UNIVERSITY OF CAMBRIDGE

Department of Physics
Cavendish Laboratory

Science & Technology Facilities Council

Double Parton Scattering Singularity in One-Loop Integrals

Jo Gaunt

RADCOR 2011, Mamallapuram, India, 29th September 2011

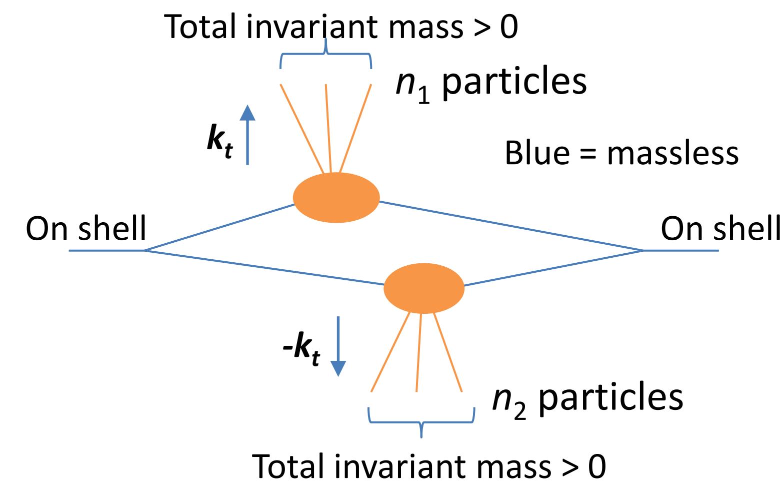
Based on JHEP 1106 (2011) 048 (JG, W J Stirling)

Introduction

- Definition of double parton scattering (DPS) singularity in one loop amplitudes
- (A) motivation for studying the DPS singularity – some unexplained behaviour in simple SM one loop diagrams close to points at which the loop contains a DPS singularity.
- Derivation of a compact analytical expression for the DPS divergent part of any one-loop diagram. I use this to explain behaviour of the loop integrals.
- I demonstrate how the work links in with double parton scattering theory – it can be used to show that there are some theoretical issues with a certain framework for describing double parton scattering.

Double Parton Scattering Singularity

Pinch singularity in Feynman Integral (Landau Singularity) that occurs in diagrams of this structure.

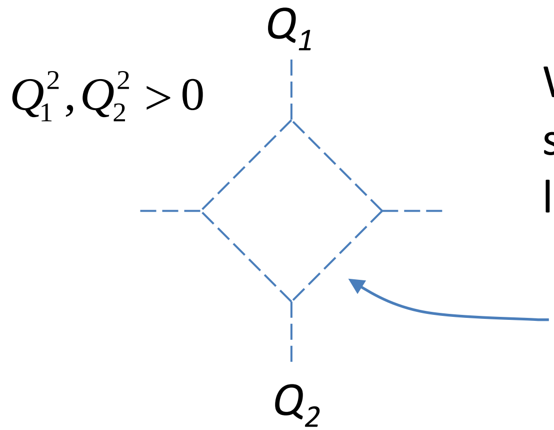


Singularity occurs when external particles attached to top (or bottom) leg of loop have zero total transverse momentum (i.e. $\boldsymbol{k_t} = 0$).

A Landau singularity corresponds to the 'resonance' of a Feynman diagram with a classical scattering process (Coleman-Norton Theorem). In this case the classical scattering process is a process where both initial state partons split and double scatter.

Nagy and Soper, Phys. Rev. D74 (2006) 093006.

Naive expectations for SM loop behaviour around DPS singularity



With no numerator structure (scalar particles), DPS singularity corresponds to an actual divergence in the loop integral (with the number of dimensions equal to 4).

e.g. scalar 'crossed box' $L_{DPS} \propto 1/\mathbf{Q}_2^2$

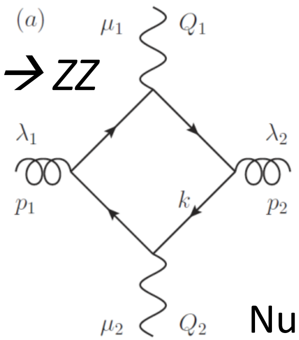
L. D. Ninh [arXiv:0810.4078].

Naively might expect SM boxes to behave similarly – they will only be less singular if the numerators in the SM loops happen to vanish at the DPS singular point.

Standard Model Box Graphs

Can extract the leading low $\boldsymbol{Q_2}$ behaviour from analytic results for some SM box amplitudes. Results are only presented for those helicity amplitudes that diverge as $\boldsymbol{Q_2} \rightarrow 0$.

gg → ZZ



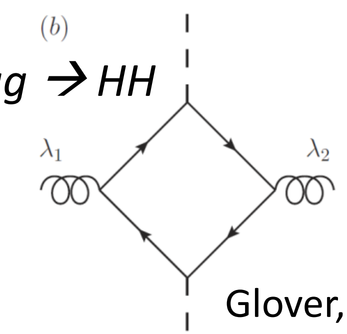
$$\overset{\lambda_1\lambda_2}{L_{DPS}(++)} = L_{DPS}(--) = -\frac{8M_H^2\pi^3\log(\boldsymbol{Q_2}^2)}{s}$$

Glover, van der Bij
Nucl.Phys. B309 (1988) 282.

$$\overset{\lambda_1\lambda_2\mu_1\mu_2}{L_{DPS}(++++)} = L_{DPS} = (----) = \frac{8\pi^3\left[s - 2M_Z^2 + s\sqrt{1-4M_Z^2/s}\right]\log(\boldsymbol{Q_2}^2)}{s}$$

$$L_{DPS}(++--) = L_{DPS} = (--++) = \frac{8\pi^3\left[s - 2M_Z^2 - s\sqrt{1-4M_Z^2/s}\right]\log(\boldsymbol{Q_2}^2)}{s}$$

gg → HH



Glover, van der Bij
Nucl.Phys. B321 (1989) 561.

