NNPDF in the LHC Era

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Part I

Introduction

Issues in Standard PDF Determination

- Extraction of a set of functions with errors from a set of data points.
- We need an error band, i.e. a **probability density** $\mathcal{P}[\mathbf{f}(\mathbf{x})]$ in the space of PDFs :

$$\langle \mathcal{O} \rangle = \int \mathcal{D}f \, \mathcal{P}[f] \mathcal{O}[f]$$

$$\sigma_{\mathcal{O}}^2 = \int \mathcal{D}f \, \mathcal{P}[f] \left(\mathcal{O}[f] - \langle \mathcal{O} \rangle \right)^2$$

Standard approach:

1. Choose a specific functional form:

$$q_i(x, Q_0^2) = A_i x^{b_i} (1 - x)^{c_i} (1 + \ldots).$$

2. Errors determined by means of linear error propagation.

But:

1. Is the **parametrization** flexible enough?
2. What is the **error** associated to any particular choice?
3. Need to rely on **linear propagation** of errors.

NNPDF Methodology (Shortly)

- Generate **Monte Carlo replicas** of the experimental data:
  - **Generation** through Monte Carlo sampling of data,
  - **Validation** against experimental data.

$\Rightarrow$ No need to rely on **linear propagation** of errors,
$\Rightarrow$ possibility to test for **non-gaussian behaviour** in fitted PDFs.

- Fit PDFs with a set of **Neural Networks** on each replica:
  - **Redundant Parametrization**: 7 independent PDFs $\Rightarrow$ 259 free parameters.
  - **Dynamical Stopping Criterion**: Cross-Validation method.

$\Rightarrow$ Neural Networks provide an **unbiased** parametrization.

- **Expectation values** for observables are **Monte Carlo integrals**:

$$\langle \mathcal{O}[f] \rangle = \frac{1}{N} \sum_{k=1}^{N} \mathcal{O}[f_k]$$

... and the same is true for errors, correlations, etc.

Part II

NNPDF2.1 LO, NLO and NNLO sets

NNPDF2.1 is an ensemble of PDF sets presently avalilable at **LO, NLO** and **NNLO**.

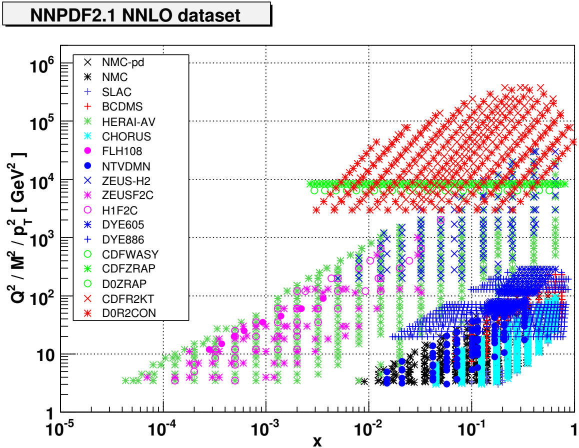
The NNPDF Collaboration, R.D. Ball et al., [arXiv:1107.2652]

Features:

- **Global Fit**: DIS + DY + JET data.
- **Heavy quark** mass effects included using the **FONLL** method up to NNLO,
  S. Forte et al., Nucl. Phys. B834 (2010) 116, [arXiv:1001.2312]
- **FastKernel** method for the inclusion of the higher order corrections:
  - **DIS** up to **NNLO**,
  - **DY and JET** up to **NLO**.

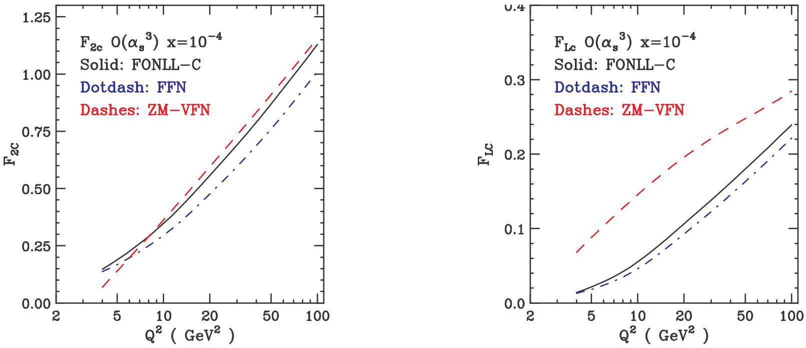
  The NNPDF Collaboration, R.D. Ball et al., Nucl. Phys. B838 (2010) 136, [arXiv:1002.4407]
- **NNLO** corrections to **DY** included by means of K-**factors**,
- **NNLO** corrections to inclusive **JET** implemented using **FastNLO**:
  - approximated NNLO corrections based on threshold resummation.

  T. Kluge, K. Rabbertz and M. Wobisch, (2006), [hep-ph/0609285]



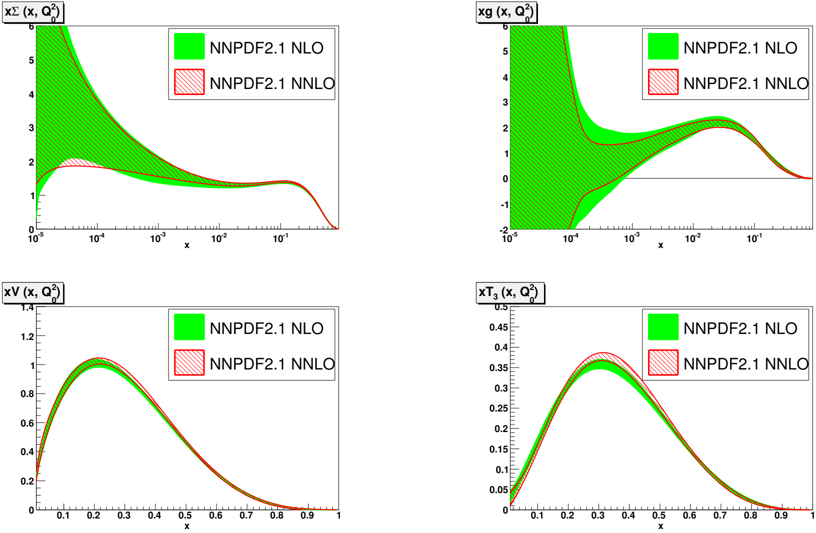
- Inclusion of the HERA F_2^c data.
- Inclusion of **systematics**.

