Theoretical improvements for luminosity monitoring at low energies

Janusz Gluza (Silesia U.)

29 September 2011
Mamallapuram, India

Himalaya mountains

Mamallapuram, Temple Bay

Mamallapuram, Temple Bay

[not less] interesting physics

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T. Riemann, (DESY, Zeuthen, Germany)
M. Worek, (Wuppertal U., Germany)

- Introduction
- Bhabha NNLO corrections
  - Figures: scan over additional events selection
  - Table
  - Some technical details on hadronic contributions
  - Conclusions
- $e^+e^- \to \mu^+\mu^-\gamma$ process
- Conclusions


Indian Heroes
..iloveindia

- Precise calculations of higher order corrections for the process of Bhabha scattering ($e^+e^- \to e^+e^-$) are necessary for determine colliders luminosity with high accuracy.

$$L_{tot} = \frac{N}{\sigma_{theory}}$$

- High accuracy of luminosity in low energy region is necessary to research low energy hadron cross section from $e + e-$ annihilation process.

$$\sigma_{had} = \frac{N_{had}}{L_{tot}}$$

The muon pair production with real photon emission $e^+e^- \to \mu^+\mu^-\gamma$ is an important background and normalization reaction in the measurement of the pion form-factor:

$$R_{exp} = \frac{\sigma(e^+e^- \to \pi\pi\gamma)}{\sigma(e^+e^- \to \mu^+\mu^-\gamma)}$$

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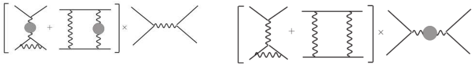
KLOE-2 uses both Bhabha and muon pair normalizations, Babar only radiative return

The complete NNLO $N_f = 1, 2$ corrections to Bhabha scattering consist of three parts:

$$\frac{d\sigma^{\text{NNLO}}_{N_f}}{d\Omega} = \frac{d\sigma^{\text{NNLO}}_{virt}}{d\Omega}^{1} + \frac{d\sigma^{\text{NLO}}_{\gamma}}{d\Omega}^{2} + \frac{d\sigma^{\text{LO}}_{real}}{d\Omega}^{3}$$
$$= \frac{d\sigma_{e^+e^-}}{d\Omega} + \frac{d\sigma_{\mu^+\mu^-}}{d\Omega} + \frac{d\sigma_{\tau^+\tau^-}}{d\Omega} + \frac{d\sigma_{had}}{d\Omega}.$$

1 - bha_nnlo_hf
2 - BHAGHEN–1PH+...,bha_nnlo_hf
3 - HELAC–PHEGAS,EKHARA

- the $\sigma_{virt}^{\mathrm{NNLO}}$ consists of virtual two-loop corrections $\sigma_{2L}^{\mathrm{NNLO}}$ and loop-by-loop corrections $\sigma_{1L1L}^{\mathrm{NNLO}}$



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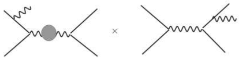

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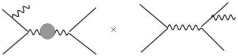
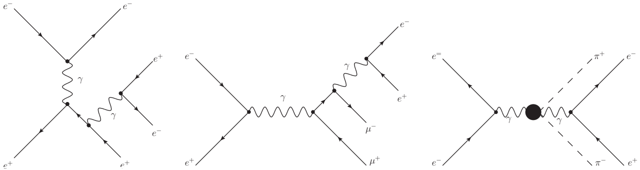


- contributions with real pair or hadron emission
  $\sigma^{\rm LO}_{real} = \sigma^{\rm LO}_{e^+e^-(e^+e^-)} + \sigma^{\rm LO}_{e^+e^-(f^+f^-)} + \sigma^{\rm LO}_{e^+e^-(hadrons)}$



- NNLO virtual corrections linked with real corrections and realistic experimental cuts for low energy machines: Φ factory Dafne at Frascati, B factories PEP-II (SLAC) and Belle (KEK) and at the charm/τ factory BEPC II, Beijing

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- comparison complete calculations with approximate ones realized in the MC generator BabaYaga: C.C.Calame, C. Lunardini, G. Montagna, O. Nicrosini, F. Piccinini

"NNLO leptonic and hadronic corrections to Bhabha scattering and luminosity monitoring at meson factories."

Carloni Calame, H. Czyz, JG, M. Gunia , G. Montagna, O. Nicrosini, F. Piccinini, T. Riemann, M. Worek, Published in

JHEP 1107:126,2011

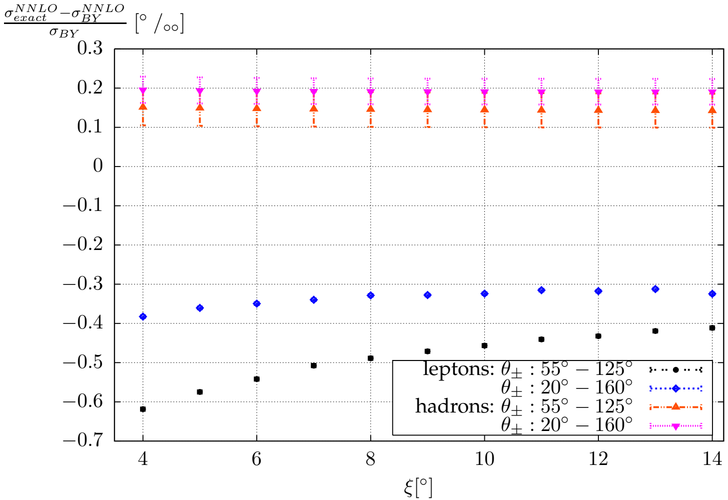
1.Φ factories KLOE/DAΦNE (Frascati)

