The top-pair total cross section at NNLL order

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M. Beneke, PF, S. Klein, C. Schwinn [arXiv:1109.1536 [hep-ph]]

The experimental top-pair production cross section

Total $t\bar{t}$ cross section measured at Tevatron with $\Delta\sigma/\sigma \pm 7-8\%$... **LHC is catching up quickly!**

- Atlas ($0.7\,\mathrm{fb}^{-1}$): $179 \pm 12 (\Delta\sigma/\sigma \pm 6.6\%)$
- CMS ($36\,\mathrm{pb}^{-1}$): $154 \pm 18 (\Delta\sigma/\sigma \pm 12\%)$

Precise measurements of $\sigma_{t\bar{t}}$ relevant for

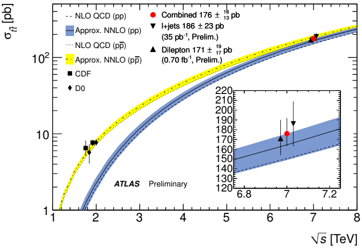
- testing **SM** and **new-physics** models
- constraining **gluon PDF** in the proton
- theoretically clean extraction of the **top quark mass**

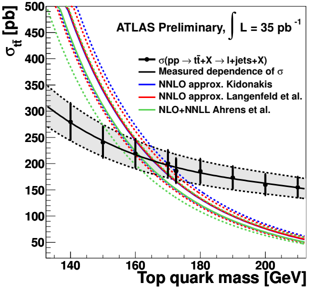
require that theoretical uncertainties are well understood and under control, $\Delta\sigma^{\mathrm{th}} \lesssim \Delta\sigma^{\mathrm{exp}}$:

$$\sigma_{t\bar{t}}^{\mathrm{NLO}} = 162^{+24}_{-26}\,\mathrm{pb}$$

$\Delta\sigma^{\mathrm{NLO}}/\sigma^{\mathrm{NLO}} \pm 15\%$ (scale+PDF+α_s)

$\Rightarrow$ **need better prediction than fixed-o. NLO!**



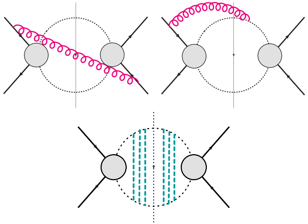


Theory status

- **NLO**: QCD corrections (Nason, Dawson, Ellis '88;...) EW corrections (Bernreuther Fücker, Si '06; Kühn, Scharf, Uwer '06) finite-width effects (Denner et al. '10; Bevilacqua et al. '10)
- **Full NNLO result**: several ingredients already known (Czakon '08; Bonciani et al. '09/'10; Körner et al. '08; Anastasiou, Aybat '08;...)
- **LL/NLL resummations**: Laenen et al. '92; Catani et al. '96; Berger, Contopanagos '96; Kidonakis, Smith '96; Bonciani et al. '98; Kidonakis et al. '01
- **NNLL resummation/NNLO approximated results**:
  - IR structure of QCD amplitudes (Neubert, Becher '09) soft-Coulomb factorization (Beneke, PF, Schwinn '10) 2-loop anomalous dimensions (Beneke et al. '09; Czakon et al. '09)
  - NNLO approximations for total cross section (HATHOR , Aliev et al. '11; Beneke et al. '09/'10) and differential cross section in 1PI/PIM kinematics (in Mellin and SCET formalism Kidonakis '09-'11; Ahrens et al. '10/'11;)
  - NNLL resummation for 1PI/PIM cross section in SCET (Ahrens et al. '10/'11) combined soft and Coulomb resummation in SCET/NRQCD (Beneke, PF, Klein, Schwinn '11)

Soft-gluon and Coulomb corrections

Total NLO **partonic cross sections** enhanced near **threshold**, $\beta \equiv \sqrt{1 - 4m_t^2/\hat{s}} \to 0$