- NNPDF2.1 implements the **FONLL method**:
  prescription for combining **massive quarks** in the decoupling scheme $N_F = 3$ and **massless quarks** in the $\overline{MS}$ scheme $N_F = 4$, at any given order, **avoiding double counting**.
  S.Forte, E.Laenen, P.Nason, J. Rojo [ArXiv:1001.2312]



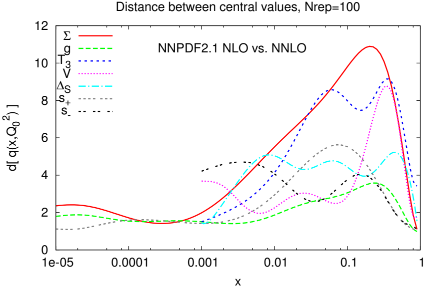
FONLL interpolates smoothly between massive (low Q^2) and massless (high Q^2) scheme.

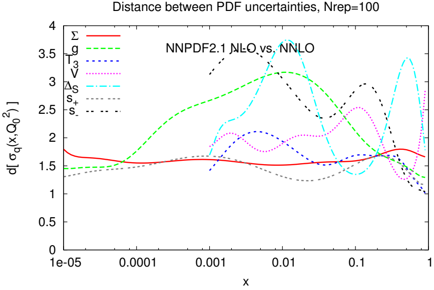
Stability going from NLO to NNLO



In order to quantify the impact on the NNLO corrections, one defines the **distance**:

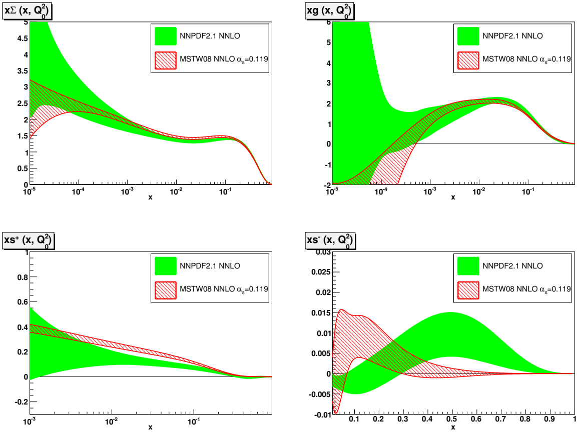
$$d^2 = \sum_{i=1}^{N_{rep}} \frac{(\mathcal{O}^{(i)}_{\rm NLO} - \mathcal{O}^{(i)}_{\rm NNLO})^2}{\sigma^2_{\rm NLO} + \sigma^2_{\rm NNLO}} \quad \Longrightarrow \quad \text{for } N_{rep}{=}100,\; d \sim \begin{cases} 1 & \text{Statistical equivalence} \\ \sqrt{50} \simeq 7 & 1\text{-}\sigma \text{ shift} \end{cases}$$





- NLO and NNLO PDFs very similar at small-x,
- Largest distances for quarks at $x \sim 0.1 - 0.2$,
- very small distances for PDF uncertainties:
  - same (and only) experimental uncertainties for both NLO and NNLO fit.

- Apart from NNPDF2.1, MSTW08 is presently the only NNLO PDF set available for different values of α_s.
- Comparison performed with a **common value of $\alpha_s(M_Z) = 0.119$** (NNPDF2.1 default value).

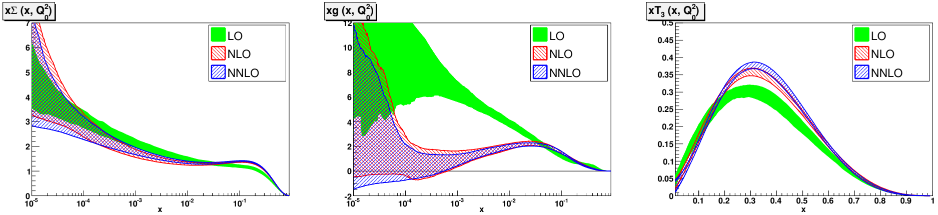


- **Reasonable agreement** between central values,
- MSTW08 uncertainties **unusually small**,
- MSTW08 gluon unstable at small x:
  - it becomes markedly negative.

- Sizable differences in the **Strange distributions**,
- restictive MSTW parametrization for s^+ and s^- (only 4 parameters).

NNPDF2.1 now available at **LO, NLO and NNLO** $\Longrightarrow$

study of **Perturbative Stability**



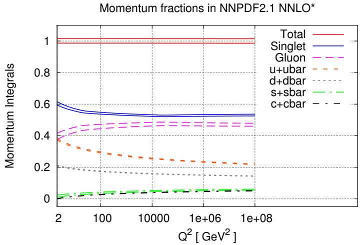
- **Excellent convergence** of the perturbative expansion,
- NLO and NNLO always agree within uncertainties.