DPS divergence in SM graphs seems to be demoted from a power to a logarithm at most. What is the physical mechanism behind this?

Why do some external helicity configurations give a divergence at $\boldsymbol{Q_2} \rightarrow 0$ but not others?

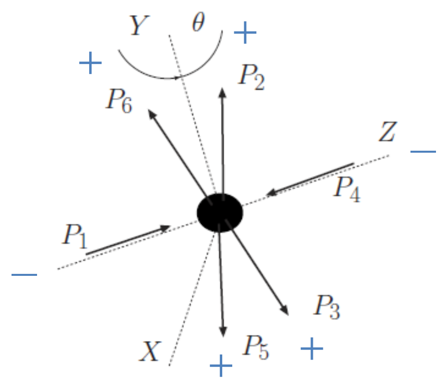
Six Photon Amplitude

Very thorough numerical study of certain six photon helicity amplitudes in the region of phase space around a point corresponding to a DPS singularity for some graphs was performed by Bernicot and Guillet.

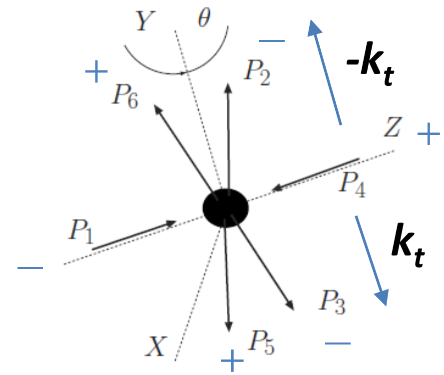
C. Bernicot [arXiv:0804.1315]
Z. Bern et al., [arXiv:0803.0494]

Amplitudes Considered:

MHV:

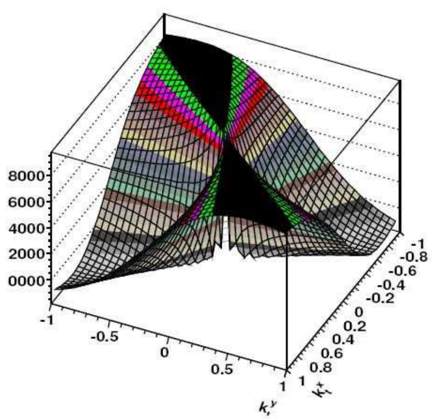


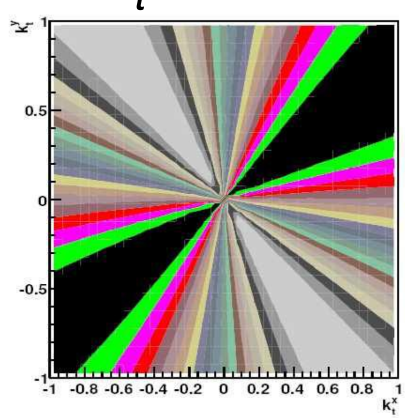
NMHV:



[Helicity labels given are relative to incoming particles]

2D approach to a point with $\boldsymbol{k_t} = 0$:



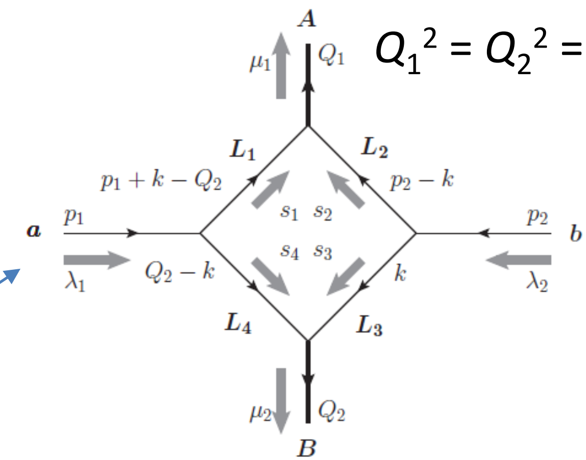


Conclusion: both MHV and NMHV amplitude do not diverge at $\boldsymbol{k_t} = 0$. Why is this?

Analytic Expression for DPS Divergence

$Q_1^2 = Q_2^2 = Q^2 > 0$

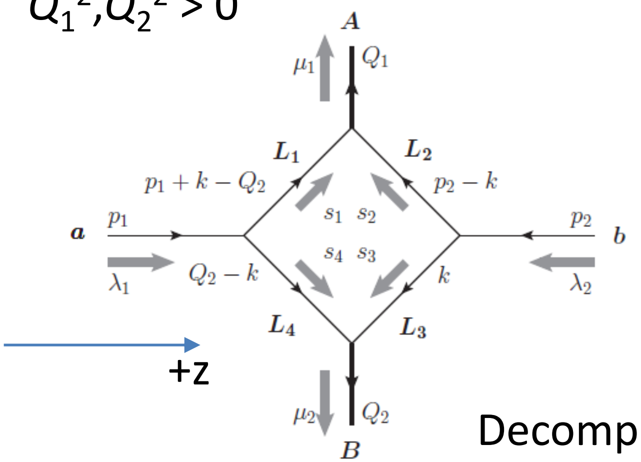
To answer these questions we performed an analytical study of the DPS divergence in one loop integrals. The simplest loop that can contain DPS singularity is this crossed box – we studied this first.



