

# NNLL resummation for squark-antisquark production

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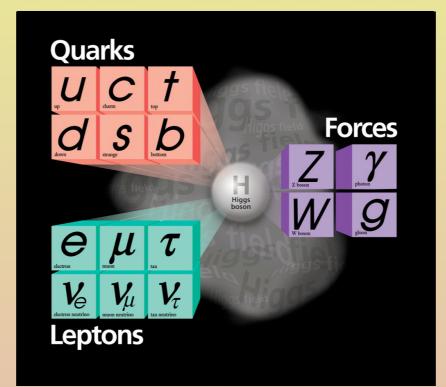


## Outline

- I. Motivation
- 2. Resummation
- 3. Ingredients for NNLL resummation
- 4. Results

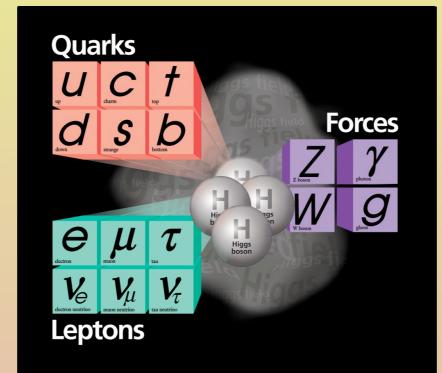


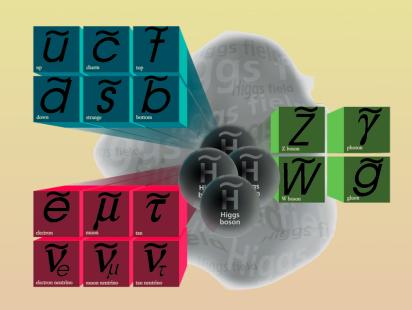
- Hierarchy problem
- Gauge coupling unification
- Dark matter





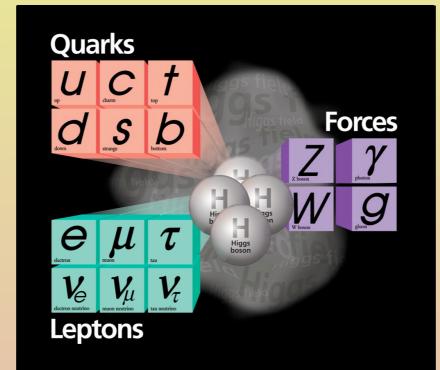
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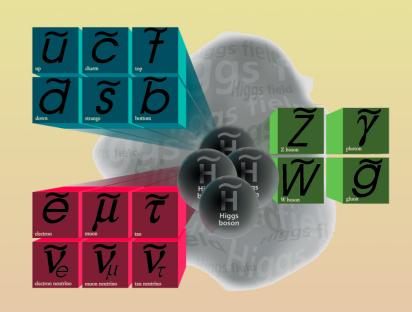






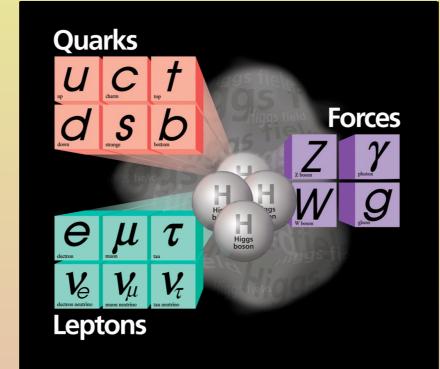
 ✓ Hierarchy problem
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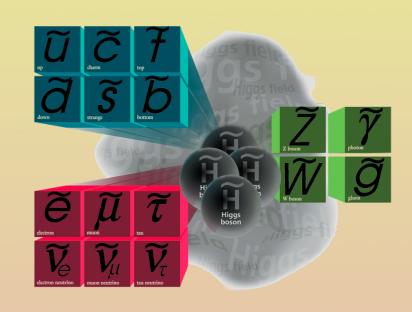






 ✓ Hierarchy problem
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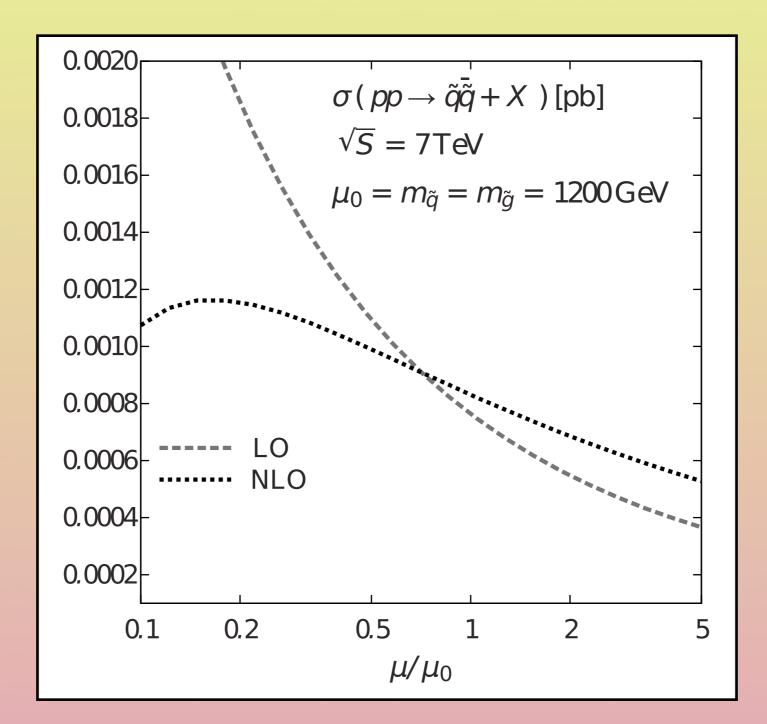




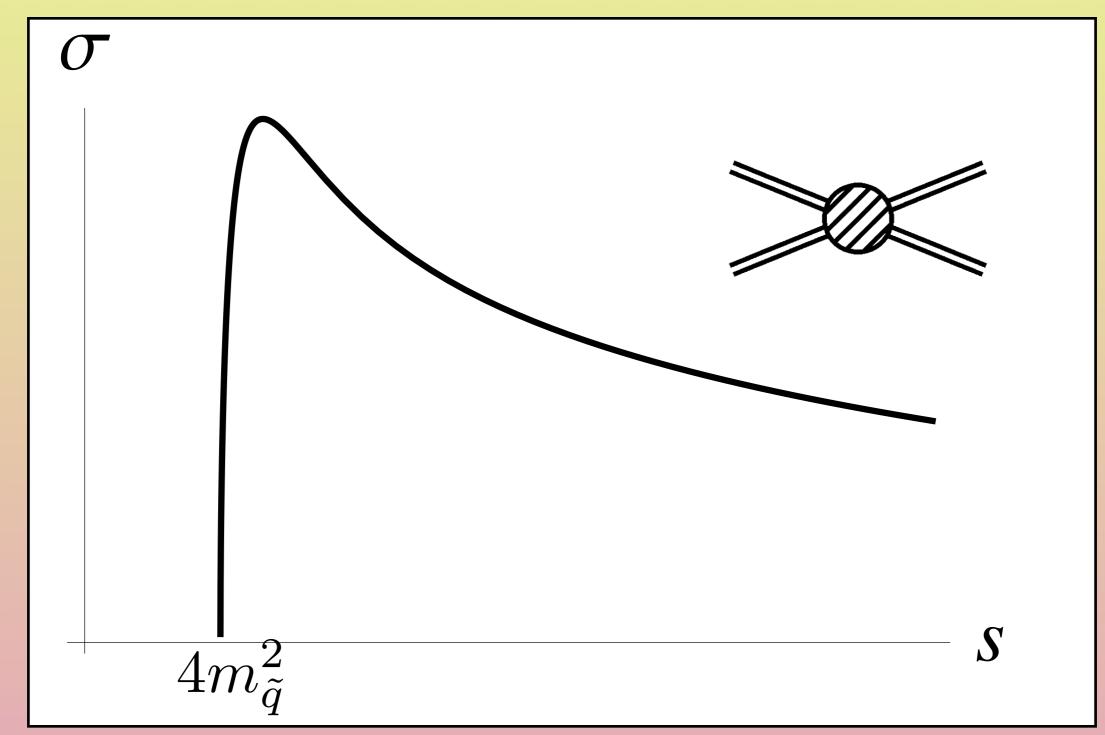
- SUSY particles are heavy
- Squark-antisquark production