Total momentum carried by partons:

$$[q](Q^2) \equiv \int_0^1 dx\, xq(x, Q^2) \quad \Rightarrow \quad [M] = [\Sigma] + [g] \overset{!}{=} 1$$

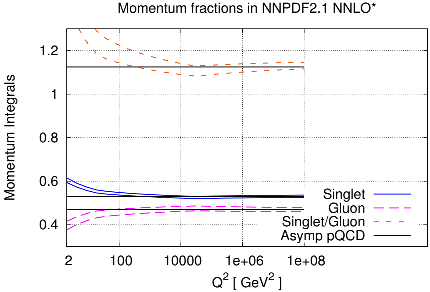
strong **consistency check** of global analysis framework.

- Default NNPDF2.1 sets produced imposing the momentum sum rule (MSR).
- Sets NNPDF2.1 LO*, NLO* and NNLO* produced relaxing MSR:
  - the fit quality doens't change significantly.

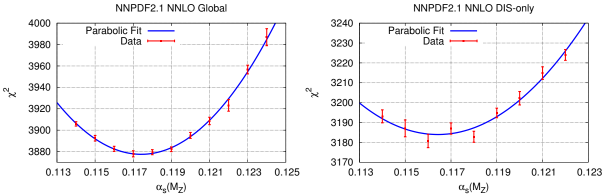


$$[M]_{LO*} = 1.161 \pm 0.032^{exp}$$
$$[M]_{NLO*} = 1.011 \pm 0.018^{exp}$$
$$[M]_{NNLO*} = 1.002 \pm 0.014^{exp}$$

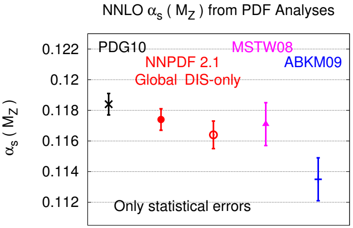
$[\Sigma](Q^2)/[g](Q^2)$



Parabolic fit of the **global** χ^2 as a function of the **input parameter** $\alpha_s(M_z)$:



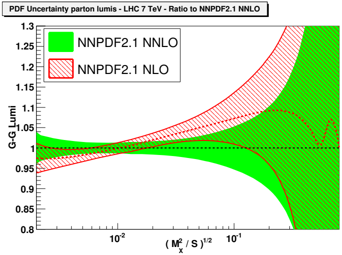
	$\alpha_s(M_Z)$	$\chi^2_{\rm par}/N_{\rm dof}$
NNPDF2.1 NNLO	$0.1174 \pm 0.0006^{\rm exp}$	0.6
NNPDF2.1 NNLO DIS–only	$0.1164 \pm 0.0008^{\rm exp}$	1.1

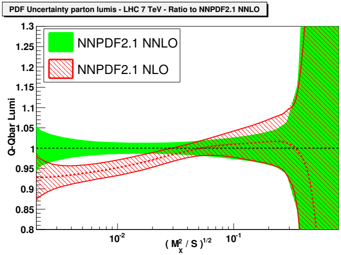


- **NNPDF2.1 global** determination consistent with the **PDG** determination at 1-σ level,
- NNPDF2.1 global and DIS-only determinations agree within uncertainties,
- MSTW08 and ABKM09 values of α_s extracted as a **fitting parameter**,
- NNPDF2.1 and MSTW08 determinations in **good agreement**.

At LHC observables depend on PDFs through **Parton Luminosities**:

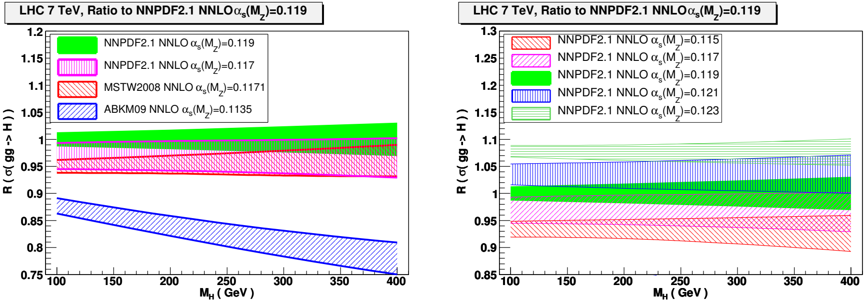
$$\Phi_{ij}(\tau) = \frac{1}{s}\int_{\tau}^{1} \frac{dx}{x} f_i(x, M_X^2) f_j(\tau/x, M_X^2) \quad \text{with} \quad \tau = \frac{M_X^2}{s}$$





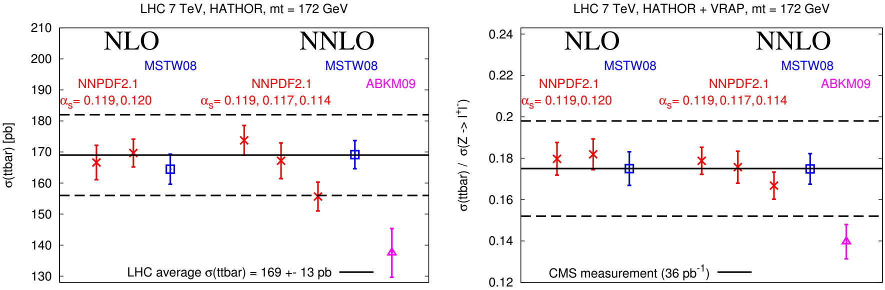
- All luminosities **reasonably compatible**.
- Gluon-gluon luminosity relevant for the **Higgs production**:
  - particularly stable in the "standard Higgs" region ($\sqrt{\tau} \simeq 2 \times 10^{-2}$).
- Quark-antiquark luminosity relevant for the **W/Z production**:
  - NNLO significantly larger than NLO in the W/Z mass region ($\sqrt{\tau} \simeq 10^{-2}$).

