Hard multi-particle processes at NLO QCD

Andreas van Hameren
IFJ-PAN, Kraków, Poland

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Motivation

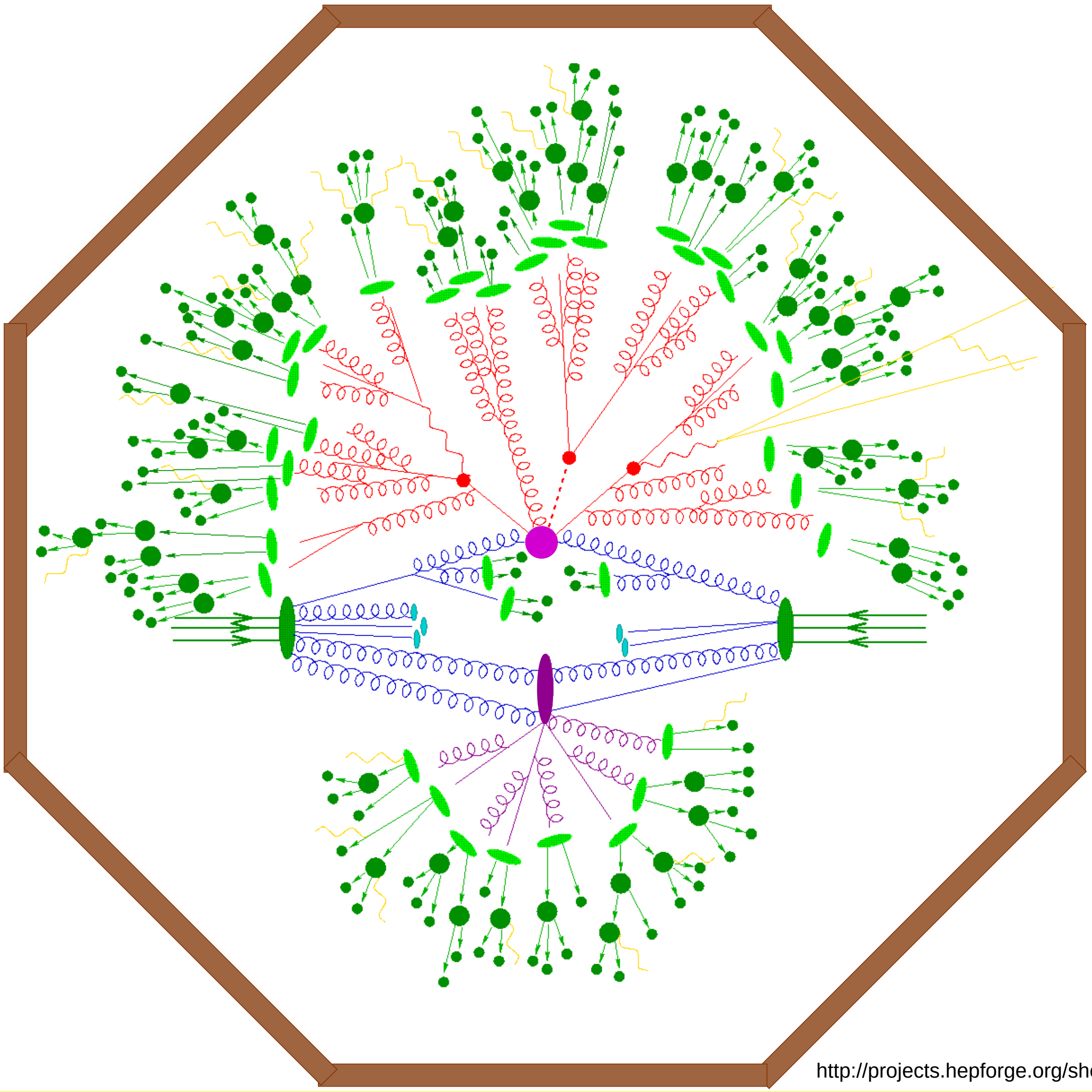
- Precise theoretical control over backgrounds and signals is essential for physics at LHC.
- This implies computational control over multi-particle/multi-jet processes ...
- ... to at least NLO QCD.
  - reducing scale dependence;
  - normalization of distributions;
  - shape of distributions.

Recent activity in NLO calculations

Recent calculations of $2 \to 4$ processes since (2008):

- $pp \to t\bar{t}\,b\bar{b}$ Bredenstein,Denner,Dittmaier,Pozzorini
  Bevilacqua,Czakon,Papadopoulos,Pittau,Worek
- $pp \to VV + 2j$ VBF: Bozzi,Jäger,Oleari,Zeppenfeld
- $pp \to W + 3j$ Ellis,Melnikov,Zanderighi
- $pp \to V + 3j,\ pp \to W + 4j$
  Berger,Bern,Dixon,Febres Cordero,Forde,Gleisberg,Ita,Kosower,Maître
- $pp \to b\bar{b}b\bar{b}$ Binoth,Greiner,Guffanti,Reuter,Guillet,Reiter
- $pp \to W^+W^+ + 2j,\ pp \to W^+W^- + 2j$ Melia,Melnikov,Rontsch,Zanderighi
- $pp \to W^+W^-\,b\bar{b}$ Denner,Dittmaier,Kallweit,Pozzorini
- $pp \to W^+W^-\,b\bar{b} \to 4\ell\,b\bar{b}$ Bevilacqua,Czakon,AvH,Papadopoulos,Worek
- $pp \to Z + 4j$ Bern,Dixon,Febres Cordero,Ita,Kosower,Maître
- $pp \to t\bar{t} + 2j$ Bevilacqua,Czakon,Papadopoulos,Worek