(a) $\sqrt{s} = 1.02$ GeV
(b) $E_{min} = 0.4$ GeV
(c) For $\theta\pm$ two selections have to be checked
i. tighter selection $55^o < \theta\pm < 125^o$
ii. wider selection $20^o < \theta\pm < 160^o$
(d) ζ_{max}=4,5,6,7,8,...,14 deg., with reference value $\zeta_{max} = 9^o$

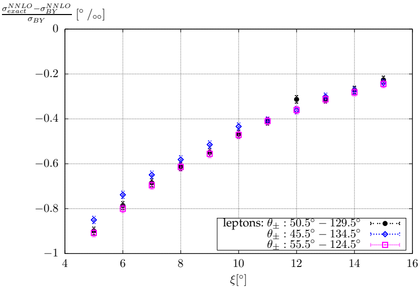
2. B-factories BABAR/PEP-II (SLAC) & BELLE/KEKB (KEK)

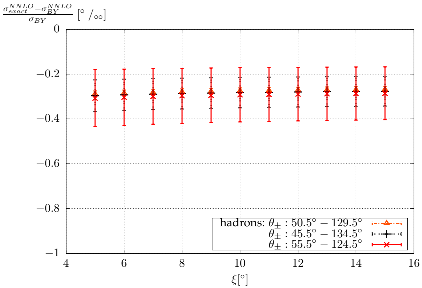
(a) $\sqrt{s} = 10.56$ GeV
(b) $|\vec{p}_+|/E_{beam} > 0.75$ and $|\vec{p}_-|/E_{beam} > 0.50$
or $|\vec{p}_-|/E_{beam} > 0.75$ and $|\vec{p}_+|/E_{beam} > 0.50$
(c) For $|cos(\theta\pm)|$ the following selections have to be checked
i. $|cos(\theta\pm)| < 0.65$ and $|cos(\theta+)| < 0.60$ or $|cos(\theta-)| < 0.60$
ii. $|cos(\theta\pm)| < 0.70$ and $|cos(\theta+)| < 0.65$ or $|cos(\theta-)| < 0.65$
iii. $|cos(\theta\pm)| < 0.60$ and $|cos(\theta+)| < 0.55$ or $|cos(\theta-)| < 0.55$
(d) ζ^{3d}_{max}= 20,22,24,...,40 deg., with reference value $\zeta^{3d}_{max} = 30^o$

KLOE - relative difference in per-mile



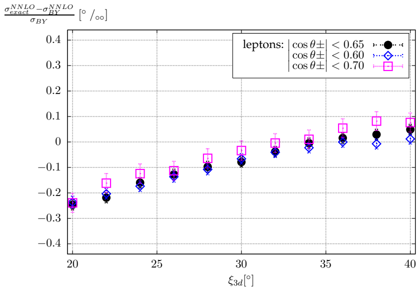
Belle - relative difference in per-mile

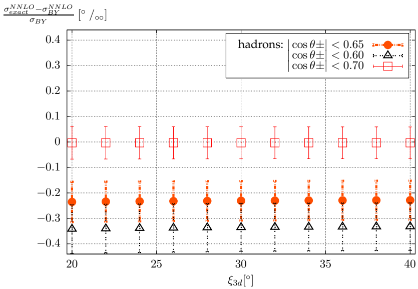




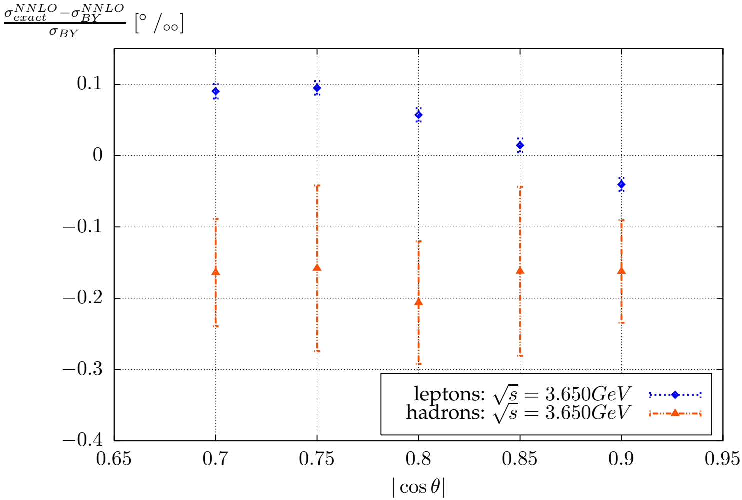
Contributions of leptons and hadrons to NNLO Bhabha process can be constructive (Belle) or destructive (Kloe), they also depends strongly for some colliders/detectors on kinematical cuts.

BaBar - relative difference in per-mile





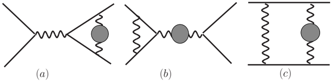
BES III - relative difference in per-mile



	$\sqrt{s}$		$\sigma_{\rm BY}$	$S_{e^+e^-}$	$S_{lep}[10^{-3}]$	S_{had}	S_{tot}
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BY_{NLO}	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.097	NNLO		-2.246(8)	-2.771(8)	-	-
		BY_{NLO}	158.23	-2.019(3)	-2.548(3)	-	-
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BY_{NLO}	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BES	3.686	NNLO		-1.435(8)	-1.873(8)	-	-
		BY_{NLO}	114.27	-1.502(4)	-1.947(4)	-	-
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BY_{NLO}	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BY_{NLO}	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

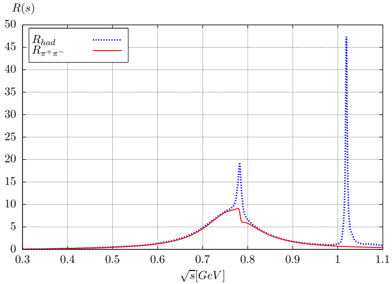
The $\sigma_{\rm BY}$ is the cross section in nb from BabaYaga(at)NLO, and $S_x = \frac{\sigma_x^{\rm NNLO}}{\sigma_{\rm BY}}$ in per-milles with $x = e^+e^-, lep, tot$, where tot stands for leptonic (lep) + hadronic corrections.