We desire an analytical expression for the **DPS divergent part** of a crossed box integral = contribution to crossed box loop integral at small external transverse momentum coming from region of loop integration around DPS singular point (i.e. loop particles having small transverse momentum and virtualities).

DPS Divergence in Crossed Box

$Q_1^2, Q_2^2 > 0$



Numerator – depends on nature of particles in diagram

$$L = \int d^d k \frac{\mathcal{N}}{[k^2 + i\epsilon][(k - Q_2)^2 + i\epsilon][(p_1 + k - Q_2)^2 + i\epsilon][(p_2 - k)^2 + i\epsilon]}$$

Loop propagator denominators – universal to all crossed boxes

Decompose all vectors in terms of a light cone basis defined using p_1 and p_2 as basis vectors.

Perform k^- integral followed by k^+ integral using contour methods, throwing away terms that are negligible in region around DPS singularity $|k^- - Q_2^-|, k^+, |\mathbf{k} - \mathbf{Q_2}|, |\mathbf{k}| << Q_i^+, Q_i^-$

$$L_{DPS} \simeq \frac{(2\pi i)^2}{2s} \int_{|\boldsymbol{k}| \ll Q_i^+, Q_i^-} \frac{d^{d-2}\boldsymbol{k}\, \mathcal{N}\,|_{k^- = Q_2^-, k^+ = 0}}{(\boldsymbol{k} - \boldsymbol{Q_2})^2 \boldsymbol{k}^2}$$

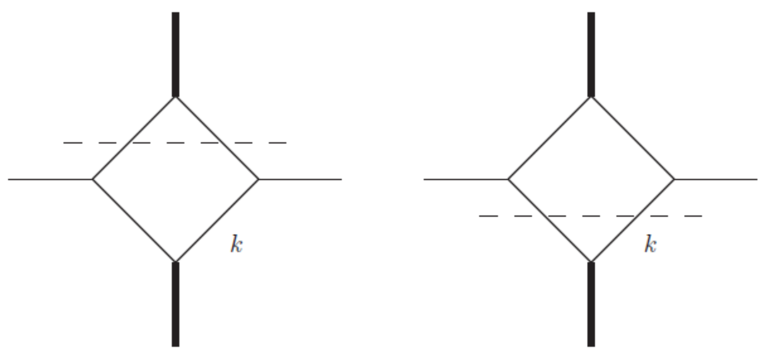
Compact expression!

Cutkosky cuts of the box

$$L_{DPS} \simeq \frac{(2\pi i)^2}{2s} \int_{|\boldsymbol{k}| \ll Q_i^+, Q_i^-} \frac{d^{d-2}\boldsymbol{k}\, \mathcal{N}\,|_{k^-=Q_2^-, k^+=0}}{(\boldsymbol{k}-\boldsymbol{Q_2})^2 \boldsymbol{k}^2}$$

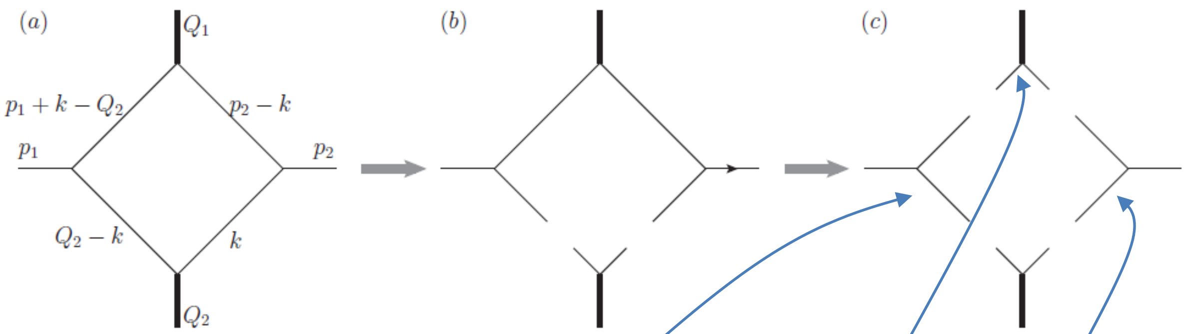
DPS divergence is in the real part of box integral – i.e. imaginary part of box amplitude since $i\mathcal{M} \propto L$

➡ DPS divergent part of loop integral can also be found by taking sum of cuts in limit where external transverse momenta are small and internal particles are almost on shell.



Two cuts give the same contribution.