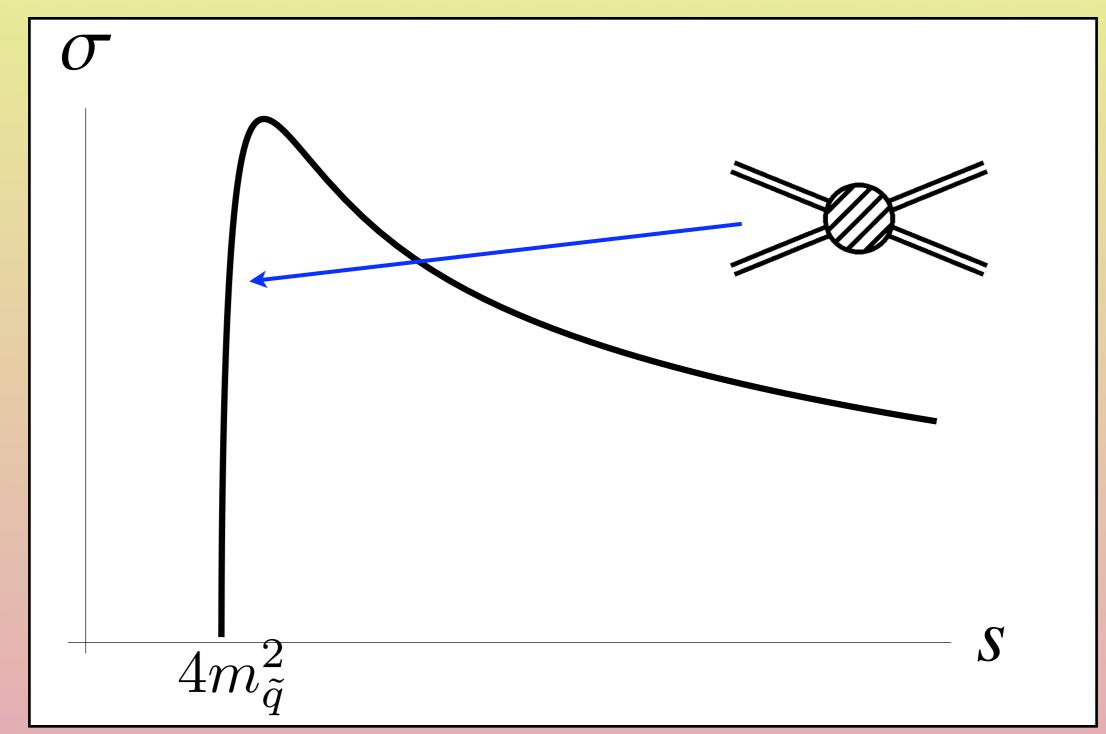
## Scale dependence



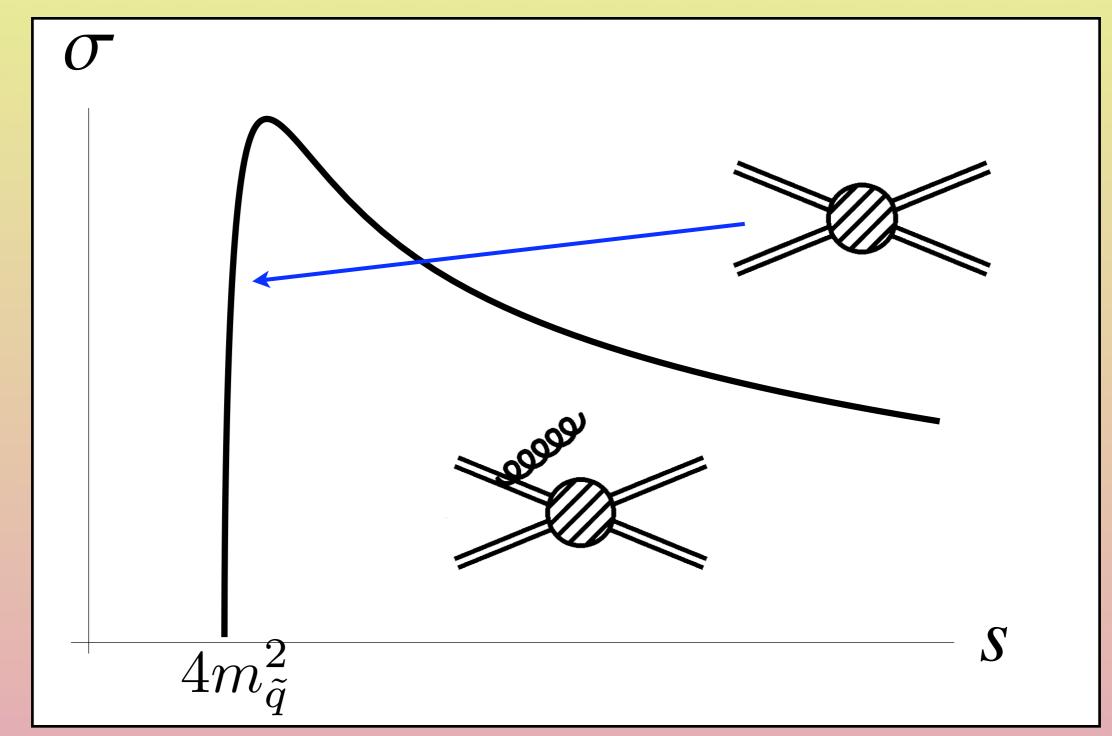




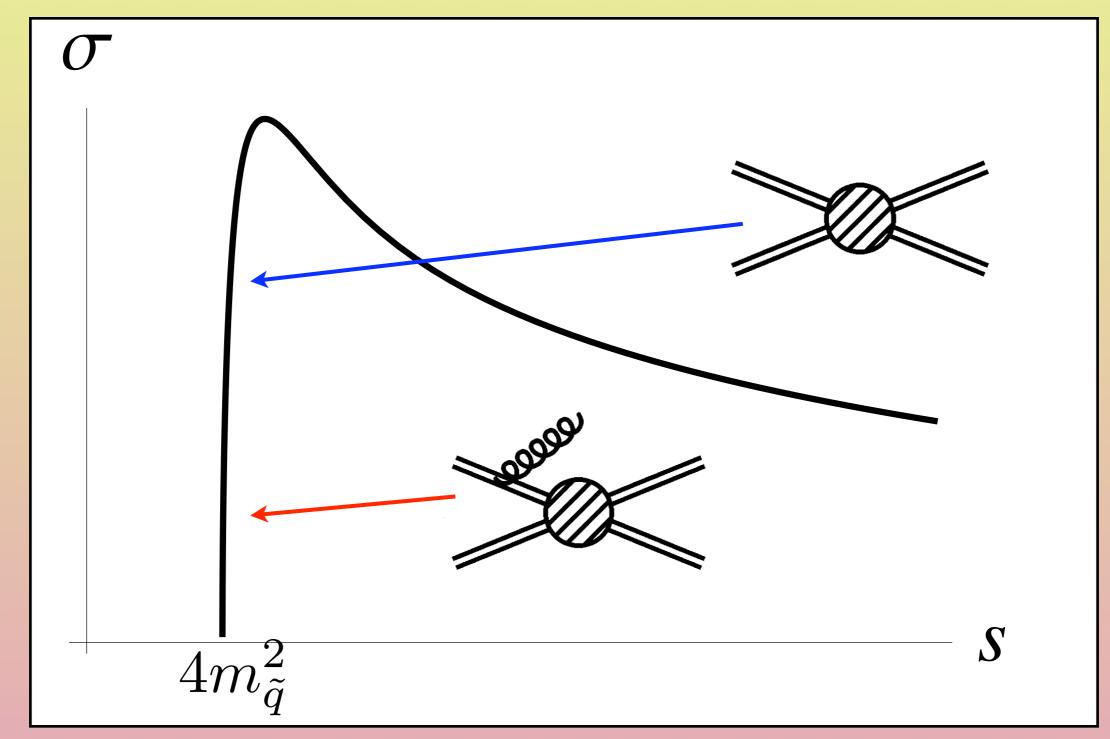














# $LO \qquad 1$ $NLO \qquad \alpha_s L^2 \quad \alpha_s L \qquad \alpha_s$ $NNLO \qquad \alpha_s^2 L^4 \quad \alpha_s^2 L^3 \quad \alpha_s^2 L^2 \quad \alpha_s^2 L \quad \alpha_s^2$ $N^3 LO \qquad \alpha_s^3 L^6 \quad \alpha_s^3 L^5 \quad \alpha_s^3 L^4 \quad \alpha_s^3 L^3 \quad \alpha_s^3 L^2 \quad \alpha_s^3 L \quad \alpha_s^3$ $N^4 LO \qquad \cdots$

$$L = \log(8\beta^2) \qquad \beta = \sqrt{1-\rho} \qquad \rho = \frac{4m_{\tilde{q}}^2}{\hat{s}}$$



# $LO \qquad 1$ $NLO \qquad \alpha_s L^2 \quad \alpha_s L \qquad \alpha_s$ $NNLO \qquad \alpha_s^2 L^4 \quad \alpha_s^2 L^3 \quad \alpha_s^2 L^2 \quad \alpha_s^2 L \quad \alpha_s^2$ $N^3 LO \qquad \alpha_s^3 L^6 \quad \alpha_s^3 L^5 \quad \alpha_s^3 L^4 \quad \alpha_s^3 L^3 \quad \alpha_s^3 L^2 \quad \alpha_s^3 L \quad \alpha_s^3$ $N^4 LO \qquad \cdots$

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho)$$

 $L = \log(8\beta^2) \qquad \beta = \sqrt{1-\rho} \qquad \rho = \frac{4m_{\tilde{q}}^2}{\hat{s}}$ 



# $LO \qquad 1$ $NLO \qquad \alpha_s L^2 \quad \alpha_s L \qquad \alpha_s$ $NNLO \qquad \alpha_s^2 L^4 \quad \alpha_s^2 L^3 \quad \alpha_s^2 L^2 \quad \alpha_s^2 L \quad \alpha_s^2$ $N^3 LO \qquad \alpha_s^3 L^6 \quad \alpha_s^3 L^5 \quad \alpha_s^3 L^4 \quad \alpha_s^3 L^3 \quad \alpha_s^3 L^2 \quad \alpha_s^3 L \quad \alpha_s^3$ $N^4 LO \qquad \cdots$

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \qquad L \to \log(N)$$

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# $LO \qquad 1$ $NLO \qquad \alpha_s L^2 \quad \alpha_s L \qquad \alpha_s$ $NNLO \qquad \alpha_s^2 L^4 \quad \alpha_s^2 L^3 \quad \alpha_s^2 L^2 \quad \alpha_s^2 L \quad \alpha_s^2$ $N^3 LO \qquad \alpha_s^3 L^6 \quad \alpha_s^3 L^5 \quad \alpha_s^3 L^4 \quad \alpha_s^3 L^3 \quad \alpha_s^3 L^2 \quad \alpha_s^3 L \quad \alpha_s^3$ $N^4 LO \qquad \cdots$

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \qquad L \to \log(N)$$

 $\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}} e^{LP_1(\alpha_s L)} e^{P_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)}$ 



#### 

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \qquad L \to \log(N)$$

$$\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}} e^{\frac{LP_1(\alpha_s L)}{L}} e^{P_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)}$$



#### 

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \qquad L \to \log(N)$$

$$\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}} e^{LP_1(\alpha_s L)} e^{P_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)}$$

$$\downarrow LL \qquad \text{NLL}$$



LO

## Resummation

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \qquad L \to \log(N)$$

$$\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}}_{\text{LO}} e^{LP_1(\alpha_s L)} e^{P_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)} e^{\Gamma_1(\alpha_s L)} e^{\Gamma_2(\alpha_s L)} e^{\Gamma$$



 $T \cap$ 

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## Resummation

$$\tilde{f}(N) = \int_0^1 d\rho \rho^{N-1} f(\rho) \qquad L \to \log(N)$$

$$\tilde{\sigma}^{\text{resum}} = \tilde{\sigma}^{\text{thr}}_{\mathsf{NLO}} e^{LP_1(\alpha_s L)} e^{P_2(\alpha_s L)} e^{\alpha_s P_3(\alpha_s L)} \\ \begin{array}{c} \mathsf{LL} & \mathsf{NLL} & \mathsf{NLL} \\ \mathsf{NLO} & \mathsf{NLL} \end{array}$$