Higgs production from gluon-gluon fusion: **Higgs exclusion bounds**.



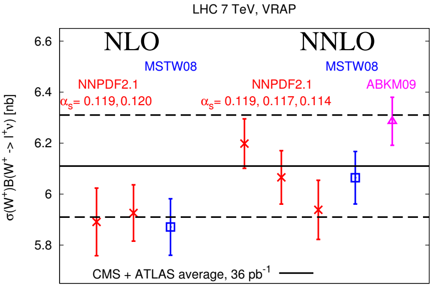
- Strong dependence on α_s.
- NNPDF2.1 and MSTW08 in **excellent agreement**, provided the same value of α_s.
- Sizable differences between NNPDF2.1 and ABKM09:
  - partially accounted by the **different value of α_s** (right plot).

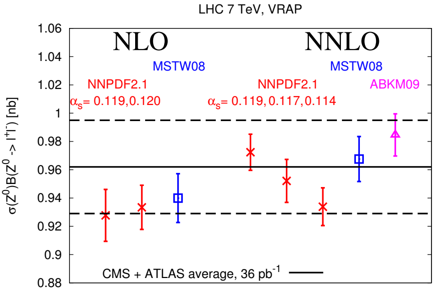
$t\bar{t}$ cross-section: **senstive probe of the gluon distribution**.



- Comparison with **CMS** and **ATLAS** measurements:
  - **Discrimination** between PDF sets,
  - NNPDF2.1 and MSTW08 in good agreement with LHC data,
  - ABKM09 does not agree with the present LHC measurements.
- NNPDF2.1 and MSTW08 in **good agreement** with each other:
  - consistently, using the **same value of α_s improves the agreement**.

W and *Z* production: light flavour decomposition.





- **Weaker dependence on α_s** for these processes:
  - but higher order correction not negligible.
- **Less significant differences** between PDF sets.

Part III

The first PDF set including LHC data: NNPDF2.2

The NNPDF Collaboration, R.D. Ball et al., [arXiv:1108.1758]

NNPDF2.2 NLO:

$\Downarrow$

PDF set including D0 and LHC data

Reweighting NNPDF2.1 NLO using **D0**, **ATLAS** and **CMS** W asymmetry data:

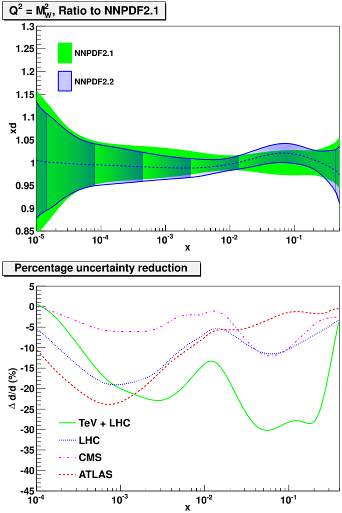
$$A_W^l = \frac{d\sigma_{l^+}/d\eta_l - d\sigma_{l^-}/d\eta_l}{d\sigma_{l^+}/d\eta_l + d\sigma_{l^-}/d\eta_l}$$

Experiment	$N_{\rm dat}$	χ^2(NNPDF2.1)	χ^2(NNPDF2.2)
ATLASmuASY	11	[0.77]	1.07
CMSeASY	6	[1.83]	1.08
CMSmuASY	6	[1.24]	0.56
D0eASY	12	[4.39]	1.38
D0muASY	10	[1.48]	0.35
Total		1.165	1.157

*In NNPDF2.2 no deterioration in the χ^2 of the other experiments.

Light flavour PDFs more affected by the W asimmetry data:

- d distribution and light sea asymmetry $\Delta_s = (\bar{d} - \bar{u})$ most affected.



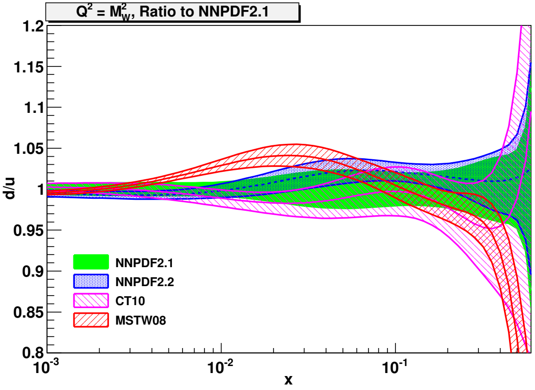
Most noticeable effect in **two separate regions**:

- $x \sim 10^{-3}$,
  mostly affected by the ATLAS data.
- $x \sim 10^{-2} - 10^{-1}$,
  mostly affected by the CMS and D0 data.
- Deviation up to 1-σ in the high x region:
  - mostly due to D0 data.

Sizable **uncertainties reduction**:

- $\sim$ 20% in the low-x region,
- up to 30% at the higher x.

- NNPDF2.1 and MSTW08 do not include **any W asymmetry** data,
- CT10 includes **only D0 data**,
- NNPDF2.2 includes **D0, ATLAS and CMS data** (⇒ most reliable).



Rather good agreement among NLO global PDFs, but:

- MSTW08 prediction too high at medium x and too low at large x.

Part IV

Conclusions

Conclusions

NNPDF Methodology:

- **Monte Carlo** sampling of data $\Rightarrow$ No need for linear propagation of errors,
- **Neural Networks** as interpolating functions $\Rightarrow$ Unbiased parametrization.