Decomposition of DPS divergent part of Crossed Box



$$L_{DPS}(\lambda_1\lambda_2\mu_1\mu_2) = \sum_{s_i,L_i} \int d^dk\,\delta(k^2)\delta((k-Q_2)^2)\Phi^{\lambda_2\to s_2 s_3}_{b\to L_2 L_3}(p_2; p_2-k, k)$$
$$\times \Phi^{\lambda_1\to s_1 s_4}_{a\to L_1 L_4}(p_1; p_1+k-Q_2, Q_2-k)\mathcal{M}^{s_3 s_4\to\mu_2}_{L_3 L_4\to B}(k, Q_2-k; Q_2)$$
$$\times \mathcal{M}^{s_1 s_2\to\mu_1}_{L_1 L_2\to A}(p_1+k-Q_2, p_2-k; Q_1)\left(\times\sqrt{Q_1^2/Q_2^2}\right)$$

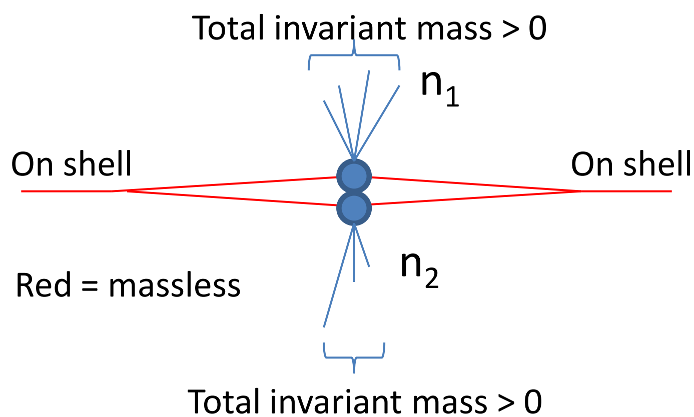
Hard matrix elements – can evaluate with incoming off-shellness and transverse momentum = 0

‘Light-cone wavefunction to find L_2L_3 in b’

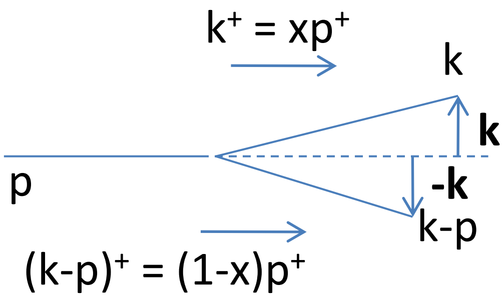
DPS divergent part of arbitrary one-loop integral

For crossed box: $$L_{DPS}(\lambda_1\lambda_2\mu_1\mu_2) = \sum_{s_i,L_i}\int d^dk\delta(k^2)\delta((k-Q_2)^2)\Phi^{\lambda_2\to s_2s_3}_{b\to L_2L_3}(p_2;p_2-k,k)$$
$$\times\Phi^{\lambda_1\to s_1s_4}_{a\to L_1L_4}(p_1;p_1+k-Q_2,Q_2-k)\mathcal{M}^{s_3s_4\to\mu_2}_{L_3L_4\to B}(k,Q_2-k;Q_2)$$
$$\times\mathcal{M}^{s_1s_2\to\mu_1}_{L_1L_2\to A}(p_1+k-Q_2,p_2-k;Q_1)\left(\times\sqrt{Q_1^2/Q_2^2}\right)$$

To obtain DPS divergent part of an arbitrary one-loop diagram (of the appropriate character), replace 2→1 matrix elements by 2→n_1, 2→n_2 matrix elements above.



Light-cone wavefunctions



$$\Phi_{a\to bc}^{\lambda_1\to s_1 s_2}(p;k,k-p) = X_{a\to bc}^{\lambda_1\to s_1 s_2}(x) K_{a\to bc}^{\lambda_1\to s_1 s_2}(\mathbf{k})$$

Square root of helicity dependent splitting function.

Transverse momentum dependent factor *K* contains a $1/\mathbf{k}^2$ factor from propagator denominator, multiplied by a further factor coming from splitting matrix element.

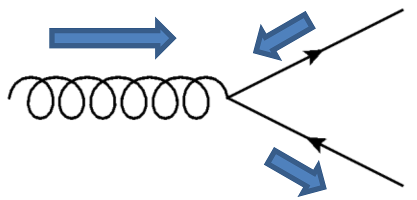
Scalar ϕ^3 theory : splitting matrix element doesn't depend on **k.** For an arbitrary loop:

$$L_{DPS,\phi^3} \sim \int d^{d-2}\mathbf{k}\, K(\mathbf{k}-\mathbf{Q}_2) K(\mathbf{k}) \propto \int \frac{d^{d-2}\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{Q}_2)^2} \propto \frac{1}{\mathbf{Q}_2^2} \quad \text{when d=4}$$

Light-cone wavefunctions

For any Standard Model massless particle splitting, matrix element is proportional to **k**.