Computational challenges in jet events



Detector response

Hadron decay

Photon emission

Hadronization

Final−state parton shower

Hard process

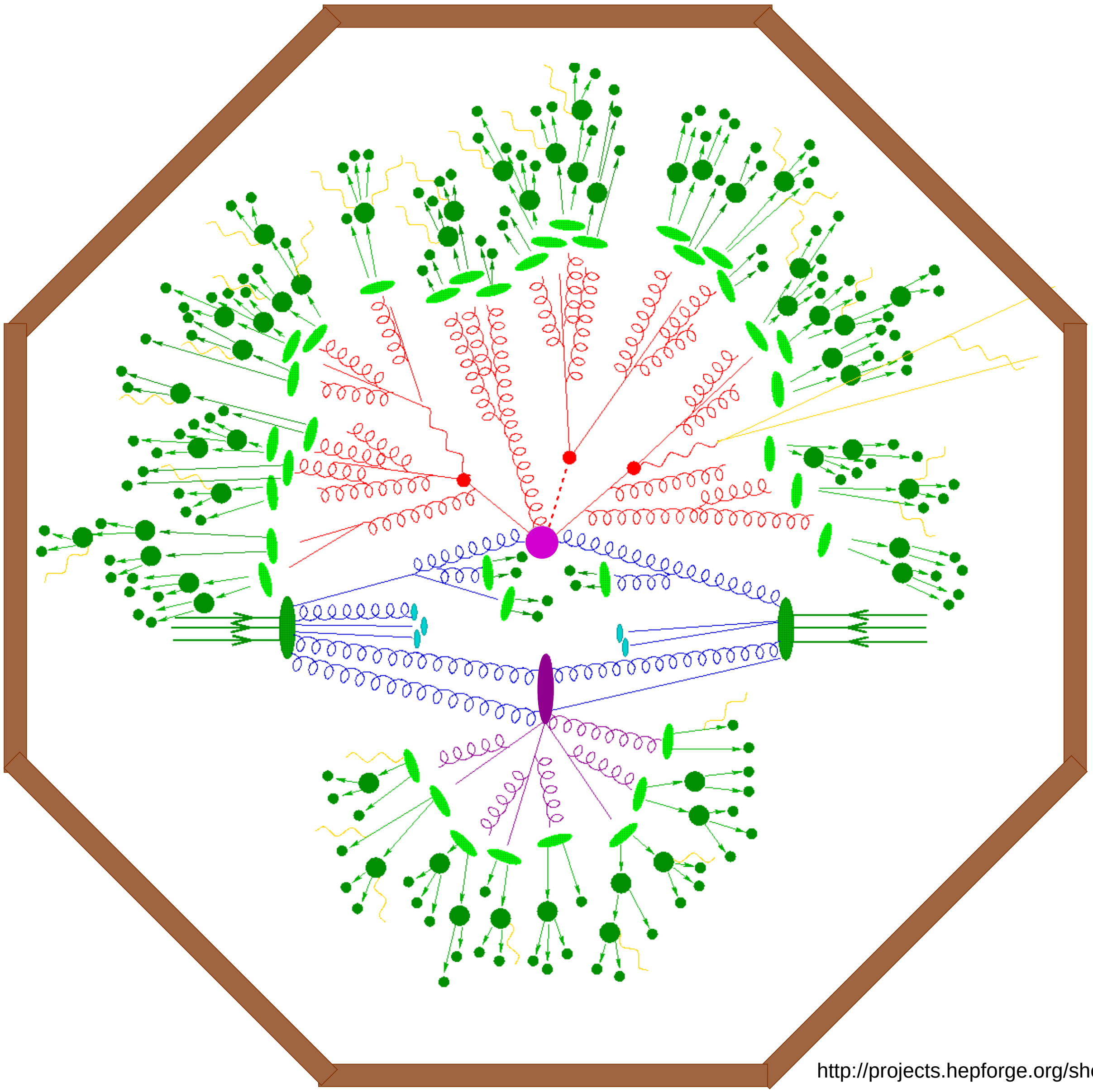
Initial−state parton shower

Beam remnants

Underlying event

http://projects.hepforge.org/sherpa/docuwiki/lib/exe/detail.php?media=sketch.gif

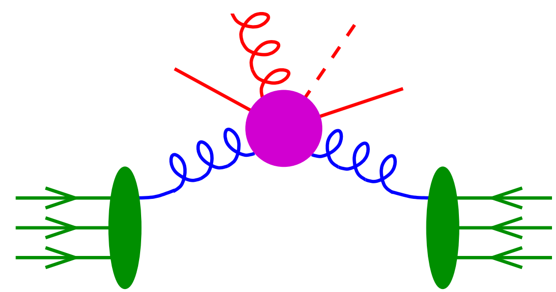
Computational challenges in jet events



Fragmentation:

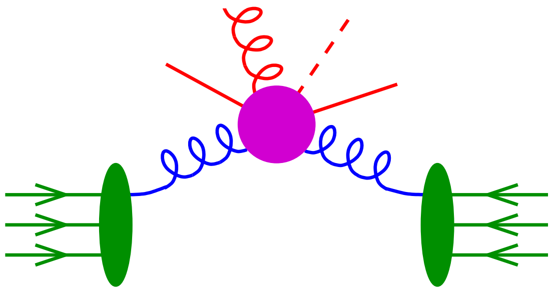
Probability for event to happen is already given by the

Hard process



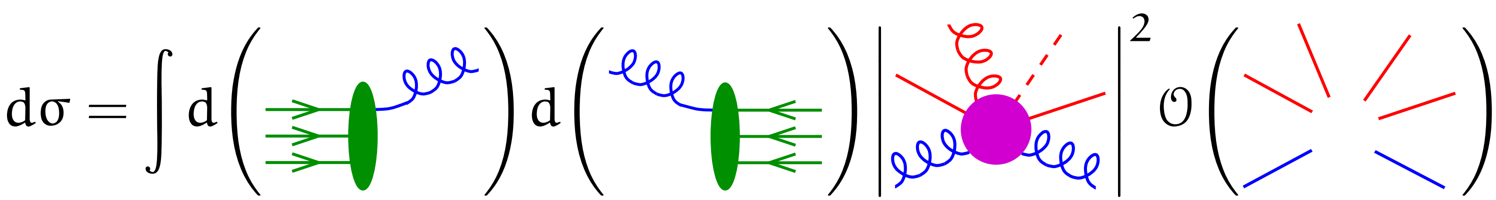
http://projects.hepforge.org/sherpa/docuwiki/lib/exe/detail.php?media=sketch.gif

Factorization in jet processes



The hard scattering cross section can be written as a convolution of process-independent Parton Density Functions and a partonic cross section.

$$d\sigma(\,h_1(p_1)h_2(p_2) \to X\,) = \sum_{k,l} \int dx_1 f_{1,k}(x_1, \mu_F)\; dx_2 f_{2,l}(x_2, \mu_F)$$
$$\times \sigma_{\text{hard}}(\,\phi_k(x_1 p_1)\phi_l(x_2 p_2) \to X\,;\,\mu_F\,)$$

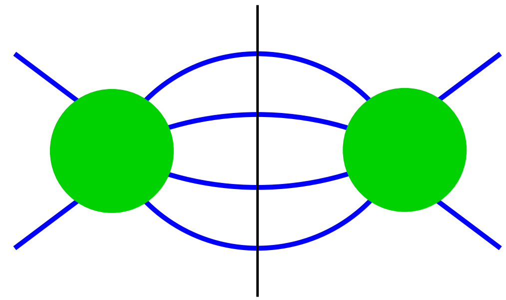


Formal independence of the factorization scale μ_F, but not perturbatively term by term.

Ingredients for NLO calculations

LO calculation

$$\langle O\rangle^{\mathrm{LO}} = \int d\Phi_n \, |\mathcal{M}_n^{(0)}|^2 \, O_n^{\mathrm{LO}} \quad =$$

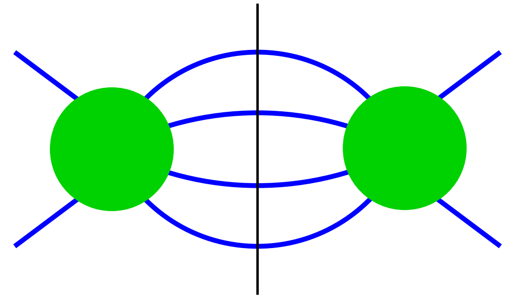


- Observable O_n^{LO} represents some interesting distribution, and includes phase space cuts;
- $\mathcal{M}_n^{(0)}$ is the Born (tree-level) matrix element.