## NNLL Resummation

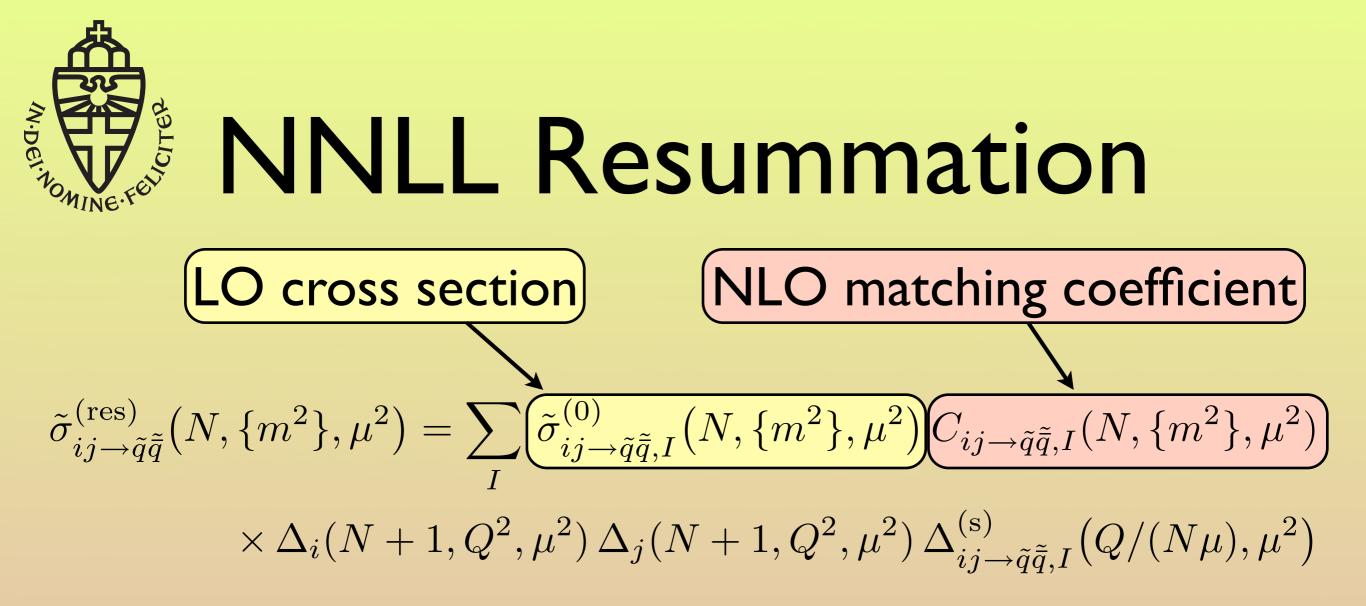
$$\tilde{\sigma}_{ij\to\tilde{q}\tilde{\tilde{q}}}^{(\text{res})}(N,\{m^2\},\mu^2) = \sum_{I} \tilde{\sigma}_{ij\to\tilde{q}\tilde{\tilde{q}},I}^{(0)}(N,\{m^2\},\mu^2) C_{ij\to\tilde{q}\tilde{\tilde{q}},I}(N,\{m^2\},\mu^2) \\ \times \Delta_i(N+1,Q^2,\mu^2) \Delta_j(N+1,Q^2,\mu^2) \Delta_{ij\to\tilde{q}\tilde{\tilde{q}},I}^{(\text{s})}(Q/(N\mu),\mu^2)$$

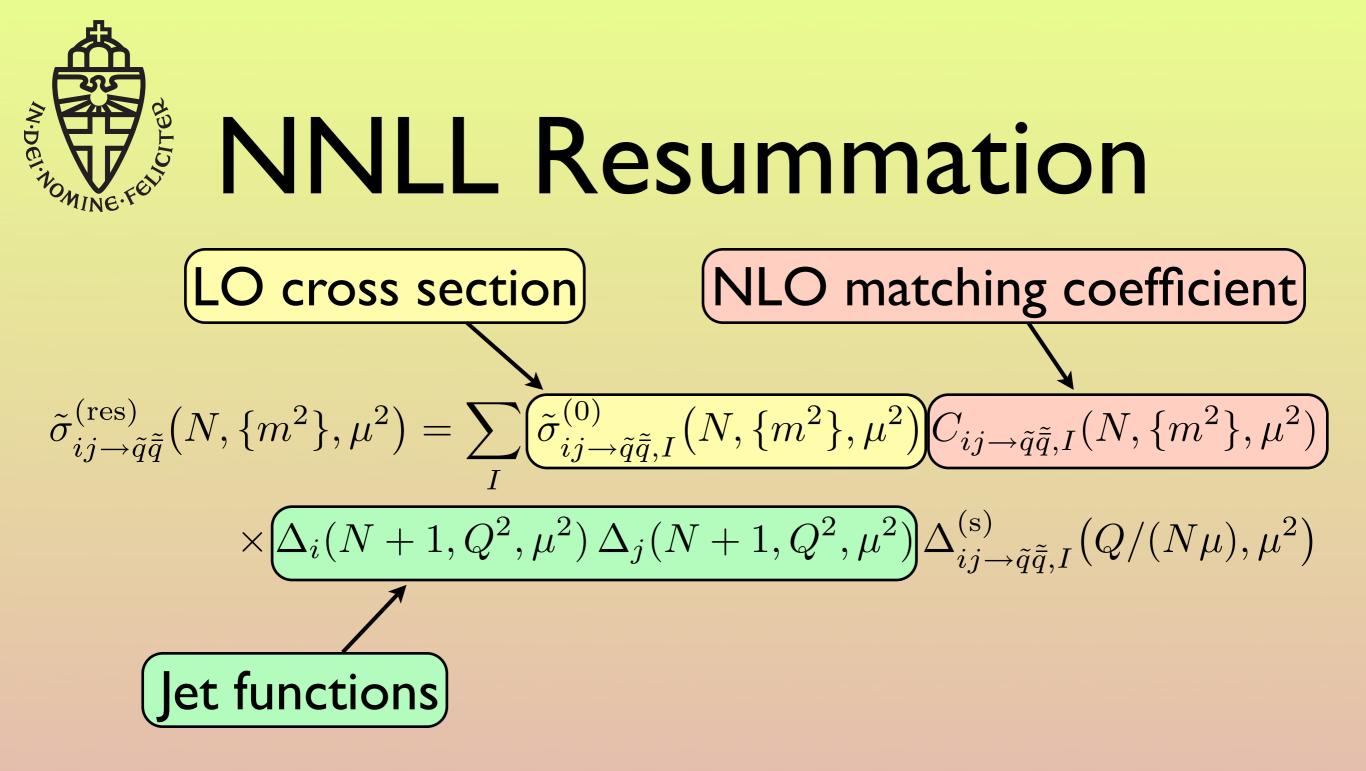
$$\widetilde{\sigma}_{ij \to \tilde{q}\tilde{\tilde{q}}}^{\text{(res)}} NNLL Resumation$$