- **Threshold logarithms**: $\sim \alpha_s^n \ln^m \beta$
  $\Leftrightarrow$ suppression of **soft-gluon emission**
- **Coulomb corrections**: $\sim (\alpha_s/\beta)^n$
  $\Leftrightarrow$ **potential interactions** of non-relativistic particles

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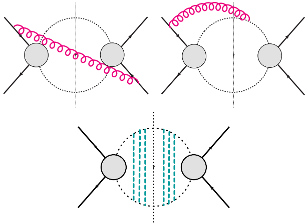


if **hadronic cross section** is dominated by partonic threshold resummation of soft logs and Coulomb singularities leads to improved predictions and reduced theoretical uncertainties

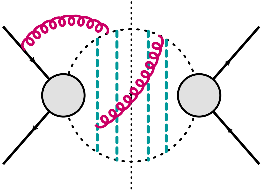
Counting scheme: $\alpha_s/\beta \sim \alpha_s \ln \beta \sim 1$

$$\hat{\sigma}_{pp'} \quad \propto \quad \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \left\{1\,\text{(LL,NLL)};\ \alpha_s, \beta\,\text{(NNLL)};\ \alpha_s^2, \alpha_s\beta, \beta^2\,\text{(NNNLL)}; \dots\right\}$$

Factorisation of pair production near threshold at NNLL

- **non-relativistic H, H' and Coulomb gluons:**
  $E \sim m_H \beta^2$, $|\vec{p}| \sim m_H \beta$
- **soft gluons:** $q_s \sim m_H \beta^2$



potential and soft modes have the same energy and can "talk" to each other

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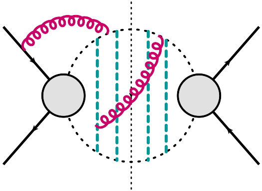
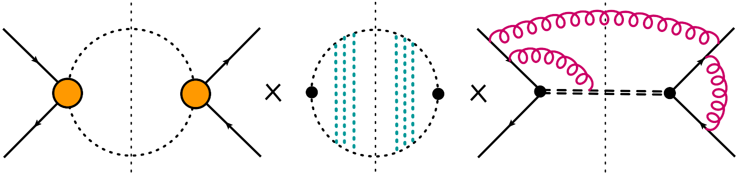


Effective-theory description of pair production near threshold $\hat{s} \sim (m_H + m_{H'})^2$
[Beneke, PF, Schwinn, '09/'10] $\Rightarrow$ factorization of hard, Coulomb and soft contributions

$$\hat{\sigma}_{pp'}(\hat{s}, \mu_f) = \sum_i H_i(M, \mu_f) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega, \mu_f)$$



Soft radiation couples to total colour charge of the pair!

Resummation of soft/hard functions in momentum space

RG evolution equations for the soft function $W_i^{R_\alpha}$ and the hard function $H_i^{R_\alpha}$ follow from IR structure of QCD amplitudes and scale-invariance of the hadronic cross section (generalisation of DY result [Becher, Neubert, Xu '07] to **arbitrary** R_α)

$$\frac{d}{d\ln\mu_f} W_i^{R_\alpha}(\omega,\mu_f) = -2\left[(C_r + C_{r'})\,\Gamma_{\rm cusp}\ln\left(\frac{\omega}{\mu_f}\right) + 2\gamma_{H,s}^{R_\alpha} + 2\gamma_s^{r} + 2\gamma_s^{r'}\right] W_i^{R_\alpha}(\omega,\mu_f)$$
$$-2\,(C_r + C_{r'})\,\Gamma_{\rm cusp}\int_0^\omega d\omega'\,\frac{W_i^{R_\alpha}(\omega',\mu_f) - W_i^{R_\alpha}(\omega,\mu_f)}{\omega-\omega'}$$

and similar for hard function $H_i(M,\mu_f)$

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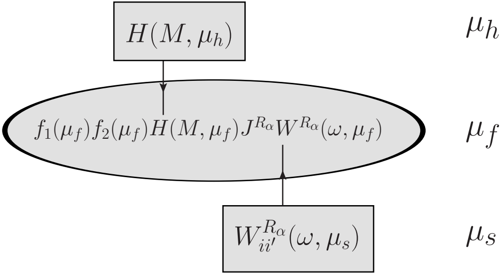
Resummation strategy