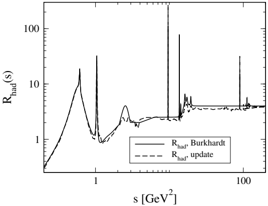
The vacuum polarisation function:



$$\Pi(q^2) = \frac{\alpha q^2}{3\pi} \int_{m_{\Pi 0}^2}^{\inf} \frac{dz}{z} \frac{R(z)}{q^2 - z + i\epsilon}$$

For leptons VP analytical expresions were used.
For pions VP numerical calculations of the integral were used.
For hadrons program VPHLMNT (T.Teubner et all.) was used.





"rest": genuine massive QED NNLO virtual corrections

$$
\begin{aligned}
\frac{d\sigma_{\rm rest}}{d\Omega} &= \frac{\alpha^4}{\pi^2 s}\Bigg[\int_{M_0^2}^{\infty} dz\,\frac{R(z)}{z}\,\frac{1}{t-z}\,F_1(z) \\
&+ \int_{M_0^2}^{\infty} dz\,\frac{1}{z\,(s-z)}\Big\{R(z)F_2(z) - R(s)F_2(s) + [R(z)F_3(z) \\
&+ \frac{R(s)}{s}\Big\{F_2(s)\,\ln\Big(\frac{s}{M_0^2}-1\Big) - 6\,\zeta_2\,F_4(s) \\
&+ F_3(s)\Big[2\,\zeta_2 + \frac{1}{2}\,\ln^2\Big(\frac{s}{M_0^2}-1\Big) + \mathsf{Li}_2\Big(1-\frac{s}{M_0^2}\Big)\Big]\Big\}\Bigg].
\end{aligned}
$$

F_3, the shortest of auxiliary functions (Actis, Czakon, JG, Riemann, PRL,PRD, 2008:

$$
\begin{aligned}
F_3(z) \quad = \quad & \frac{1}{3}\left\{\left[\frac{z^2}{s} - 2\,z\left(1+\frac{t}{s}\right) + 4\,\frac{t^2}{s} + 2\,\frac{s^2}{t} + 7\,s + 8\,t\right]\ln\left(1+\frac{t}{s}\right)\right. \\
- \quad & \left[z^2\left(\frac{1}{s}+\frac{1}{t}\right) + 2\,z\left(1+\frac{t}{s}\right) + 4\,\frac{t^2}{s} + \frac{s^2}{t} + 3\,s + 4\,t\right]\ln\left(-\frac{t}{s}\right) \\
- \quad & \left.\left[z^2\left(\frac{1}{t}+\frac{2}{s}+2\,\frac{t}{s^2}\right) - 2\,z\left(2+\frac{s}{t}+2\,\frac{t}{s}\right) + \frac{s^2}{t} + 2\left(s+t\right)\right]\right\}
\end{aligned}
$$

resonance	M_{res} [GeV]	$\Gamma_{\mathrm{res}}^{e^+e^-}$ [keV]
J/ψ(1S)	3.096916	5.55
ψ(2S)	3.686093	2.33
Υ(1S)	9.46030	1.34
Υ(2S)	10.02326	0.612
Υ(3S)	10.3552	0.443
Υ(4S)	10.5794	0.272
Υ(5S)	10.865	0.31
Υ(6S)	11.019	0.13

	$\sqrt{s}$	$\sigma^{\rm NNLO}_{\rm rest,res}$	$\sigma^{\rm NNLO}_{\rm rest,res'}$	σ_B
KLOE	1.020	[all n.r.] -0.04538	[n.r. without J/ψ(1S)] -0.0096	529.5
BES	3.097	[all n.r.] 228.08	[n.r. without J/ψ(1S)] -0.0258	14.75
BES	3.650	[all n.r.] -0.1907	[n.r. without ψ(2S)] -0.023668	123.94
BES	3.686	[all n.r.] -62.537	[n.r. without ψ(2S)] -0.0254	121.53
BaBar	10.56	[all n.r.] -0.0163	[n.r. without Υ(4S)] -0.01438	6.744
Belle	10.58	[all n.r.] 0.04393	[n.r. without Υ(4S)] -0.0137	6.331

$$R_{res}(z) = \frac{9\pi}{\alpha^2} M_{res} \Gamma_{res}^{e^+e^-} \delta(z - M_{res}^2) \,.$$

$$\frac{d\sigma_{\rm rest}}{d\Omega} = \frac{9\alpha^2}{\pi\ s} \frac{\Gamma^{e^+e^-}_{\rm res}}{M_{\rm res}} \left\{ \frac{F_1(M^2_{\rm res})}{t - M^2_{\rm res}} + \frac{1}{s - M^2_{\rm res}} \left[F_2(M^2_{\rm res}) + F_3(M^2_{\rm res}) \ln \left| 1 - \frac{M^2_{\rm res}}{s} \right| \right] \right\}$$

adaptive VEGAS is able to identify narrow resonances!
we used it instead of above approximation in numerical calculations

Comparison of hadronic contributions modelled by $R_{\pi^+\pi^-}$ and R_{had}. For hadrons, real emission is restricted to pions only

	KLOE	BES	BaBar
σ_{S+V}, $R_{\pi^+\pi^-}$	-1.36	-0.818	-0.0533
σ_{S+V}, R_{had}	-1.06	-1.81	-0.1888
σ_{S+V+H}, $R_{\pi^+\pi^-}$	-0.186	-0.0447	-0.00229
σ_{S+V+H}, R_{had}	0.47	-0.15	-0.0088

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- NNLO massive corrections are relevant for precision luminosity measurements with 10^{-3} accuracy. The electron pair contribution is the largely dominant part of the correction. The muon pair and hadronic corrections are the next-to-important effects and quantitatively on the same grounds. The tau pair contribution is negligible for the energies of meson factories.