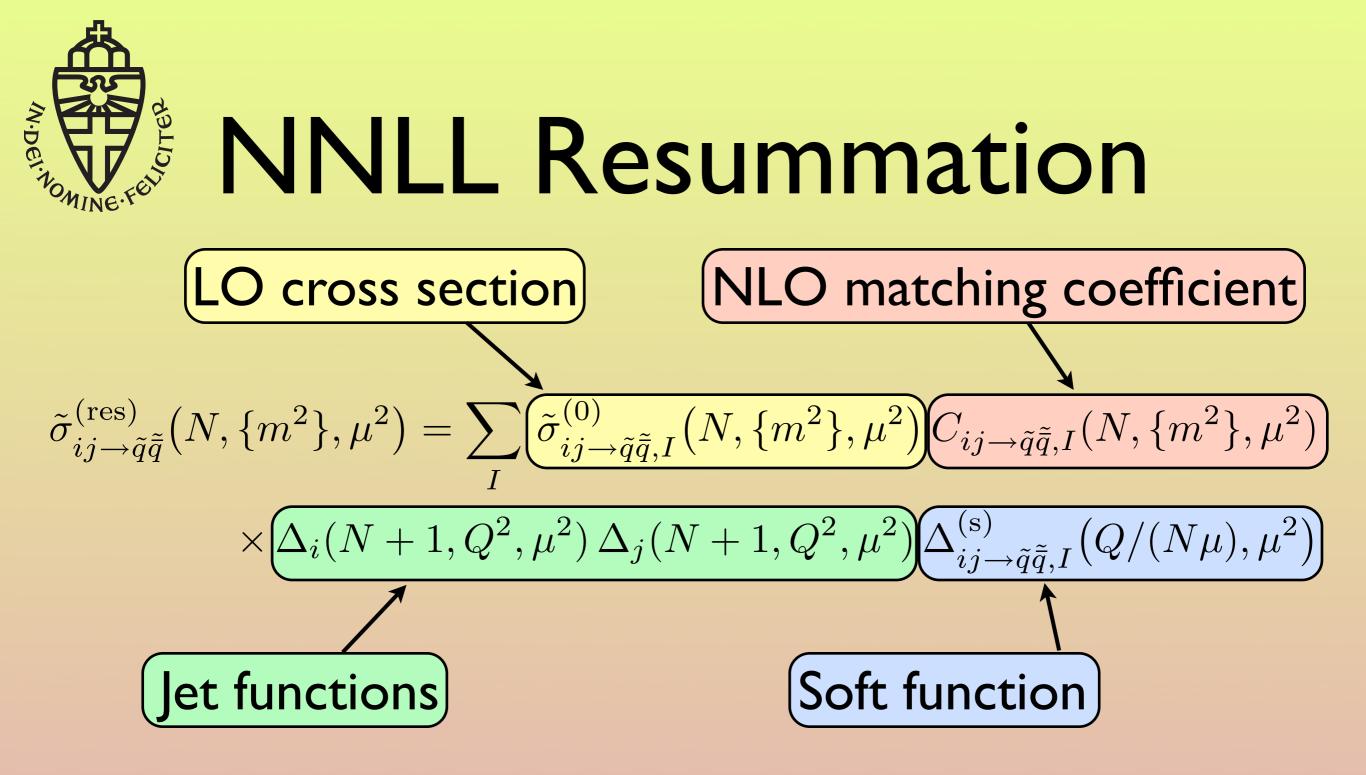
$$LO \text{ cross section}$$

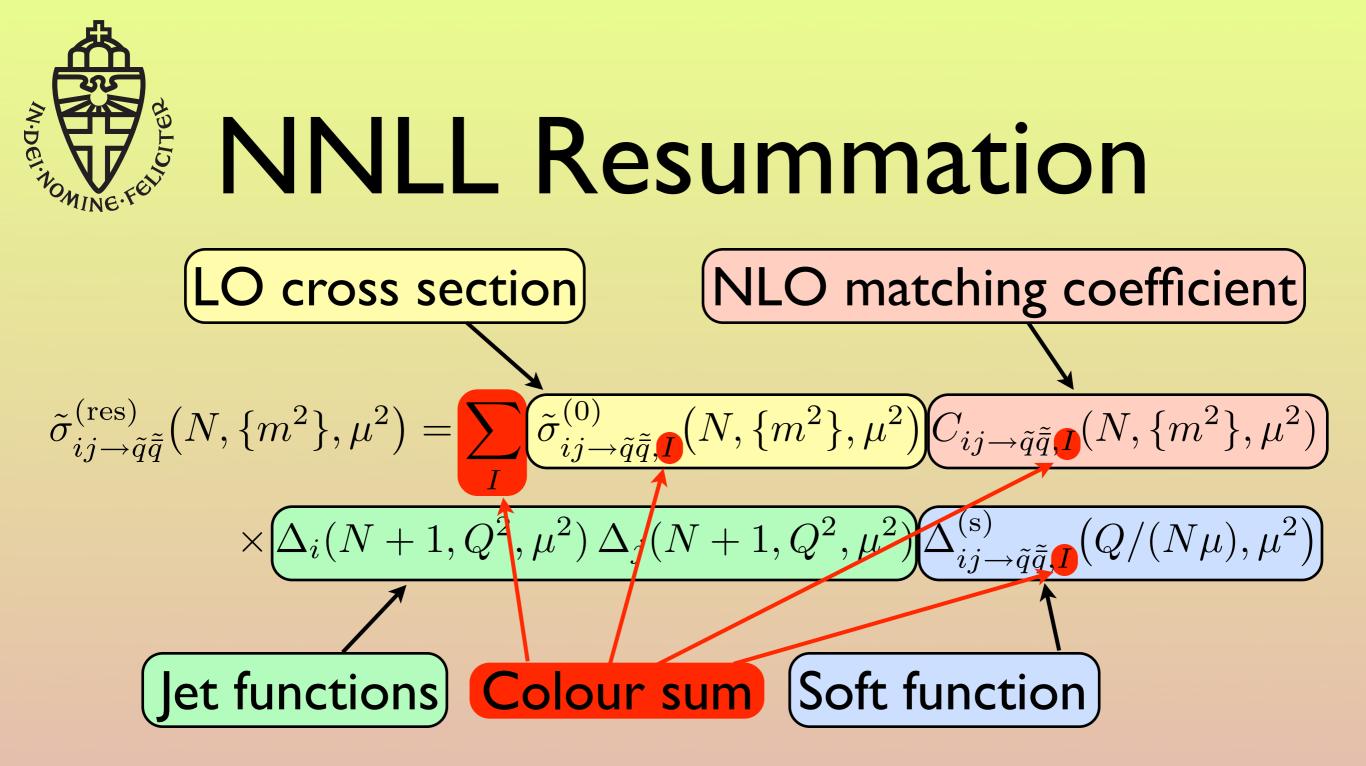
$$\widetilde{\sigma}_{ij \to \tilde{q}\tilde{\tilde{q}}}^{(\text{res)}} (N, \{m^2\}, \mu^2) = \sum_{I} \underbrace{\widetilde{\sigma}_{ij \to \tilde{q}\tilde{\tilde{q}}, I}^{(0)} (N, \{m^2\}, \mu^2)}_{I} C_{ij \to \tilde{q}\tilde{\tilde{q}}, I} (N, \{m^2\}, \mu^2)$$

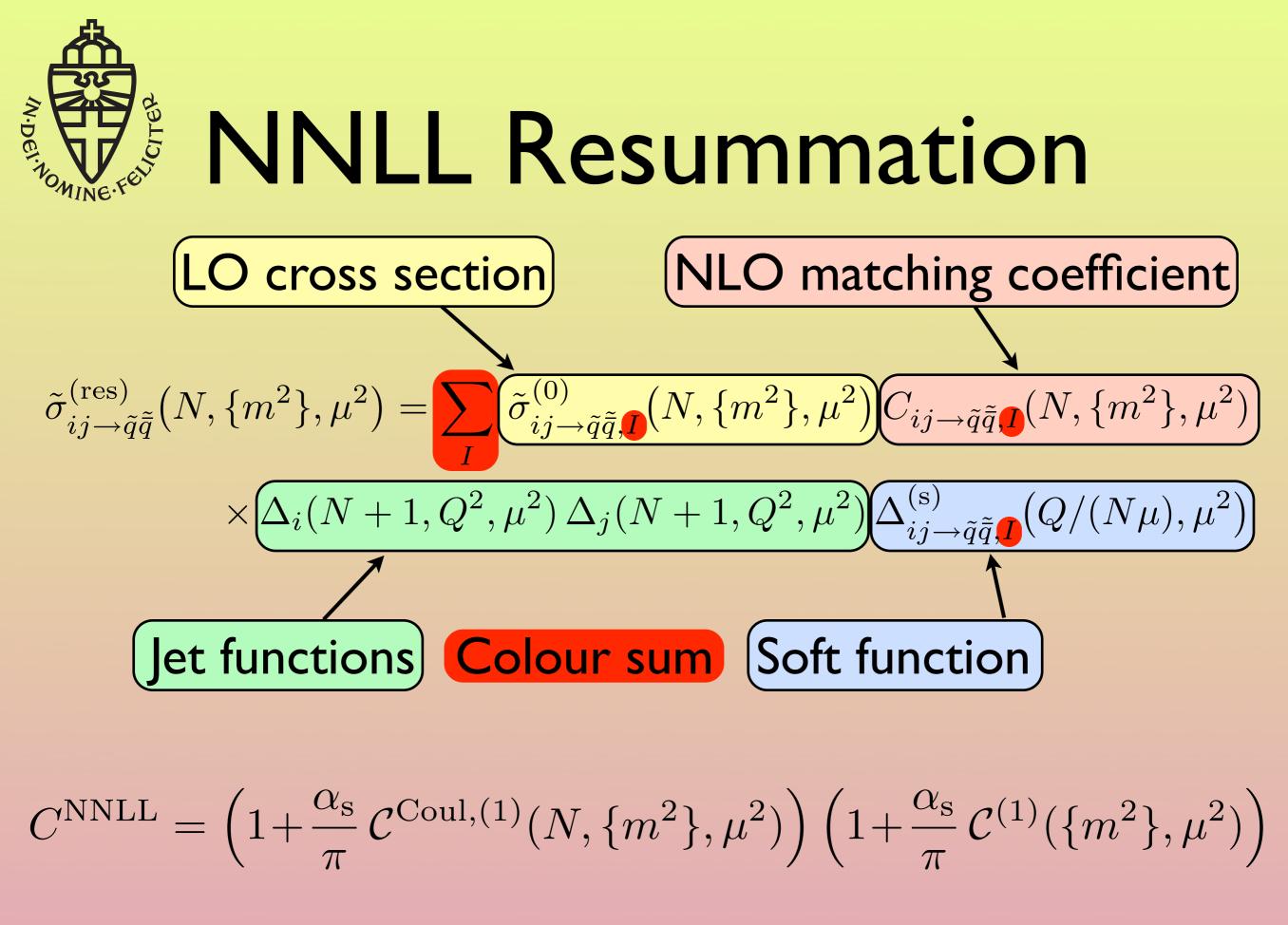
$$\times \Delta_i (N+1, Q^2, \mu^2) \Delta_j (N+1, Q^2, \mu^2) \Delta_{ij \to \tilde{q}\tilde{\tilde{q}}, I}^{(s)} (Q/(N\mu), \mu^2)$$