- solve evolution equation in momentum space
- evolve the function H_i from the hard scale $\mu_h \sim m_t$ to μ_f
- evolve soft function $W_i^{R_\alpha}$ from a low scale $\mu_s \sim m_t\beta^2$ to μ_f.



Resummation of Coulomb corrections

β^{-n} singularities arise from non-local 4-fermion interaction mediated by Coulomb potential:

$$V_C(r) = \frac{D_{R_\alpha}\alpha_s(\mu_C)}{r}\left[1+\frac{\alpha_s}{4\pi}\left(\hat{a}_1+2\beta_0\ln(\mu_C r)\right)+\ldots\right] \qquad D_{R_\alpha} = \frac{1}{2}(C_{R_\alpha}-C_R-C_{R'})$$

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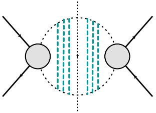
Resummation of Coulomb effects well understood from **PNRQCD** and quarkonia physics:

$$J_{R_\alpha}(E) = 2\mathrm{Im}\left[G^{(0)}_{C,R_\alpha}(0,0;E)\Delta_{\mathrm{nc}}(E) + G^{(1)}_{C,R_\alpha}(0,0;E) + \ldots\right]$$

$G^{(0)}_{C,R_\alpha} \quad \Leftrightarrow$



$$= -\frac{m_t^2}{4\pi}\left\{\sqrt{-\frac{E}{m_t}} + \alpha_s(-D_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{4\,m_t E}{\mu_C^2}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 - \frac{\alpha_s(-D_{R_\alpha})}{2\sqrt{-E/m_t}}\right)\right]\right\}$$

Includes bound-states below threshold (E<0)

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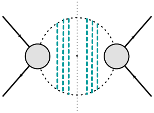
$G^{(0)}_{C,R_\alpha}$ $\Leftrightarrow$ 

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Includes bound-states below threshold (E<0)

- NLO Coulomb potential: $G^{(1)}_{C,R_\alpha} = \mathcal{O}(\alpha_s^2/\beta, \alpha_s^2 \ln\beta/\beta) + \ldots$
- Non-Coulomb potentials: $\Delta_{\rm nc} = 1 + \mathcal{O}(\alpha_s^2 \ln\beta) + \ldots$

$t\bar{t}$ production at NNLL/NNLO

All ingredients for NNLL resummation of $t\bar{t}$ cross section are known:

- 1-loop colour-separated hard functions $H_i^{(1)}$ [Czakon, Mitov '09]
- 2**-loop soft anomalous dimension** [Beneke, PF, Schwinn '09; Czakon, Mitov, Sterman '09]

$$\gamma_{H,s}^{R_\alpha,(1)} = C_{R_\alpha}\left[-C_A\left(\frac{98}{9}-\frac{2\pi^2}{3}+4\zeta_3\right)+\frac{20}{9}n_f\right]$$

- NLO Coulomb and non-Coulomb potentials [Beneke, Signer, Smirnov '99]

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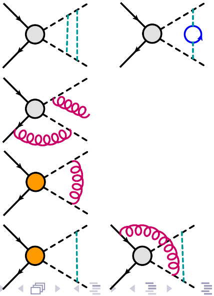
Can be used to construct approx. NNLO containing all terms singular in β
[Beneke, PF, Czakon, Mitov, Schwinn '09; HATHOR Aliev et al. '10]