Can show where this comes from for e.g. g → q$\bar{q}$ graph:



Helicity conservation ➔ J_z of final state = 0 in collinear limit

J_z of initial state $=\pm 1$ ➔ splitting must be suppressed in collinear limit.

$$L_{DPS,SM} \sim \int d^{d-2}\mathbf{k}\, K(\mathbf{k}-\mathbf{Q_2})K(\mathbf{k}) \sim \int_0^{Q^2} \frac{d^{d-2}\mathbf{k}\,\mathbf{k}^i(\mathbf{k}-\mathbf{Q_2})^j}{\mathbf{k}^2(\mathbf{k}-\mathbf{Q_2})^2} \sim \log\left(\frac{\mathbf{Q}_2^2}{Q^2}\right)$$

Strongly related to logarithmic scaling violations of parton distributions

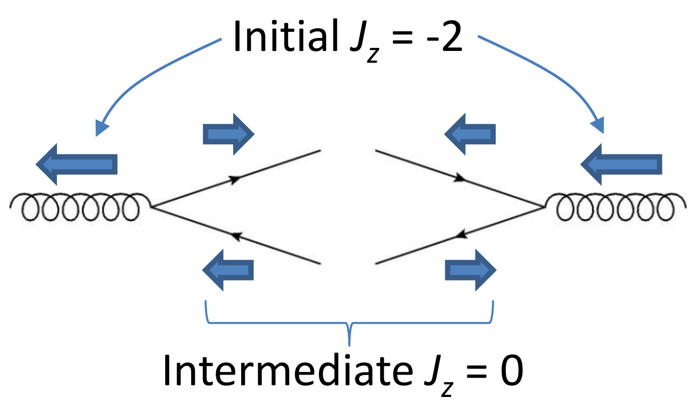
➔ DPS divergence in SM graphs cannot be stronger than a logarithm of $\boldsymbol{Q_2}$.

SM crossed boxes – why are some helicity configurations not divergent?

1. Suppression of divergence from the wavefunction factors.

For a gg → q$\bar{q}$q$\bar{q}$ → AB box, if the helicities of initial state gluons are different, then the product of lightcone wavefunctions vanishes upon integration over $\boldsymbol{k}$ when $\boldsymbol{Q_2}$ =0 **.**

Physical explanation: total J_z nonconservation in collinear limit.

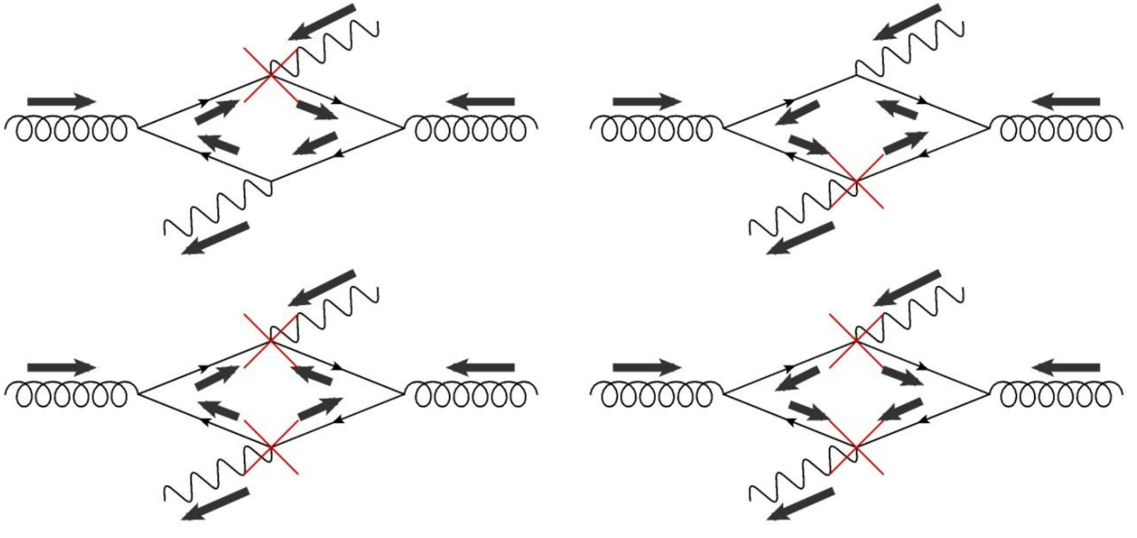


SM crossed boxes – why are only some helicity configurations divergent?

2. Suppression of divergence from matrix element factors.

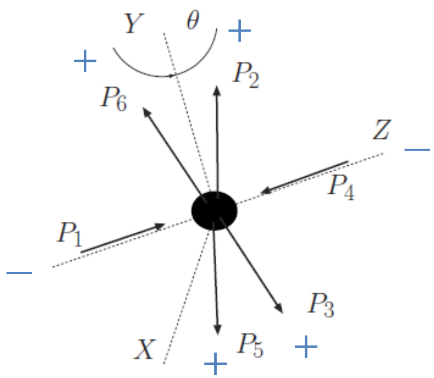
For the *gg* → *ZZ* crossed box, if the helicities of the final state bosons are not the same, at least one of the $q\bar{q}$ → Z matrix elements vanish for all internal helicity configurations allowed by g → $q\bar{q}$ wavefunctions.

Physical explanation: no configuration of internal helicity in the loop which simultaneously ensures helicity conservation at every external vertex, and conserves J_z in both $q\bar{q}$ → Z processes in the collinear limit.



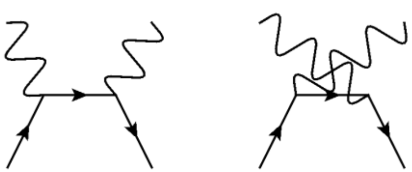
Six photon helicity amplitudes

Similar arguments can be used to explain why MHV and NMHV amplitudes studied by Bernicot and Guillet are finite at $\boldsymbol{k_t} = 0$.



MHV amplitude: suppression in matrix element factors.