NNPDF2.1:

- **global fit** including **heavy quark** mass effects,
- available at **LO**, **NLO** and **NNLO**,

NNPDF2.2:

- Presently, the only PDF set including **ATLAS and CMS data**.

All the NNPDF PDF sets are available either on the web site:

`http://sophia.ecm.ub.es/nnpdf`

or through the LHAPDF inteface.

Part V

Backup Slides

OBS	Data sets
DIS	
F_2^p	NMC,SLAC,BDCMS
F_2^d	SLAC,BCDMS
F_2^d/F_2^p	NMC-pd
σ_{NC}	HERA-I AV, ZEUS-H2
σ_{CC}	HERA-I AV, ZEUS-H2
F_L	H1
$\sigma_\nu, \sigma_{\bar{\nu}}$	CHORUS
dimuon prod.	NuTeV
F_2^c	**ZEUS (99,03,08,09)**
F_2^c	**H1 (01,09,10)**
DY	
$d\sigma^{\mathrm{DY}}/dM^2dy$	E605
$d\sigma^{\mathrm{DY}}/dM^2dx_F$	E886
W asymmetry	CDF
Z rap. distr.	CDF,D0
JET	
incl. $\sigma^{(\mathrm{jet})}$	D0(cone) Run II
incl. $\sigma^{(\mathrm{jet})}$	CDF(k_T) Run II

NLO cuts:

- $W^2 = Q^2(1-x)/x >$12.5 GeV^2,
- $Q^2 > 3$ GeV^2 + further cuts on F_2^c:
  - no data below m_c ($m_c^2 = 2, 2.25, 2.56, 2.89$ GeV^2),
- no cuts on hadronic data.

LO cuts:

- NLO cuts,
- F_L data removed (null at LO).

NNLO cuts:

- NLO cuts,
- further cuts on F_2^c removed,
- conversion of the E866 data from x_F to y distributions,
- NMC proton data included as reduced cross-sections, rather than structure functions.

- NNPDF2.1 implements the **FONLL method**:
  prescription for combining **massive quarks** in the decoupling scheme $N_F = 3$ and **massless quarks** in the $\overline{MS}$ scheme $N_F = 4$, at any given order, **avoiding double counting**.
  Reference: S.Forte, E.Laenen, P.Nason, J. Rojo [ArXiv:1001.2312]

- Definition of FONLL structure function:

$$F^{\text{FONLL}}(x, Q^2) = F^{(n_f+1)}(x, Q^2) + F^{(n_f)}(x, Q^2) - F^{(n_f,0)}(x, Q^2)$$

$F^{(n_f+1)}(x, Q^2)$: massless-scheme structure function
$F^{(n_f)}(x, Q^2)$: massive-scheme structure function $\Rightarrow$ inclusion of the mass suppressed terms
$F^{(n_f,0)}(x, Q^2)$: massless limit of $F^{(n_f)}(x, Q^2)$ $\Rightarrow$ subtraction of the double counting terms

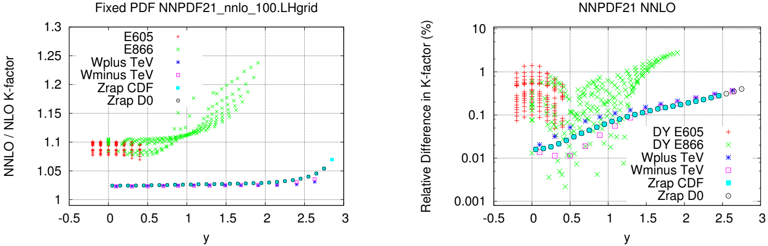
- At the moment, three possibilities:
  - FONLL-A: $\mathcal{O}(\alpha_s)$ PDFs + $\mathcal{O}(\alpha_s)$ coefficient functions (NNPDF2.1 NLO set),
  - FONLL-B: $\mathcal{O}(\alpha_s)$ PDFs + $\mathcal{O}(\alpha_s^2)$ coefficient functions,
  - FONLL-C: $\mathcal{O}(\alpha_s^2)$ PDFs + $\mathcal{O}(\alpha_s^2)$ coefficient functions (NNPDF2.1 NNLO set).

- $\mathcal{O}(\alpha_s^2)$ coefficient functions available only in the NC and not the CC sector.

- **NNPDF2.1** is presently available in **all** the above schemes.

- **Hadronic data** treated **consistently in pQCD up to NLO**.
- **DY NNLO** effects included by means of **K-factors**:

$$K = \frac{(d^2\sigma/dydM^2)_{\mathrm{NNLO}}}{(d^2\sigma/dydM^2)_{\mathrm{NLO}}}$$

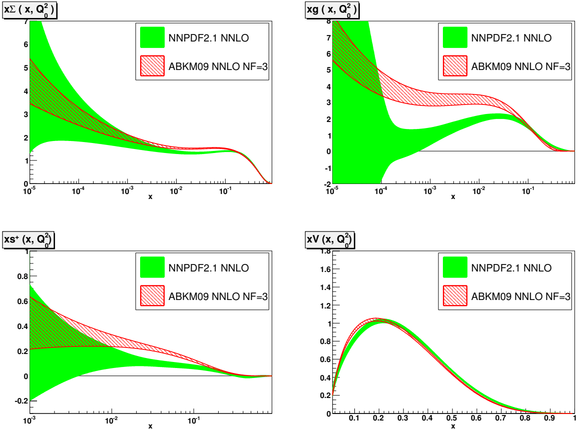


  - **Collider data**: a few percent level,
  - **fixed-target data**: up to 25% ⇒ but experimental errors $\mathcal{O}(20\%)$.
  - K-factor recomputed with NNPDF2.1 NNLO set:
    - accuracy $< 2\%$ ⇒ cross-section uncertainty $< 0.5\%$

- Inclusive **JET** data, exact NNLO corrections not known yet, but:
  - **approximated NNLO corrections** based on threshold resummation,
  - exact NNLO PDF evolution.