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- to be done: scan over c.m. energies ($\sim$1 MeV spread) near BES(3.097) and BES(3.686GeV) resonances

$e^+e^- \to \mu^+\mu^-\gamma$ — ideal benchmark process for massive tensor reduction

- Two different masses
- Large difference of scales (up to 7 orders in magnitude)
- Quasi-collinear region (due to small electron mass)
- Small number of diagrams

- Diagram generation with DIANA [Tentyukov, Fleischer]
- Algebraic processing in FORM [Vermaseren]
- Tensor reduction PJFry [VY]
- Scalar integrals OneLOop [van Hameren]
- Monte-Carlo PHOKHARA [Rodrigo, Czyż, Kühn]

Compact result for squared one loop amplitude (~ 3 ms per point).

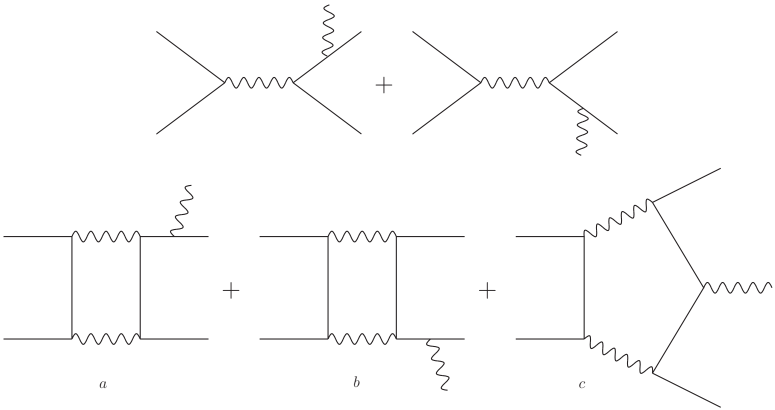
Monte-Carlo integration as a stability test

Two realistic sets of kinematical cuts

	BaBar	KLOE
E_{CMS}	10.56 GeV	1.02 GeV
$E_{\gamma,\text{min}}$	3 GeV	0.02 GeV
θ_γ	$20°$–$138°$	$0°$–$15°$, $165°$–$180°$
Q^2	0.25–50 GeV^2	0.25–1.06 GeV^2
$\theta_{\mu^\pm}$	$40°$–$140°$	$50°$–$130°$

$$m_e = 0.5109989 \cdot 10^{-3} \text{ GeV}, \; m_\mu = 0.105658367 \text{ GeV},$$
$$\alpha(0) = 1/137.03599968.$$

Phase-space cuts for KLOE and BaBar settings.
Q^2 is the invariant mass squared of the muon pair.



FSR gauge invariance between tree diagrams (upper picture), and gauge invariance among four and five point one-loop integrals (below). Here diagrams were limited to FSR cases, the same property is present for ISR amplitudes.

	KLOE	BaBar
double precision	10^{-2}	10^{-5}
quadrupole precision	10^{-12}	10^{-10}

Gauge invariance for loop diagrams of the previous slide for KLOE and BaBar setting and different real number declarations.
The numbers give relative accuracy defined as

$$max\{\frac{\sum_{i=a,b,c} Re(M^i_{\mathsf{loop}} M^\dagger_{\mathsf{tree}})}{min(Re(M^i_{\mathsf{loop}} M^\dagger_{\mathsf{tree}}))}\}$$

Indices a, b, c refer to a, b, c diagrams in previous slide.

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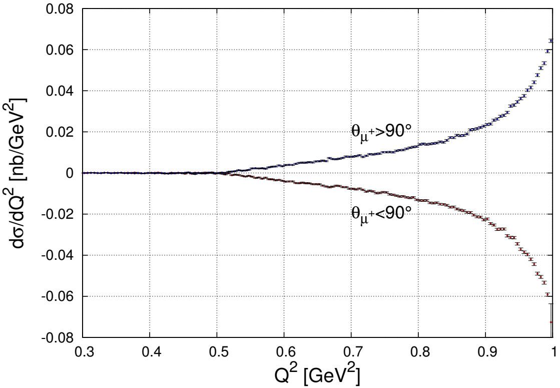
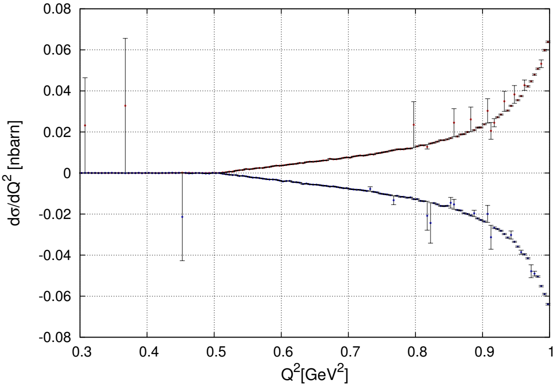
Indices a, b, c refer to a, b, c diagrams in previous slide.
so, all looks all right

results using LT and FF package, $3 \cdot 10^6$ points in Phokhara MC



KLOE NLO results for $\theta_\mu > 90^\circ$ and $\theta_\mu < 90^\circ$

however:



Muon pair invariant mass distribution, results using LT and FF package, PhD thesis of K.Kajda, 2009, 10^9 points in Phokhara MC why?