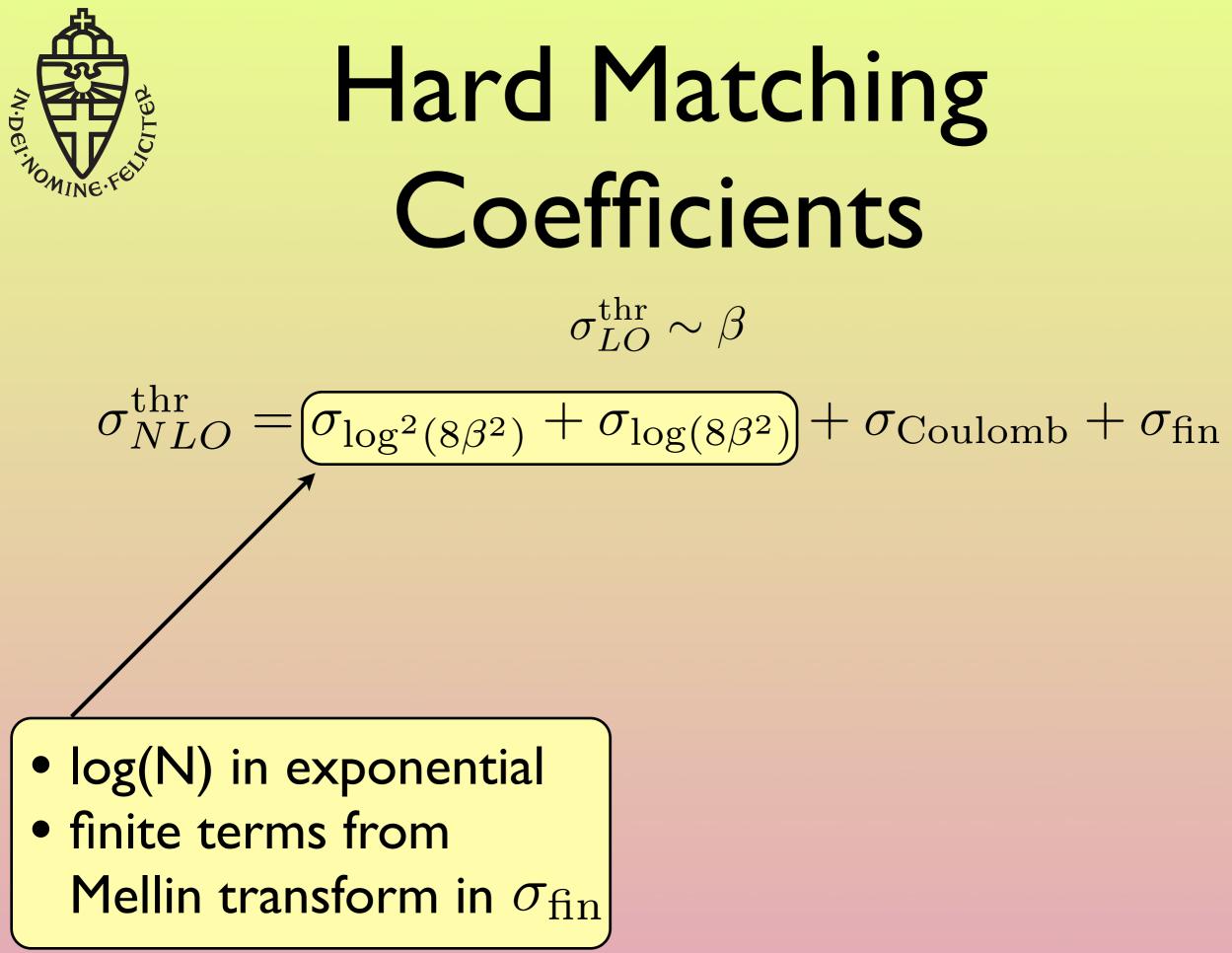


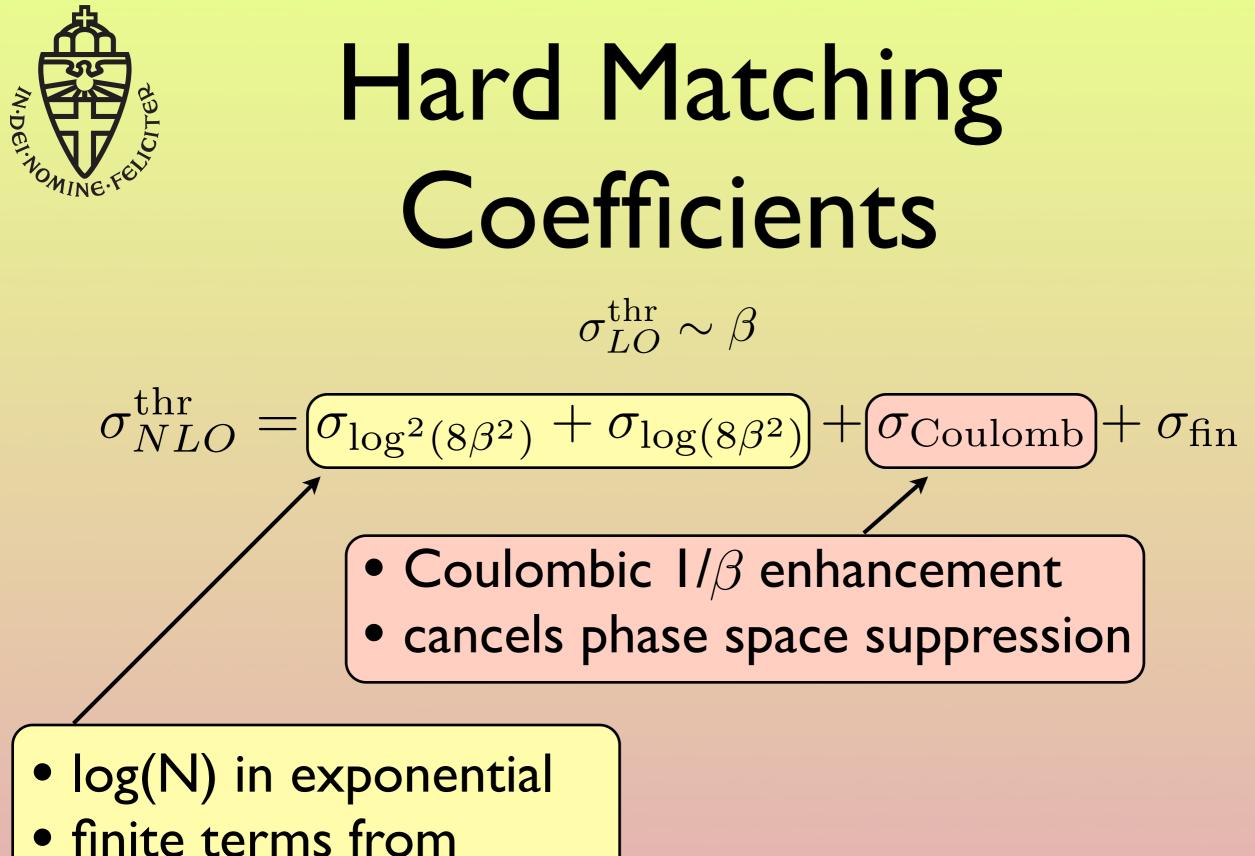


# Hard Matching Coefficients

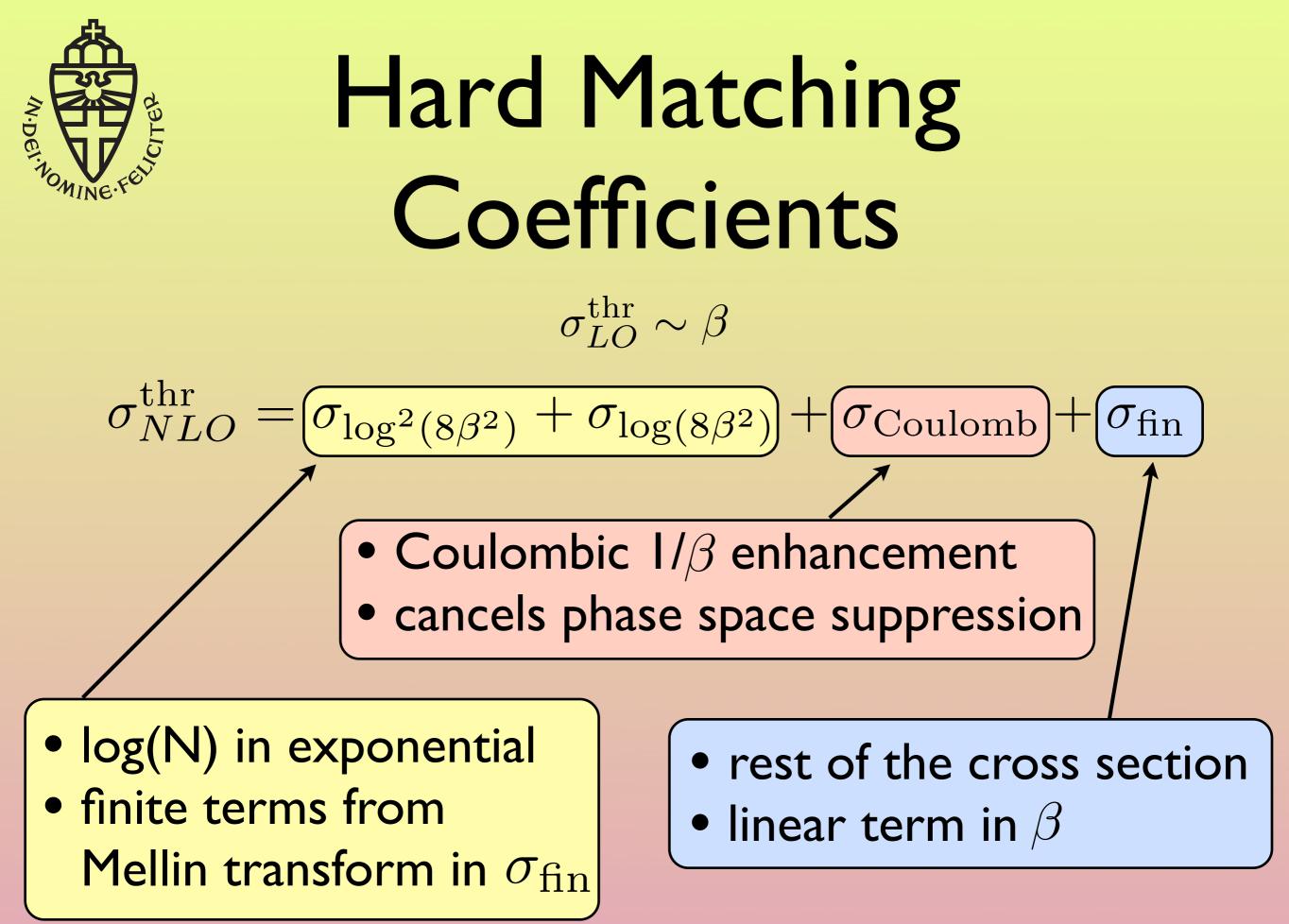
 $\sigma_{LO}^{\rm thr} \sim \beta$ 

 $\sigma_{NLO}^{\rm thr} = \sigma_{\log^2(8\beta^2)} + \sigma_{\log(8\beta^2)} + \sigma_{\rm Coulomb} + \sigma_{\rm fin}$ 



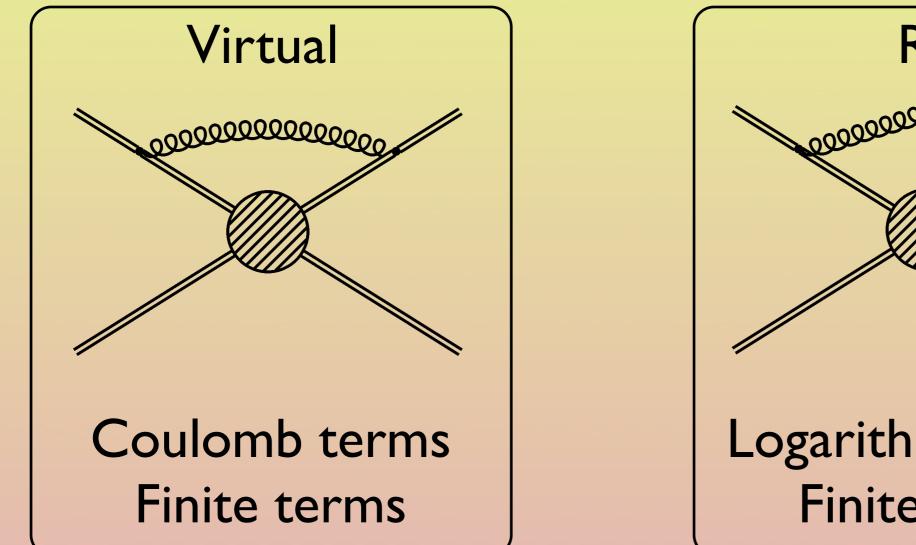


Mellin transform in  $\sigma_{\rm fin}$ 





## NLO calculation



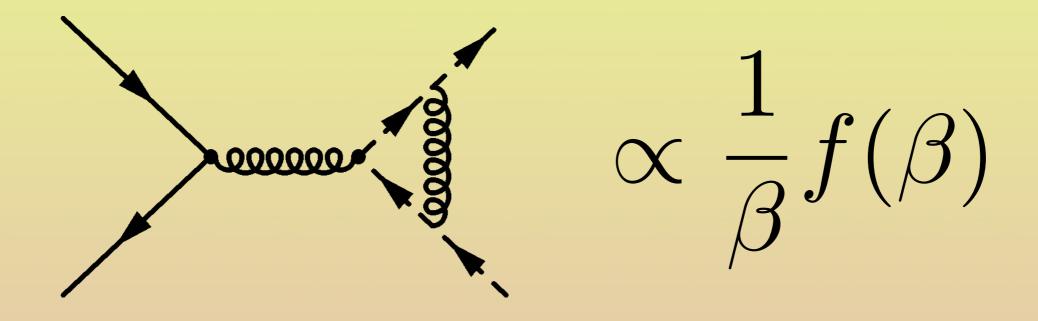
#### Logarithmic terms Finite terms

Real

#### Calculate up to $O(\beta)$

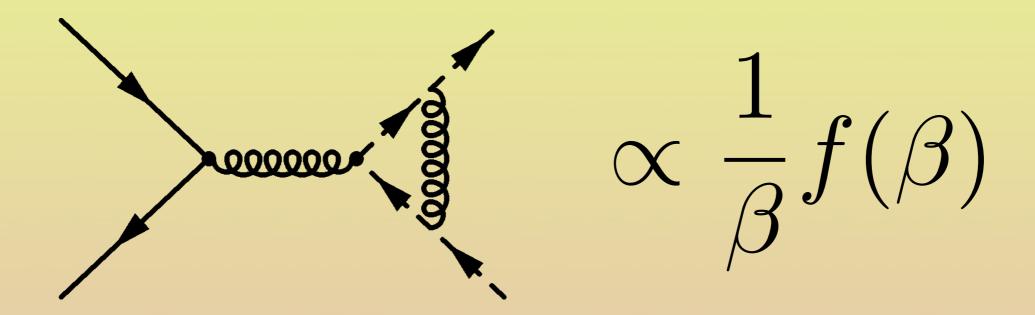


## Virtual: Coulomb





## Virtual: Coulomb

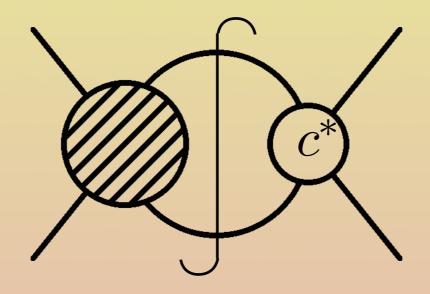


$$\sigma_{ij \to \tilde{q}\bar{\tilde{q}},I}^{\text{Coul},(1)} = -\frac{\alpha_{s}}{\pi} \frac{\pi^{2}}{2\beta} \kappa_{ij \to \tilde{q}\bar{\tilde{q}},I} \sigma_{ij \to \tilde{q}\bar{\tilde{q}},I}^{(0)}$$
$$\kappa_{1} = -\frac{4}{3} \qquad \kappa_{8} = \frac{1}{6}$$