NNLO Parton Distributions: NNPDF2.1 vs. ABKM09

- Interesting comparison with the ABKM09 ($N_f = 3$) NNLO set, but some *caveat*:
  - only one value of α_s available for the ABKM set: $\alpha_s(M_Z) = 0.1135 \pm 0.0014$,
  - the ABKM distributions account for the combined PDF+α_s error.

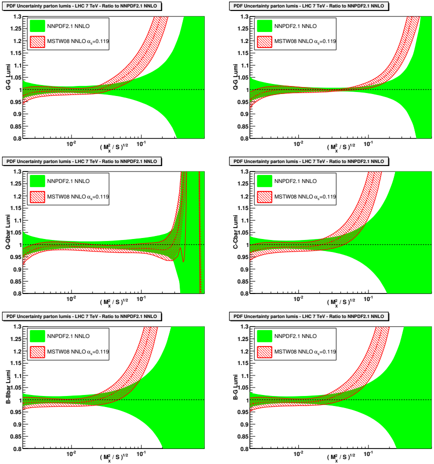


- Comparison performed with the smallest value of α_s provided by NNPDF2.1: $\alpha_s = 0.114$,
- worse agreement with respect of MSTW08.

- Reasonable agreement for Total Strangeness and Valence.

At LHC observables depend on PDFs through **Parton Luminosities**:

$$\Phi_{ij}(\tau) = \frac{1}{s}\int_{\tau}^{1}\frac{dx}{x} f_i(x, M_X^2) f_j(\tau/x, M_X^2) \quad \text{with} \quad \tau = \frac{M_X^2}{s}$$



- Same value of $\alpha_s = 0.119$.
- General good agreement in the region of τ of the typical electoweak final states at LHC ($M_x \sim 100$ GeV).
- Huge disagreement at high M_X:
  - MSTW08 NNLO gluon instability at small-x

The **NNPDF approach** allows to obtain reliable PDFs from datasets of widely varying size without having to modify any aspect of the methodology.

$\Downarrow$

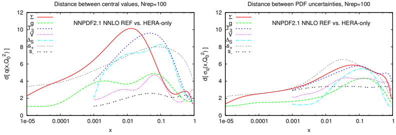
Study of the dependece of PDFs on the underlying dataset.

NNPDF provides **four new NNLO PDF set**, based on subsets of the full dataset:

1. **NNPDF2.1 NNLO HERA-only**: only HERA data
   - the HERAPDF group provides PDFs based on the same dataset.
2. **NNPDF2.1 NNLO DIS-only**: no hadron-hadron data
   - DIS data theoretically and experimentally more clean.
3. **NNPDF2.1 NNLO DIS+DY**: no JET data
   - approximated NNLO JET.
4. **NNPDF2.1 NNLO Colliders-only**: no fixed-target data
   - fixed-target data less clean: low energy and nuclear targets.

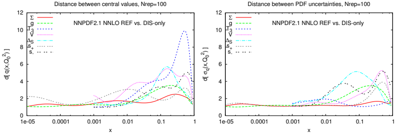
NNPDF2.1

Accuracy of the NNLO Determination: Full Set vs. Subsets



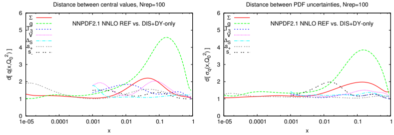
HERA-only: **general poor description of data**,

- s^+ very uncertain $\Rightarrow \chi^2$ NuTeV data very poor.
- bad Singlet-Triplet (Σ and T_3) decomposition,
- bad light sea (Δ_s) decomposition.



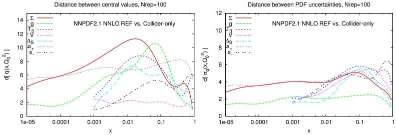
DIS-only: **reasonably accurate flavour decomposition**,

- Σ and g in perfect agreement with the global fit,
- also V, s^+ and Δ_s in good agreement,
- substantial deviation of T_3. **(???)**



DIS+DY: **quite close to the global fit**,

- almost al PDFs statistically equivalent,
- reduction of the uncertainties.



Colliders-only: **fixed-target data description very poor**,

- almost all PDFs disagree with the global fit,
- collider only dataset more consistent (global $\chi^2 = 1.02$),
- improvement expected with the upcoming HERA-II and LHC data.

Experiment	Global	HERA–only	DIS–only	DIS+DY	Collider–only
N_{dat}	3507	976	2933	3321	1232
Total	1.16	1.07	1.15	1.18	1.02
NMC-pd	0.93	[13.15]	0.88	0.94	[3.43]
NMC	1.63	[1.91]	1.69	1.69	[2.06]
SLAC	1.01	[3.17]	0.97	1.03	[1.23]
BCDMS	1.32	[2.15]	1.28	1.30	[2.22]
HERAI-AV	1.10	1.05	1.09	1.09	1.06
CHORUS	1.12	[2.63]	1.08	1.13	[1.74]
FLH108	1.26	1.32	1.27	1.26	1.26
NTVDMN	0.49	[60.51]	0.45	0.54	[23.02]
ZEUS-H2	1.31	1.21	1.26	1.28	1.30
ZEUSF2C	0.88	0.77	0.86	0.88	0.75
H1F2C	1.46	1.30	1.47	1.50	1.24
DYE605	0.81	[9.06]	[6.86]	0.82	[1.34]
DYE866	1.32	[12.41]	[2.70]	1.32	[5.76]
CDFWASY	1.65	[7.71]	[13.94]	1.64	1.07
CDFZRAP	2.12	[3.74]	[2.15]	1.91	1.22
D0ZRAP	0.67	[1.11]	[0.67]	0.65	0.61
CDFR2KT	0.74	[1.15]	[0.99]	[1.25]	0.64
D0R2CON	0.82	[1.28]	[0.88]	[1.03]	0.83

* [. . .] not fitted sets.