Ingredients for NLO calculations

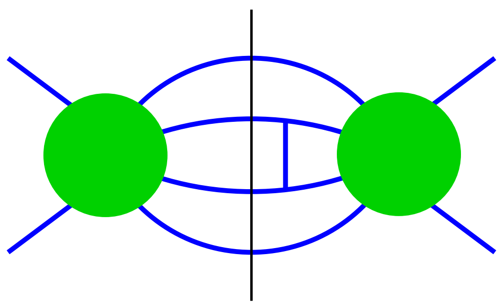
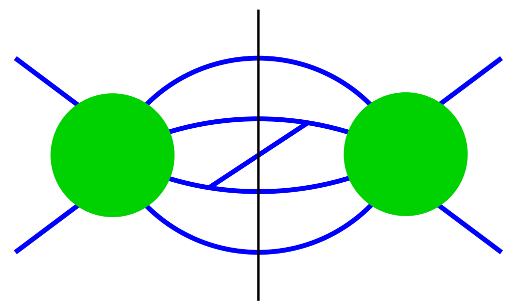
<u>LO calculation</u>

$$\langle O\rangle^{\mathrm{LO}} = \int d\Phi_n \, |\mathcal{M}_n^{(0)}|^2 \, O_n^{\mathrm{LO}} \quad =$$



<u>NLO calculation</u>: add

$$\langle O\rangle^{\mathrm{NLO}} = \int d\Phi_n \, 2\Re\big(\mathcal{M}_n^{(0)}\mathcal{M}_n^{(1)}\big) \, O_n^{\mathrm{LO}} + \int d\Phi_{n+1} \, |\mathcal{M}_{n+1}^{(0)}|^2 \, O_{n+1}^{\mathrm{NLO}}$$

$=$  $+$ 

- $\mathcal{M}_n^{(1)}$ is the one-loop amplitude
- $\mathcal{M}_{n+1}^{(0)}$ is the real-radiation (tree-level) matrix element;
- O_{n+1}^{NLO} includes a jet algorithm;

Ingredients for NLO calculations

LO calculation

$$\langle O\rangle^{\text{LO}} = \int d\Phi_n\, |\mathcal{M}_n^{(0)}|^2\, O_n^{\text{LO}}$$

NLO calculation

$$\langle O\rangle^{\text{NLO}} = \int d\Phi_n\, 2\Re\big(\mathcal{M}_n^{(0)}\mathcal{M}_n^{(1)}\big)\, O_n^{\text{LO}} + \int d\Phi_{n+1}\, |\mathcal{M}_{n+1}^{(0)}|^2\, O_{n+1}^{\text{NLO}}$$

$$= \int d\Phi_n \left[2\Re\big(\mathcal{M}_n^{(0)}\mathcal{M}_n^{(1)}\big) + \int d\Phi_1\, \mathcal{A}_{n+1}\right] O_n^{\text{LO}}$$

$$+ \int d\Phi_{n+1} \left[\, |\mathcal{M}_{n+1}^{(0)}|^2\, O_{n+1}^{\text{NLO}} - \mathcal{A}_{n+1} O_n^{\text{LO}}\, \right]$$

- get finite phase space integrals with the help of subtraction;
- demands factorization both of phase space and singularities, eg. dipole subtraction Catani, Seymour '97.

Numerical evaluation of amplitudes

Phase space integration

$$\langle O \rangle = \int d\Phi_n(\{p\}_n)\,|\mathcal{M}_n(\{p\}_n)|^2\,O_n(\{p\}_n)$$

has to be done by Monte Carlo.

Numerical evaluation of amplitudes

Phase space integration

has to be done by Monte Carlo.

Helicity and Color summation

does not involve complicated restrictions like “cuts”

$$\langle O\rangle = \int d\Phi_n(\{p\}_n) \sum_{\{\lambda\}_n} \sum_{\{a\}_n} |\mathcal{M}_n(\{p\}_n, \{\lambda\}_n, \{a\}_n)|^2 \, O_n(\{p\}_n)$$

and could be conceived to be performed algebraïcly. For many-particle final states, however, this leads to huge expressions for

$$\sum_{\{\lambda\}_n} \sum_{\{a\}_n} |\mathcal{M}_n(\{p\}_n, \{\lambda\}_n, \{a\}_n)|^2$$

Numerical evaluation of amplitudes

Phase space integration
has to be done by Monte Carlo.

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and could be conceived to be performed algebraïcly. For many-particle final states, however, this leads to huge expressions.

The alternative is to treat helicity and color summation on the same footing as phase space integration, and just evaluate

$$\mathcal{M}_n(\{p\}_n, \{\lambda\}_n, \{a\}_n)$$

numerically as function of momentum- helicity- and color-configurations.

Numerical evaluation of amplitudes

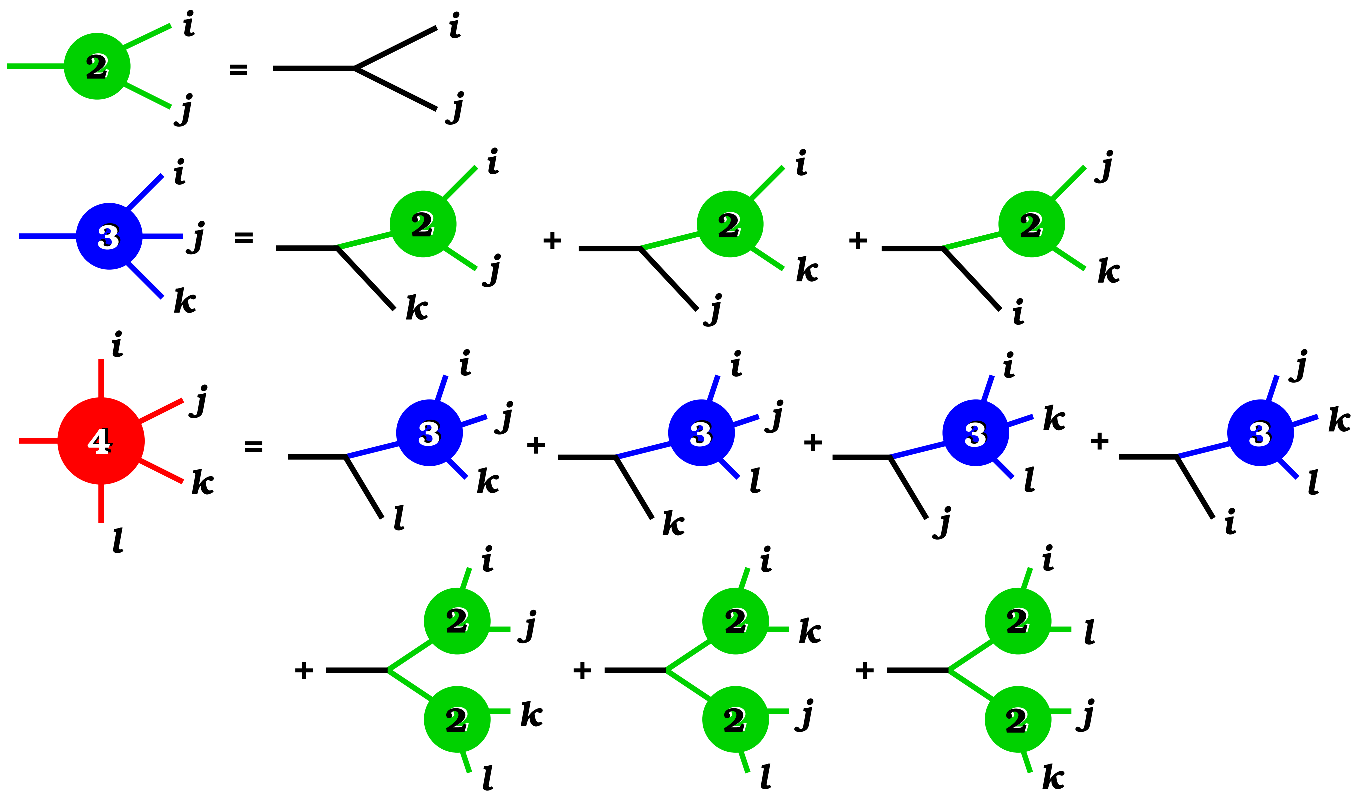
- Expressions in terms of invariants for scattering amplitudes involving several particles tend to become huge.
- Eventual goal is (just) their repeated numerical evaluation in a MC.