There are four graphs giving a DPS divergence at the point $\boldsymbol{k}_t = 0$. The matrix elements to be used in the calculation of the DPS divergent parts of the **sum** of these graphs are the sum of the following two graphs:

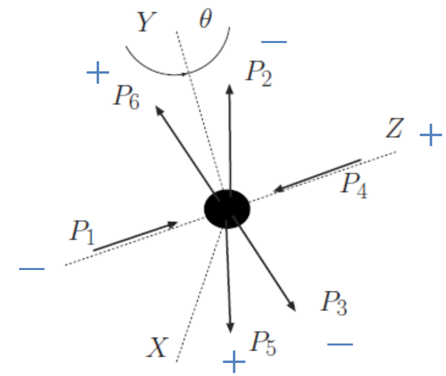


= full matrix element for $q\bar{q} \rightarrow \gamma\gamma$. For MHV amplitude studied, photons have same helicity in both matrix elements, and go to zero by MHV rules for QED.

Six photon helicity amplitudes

NMHV amplitude: suppression in light cone wavefunctions

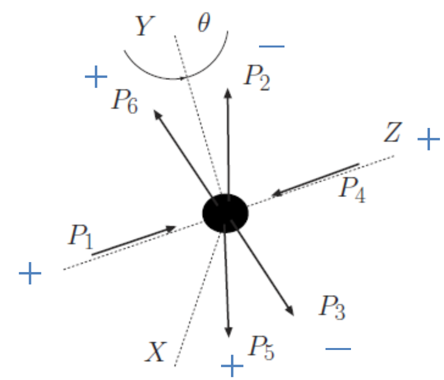
Unequal photon helicities in initial state → total J_z nonconservation between initial state and long-lived $q\bar{q}q\bar{q}$ intermediate state in collinear limit.



In general:

No NMHV six-photon amplitude can ever contain a DPS divergence - however one distributes the helicities, one always ends up either with the initial state photons having opposite helicities, or with one of the pairs of the final state photons having the same helicity

There are MHV amplitudes that do have logarithmic DPS divergences – for example:

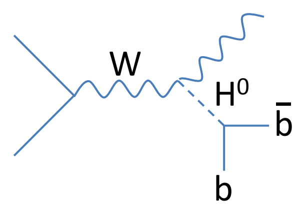


Application of the work to double parton scattering theory

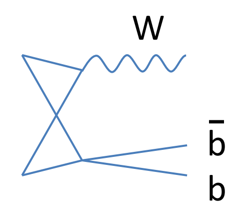
Double parton scattering is the process in which two pairs of partons participate in hard interactions in a single proton-proton collision.

DPS processes can constitute important backgrounds to Higgs and other interesting signals and can themselves be considered as interesting signal processes, since they reveal information about parton pair correlations in the proton.

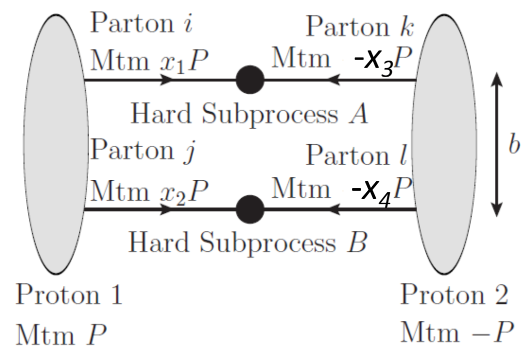
Higgs signal



DPS background



Assuming only the factorisation of the two hard processes A and B, we can write the cross section for proton-proton DPS as follows:



Two-parton GPDs

Parton-level cross sections

$$\sigma^D_{(A,B)} \propto \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, \boldsymbol{b}; Q_A^2, Q_B^2)\hat{\sigma}_{ik\to A}(\hat{s} = x_1x_3s)\hat{\sigma}_{jl\to B}(\hat{s} = x_2x_4s)$$
$$\times\, \Gamma_{kl}(x_3, x_4, \boldsymbol{b}; Q_A^2, Q_B^2)dx_1dx_2dx_3dx_4d^2\boldsymbol{b}$$

Transverse parton pair separation

Double Parton Scattering theory

In many extant studies of DPS, it is assumed that the 2pGPD can be approximately factorised into a product of a longitudinal piece and a (typically flavour and scale independent) transverse piece:

$$\Gamma_{ij}(x_1, x_2, \boldsymbol{b}; Q_A^2, Q_B^2) \simeq D_p^{ij}(x_1, x_2; Q_A^2, Q_B^2) F(\boldsymbol{b})$$

Smooth function of size R_p.