Reweighting allows to incorporate a new dataset into an PDF set **without refitting**.

For users:

1. Take an NNPDF parton ditribution set $\{f_k\}$, with $k = 1, \dots, N_{rep}$ (NNPDF2.1 for instance),
2. take the new dataset $y = \{y_1, \dots, y_n\}$ and its eventual covariance matrix σ_{ij},
3. compute the χ^2 of each replica on the new set y according to usual formula:

$$\chi^2_k = \sum_{i,j=1}^{n} (y_i - y_i[f_k]) \sigma^{-1}_{ij} (y_j - y_j[f_k])$$

   where $y_i[f_k]$ is the prediction for the experimental point y_i using the k-th replica,
4. evaluate the weights according to the formula:

$$w_k \propto (\chi^2_k)^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k} \quad \text{with} \quad \sum_{k=1}^{N_{rep}} w_k = N_{rep} \, ,$$

5. compute the new expectation value of your favourite observable as:

$$\langle \mathcal{O} \rangle_{new} = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} w_k \mathcal{O}[f_k] \, .$$

Unweighting allows to construct a standard PDF set (without weights) **statistically equivalent** to a given reweighted set.

Idea:

Given a weighted set of N_{rep} replicas,
select (eventually more than once) replicas carrying relatively hight weight
and discard replicas carrying relatively small weight.

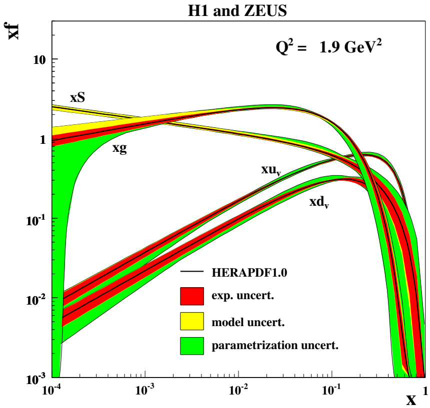
Construction of the unweighted set:

1. Decide the number of replicas N'_{rep} of the unweighted set:
   - pointless to choose $N'_{rep} > N_{rep}$: no gain of information.
2. calculate for the k-th replica of the reweighted set the **integer non negative** number:

$$w'_k = \sum_{j=1}^{N'_{rep}} \theta\left(\frac{j}{N'_{rep}} - w_{k-1}\right) \theta\left(w_k - \frac{j}{N'_{rep}}\right) \quad \left(\Rightarrow \sum_{k=1}^{N_{rep}} w'_k = N'_{rep}\right) ,$$

3. construct the unweighted set taking w'_k copies of the k-th replica, for $k = 1, \ldots, N_{rep}$.

Reference: [arXiv:0911.0884]



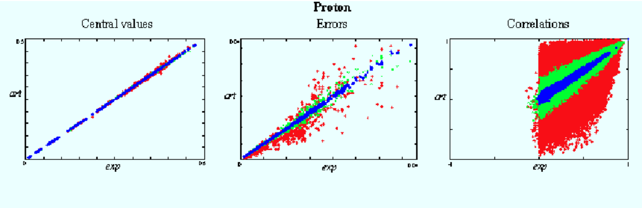
- HERAPDF analysis: computation of parametrization uncertainty on top of the model (initial scale and heavy quark mass effects) and experimental uncertainty.
- In many regions the **parametrization uncertainty** is **dominating**.

- **Generation** of artificial Monte Carlo data according to the distribution:

$$\mathcal{O}_i^{(art)(k)} = (1 + r_{norm}^{(k)}\sigma_{norm}) \left[\mathcal{O}_i^{(exp)} + r_{stat}^{(k)}\sigma_{stat} + \sum_{p=1}^{N_{sys}} r_{sys,p}^{(k)}\sigma_{sys,p} \right]$$

where $r_i^{(k)}$ are univariate (gaussianly distributed) random numbers.

- **Validation** of the Monte Carlo replicas against experimental data .



Red points: 10 replicas
Green points: 100 replicas
Blue points: 1000 replicas

- $\mathcal{O}(1000)$ replicas needed to reproduce correlations to the percent accuracy.

NNPDF Approach

Neural Networks as Redundant and Unbiased Parametrisation of PDFs

- **Unbiased basis** of functions parameterized by a very large and **redundant set of parameters** $\Rightarrow$ **Neural Networks**.
- Each one of the **7 independent PDFs**:

Gluon	$g(x)$
Singlet	$\Sigma(x) = \sum_q (q(x) + \overline{q}(x))$
Valence	$V(x) = \sum_q (q(x) - \overline{q}(x))$
Triplet	$T_3(x) = (u(x) + \overline{u}(x)) - (u(x) + \overline{u}(x))$
Sea asymmetry	$\Delta_S(x) = \overline{d}(x) - \overline{u}(x)$
Total Strangeness	$s^+(x) = s(x) + \overline{s}(x)$
Strange Valence	$s^-(x) = s(x) - \overline{s}(x)$

is parametrized at the initial scale $Q_0^2 = 2\ \text{GeV}^2$ by an individual Neural Network having architecture 2-5-3-1 $\Rightarrow$ **37 parameters**.