Avoid expressions completely!

- Essentially the only expressions involved should be the vertices and the propagators of the field theory.
- Use an algorithm to evaluate scattering amplitudes, given the numerical values of momenta, helicity and color degrees of freedom of the external particles as initial input.

Calculation of tree-level amplitudes

Recursive Dyson-Schwinger approach Berends,Giele'88; Caravaglios,Moretti'95



- Algorithm comes directly from field theory;
- Straightforward to automate for arbitrary processes;
- Efficient: $\mathrm{O}(n!)$ for graphs to $\mathrm{O}(3^n)$, $n =$ number of external legs;

One-loop amplitude with O_{ssola}P_{apadopoulos}P_{ittau}

Identify a set of n_{tot} denominators and write

$$\mathcal{M}^{(1)} = \sum_{\mathcal{J}\subset\{1,2,3,\dots,n_{\text{tot}}\}} \int d^\omega q \, \frac{N_{\mathcal{J}}(q)}{\prod_{i\in\mathcal{J}} D_i} \quad , \quad D_i = (q+p_i)^2 - m_i^2$$

Integrals can be expressed in terms of universal scalar-functions:

$$\int \frac{d^\omega q \; N(q)}{D_1 D_2 \cdots D_n} = \sum_{i_1>i_2>i_3>i_4} d_{i_1 i_2 i_3 i_4} \int \frac{d^\omega q}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1>i_2>i_3} c_{i_1 i_2 i_3} \int \frac{d^\omega q}{D_{i_1} D_{i_2} D_{i_3}}$$
$$+ \sum_{i_1>i_2} b_{i_1 i_2} \int \frac{d^\omega q}{D_{i_1} D_{i_2}} + \sum_{i_1} a_{i_1} \int \frac{d^\omega q}{D_{i_1}} + \text{rational terms} + \mathcal{O}(\omega - 4)$$

Rational terms are a consequence of the demand that the coefficients do not depend on ω.

One-loop amplitude with OPP

Identify a set of n_{tot} denominators and write

$$\mathcal{M}^{(1)} = \sum_{\mathcal{J}\subset\{1,2,3,\ldots,n_{\text{tot}}\}} \int d^\omega q \, \frac{N_{\mathcal{J}}(q)}{\prod_{i\in\mathcal{J}} D_i} \quad , \quad D_i = (q+p_i)^2 - m_i^2$$

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$$+ \sum_{i_1>i_2} b_{i_1 i_2} \int \frac{d^\omega q}{D_{i_1} D_{i_2}} + \sum_{i_1} a_{i_1} \int \frac{d^\omega q}{D_{i_1}} + \text{rational terms} + \mathcal{O}(\omega-4)$$

Coefficients can be determined from polynomial equations involving few more coefficients

$$\frac{N(q)}{D_1 D_2 \cdots D_n} = \sum_{i_1>i_2>i_3>i_4} \frac{d_{i_1 i_2 i_3 i_4} + \tilde{d}_{i_1 i_2 i_3 i_4}(q)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1>i_2>i_3} \frac{c_{i_1 i_2 i_3} + \tilde{c}_{i_1 i_2 i_3}(q)}{D_{i_1} D_{i_2} D_{i_3}}$$
$$+ \sum_{i_1>i_2} \frac{b_{i_1 i_2} + \tilde{b}_{i_1 i_2}(q)}{D_{i_1} D_{i_2}} + \sum_{i_1} \frac{a_{i_1} + \tilde{a}_{i_1}(q)}{D_{i_1}}$$

1 extra coefficient for $\tilde{d}$, 6 for $\tilde{c}$, 8 for $\tilde{b}$, 4 for $\tilde{a}$

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Coefficients can be determined from polynomial equations involving few more coefficients.
Rational terms can be split into $R_1 + R_2$, where

- R_1 is also delivered by the OPP-reduction,
- R_2 is a finite counterterm, tree-level with extra vertices
  Draggiotis,Garzelli,Malamos,Papadopoulos,Pittau

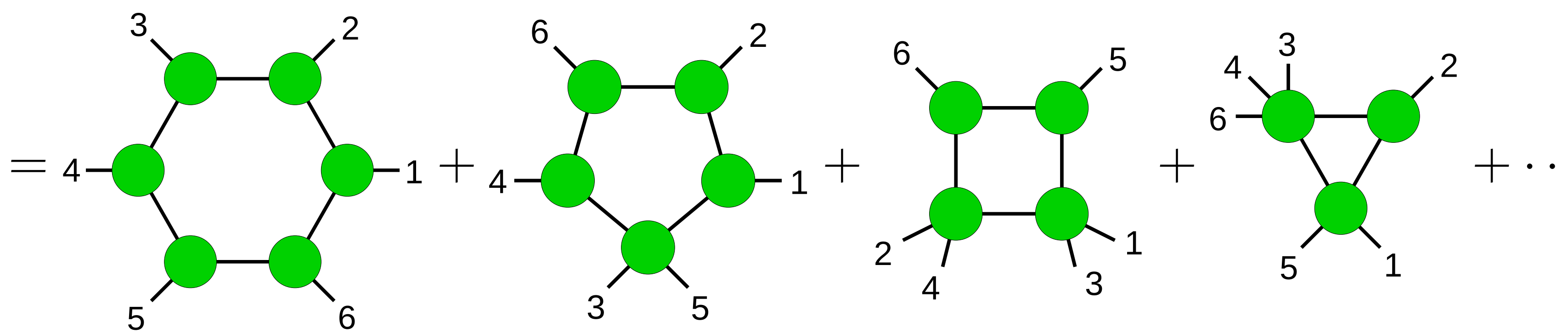
The OPP reduction is universal, needs only the numerator function and the denominator momenta/masses as input