Reducing tensor rank introduces inverse Gram determinant (5 point example, rank $R \to R-1$):

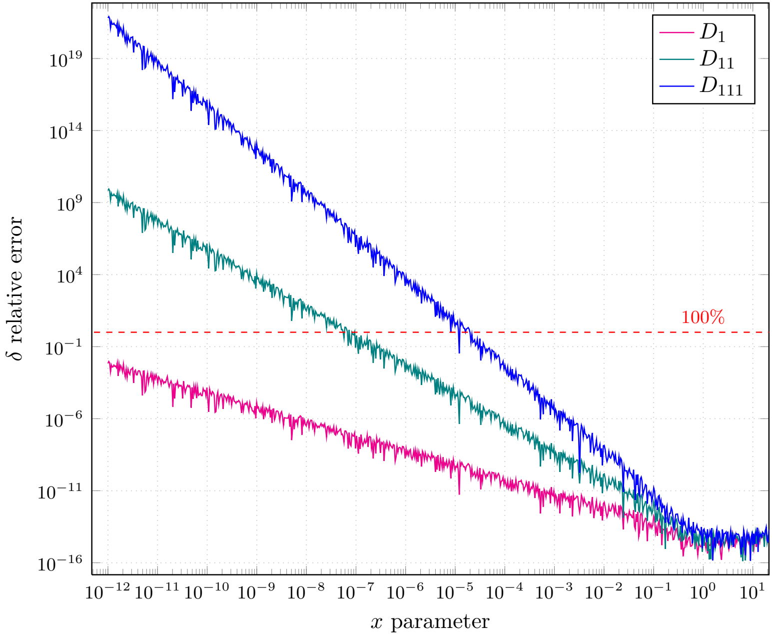
$$I_5^{\mu_1 \cdots \mu_{R-1} \mu_R} = \sum_{i=1}^{5} \frac{q_i^{\mu_R}}{|G^{(5)}|} \left(K_{0i}\, I_5^{\mu_1 \cdots \mu_{R-1}} - \sum_{s=1}^{5} K_{si}\, I_4^{\mu_1 \cdots \mu_{R-1},s} \right)$$

Gram matrix:

$$|G^{(5)}| \equiv \det G_{ik}^{(5)}, \qquad G_{ik}^{(n)} = 2\, q_i\, q_k, \qquad i,k = 1,\ldots,n-1$$

K_{0i} and K_{si} — kinematic coefficients

Passarino-Veltman reduction accuracy loss in small Gram region



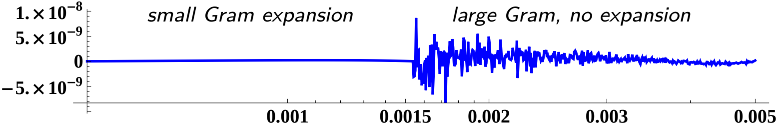
Numerical implementation of [Fleischer, Riemann 2010] algorithms:
C++ package **PJFry**

- Reduction of **5-point** 1-loop tensor integrals up to **rank 5**
  4- and 3-point tensor integrals come "for free" as a by-product
- No limitations on internal/external **masses combinations**
- Automatic selection of optimal formula for each coefficient
- Leading $|G^{(5)}|$ are eliminated in the reduction
- Small $|G^{(4)}|$ are avoided using **asymptotic expansion**
- Cache system for tensor coefficients and signed minors
- Interfaces for C, C++, FORTRAN and MATHEMATICA
- Uses QCDLoop or OneLOop for 4-dim scalar integrals
- Available from project page:
  https://github.com/Vayu/PJFry/

Average time per phase-space point on Core2 2GHz laptop for evaluation of all 81 **rank 5** tensor form-factors: **2 ms**

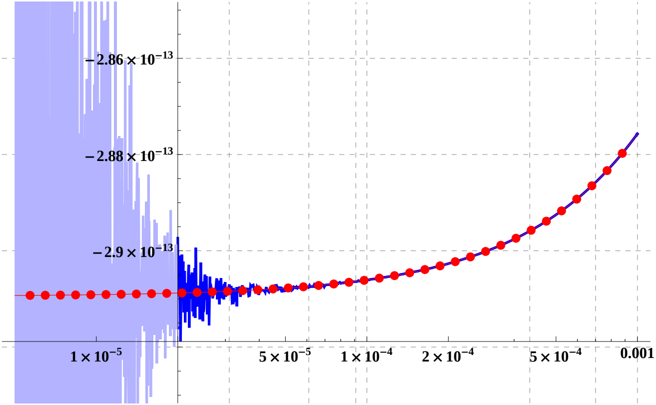
Expansion accuracy example:

Relative accuracy of E_{3333} coef. around small $|G^{(4)}|$

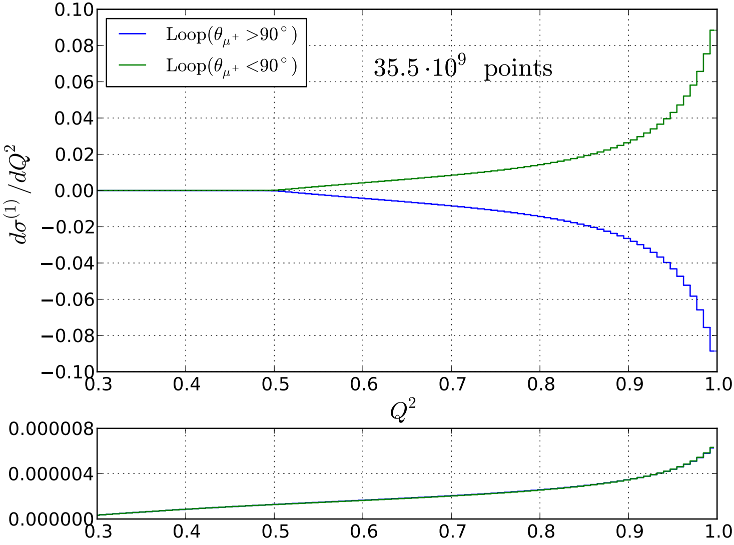


Example: E_{3333} coefficient in small $|G^{(4)}|$ region $(x = 0)$

Comparison of Regular and Expansion formulae:



$x{=}0$: $E_{3333}(0, 0, -6{\cdot}10^4(x{+}1), 0, 0, 10^4, -3.5{\cdot}10^4, 2{\cdot}10^4, -4{\cdot}10^4, 1.5{\cdot}10^4, 0, 6550, 0, 0, 8315)$



“Penta” contribution to forward-backward Q^2 of μ^+ for KLOE.
Bottom: absolute error estimate.

- Pole structure
- Gauge invariance test
- Known pieces compared to Phokhara
- Comparison with published points

  [Actis, Mastrolia, Ossola]

- Application to full one-loop corrections to $e^+e^- \to \mu^+\mu^-\gamma$ with analysis focused on KLOE-2 data (work in progress)

Final Word

There is a progress in low energy physics,
2 examples given,
see also the next 2 talks.

Shukriyaa Bahut dhanyavaad!
Thank you for your attention!

Backup slides

massless $|G^{(5)}|$ in Mandelstam variables $s_{ik} = (p_i + p_k)^2$

$$|G^{(5)}| = -s_{12}^2(s_{15} - s_{23})^2 - \big(s_{23}s_{34} + (s_{15} - s_{34})s_{45}\big)^2 + \\ + 2s_{12}\Big(s_{23}s_{34}(s_{23} - s_{45}) + s_{15}^2 s_{45} - s_{15}\big(s_{34}s_{45} + s_{23}(s_{34} + s_{45})\big)\Big)$$

Reducing $I_4^{\mu_1 \cdots \mu_{R-1}, s}$ gives five $|G^{(4)}|$ in the denominators:

$$|G^{(4)}(s,t)| = 2st(s+t)$$

$$|G^{(4)}(s_{12}, s_{23})|, \quad |G^{(4)}(s_{23}, s_{34})|, \quad |G^{(4)}(s_{34}, s_{45})|, \\ |G^{(4)}(s_{45}, s_{15})|, \quad |G^{(4)}(s_{15}, s_{12})|$$

Zero Gram determinant is not a physical singularity

It is an artefact of the reduction procedure

Numerator and denominator go to 0 simultaneously

Leading to large cancellations and loss of accuracy

One loop tensor integrals

1. core of traditional Feynman diagram approach
   $2 \to 3$ and $2 \to 4$ NLO calculations by
   [Binoth, Bredenstein, Denner, Dittmaier, Pozzorini, Roth, Wieders, . . .]

2. element of alternative methods
   tensorial reconstruction at the integrand level
   [Heinrich, Ossola, Reiter, Tramontano 2010]

   loop level recursion
   [van Hameren 2009]

Scalar integrals: *No problems here*

- QCDLoop/FF $(n \leq 4)$ [Ellis, Zanderighi 2007; van Oldenborgh 1990]
  dim-reg, real masses
- OneLOop $(n \leq 4)$ [van Hameren 2010]
  dim-reg, complex masses

Tensor integrals:

- LoopTools/FF $(n \leq 5,\ R \leq 4)$ [Hahn 2006; van Oldenborgh 1990]
  no $1/\epsilon^2$, no $R{=}5$, unstable for small Gram determinants
- Golem95 $(n \leq 6)$ [Binoth, Guillet, Heinrich, Pilon, Reiter 2008]
  massless is OK, massive is unstable for small Gram determinants *(work in progress)*
- private codes by various groups

Goal:

- *stable and fast public implementation of tensor reduction*
- *suitable for any physically relevant kinematics*

Tensor form-factors (rank 3 example):

$$I_n^{\mu_1\mu_2\mu_3} = \sum_{\substack{i,j,k=1 \\ i\le j\le k}}^{n-1} q_i{}^{[\mu_1} q_j{}^{\mu_2} q_k{}^{\mu_3]} F_{ijk}^{(n)} + \sum_{i=1}^{n-1} g^{[\mu_1\mu_2} q_i{}^{\mu_3]} F_{00i}^{(n)}$$

Standard naming convention:

$F_{\dots}^{(1)} = A_{\dots},\ F_{\dots}^{(2)} = B_{\dots},\ F_{\dots}^{(3)} = C_{\dots},\ F_{\dots}^{(4)} = D_{\dots},\ F_{\dots}^{(5)} = E_{\dots},$ *etc*

$e^+e^- \to \mu^+\mu^-\gamma$ KLOE Pentagons Q^2

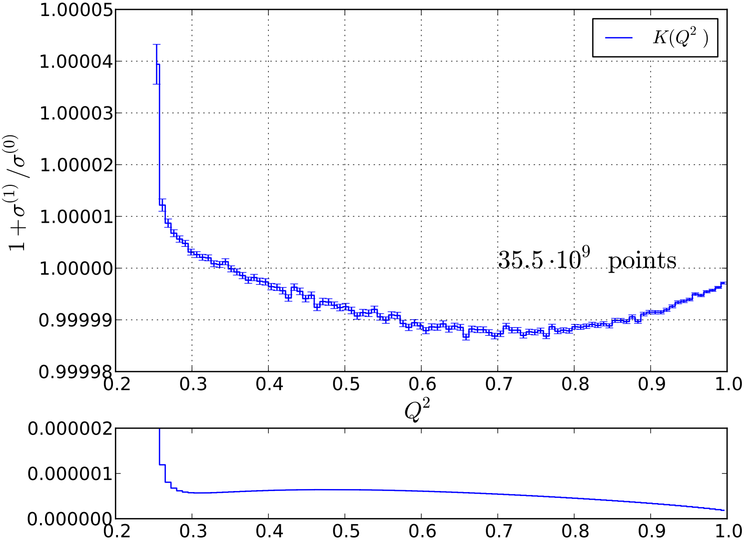


Figure: "Penta" contribution to muon pair Q^2 distribution for KLOE. Bottom: absolute error estimate.

