.031 Just as General Herwig6.5, No Tweaking, Should Be Better in IR Due to Bloch-Nordsiek Effect -- Try it!
* Real Progress on Λ in QFT (Resummed Quantum Gravity Realization of Feynman's Approach)

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