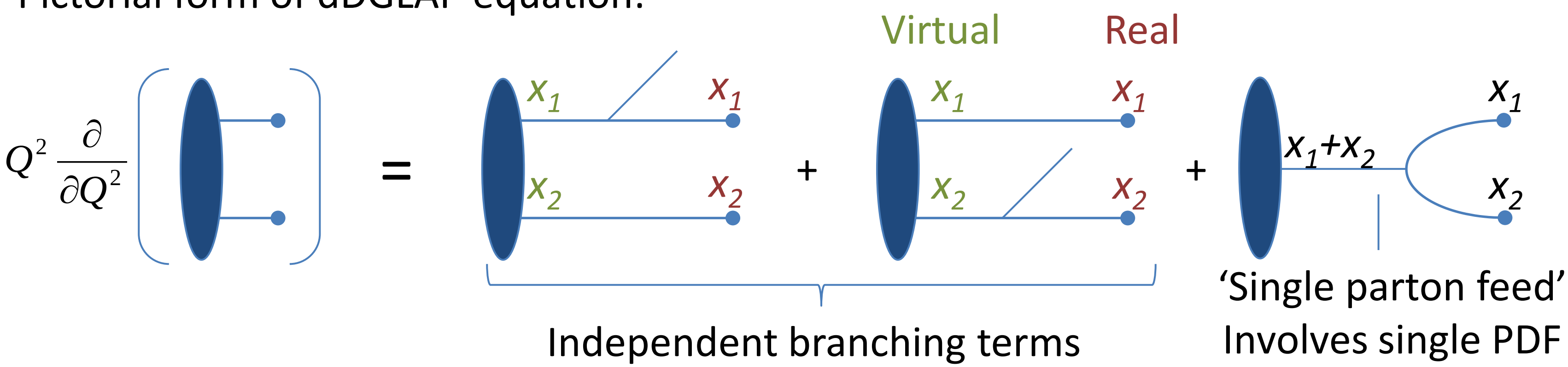
Then, introducing σ_{eff} via $\sigma_{eff} \equiv 1 / \int F(\boldsymbol{b})^2 d^2\boldsymbol{b}$ we find that the DPS cross section can be written as follows:

$$\sigma^D_{(A,B)} \propto \frac{1}{\sigma_{eff}} \sum_{i,j,k,l} \int \prod_{a=1}^{4} dx_a D_p^{ij}(x_1, x_2; Q_A^2, Q_B^2) D_p^{kl}(x_3, x_4; Q_A^2, Q_B^2) \times \hat{\sigma}_{ik \to A}(\hat{s} = x_1 x_3 s) \hat{\sigma}_{jl \to B}(\hat{s} = x_2 x_4 s)$$

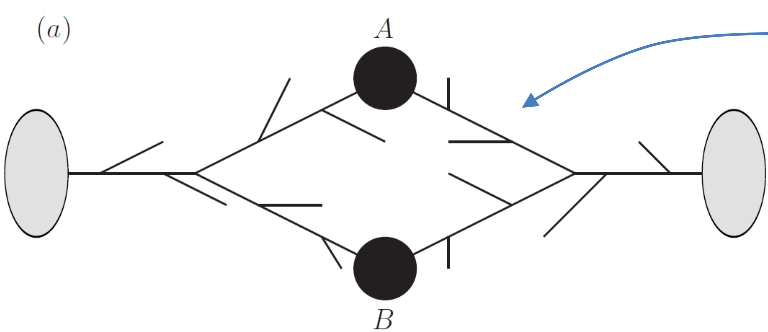
A quantity denoted as $D_h^{j1j2}(x_1,x_2,Q^2)$ (the double PDF, or dPDF) was introduced in 1982 by Shelest, Snigirev and Zinovjev [Phys. Lett. B 113:325], and an evolution equation for this quantity was given (dDGLAP equation). Snigirev suggested afterwards [hep-ph/0304172] that this quantity is equal to the factorised longitudinal piece of the 2pGPD for the case where $Q_A = Q_B \equiv Q$.

Double PDF framework for calculating DPS

Pictorial form of dDGLAP equation:



$Q_A^{\ 2} = Q_B^{\ 2} = Q^2 > 0$



Given the inclusion of single feed term, dPDF framework predicts that part of these ‘double perturbative splitting’ graphs should be included as DPS. At the cross section level the part that should be included is proportional to:

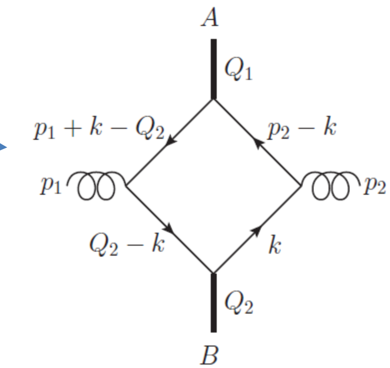
$$[\log(Q^2/\Lambda^2)]^n/\sigma_{eff}$$

n = total number of QCD branching vertices on either side of diagram.

This part should be associated with QCD branchings on either side of the diagram being strongly ordered in transverse momenta.

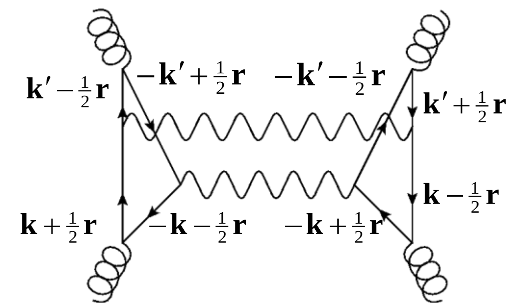
‘Double Perturbative Splitting’ graphs

Is there such a structure in these ‘Double Perturbative Splitting’ graphs? Let’s see for the simple box graph:



We expect the $[\log(Q^2/\Lambda^2)]^2/\sigma_{eff}$ piece in this graph

to be contained in the region of cross section integration around the DPS singularity ➔ insert our analytic expression for DPS singular part of loop into standard 2 → 2 cross section formula:



r is Fourier conjugate of parton pair separation ***b***

$$\sigma_{DPS,\text{fig 1(b)}} \propto \int \prod_{i=1}^{2} dx_i d\bar{x}_i \hat{\sigma}_{q\bar{q}\to A}(\hat{s} = x_1\bar{x}_1 s)\hat{\sigma}_{q\bar{q}\to B}(\hat{s} = x_2\bar{x}_2 s)$$