$$\begin{aligned}\hat{\sigma}^{\text{NNLO}}_{\text{approx.}} = & \frac{k_{\text{LO}}^2}{\beta^2}+\frac{1}{\beta}\left[k_{\text{NLO},1}\ln\beta+k_{\text{NLO},0}\right]+k_{\text{n-C}}\ln\beta \\ & +c_{S,4}^{(2)}\ln^4\beta+c_{S,3}^{(2)}\ln^3\beta+c_{S,2}^{(2)}\ln^2\beta+c_{S,1}^{(2)}\ln\beta \\ & +H^{(1)}\left[c_{S,2}^{(1)}\ln^2\beta+c_{S,1}^{(1)}\ln\beta\right] \\ & +\frac{k_{\text{LO}}}{\beta}\left[c_{S,2}^{(1)}\ln^2\beta+c_{S,1}^{(1)}\ln\beta+c_{S,0}^{(1)}+H^{(1)}\right]\end{aligned}$$



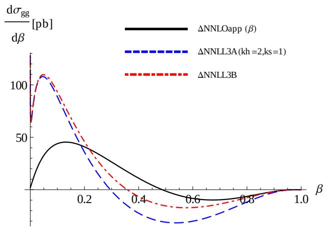
N^3LO approximations

Resummed cross sections can be re-expanded to even higher order in α_s
$\Rightarrow$ information on residual theoretical uncertainties

$$f^{(3,0)}_{gg(1)} = 147456.\ln^6\beta - 59065.6\ln^5\beta - 286099.\ln^4\beta + 349463.\ln^3\beta + \frac{1}{\beta}\left[121278.\ln^4\beta + 103557.\ln^3\beta - 164944.\ln^2\beta + 56418.5\ln\beta\right] + \frac{1}{\beta^2}\left[22166.\ln^2\beta + 39012.1\ln\beta - 2876.61\right] + \tilde{f}^{(3,0)}_{gg(1)} + \mu_f - \text{dep. terms}$$

Not all singular terms known exactly at $\mathcal{O}(\alpha_s^5)$ $\Leftrightarrow \tilde{f}^{(3,0)}_{gg(1)}$

- **N^3LO_A**: keep all terms generated by resummation at $\mathcal{O}(\alpha_s^5)$
- **N^3LO_B**: keep only terms known exactly $\rightarrow \tilde{f}^{(3,0)}_{pp'} = 0$



Matching and scales

Resumed cross sections are matched to fixed-order results

Two possible prescriptions:

- $\text{NNLL}_1 = \text{NNLL} - \text{NNLL}(\alpha_s) + \text{NLO}$
- $\text{NNLL}_2 = \text{NNLL} - \text{NNLL}(\alpha_s^2) + \text{NNLO}_{\text{approx}}$

Choose NNLL_2 as default choice in the following

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NNLL_1, NNLL_2 (and $\text{N}^3\text{LO}_\text{A}$) depend on several scales, μ_f, μ_h, μ_s, μ_C.

What are reasonable choices for μ_s, μ_h and μ_C?

- **Hard scale**: $\mu_h = 2m_t$
- **Coulomb scale**: set by typical virtuality of Coulomb gluons $\sqrt{|q^2|} \sim m_t\beta \sim m_t\alpha_s$

$$\Rightarrow \mu_C = \max\{2m_t\beta, C_F m_t \alpha_s(\mu_C)\}$$

$\hookrightarrow$ twice inverse Bohr radius of first bound state

Soft scale determination

Choose running soft scale $\mu_s \sim q_s \sim m_t\beta^2$ above a certain cutoff $\beta_{\rm cut}$ (avoid **Landau pole**)

$$\mu_s^< = k_s m_t \beta_{\rm cut}^2 \qquad\qquad \mu_s^> = k_s m_t \beta^2$$

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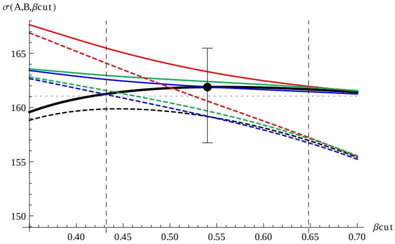
β_{cut} chosen such that resummation ambiguities are small for $\beta < \beta_{\text{cut}}$ and pertubation theory is not spoiled by large logs for $\beta > \beta_{\text{cut}}$:

$$\hat{\sigma}_{t\bar{t}}(A_<, B_>, \beta_{\text{cut}}) = \hat{\sigma}_{t\bar{t}}^{A<}\ \theta(\beta_{\text{cut}} - \beta) + \hat{\sigma}_{t\bar{t}}^{B>}\ \theta(\beta - \beta_{\text{cut}})$$