CutTools Ossola,Papadopoulos,Pittau   Samurai Mastrolia,Ossola,Reiter,Tramontano

Evaluation of the numerator for OPP

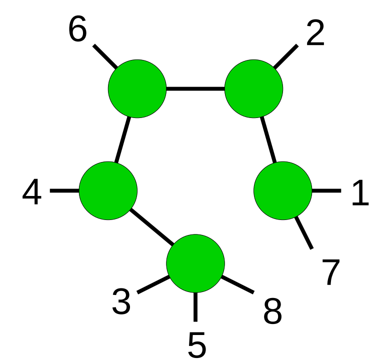
Helac-1Loop-approach: go explicitly through all possible 1PI structures:

$$\mathcal{M}^{(1)} = \sum_{\mathcal{J}\subset\{1,2,3,\dots,n_{\text{tot}}\}} \int d^{\omega}q \, \frac{N_{\mathcal{J}}(q)}{\prod_{i\in\mathcal{J}} D_i}$$



All blobs are tree-level, without denominators containing the integration momentum.

The numerator can be obtained for each term from a tree-level amplitude with 2 more particles, and restricted such that it contains the necessary propagators.



7 has momentum q
8 has momentum −q

Phase space integration

The real-subtracted integral within dipole-subtraction:

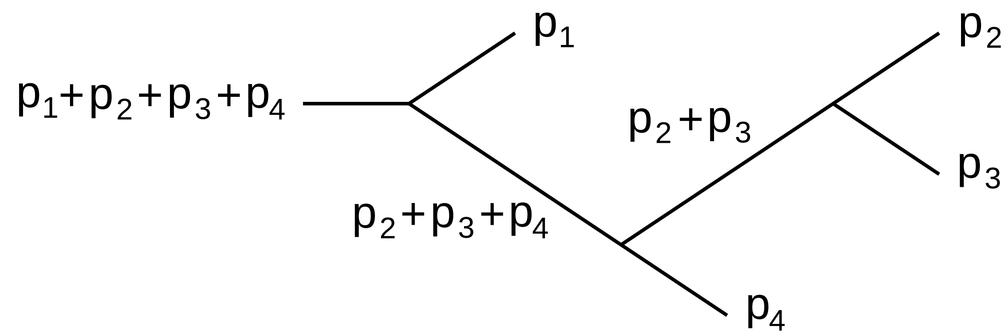
$$\int d\Phi_{n+1}\left[\,|\mathcal{M}^{(0)}_{n+1}|^2\,O^{\mathrm{NLO}}_{n+1} - \sum_{j}\sum_{i\neq j\neq k}\left(\mathcal{D}^{(ijk)}_{n}\,O^{\mathrm{LO}}_{n}\right)\circ T^{(ijk)}_{n+1\to n}\,\right]$$

- Phase space generator designed just to deal with $|\mathcal{M}^{(0)}_{n+1}|$, like Phegas, appears not to be adequate.
- Kaleu is a completely independent generator, also designed to deal just with $|\mathcal{M}^{(0)}_{n+1}|$, but is more Object Oriented.

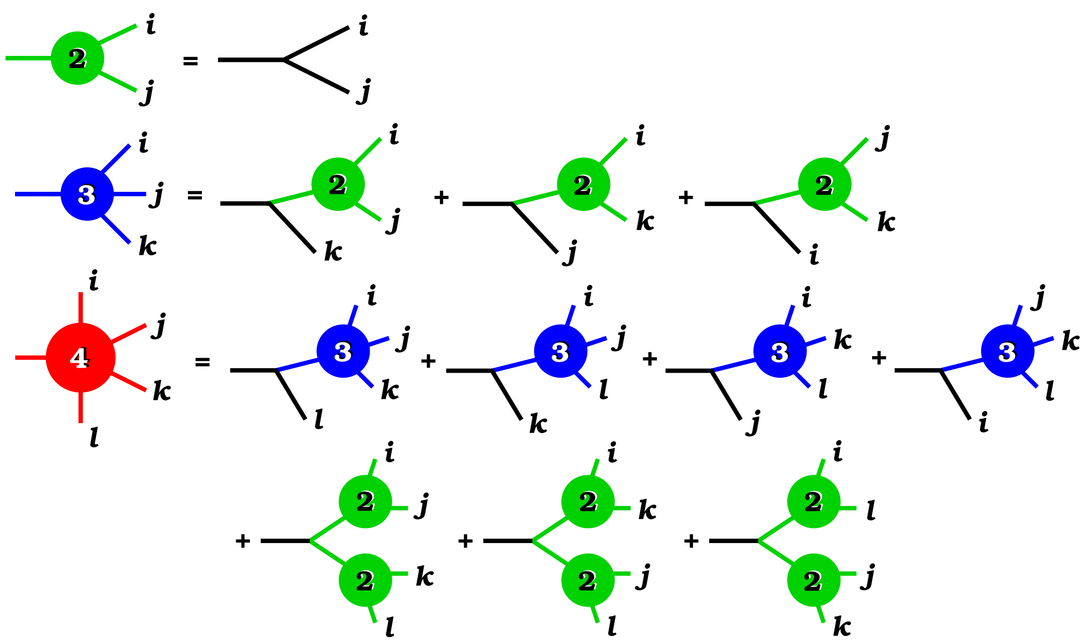
Phase space integration

Generate momenta via consecutive branchings $\Longleftrightarrow$ Feynman graph

A Feynman graph encodes the generator for optimal integration of its own square.



Full amplitude can efficiently be integrated with a "sum of graphs", via the multi-channel method. However, the curse of $n!$ returns.



Solution: Gleisberg,Höche'08
Choose branchings by performing the recursion backward.
Generator weight is given by performing recursion forward.

Easy to automate.

Phase space integration

The real-subtracted integral within dipole-subtraction:

$$\int d\Phi_{n+1}\left[\,|\mathcal{M}^{(0)}_{n+1}|^2\,O^{\text{NLO}}_{n+1} - \sum_{j}\sum_{i\neq j\neq k}\left(\mathcal{D}^{(ijk)}_{n}\,O^{\text{LO}}_{n}\right)\circ T^{(ijk)}_{n+1\to n}\,\right]$$

- Phase space generator designed just to deal with $|\mathcal{M}^{(0)}_{n+1}|$, like Phegas, appears not to be adequate.
- Kaleu is a completely independent generator, also designed to deal just with $|\mathcal{M}^{(0)}_{n+1}|$, but is more Object Oriented.
- Each dipole gets its own phase space generator, all of them working together in a multi-channel approach:

$$d\Phi_{n+1} = \alpha_0\,d\text{Kaleu}_{n+1} + \sum_{\{ijk\}}\alpha_{\{ijk\}}T^{(ijk)}_{n\to n+1}\circ dy\,dz\,d\phi\;d\text{Kaleu}^{(ijk)}_{n}$$

- Variables y, z are generated with self-adaptive grids.