‘O(α_s) g → q$\bar{\text{q}}$ 2pGPD’

$$\times \int \frac{d^2\boldsymbol{r}}{(2\pi)^2} \Gamma_{g\to q\bar{q}}(x_1, x_2, \boldsymbol{r})\Gamma_{g\to \bar{q}q}(\bar{x}_1, \bar{x}_2, -\boldsymbol{r})$$

1 → 2 splitting function

$$\Gamma_{g\to q\bar{q}}(x_1, x_2, \boldsymbol{r}) \propto \frac{\alpha_S}{2\pi}\delta(1 - x_1 - x_2)T^{ij}(x_1, x_2) \int^{\tilde{\boldsymbol{k}}^2 < \mathcal{O}(Q^2)} d^2\tilde{\boldsymbol{k}} \frac{[\tilde{\boldsymbol{k}} + \frac{1}{2}\boldsymbol{r}]^i[\tilde{\boldsymbol{k}} - \frac{1}{2}\boldsymbol{r}]^j}{[\tilde{\boldsymbol{k}} + \frac{1}{2}\boldsymbol{r}]^2[\tilde{\boldsymbol{k}} - \frac{1}{2}\boldsymbol{r}]^2}.$$

‘Double Perturbative Splitting’ graphs

$$\sigma_{DPS,\text{fig 1(b)}} \propto \int \prod_{i=1}^{2} dx_i d\overline{x}_i \hat{\sigma}_{q\overline{q}\to A}(\hat{s} = x_1\overline{x}_1 s)\hat{\sigma}_{q\overline{q}\to B}(\hat{s} = x_2\overline{x}_2 s)$$

$$\times \int \frac{d^2\boldsymbol{r}}{(2\pi)^2}\Gamma_{g\to q\overline{q}}(x_1, x_2, \boldsymbol{r})\Gamma_{g\to\overline{q}q}(\overline{x}_1, \overline{x}_2, -\boldsymbol{r})$$

$$\Gamma_{g\to q\overline{q}}(x_1, x_2, \boldsymbol{r}) \propto \frac{\alpha_S}{2\pi}\delta(1 - x_1 - x_2)T^{ij}(x_1, x_2) \int^{\tilde{\boldsymbol{k}}^2 < \mathcal{O}(Q^2)} d^2\tilde{\boldsymbol{k}} \frac{[\tilde{\boldsymbol{k}}+\frac{1}{2}\boldsymbol{r}]^i[\tilde{\boldsymbol{k}}-\frac{1}{2}\boldsymbol{r}]^j}{[\tilde{\boldsymbol{k}}+\frac{1}{2}\boldsymbol{r}]^2[\tilde{\boldsymbol{k}}-\frac{1}{2}\boldsymbol{r}]^2}.$$

Obtain a result that is consistent with the double PDF framework if one considers the portion of the integral with $|\boldsymbol{r}| < \Lambda_S$ as DPS, where Λ_S is a specific choice of cut-off of the order of Λ_{QCD}. But why should we consider this piece specifically as DPS?

Same issues are encountered for an arbitrary double perturbative splitting graph. There is no distinct piece of the arbitrary double splitting graph that contains a natural scale of order Λ_{QCD} and is associated with the transverse momenta inside the loop being strongly ordered on either side of the diagram. Most of the contribution to the cross section expression for this graph comes from the region of integration in which the transverse momenta of particles inside the loop are of $O(\sqrt{Q^2})$.

Perhaps, then, we shouldn’t include any of this graph as DPS. This has the advantage of avoiding potential double counting between DPS and SPS.

‘Double Perturbative Splitting’ graphs

There are clearly theoretical issues with the double PDF framework. Can gain some insight into the root of the problems in the dPDF framework by Fourier transforming the ***r***-space perturbative splitting 2pGPD we obtained before into ***b*** space. We find:

$$\Gamma_{qq}\Big|_{g\to q\bar{q}}(x_1, x_2, \mathbf{b}) \sim 1/\mathbf{b}^2$$

Power law behaviour – very different from smooth function of size R_p expected from double PDF framework. A key error then in the formulation of the dPDF framework seems to be the assumption that all 2pGPDs can be approximately factorised into dPDFs and smooth transverse functions of size R_p.

A sound theoretical framework for describing proton-proton DPS needs to carefully take account of the different ***b*** dependence of pairs of partons emerging from perturbative splittings, whilst simultaneously avoiding double counting between SPS and DPS.

See also Diehl and Schafer [arXiv:1102.3081].

Summary

- We have derived a compact analytical expression for the DPS divergence in an arbitrary one-loop diagram.
- This expression was used to show that no Standard Model loop can have a DPS singularity worse than a logarithm of the transverse momentum of particles on the top/bottom leg of the loop.
- We explained why certain amplitudes studied by the NLO multileg community do not have a DPS divergence – e.g. six photon MHV and NMHV helicity amplitudes studied by Bernicot and Guillet.
- Relevance of the work to double parton scattering theory – used analytical expression for DPS singularity to show that that the treatment of the double parton splitting diagrams in the double PDF framework appears to be unsatisfactory.