- $A_< = \text{NNLL}_1,\ \text{NNLL}_2$
- $B_> = \text{NNLL}_2,\ \text{NNLO}_{\text{approx}},\ \text{N}^3\text{LO}_\text{A},\ \text{N}^3\text{LO}_\text{B}$



β_{cut} determined by minimisation of the envelope of the 8 curves

β_{cut} :	0.35 (Tevatron),	0.54 (LHC7),	0.55 (LHC14)
$\mu_s^<$:	42.5GeV (Tevatron),	101GeV (LHC7),	105GeV (LHC14)

Estimate of theoretical uncertainties

Resummation uncertainties

- β_{cut}: vary β_{cut} by $\pm 20\%$ and take width of envelope of the 8 curves
- $\mu_s = k_s m_t \beta^2$: choose default value as $k_s = 2$ and vary between $k_s = 1$ and $k_s = 4$
- power-suppressed corrections: consider difference between $E = m_t \beta^2$ and $E = \sqrt{s} - 2m_t$

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Scale uncertainties

- for NLO and NNLO vary $m_t/2 < \mu_F, \mu_R < 2m_t$ with $1/2 < \mu_R/\mu_F < 2$
- for NNLL vary μ_C in the interval $[\tilde{\mu}_C/2, 2\tilde{\mu}_C]$.
  μ_F, μ_H are varied simultaneously with the constraint $1 < \mu_H/\mu_F < 4$

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$O(\alpha_s^2)$ constant

- choose $C_{pp'}^{(2)} = 0$ as default in $\text{NNLO}_{\text{approx}}$
- vary by $\pm {C_{pp'}^{(1)}}^2$, where $C_{pp'}^{(1)}$ is the $\mathcal{O}(\alpha_s)$ constant for the partonic channel pp'

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PDF+α_s uncertainty

- MSTW2008 with 90% CL sets
- $\alpha_s(M_Z) = 0.1171 \pm 0.0034$

NNLL/NNLO total $t\bar{t}$ cross section

$m_t = 173.3$ GeV, $\mu_F = m_t$, MSTW2008NLO/NNLO
Beneke, PF, Klein, Schwinn [arXiv:1109.1536[hep-ph]]

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC@7	LHC@14
NLO	$6.68^{+0.36+0.51}_{-0.75-0.45}$	$158.1^{+18.5+13.9}_{-21.2-13.1}$	$884^{+107+65}_{-106-58}$
NNLO$_{\text{app}}$	$7.06^{+0.27+0.69}_{-0.34-0.53}$	$161.1^{+12.3+15.2}_{-11.9-14.5}$	891^{+76+64}_{-69-63}
NNLL$_2$	$7.22^{+0.31+0.71}_{-0.47-0.55}$	$162.6^{+7.4+15.4}_{-7.5-14.7}$	896^{+40+65}_{-37-64}
Coul.($>\alpha_s^2$)	-0.038	0.80	2.8

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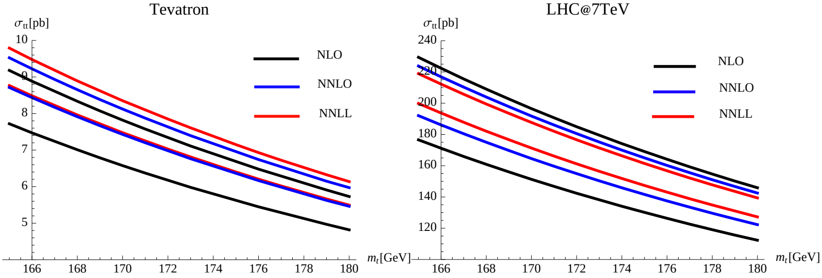
Experiment ($m_t = 172.5$):