$e^+e^- \to \mu^+\mu^-\gamma$ KLOE Pentagons forward-backward Q^2

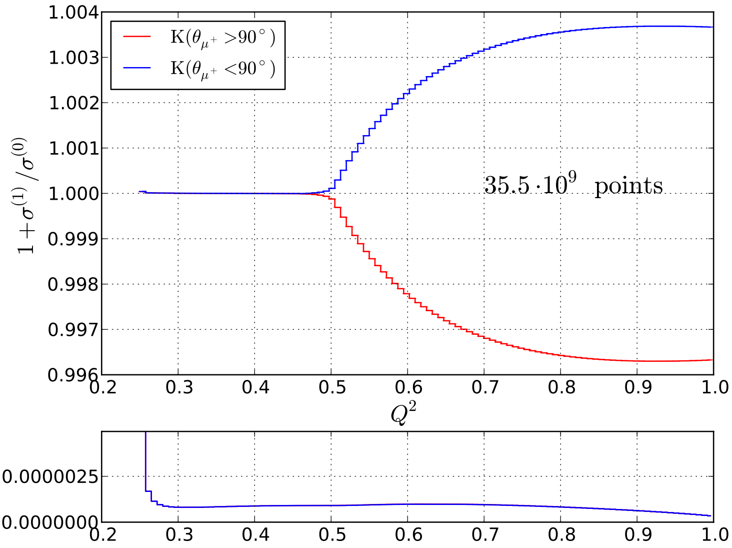


Figure: "Penta" contribution to forward-backward Q^2 of μ^+ for KLOE. Bottom: absolute error estimate.

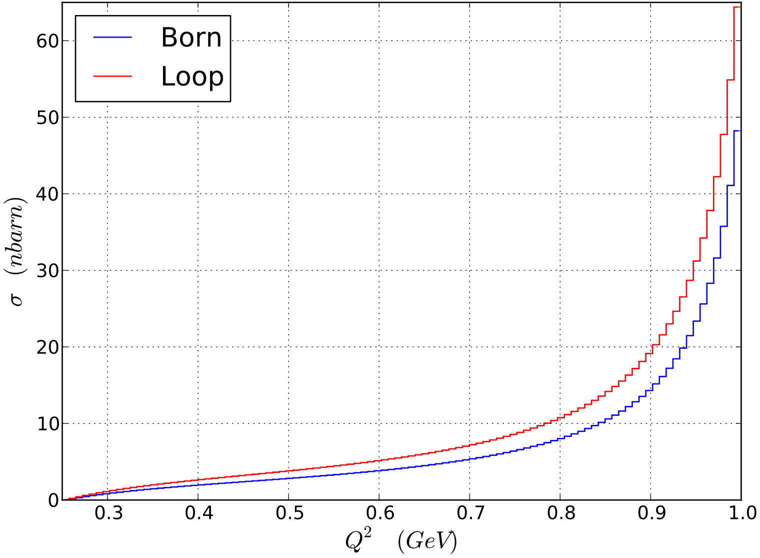


Figure: Muon pair invariant mass distribution for KLOE

$e^+e^- \to \mu^+\mu^-\gamma$ KLOE Pentagons angular distributions

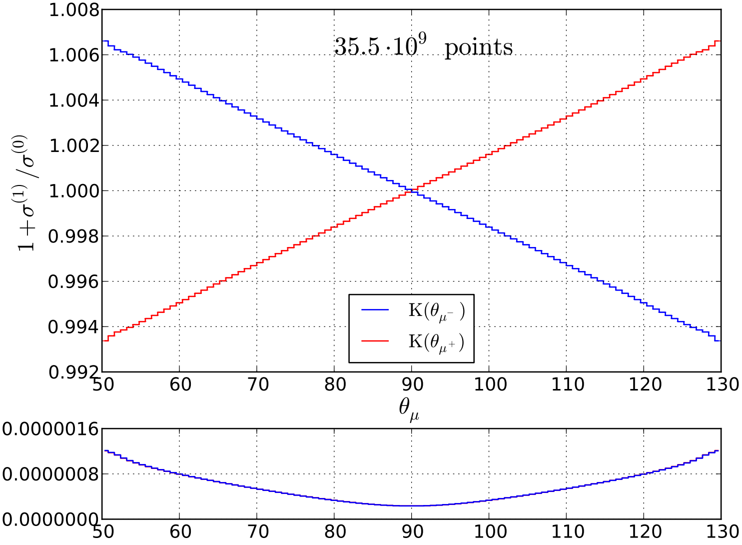


Figure: “Penta” contribution to muon angular distributions for KLOE. Bottom: absolute error estimate.