## Virtual corrections

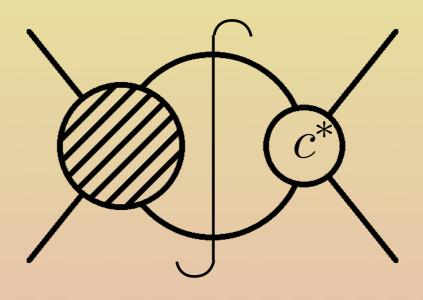
#### Colour decomposition: only LO





## Virtual corrections

#### Colour decomposition: only LO



Linear term in  $\beta$ :

- Most scalar integrals:  $\beta=0$
- Coulomb integrals: expand in  $\beta$



## Real Corrections

 $\sigma_{NLO} = \sigma^R + \sigma^V + \sigma^C$ 



$$\sigma_{NLO} = \sigma^{R} + \sigma^{V} + \sigma^{C}$$
$$= \int_{3} \left[ d\sigma^{R} - d\sigma^{A} \right]_{\epsilon=0} + \int_{2} \left[ d\sigma^{V} + \int_{1} d\sigma^{A} \right]_{\epsilon=0} + \sigma^{C}$$



$$\sigma_{NLO} = \sigma^R + \sigma^V + \sigma^C$$
  
= 
$$\int_3 \left[ d\sigma^R - d\sigma^A \right]_{\epsilon=0} + \int_2 \left[ d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0} + \sigma^C$$
  
= 
$$\sigma^{\{3\}} + \sigma^{\{2\}} + \sigma^C$$



$$\sigma_{NLO} = \sigma^R + \sigma^V + \sigma^C$$
  
= 
$$\int_3 \left[ d\sigma^R - d\sigma^A \right]_{\epsilon=0} + \int_2 \left[ d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0} + \sigma^C$$
  
= 
$$\sigma^{\{3\}} + \sigma^{\{2\}} + \sigma^C$$

For a finite function: 
$$\int_{1-\beta^2}^1 f(x) dx \propto \beta^2$$



$$\sigma_{NLO} = \sigma^{R} + \sigma^{V} + \sigma^{C}$$

$$= \int_{3} [d\sigma^{R} - d\sigma^{A}]_{\epsilon=0} + \int_{2} [d\sigma^{V} + \int_{1} d\sigma^{A}]_{\epsilon=0} + \sigma^{C}$$

$$= \sigma^{\{3\}} + \sigma^{\{2\}} + \sigma^{C}$$

$$For a finite function:$$

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$$\sigma_{NLO} = \sigma^{R} + \sigma^{V} + \sigma^{C}$$

$$= \int_{3} \left[ d\sigma^{R} - d\sigma^{A} \right]_{\epsilon=0} + \int_{2} \left[ d\sigma^{V} + \int_{1} d\sigma^{A} \right]_{\epsilon=0} + \sigma^{C}$$

$$= \overbrace{}^{} + \sigma^{\{2\}} + \sigma^{C}$$
For a finite function:
$$\int_{1-\beta^{2}}^{1} f(x) dx \propto \beta^{2}$$



$$\sigma_{NLO} = \sigma^{R} + \sigma^{V} + \sigma^{C}$$

$$= \int_{3} [d\sigma^{R} - d\sigma^{A}]_{\epsilon=0} + \int_{2} [d\sigma^{V} + \int_{1} d\sigma^{A}]_{\epsilon=0} + \sigma^{C}$$

$$= I + \sigma^{\{2\}} + \sigma^{C} = I + \sigma^{V} + \sigma^{A} + \sigma^{C}$$

$$For a finite function:$$

$$\int_{1-\beta^{2}}^{1} f(x) dx \propto \beta^{2}$$



# Result for $gg \to \tilde{q} \tilde{\bar{q}}$

$$\mathcal{C}_{gg \to \tilde{q}\bar{\tilde{q}},I}^{(1)} = \operatorname{Re}\left\{\pi^2 \left(\frac{5N_c}{12} - \frac{C_F}{4}\right) + \gamma_g \log\left(\frac{\mu_R^2}{\mu_F^2}\right)\right\}$$

$$-\frac{m_{\tilde{g}}^2 N_c}{2m_{\tilde{q}}^2} \log^2\left(x_{\tilde{g}\tilde{g}}(4m_{\tilde{q}}^2)\right) + C_F\left(\frac{m_+^2 m_-^2}{2m_{\tilde{q}}^4} \log\left(\frac{m_+^2}{m_-^2}\right) - \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2} - 3\right)$$

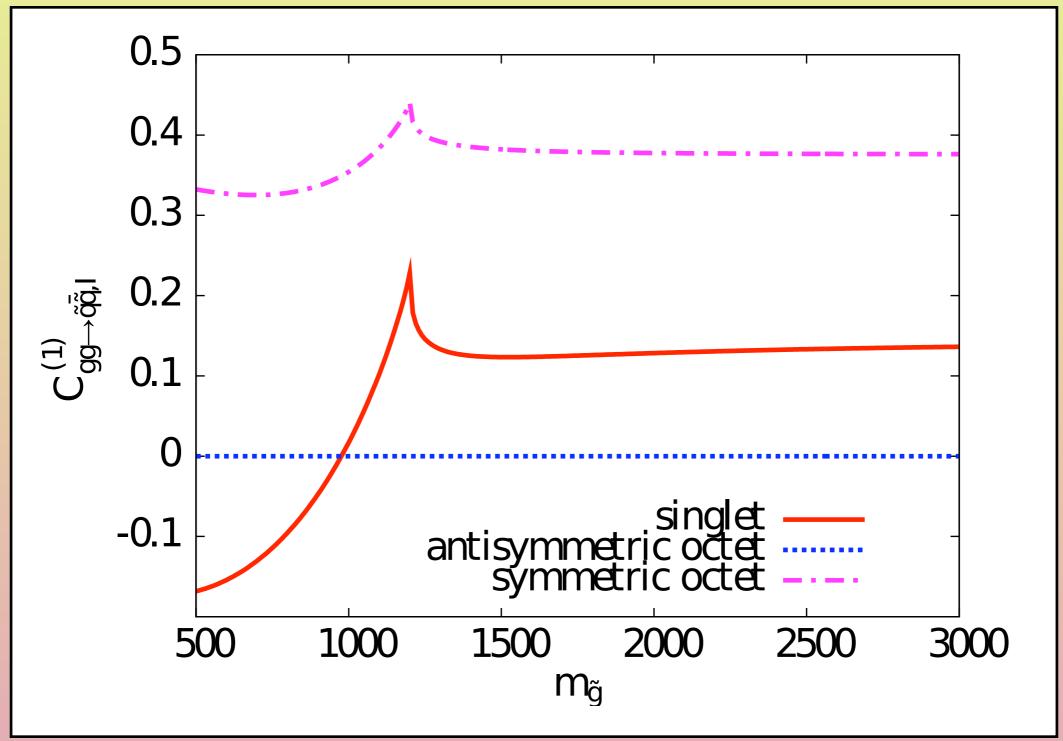
$$+\frac{m_+^2 N_c}{2m_{\tilde{q}}^2} \left( \text{Li}_2 \left( -\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2} \right) - \text{Li}_2 \left( \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2} \right) \right)$$

$$+\left[\frac{\pi^2}{8} - \frac{1}{2}\mathrm{Li}_2\left(-\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) + \frac{m_{\tilde{g}}^2}{4m_{\tilde{q}}^2}\log^2\left(x_{\tilde{g}\tilde{g}}(4m_{\tilde{q}}^2)\right)\right]C_2(I)$$

$$+ 2C_A \left(\gamma_E^2 - 2\gamma_E \log(2) + \gamma_E \log\left(\frac{\mu_F^2}{m_{\tilde{q}}^2}\right)\right) + (2 + \gamma_E) C_2(I)$$