- **CDF**: 7.5 ± 0.48
- **D0**: $7.56^{+0.63}_{-0.56}$
- **Atlas**: 179.0 ± 11.8
- **CMS**: 154 ± 18

Theory:

- Ahrens et al, 2011, NNLO($m_t = 173.1$):
  Tev.: $6.63^{+0.007}_{-0.41}$, **LHC7**: 155^{+8}_{-9}
- Kidonakis, 2011, NNLO($m_t = 173$):
  Tev.: $7.08^{+0.00}_{-0.24}$, **LHC7**: 163^{+7}_{-5}
- Ahrens et al, 2010/11, NNLL:
  Tev.: $6.55^{+0.16}_{-0.14}$ (1PI), $6.46^{+0.18}_{-0.19}$ (PIM)
  LHC7: 150^{+7}_{-7} (1PI), 147^{+7}_{-6} (PIM)

Residual theoretical uncertainties



- At **LHC** **reduction of theoretical uncertainty** from NLO → NNLO → NNLL
- At **Tevatron** NNLO has smallest uncertainty:
  - $q\bar{q}$ channel (Tevatron) VS gg channel (LHC)
  - strong dependence on the scale choice: $\text{NNLO}(\mu_R = 2m_t) = 6.73^{+0.34}_{-0.46}$

Tevatron : $\Delta\sigma^{\text{th, NNLL}}/\sigma \sim +4.5{-}6.5\%$ **LHC** : $\Delta\sigma^{\text{th, NNLL}}/\sigma \sim \pm 4.5\%$

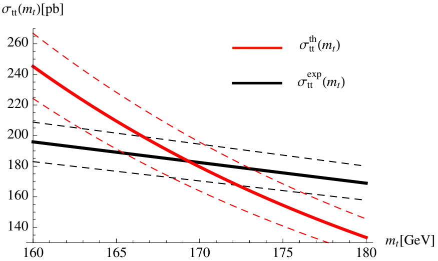
plus additional $\sim \pm 7 - 9\%$ from PDF+α_s uncertainty!

Top mass extraction

Measured cross section can be used to **extract the top-quark pole mass**

$$\sigma^{\text{exp}}_{t\bar{t}}(m_t) = (411.9 - 1.35 m_t)\ \text{pb}$$

[ATLAS-CONF-2011-121]



Extracted mass given by the maximum of $f(m_t) = \int f_{\text{th}}(\sigma|m_t) \cdot f_{\text{exp}}(\sigma|m_t) d\sigma$, with $f_{\text{th}}(\sigma|m_t)$ a gaussian likelihood function

$$m_t = 169.8^{+4.9}_{-4.7}\ \text{GeV}$$

good agreement with direct mass reconstruction at Tevatron: $m_t = 173.3 \pm 1.1$ GeV

Summary

- Fixed-order prediction for $t\bar{t}$ production improved by threshold resummations
  - $\Rightarrow$ factorization of **hard**, **soft** and **Coulomb** modes in PNRQCD+SCET
  - $\Rightarrow$ **simultaneous resummation** of threshold logarithms and Coulomb singularities in momentum space
- Size of threshold term at Tevatron and LHC
  - $\Rightarrow$ **NNLO** corrections $\sim 9.5\%$ at Tevatron and $\sim 8\%$ at LHC
  - $\Rightarrow$ **NNLL** corrections beyond NNLO not negligible at Tevatron ($\sim 3.5\%$) but small at LHC ($\sim 1\%$)
- Remaining theoretical uncertainties
  - $\Rightarrow$ Tevatron: $\Delta\sigma \sim \pm 4 - 6.5\%$
  - $\Rightarrow$ LHC: $\Delta\sigma \sim \pm 4.5\%$
  - $\Rightarrow$ PDF+α_s: $\sim \pm 7 - 9\%$
- Top mass extraction
  - $\Rightarrow$ $\Delta m_t \pm 5$ GeV
  - $\Rightarrow$ reasonable agreement with direct determination