$e^+e^- \to \mu^+\mu^-\gamma$ BaBar Q^2

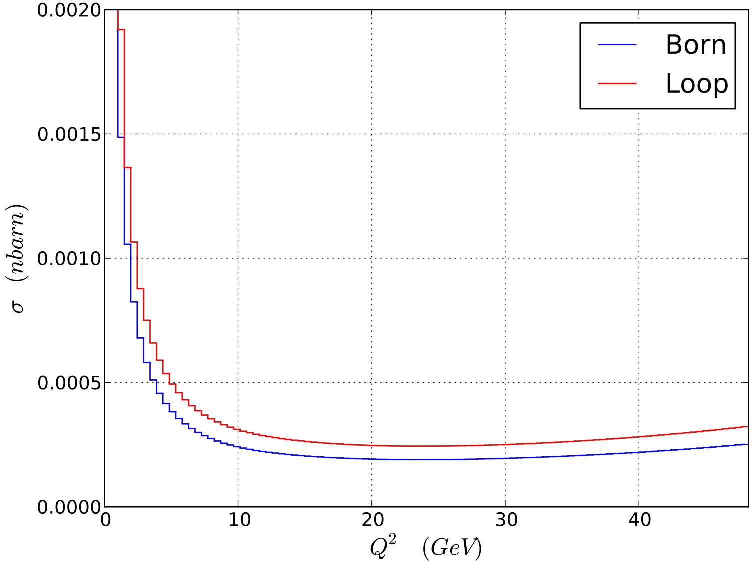


Figure: Muon pair invariant mass distribution for BaBar

$e^+e^- \to \mu^+\mu^-\gamma$ BaBar Pentagons Q^2

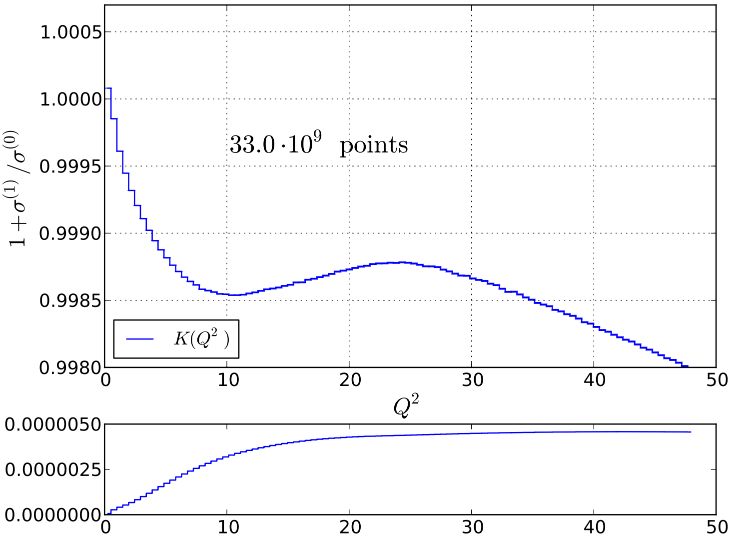


Figure: "Penta" contribution to muon pair Q^2 distribution for BaBar. Bottom: absolute error estimate.

$e^+e^- \to \mu^+\mu^-\gamma$ BaBar Pentagons angular distributions

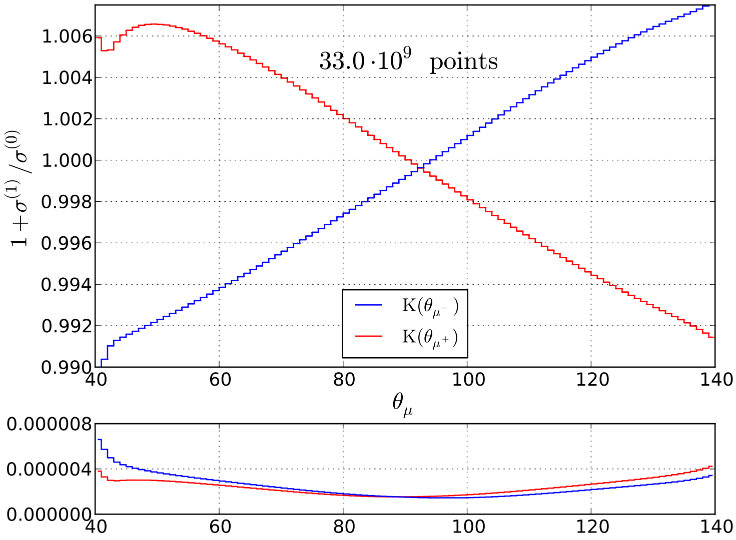


Figure: "Penta" contribution to muon angular distributions for BaBar. Bottom: absolute error estimate.

$e^+e^- \to \mu^+\mu^-\gamma$ BaBar Pentagons forward-backward Q^2

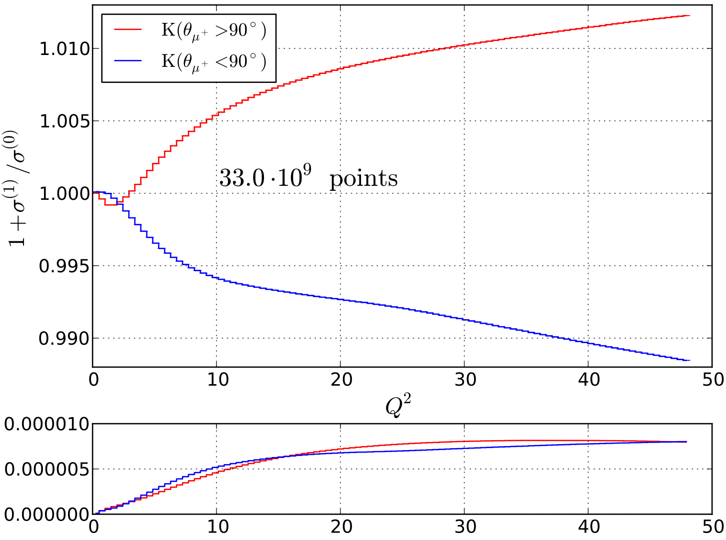


Figure: "Penta" contribution to forward-backward Q^2 of μ^+ for BaBar. Bottom: absolute error estimate.