$u\bar{u} \to b\bar{b}\, e^+\nu_e\, \mu^-\bar{\nu}_\mu\, g$

1. Kaleu with dipole-channels;
2. Kaleu without dipole channels, with adaptive grid for each invariant;
3. Kaleu without either.

α_{max}	dipoles	OPTION	$\sigma^{(+)}$	$\sigma^{(-)}$	$\sigma^{(+)} - \sigma^{(-)}$	$N^{(+)}_{eval}$	$N^{(-)}_{eval}$	N_{gnrt}	t_{cpu}
	yes	v1	316.78(.34)	159.00(.29)	157.78(.45)	7.449	2.447	26	6.6
0.01	no	v3	316.57(.58)	160.6(1.0)	156.0(1.2)	8.946	1.276	33	2.0
	no	v1	316.81(.54)	156.9(1.0)	159.9(1.2)	8.013	1.974	62	2.8
	yes	v1	286.29(.37)	305.10(.44)	-18.81(.59)	5.005	4.828	21	7.1
1.00	no	v3	286.22(.75)	304.7(1.7)	-18.5(1.9)	7.034	2.952	25	2.7
	no	v1	286.7(.9)	309.9(3.1)	-23.1(3.2)	6.509	3.347	40	2.7

Table 1: Cross sections in [fb] for the real-subtracted contribution from $u\bar{u} \to b\bar{b}\, e^+\nu_e\, \mu^-\bar{\nu}_\mu\, g$ to the process $pp \to b\bar{b}\, e^+\nu_e\, \mu^-\bar{\nu}_\mu$ at NLO. All results were obtained with KALEU. The numbers N_{gnrt} of generated and N_{eval} of accepted phase space points are multiples of 10^6. The computing times t_{cpu} are in hours on a 2.80GHz Intel Xeon processor. The superscripts $(+)$ and $(-)$ respectively refer to positive and negative weight contributions. The values of N_{gnrt} were chosen to reach comparable $N^{(+)}_{eval} + N^{(-)}_{eval}$.

Helac-NLO

- based on automatic LO platform Helac/Phegas
  Cafarella,Kanaki,Papadopoulos,Worek
- real-radiation with dipole-subtraction with Helac-Dipoles
  Czakon,Papadopoulos,Worek
- virtual contribution with Helac-1Loop AvH,Papadopoulos,Pittau
  - OPP reduction with CutTools Ossola,Papadopoulos,Pittau
  - scalar integrals with OneLOop AvH
  - rational contribution Draggiotis,Garzelli,Malamos,Papadopoulos,Pittau
- phase space integration of real-subtracted with Kaleu AvH
- virtual contribution for “any” process (presented up to hexagons so far).
- so far, produced full differential distributions for
  - $pp \to t\bar{t}\,b\bar{b}$ Bevilacqua,Czakon,Papadopoulos,Pittau,Worek
  - $pp \to t\bar{t} + 2j$ Bevilacqua,Czakon,Papadopoulos,Worek
  - $pp \to W^+W^-b\bar{b} \to 4\ell\,b\bar{b}$ Bevilacqua,Czakon,AvH,Papadopoulos,Worek

$pp(p\bar{p}) \to W^+W^-b\bar{b} \to 4\ell\, b\bar{b}$ at NLO QCD

$t\bar{t}$ production is relevant for the physics program of both Tevatron and LHC, and has been widely investigated inclusively (NLO, NNLO, NLL, NNLL) and exclusively at NLO:

$t\bar{t}H$: Beenakker,Dittmaier,Kramer,Plumper,Spira,Zerwas '01,'03,
Reina,Dawson '01, Reina,Dawson,Wackeroth '02,'03,
Dawson,Jackson,Orr,Reina,Wackeroth '03,

$t\bar{t}j$: Dittmaier,Uwer,Weinzierl '07,'09, Melnikov,Schulze '10,

$t\bar{t}Z$: Lazopoulos,McElmurry,Melnikov,Petriello '08,

$t\bar{t}\gamma$: Duan P.F.,Ma W.G.,Zhang R.Y.,Han L.,Guo L.,Wang S.M. '09,

$t\bar{t}b\bar{b}$: Bredenstein,Denner,Dittmaier,Pozzorini '08,'09,'10,
Bevilacqua,Czakon,Papadopoulos,Pittau,Worek '09,

The top quarks are usually taken on-shell, and decays treated in the narrow-width approximation (NWA). Now also off-shell calculations:

$t\bar{t} \to W^+W^-\,b\bar{b}$: Denner,Dittmaier,Kallweit,Pozzorini '11,

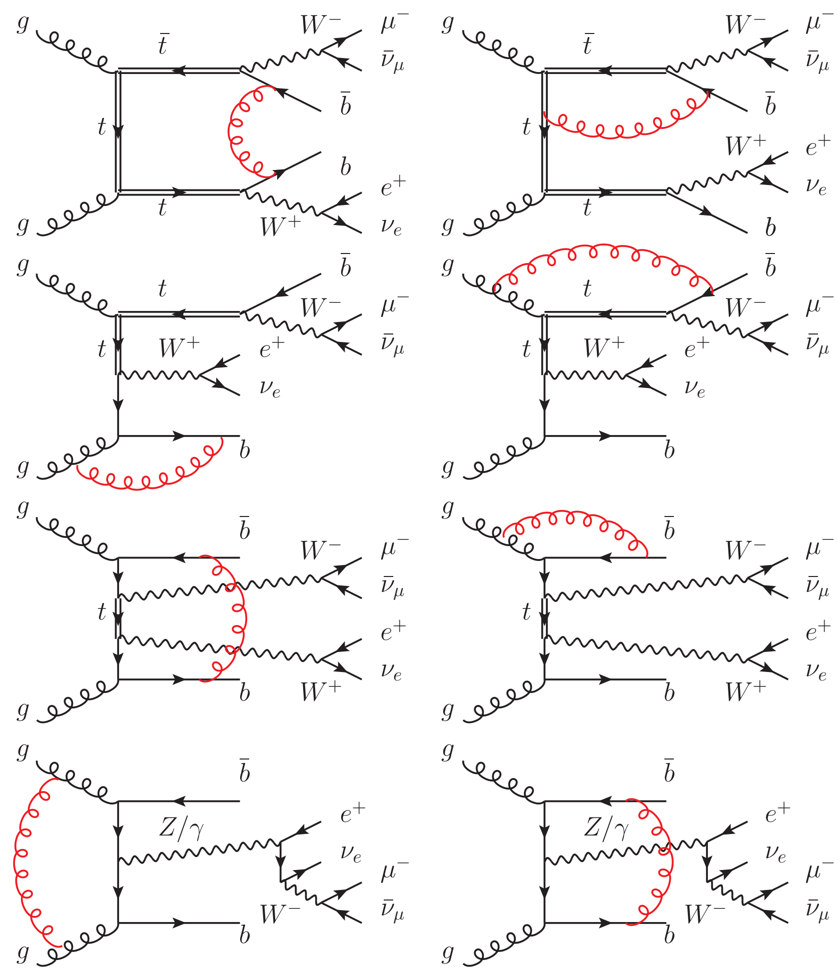
$t\bar{t} \to W^+W^-\,b\bar{b} \to 4\ell\,b\bar{b}$: Bevilacqua,Czakon,AvH,Papadopoulos,Worek '11