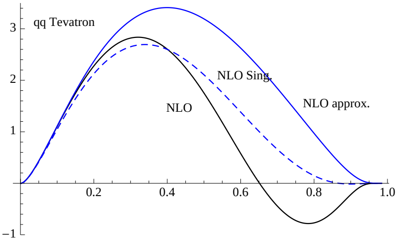
Backup slides

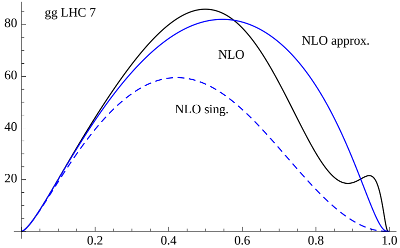
Contribution of threshold-enhanced terms

$\sqrt{s} \gg 2m_t \Rightarrow$ **How good is the threshold approximation?**
can study the approximation at the NLO level...

Plot $8\beta m_t^2/(s(1-\beta^2)^2)\mathcal{L}_{gg}(\beta)\hat{\sigma}_{t\bar{t}}(\beta)$:

- **NLO**: exact NLO result
- **NLO sing.**: only singular terms in β
- **NLO approx.**: singular terms + O(1) term ($\Leftrightarrow H_i^{(1)}$)





NLO sing. is good approximation only up to $\beta \sim 0.3$
However: expect NNLO approximation to be better (more singular terms at $O(\alpha_s^2)$...)

Theoretical uncertainties

Scale, resummation, NNLO constant and PDF+α_s uncertainties

	Tevatron	LHC ($\sqrt{s} = 7$ TeV)	LHC ($\sqrt{s} = 14$ TeV)
NLL_2	$7.31\,{}^{+0.25}_{-0.03}\,{}^{+0.30}_{-0.53}\,{}^{+0.10}_{-0.10}\,{}^{+0.57}_{-0.54}$	$172.8\,{}^{+14.8}_{-\;0.8}\,{}^{+13.0}_{-14.7}\,{}^{+4.7}_{-4.7}\,{}^{+15.9}_{-14.6}$	$954\,{}^{+85}_{-\;5}\,{}^{+65}_{-71}\,{}^{+28}_{-28}\,{}^{+74}_{-66}$
$NNLL_2$	$7.22\,{}^{+0.21}_{-0.41}\,{}^{+0.20}_{-0.21}\,{}^{+0.10}_{-0.10}\,{}^{+0.71}_{-0.55}$	$162.6\,{}^{+4.2}_{-1.9}\,{}^{+3.9}_{-5.6}\,{}^{+4.7}_{-4.7}\,{}^{+15.4}_{-14.7}$	$896\,{}^{+22}_{-\;5}\,{}^{+18}_{-23}\,{}^{+28}_{-28}\,{}^{+65}_{-64}$

Resummation uncertainties

Collider	i) E	ii) k_s	iii) $\beta_{\rm cut}$
Tevatron	${}^{+0.01}_{-0.00}\,\left\{{}^{+0.1\%}_{-0.0\%}\right\}$	${}^{+0.10}_{-0.10}\,\left\{{}^{+1.4\%}_{-1.4\%}\right\}$	${}^{+0.17}_{-0.18}\,\left\{{}^{+2.4\%}_{-2.5\%}\right\}$
LHC ($\sqrt{s} = 7$ TeV)	${}^{+1.0}_{-0.0}\,\left\{{}^{+0.6\%}_{-0.0\%}\right\}$	${}^{+0.8}_{-2.2}\,\left\{{}^{+0.5\%}_{-1.4\%}\right\}$	${}^{+3.7}_{-5.1}\,\left\{{}^{+2.3\%}_{-3.1\%}\right\}$
LHC ($\sqrt{s} = 14$ TeV)	${}^{+5}_{-0}\,\left\{{}^{+0.6\%}_{-0.0\%}\right\}$	${}^{+\;0}_{-10}\,\left\{{}^{+0.0\%}_{-1.1\%}\right\}$	${}^{+17}_{-21}\,\left\{{}^{+1.9\%}_{-2.3\%}\right\}$