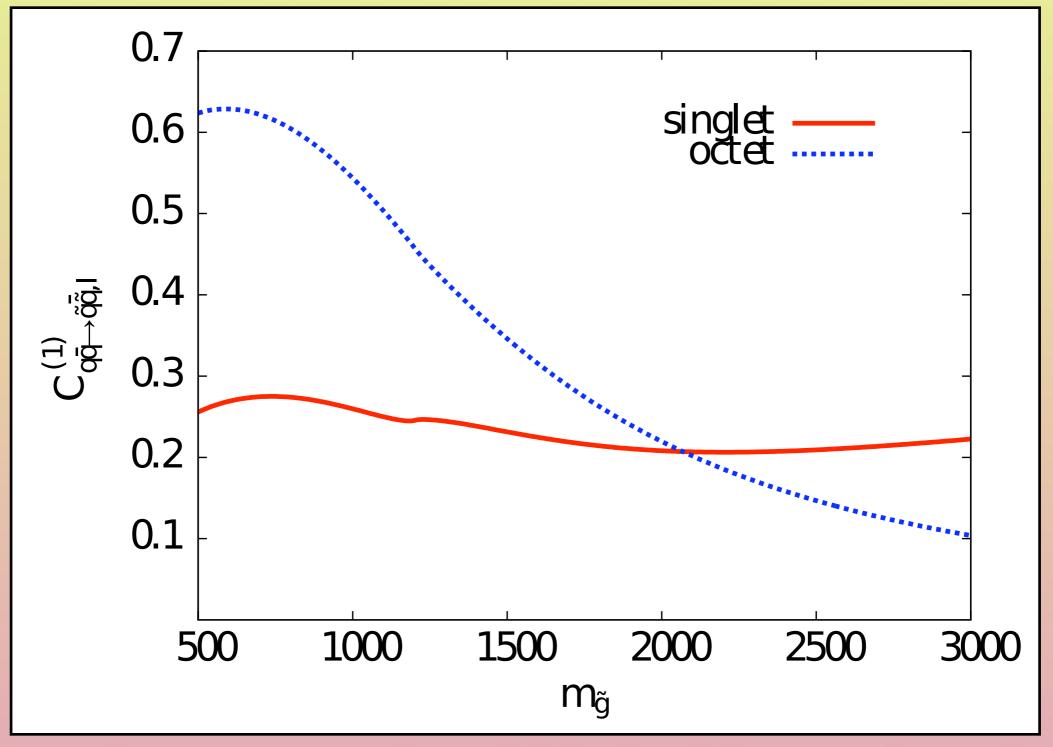


# $gg \to \tilde{q}\bar{\tilde{q}}$





# $q\bar{q} \to \tilde{q}\bar{\tilde{q}}$





$$\begin{aligned} &\sigma_{h_{1}h_{2}\to\tilde{q}\bar{\tilde{q}}}^{(\text{NNLL+NLO matched})}\left(\rho,\{m^{2}\},\mu^{2}\right) = \sigma_{h_{1}h_{2}\to\tilde{q}\bar{\tilde{q}}}^{(\text{NLO})}\left(\rho,\{m^{2}\},\mu^{2}\right) \\ &+ \sum_{i,j} \int_{\text{CT}} \frac{dN}{2\pi i} \,\rho^{-N} \,\tilde{f}_{i/h_{1}}(N+1,\mu^{2}) \,\tilde{f}_{j/h_{2}}(N+1,\mu^{2}) \\ &\times \left[\tilde{\sigma}_{ij\to\tilde{q}\bar{\tilde{q}}}^{(\text{res,NNLL})}\left(N,\{m^{2}\},\mu^{2}\right) - \tilde{\sigma}_{ij\to\tilde{q}\bar{\tilde{q}}}^{(\text{res,NNLL})}\left(N,\{m^{2}\},\mu^{2}\right)|_{(\text{NLO})}\right] \end{aligned}$$



$$\begin{split} &\sigma_{h_{1}h_{2}\to\tilde{q}\bar{\tilde{q}}}^{(\text{NNLL+NLO matched})}\left(\rho,\{m^{2}\},\mu^{2}\right) = \left[\sigma_{h_{1}h_{2}\to\tilde{q}\bar{\tilde{q}}}^{(\text{NLO})}\left(\rho,\{m^{2}\},\mu^{2}\right)\right] \\ &+ \sum_{i,j} \int_{\text{CT}} \frac{dN}{2\pi i} \,\rho^{-N} \,\tilde{f}_{i/h_{1}}(N+1,\mu^{2}) \,\tilde{f}_{j/h_{2}}(N+1,\mu^{2}) \\ &\times \left[\tilde{\sigma}_{ij\to\tilde{q}\bar{\tilde{q}}}^{(\text{res,NNLL})}\left(N,\{m^{2}\},\mu^{2}\right) - \tilde{\sigma}_{ij\to\tilde{q}\bar{\tilde{q}}}^{(\text{res,NNLL})}\left(N,\{m^{2}\},\mu^{2}\right)|_{(\text{NLO})} \end{split}$$



$$\sigma_{h_1 h_2 \to \tilde{q}\bar{\tilde{q}}}^{(\text{NNLL+NLO matched})} \left(\rho, \{m^2\}, \mu^2\right) = \left(\sigma_{h_1 h_2 \to \tilde{q}\bar{\tilde{q}}}^{(\text{NLO})} \left(\rho, \{m^2\}, \mu^2\right)\right) + \left(\sum_{i,j} \int_{\text{CT}} \frac{dN}{2\pi i} \rho^{-N} \tilde{f}_{i/h_1}(N+1, \mu^2) \tilde{f}_{j/h_2}(N+1, \mu^2)\right)$$

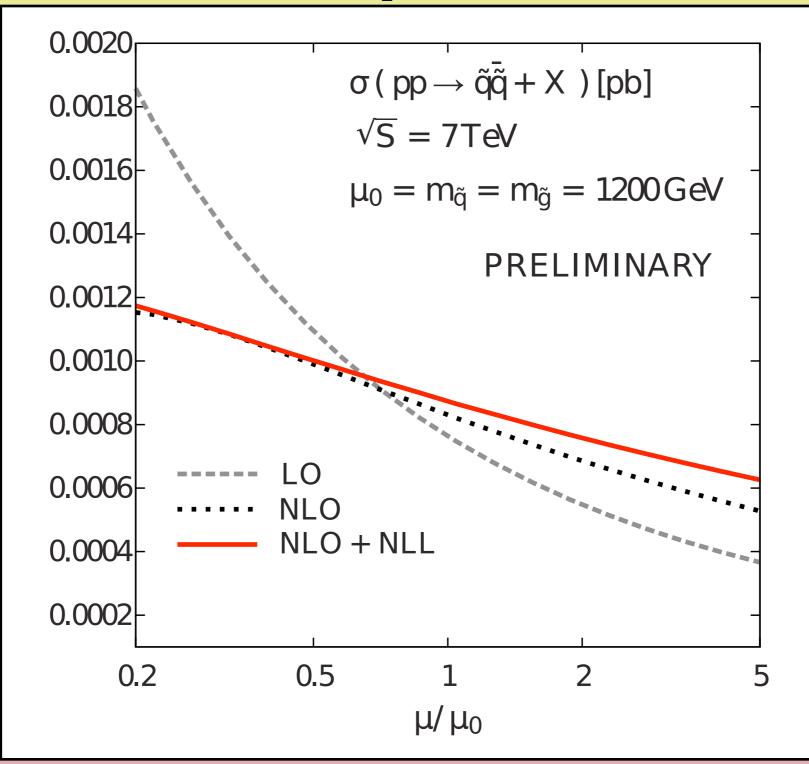
$$\times \left[ \tilde{\sigma}_{ij \to \tilde{q}\bar{\tilde{q}}}^{(\text{res,NNLL})} \left( N, \{m^2\}, \mu^2 \right) - \tilde{\sigma}_{ij \to \tilde{q}\bar{\tilde{q}}}^{(\text{res,NNLL})} \left( N, \{m^2\}, \mu^2 \right) |_{(\text{NLO})} \right]$$



$$\sigma_{h_{1}h_{2}\to\tilde{q}\bar{\tilde{q}}}^{(\text{NNLL+NLO matched})}\left(\rho,\{m^{2}\},\mu^{2}\right) = \left[\sigma_{h_{1}h_{2}\to\tilde{q}\bar{\tilde{q}}}^{(\text{NLO})}\left(\rho,\{m^{2}\},\mu^{2}\right)\right] \\ + \left[\sum_{i,j}\int_{\text{CT}}\frac{dN}{2\pi i}\rho^{-N}\tilde{f}_{i/h_{1}}(N+1,\mu^{2})\tilde{f}_{j/h_{2}}(N+1,\mu^{2})\right] \\ \times \left[\tilde{\sigma}_{ij\to\tilde{q}\bar{\tilde{q}}}^{(\text{res,NNLL})}\left(N,\{m^{2}\},\mu^{2}\right) - \left[\tilde{\sigma}_{ij\to\tilde{q}\bar{\tilde{q}}}^{(\text{res,NNLL})}\left(N,\{m^{2}\},\mu^{2}\right)|_{(\text{NLO})}\right] \right]$$