259 parameters
Standard fits have $\sim$ 25 parameters in total

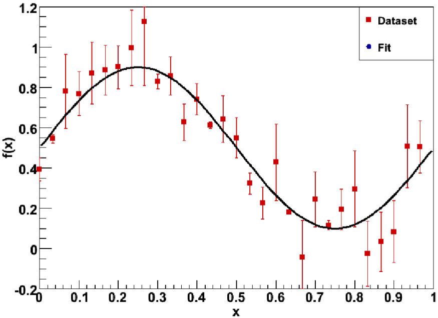
$$\xi_i = g\left(\sum_j \omega_{ij}\xi_j + \theta_i\right), \qquad g(x) = \frac{1}{1+e^{-x}}$$

* Neural Networks are just functions, which adapt well to any functional behaviour.
For instance, a (1-2-1) NN is:

Drawback:

A redundant parametrization might fit not only to physical behavior but also to random statistical fluctuations of data.

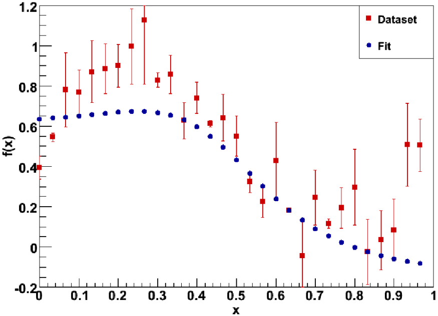
UNDERLYING PHYSICAL LAW



Drawback:

A redundant parametrization might fit not only to physical behavior but also to random statistical fluctuations of data.

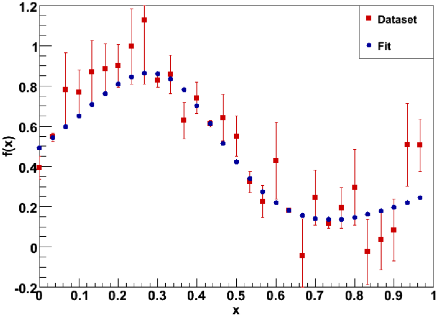
UNDERLEARNING



Drawback:

A redundant parametrization might fit not only to physical behavior but also to random statistical fluctuations of data.

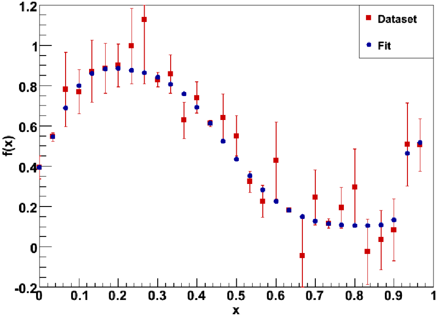
PROPER LEARNING



Drawback:

A redundant parametrization might fit not only to physical behavior but also to random statistical fluctuations of data.

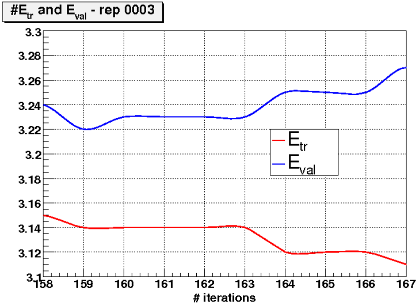
OVERLEARNING



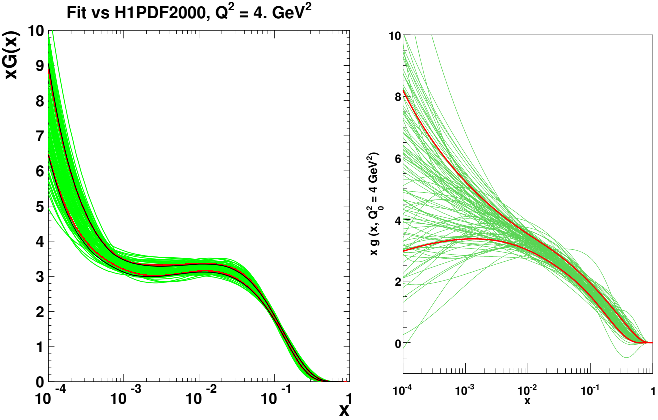
Need for a suitable stopping criterion!

Cross-validation method:

* Divide data in two sets: training and validation (for each experiment).
* Random division for each replica (tipically $f_t = f_v = 0.5$).
* Minimisation performed only on the training set. Meantime, the validation χ^2 is monitored.
* When the training χ^2 still decreases while the validation χ^2 stops decreasing → STOP.



Simple Functional Froms vs. Neural Networks



- Simple functional forms $q(x) = Ax^b(1-x)^c P(x)$ (CT, MSTW, ABKM, HERAPDF)
  $\rightarrow$ **systematic underestimation of uncertainties**
- Artificial Neural Networks as universal interpolants (NNPDF)
  $\rightarrow$ avoid theoretical bias from choice of **PDF functional form**

- We use **Neural Networks** as functions to represent PDFs at the starting scale.

- We employ Multilayer Feed-Forward Neural Networks trained using a **Genetic Algorithm**

- Activation determined by weights and thresholds:

$$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right), \qquad g(x) = \frac{1}{1+e^{-x}}$$

For instance, a (1-2-1) NN is:

$$\xi_1^{(3)} = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - \xi_1^{(1)}\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - \xi_1^{(1)}\omega_{21}^{(1)}}}}}$$

- They provide a parametrization which is redundant and robust against variations.