Fixed soft scale

Fixed soft scale: chosen such that one-loop soft corrections to the hadronic cross section are minimised [Becher, Neubert, Xu '07]

$$\frac{d}{d\bar{\mu}_s}\sum_{p,p'}\int_{\tau_0}^{1} d\tau L(\tau,\bar{\mu}_s)\frac{\hat{\sigma}^{(1)}_{pp',\mathrm{soft}}(\tau s,\bar{\mu}_s)}{\sigma^{(0)}_{N_1N_2}(s,\bar{\mu}_s)} = 0$$

choice motivated by requirement of good perturbative convergence at the scale μ_s

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choice motivated by requirement of good perturbative convergence at the scale μ_s

Choice of fixed soft scale not completely satisfactory:

- **Kinematic ambiguities**: $E = \sqrt{s} - 2m_t \Leftrightarrow m_t\beta^2$

$$\begin{aligned} \ln(m_t\beta^2/\mu_s):&\quad \tilde{\mu}_s = 35\mathrm{GeV}(\mathrm{Tevatron}), \quad 58\,\mathrm{GeV}\,(\mathrm{LHC7}), \quad 65\,\mathrm{GeV}\,(\mathrm{LHC14}),\\ \ln(E/\mu_s):&\quad \tilde{\mu}_s = 52\,\mathrm{GeV}\,(\mathrm{Tevatron}), \quad 99\,\mathrm{GeV}\,(\mathrm{LHC7}), \quad 120\,\mathrm{GeV}\,(\mathrm{LHC14}),\end{aligned}$$

- A fixed soft scale does not correctly reproduce all the threshold logs at NNLO ($\ln\beta$ terms resummed only in an average sense...)

Fixed soft scale VS Running soft scale

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC@7	LHC@14
NLO	$6.68^{+0.36(5.3\%)}_{-0.75(11\%)}$	$158.1^{+18.5(12\%)}_{-21.2(13\%)}$	$884^{+107(12\%)}_{-106(12\%)}$
NLL_2	$7.31^{+0.39(5.3\%)}_{-0.53(7.3\%)}$	$172.8^{+19.7(11\%)}_{-14.7(12\%)}$	$954^{+107(11\%)}_{-71(7.4\%)}$
NLL_2(fixed μ_s)	$6.90^{+0.32(4.6\%)}_{-0.42(6.1\%)}$	$157.6^{+23.3(15\%)}_{-20.2(13\%)}$	$879^{+136(15\%)}_{-113(13\%)}$
NNLO_{app}	$7.06^{+0.25(3.5\%)}_{-0.33(4.7\%)}$	$161.1^{+11.4(7.1\%)}_{-10.9(6.8\%)}$	$891^{+71(8.0\%)}_{-63(7.1\%)}$
NNLL_2	$7.22^{+0.29(4.0\%)}_{-0.46(6.4\%)}$	$162.6^{+5.7(3.5\%)}_{-5.9(3.6\%)}$	$896^{+29(3.2\%)}_{-24(2.7\%)}$
NNLL_2(fixed μ_s)	$7.08^{+0.16(2.3\%)}_{-0.28(3.6\%)}$	$157.4^{+10.3(6.5\%)}_{-3.6(2.3\%)}$	$868^{+69(7.9\%)}_{-21(2.4\%)}$