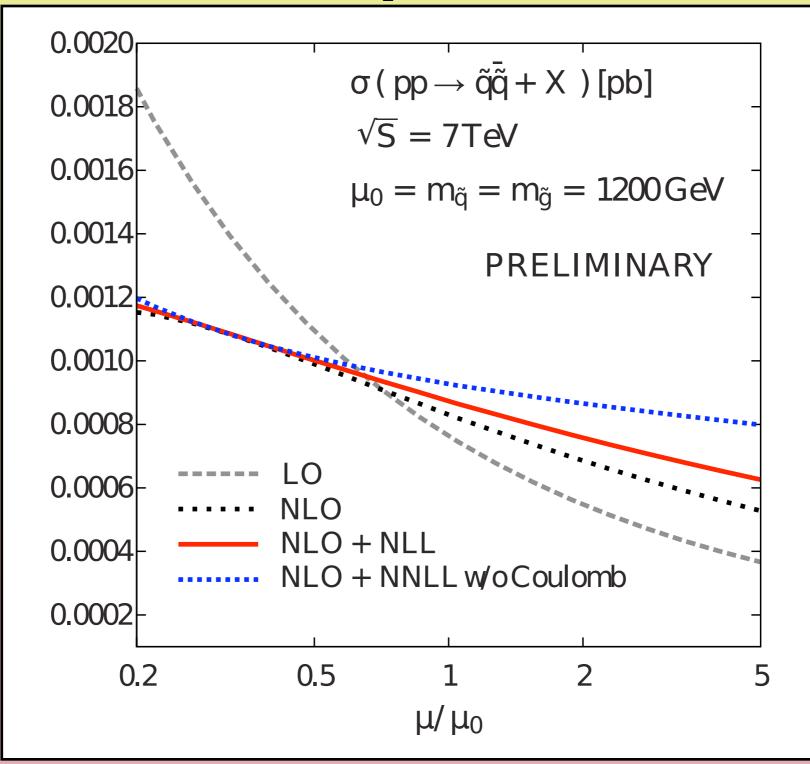


### Scale dependence



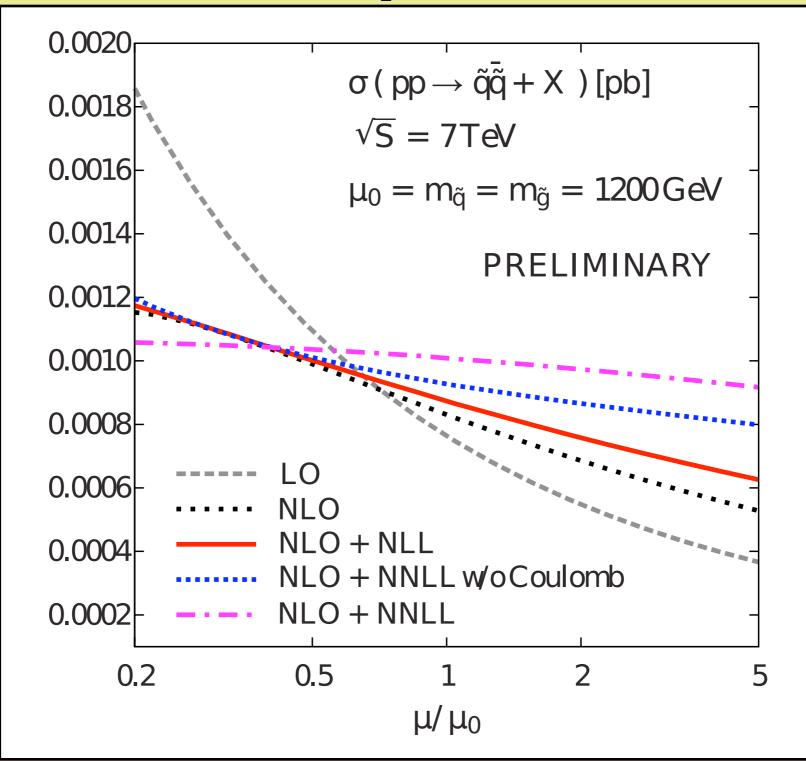


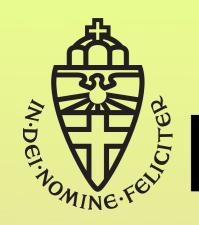
### Scale dependence



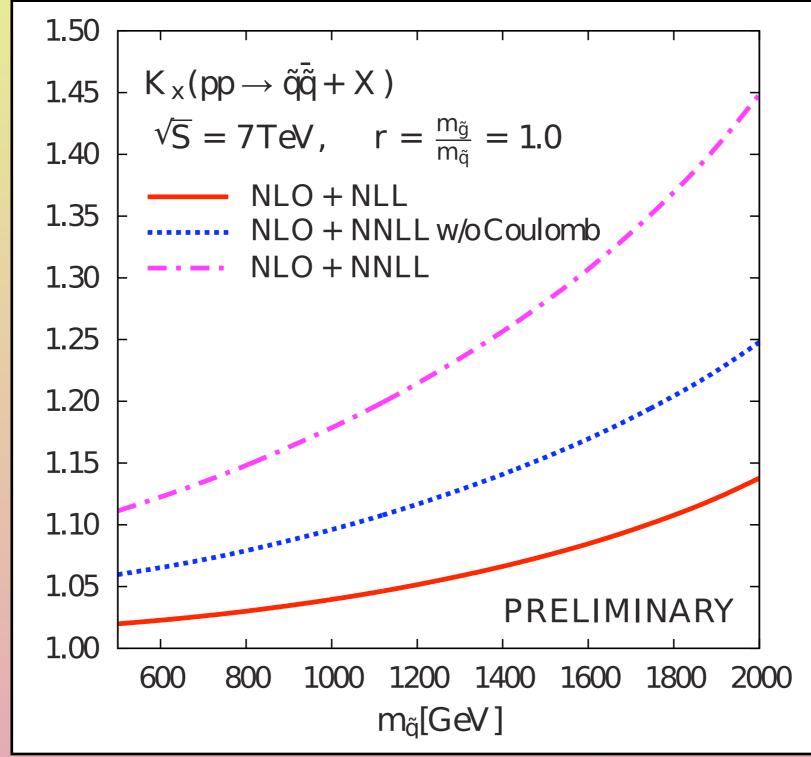


### Scale dependence





## K factor@central scale





#### Conclusion

NNLL resummation for squark-antisquark production performed

- Scale dependence reduced
- Cross section increased at central scale



#### Conclusion

NNLL resummation for squark-antisquark production performed

- Scale dependence reduced
- Cross section increased at central scale

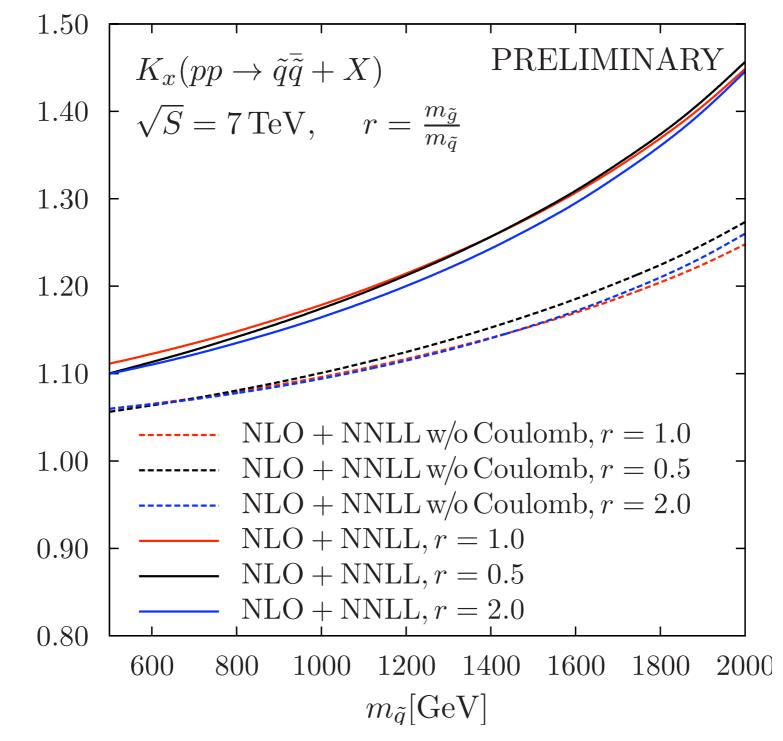
#### Still to do:

- Include other processes
- Apply this to exclusion bounds



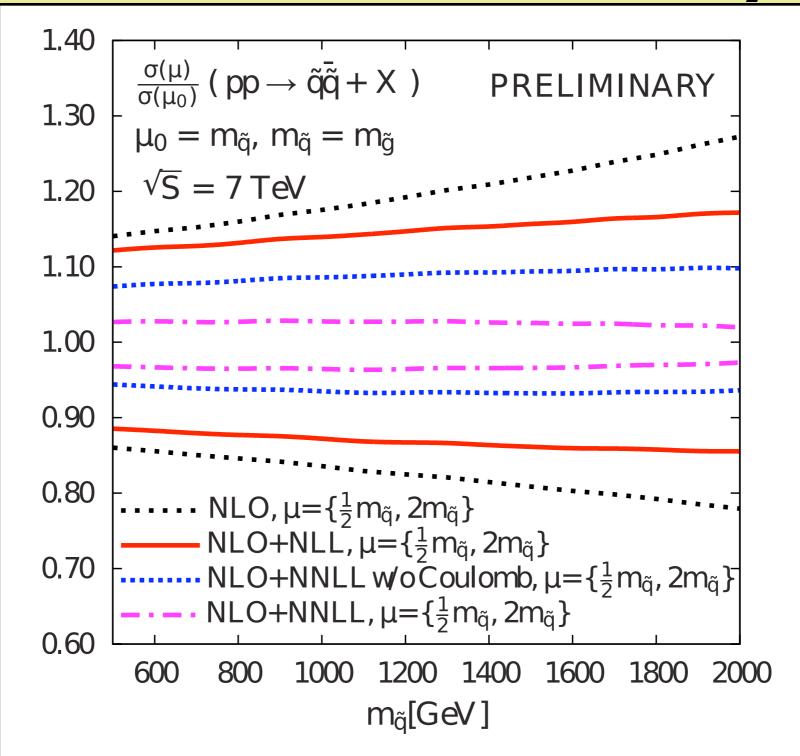
## Backup





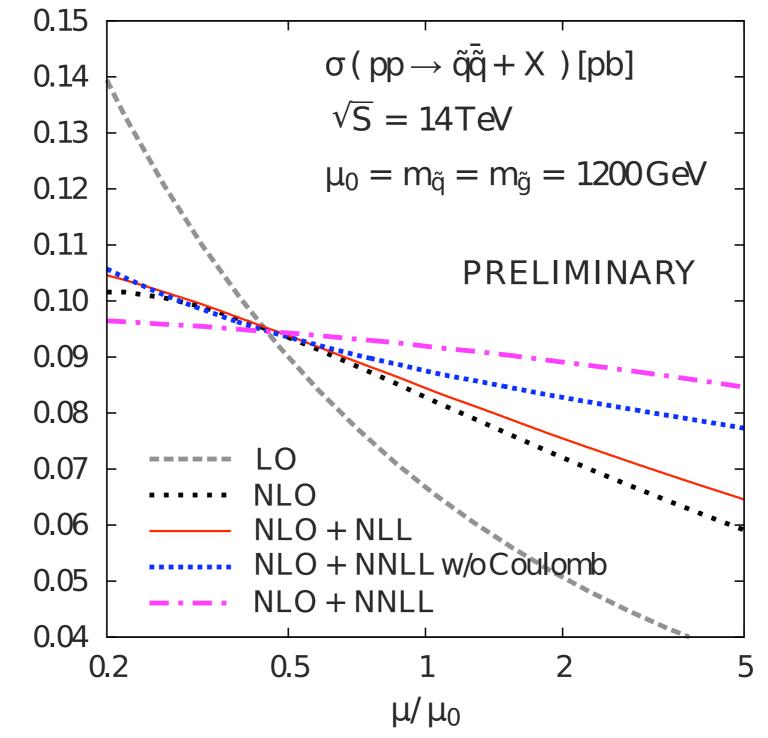


#### Scale uncertainty



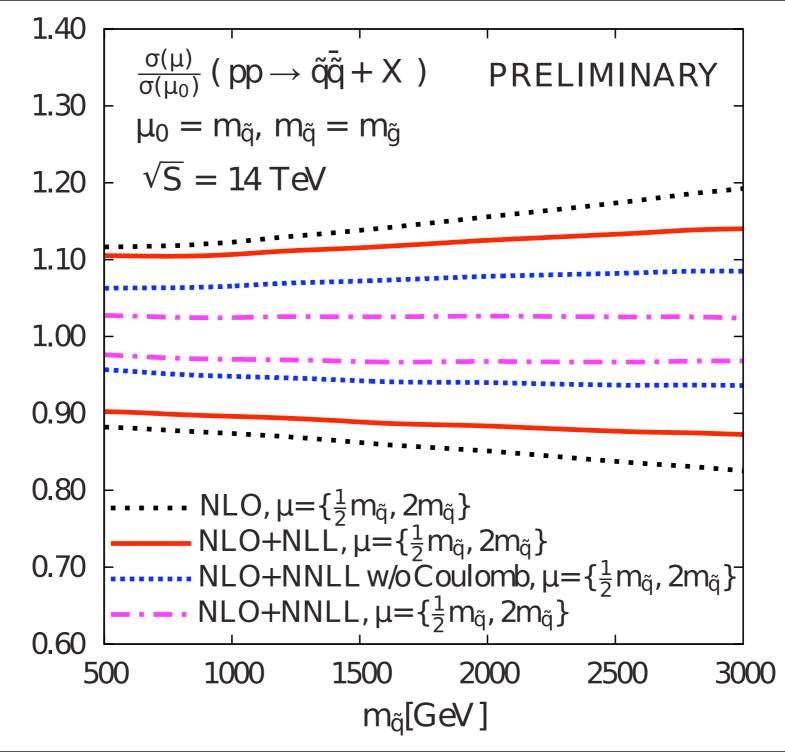
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# Scale uncertainty 14 TeV



RADCOR 2011 - NNLL resummation for squark-antisquark production - Irene Niessen - backup



## K factor 14 TeV