$pp(p\bar{p}) \to W^+W^-b\bar{b} \to 4\ell\, b\bar{b}$ at NLO QCD

<u>"New" computational challenges:</u>

- consistent treatment of top-quarks with non-zero width requires the application of the complex-mass scheme
  Denner,Dittmaier,Roth,Wackeroth '99,
  Denner,Dittmaier,Roth,Wieders '05.

  This demands one-loop scalar functions with complex internal masses
  't Hooft,Veltman '79,
  Dao Thi Nhung,Le Duc Ninh '09,
  Denner,Dittmaier '11



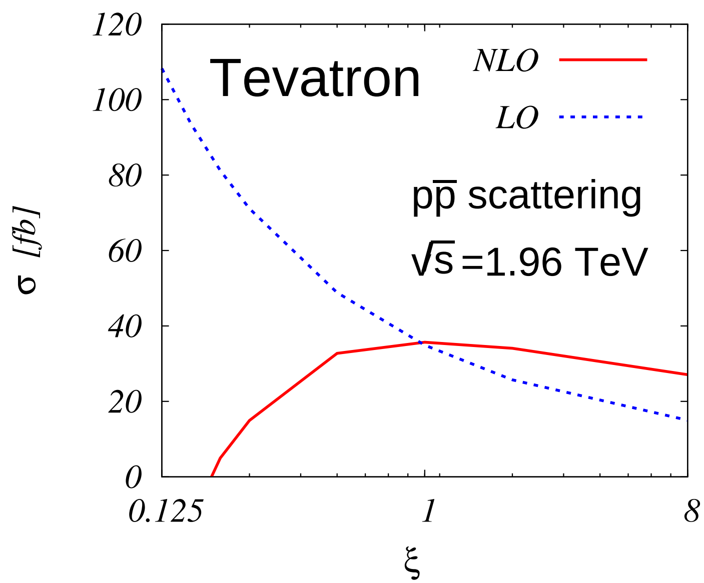
- real-subtracted phase space integral concerns 7-particle final state with non-trivial peak-structure.

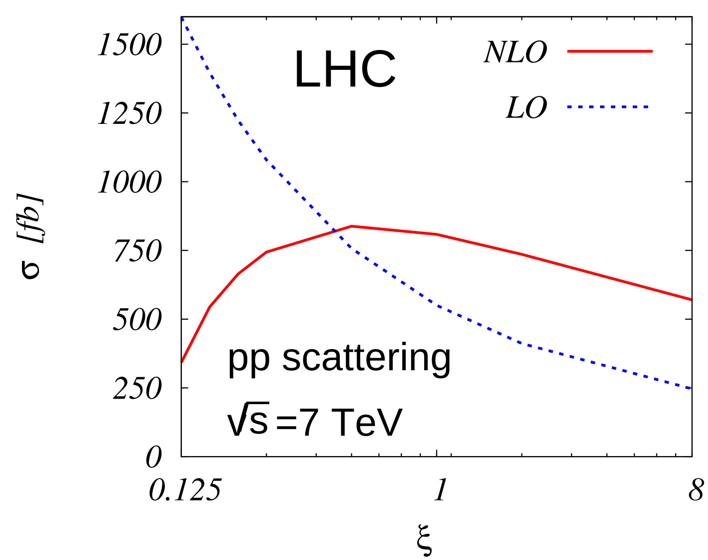
Cross sections

Inclusive cuts: $p_T(b) > 20\,\text{GeV} \quad p_T(\ell^\pm) > 20\,\text{GeV} \quad \not{p}_T > 30\,\text{GeV}$
$|y(b)| < 4.5 \quad |y(\ell^\pm)| < 2.5 \quad \Delta R(jj) > 0.4 \quad \Delta R(j\ell^\pm) > 0.4$

Three jet-algorithms: k_T, anti-k_T, Cambridge-Aachen

Total cross section as function of $\mu_R = \mu_F = \xi m_t$:



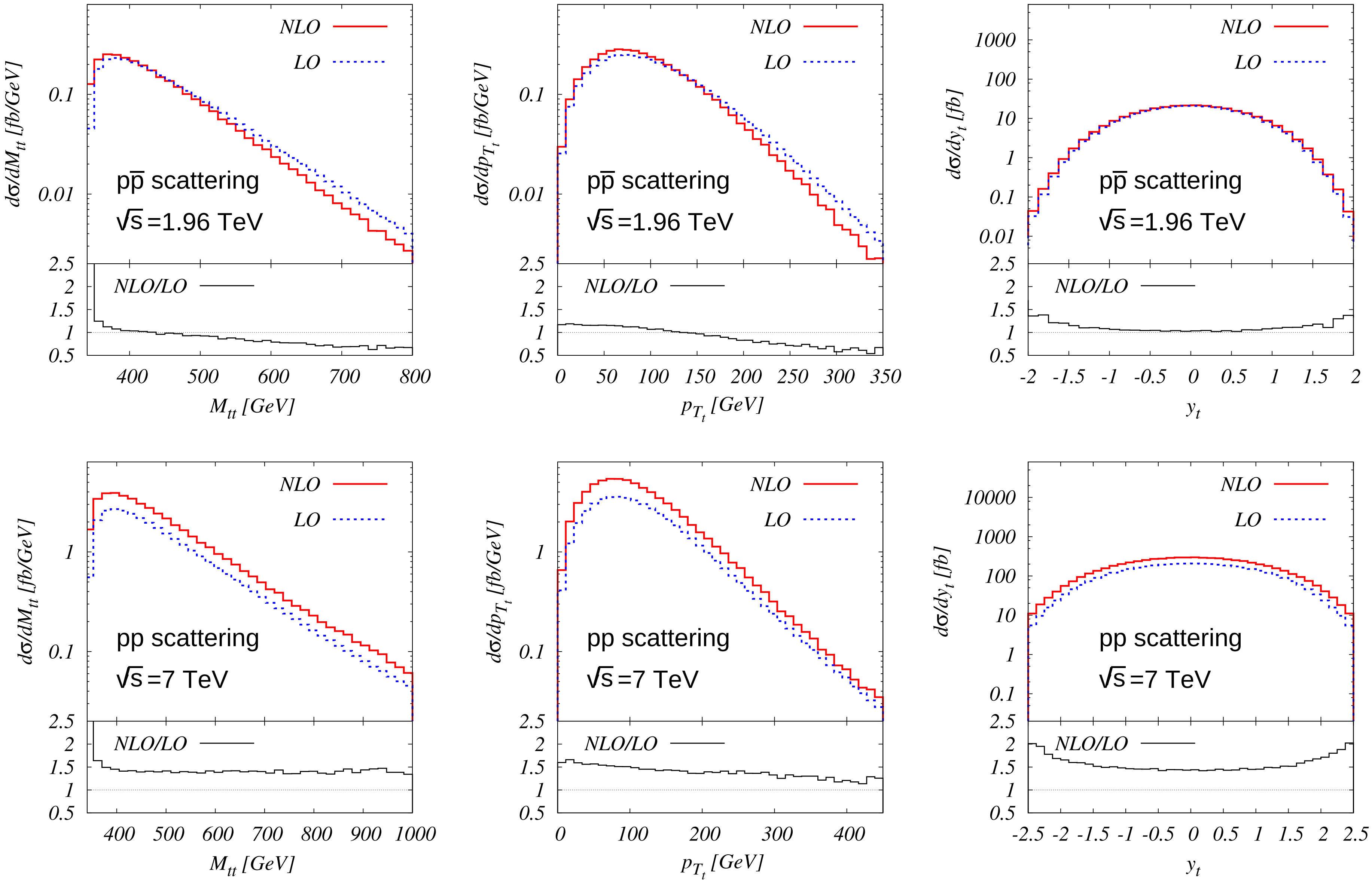


Restricting $\frac{1}{2} \leq \xi \leq 2$:

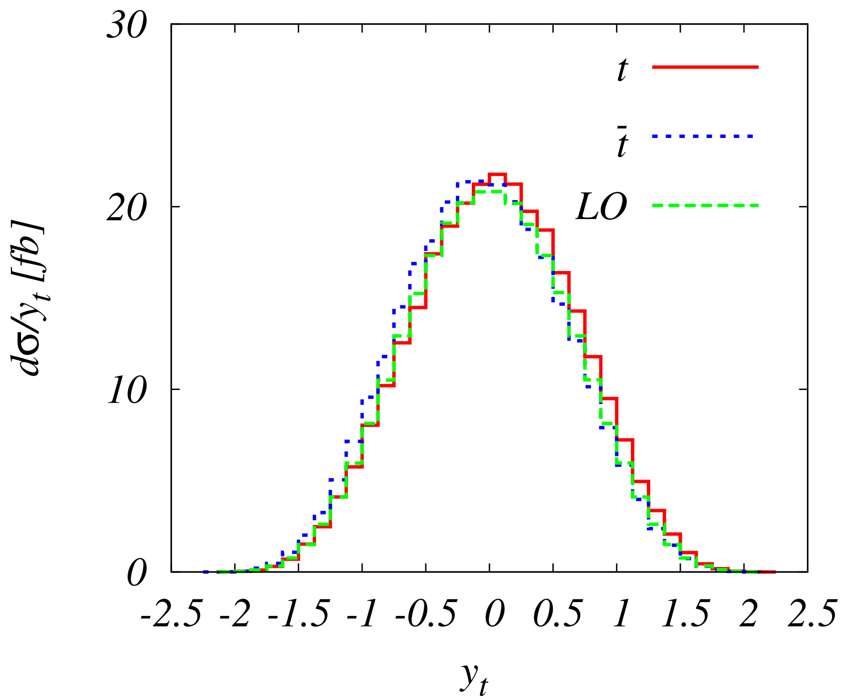
Tevatron: LO: $\sigma = 34.92\,^{+40\%}_{-26\%}\,\text{fb}$ NLO: $\sigma = 35.73\,^{+0\%}_{-8\%}\,\text{fb}$

LHC: LO: $\sigma = 550.54\,^{+37\%}_{-25\%}\,\text{fb}$ NLO: $\sigma = 808.67\,^{+4\%}_{-9\%}\,\text{fb}$

Differential cross sections



$t\bar{t}$ forward-backward asymmetry



Appears in $q\bar{q} \to t\bar{t}+X$ at NLO QCD

$$A^{t\bar{t}} = \frac{\int_{y>0} N_t(y) - \int_{y<0} N_t(y)}{\int_{y>0} N_t(y) + \int_{y<0} N_t(y)}$$

$$N_t(y) = \frac{d\sigma_{t\bar{t}}}{dy_t}(y) = \frac{d\sigma_{t\bar{t}}}{dy_{\bar{t}}}(-y)$$

we find: $A^{t\bar{t}} = 0.051 \pm 0.0013$

$t\bar{t}$-production$\times$decay: $A^{t\bar{t}} = 0.051 \pm 0.006$ Antunano,Kühn,Rodrigo '08

Tevatron:

$A^{t\bar{t}} = 0.08 \pm 0.04^{\text{stat}} \pm 0.01^{\text{syst}}$ D0-Conf-Note-6062 (2010)

$A^{t\bar{t}} = 0.150 \pm 0.050^{\text{stat}} \pm 0.024^{\text{syst}}$ CDF-Conf-Note-10185 (2010)

$A^{t\bar{t}}(m_{t\bar{t}} > 450\text{GeV}) = 0.475 \pm 0.114$ CDF collaboration Phys.Rev.D83(2011)112003

$pp(p\bar{p}) \to t\bar{t}jj$ **Bevilacqua,Czakon,Papadopoulos,Worek ’11**

Important background for Higgs searches at Tevatron and LHC

- $H \to WW^*$ via weak boson fusion.
- $H \to b\bar{b}$ associated with a $t\bar{t}$ pair.

Computational challenge: it’s big.

subprocess	#1-loop graphs	subprocess	#graphs	#dipoles
$gg \to t\bar{t}gg$	4510	$gg \to t\bar{t}ggg$	1240	75
$gg \to t\bar{t}q\bar{q}$	1100	$gg \to t\bar{t}q\bar{q}g$	341	55
$q\bar{q} \to t\bar{t}gg$	1100	$q\bar{q} \to t\bar{t}ggg$	341	75
$gq \to t\bar{t}qg$	1100	$gq \to t\bar{t}q'\bar{q}'q$	64	25
$qg \to t\bar{t}qg$	1100	$gq \to t\bar{t}qgg$	341	65
$qq' \to t\bar{t}qq'$	205	$qg \to t\bar{t}q'\bar{q}'q$	64	25
$q\bar{q} \to t\bar{t}q'\bar{q}'$	205	$qg \to t\bar{t}qgg$	341	65
$q\bar{q} \to t\bar{t}q\bar{q}$	410	$qq' \to t\bar{t}qq'g$	64	40
		$q\bar{q} \to t\bar{t}q'\bar{q}'g$	64	35
		$q\bar{q} \to t\bar{t}q\bar{q}g$	128	45

$pp(p\bar{p}) \to t\bar{t}jj$ Bevilacqua,Czakon,Papadopoulos,Worek ’11

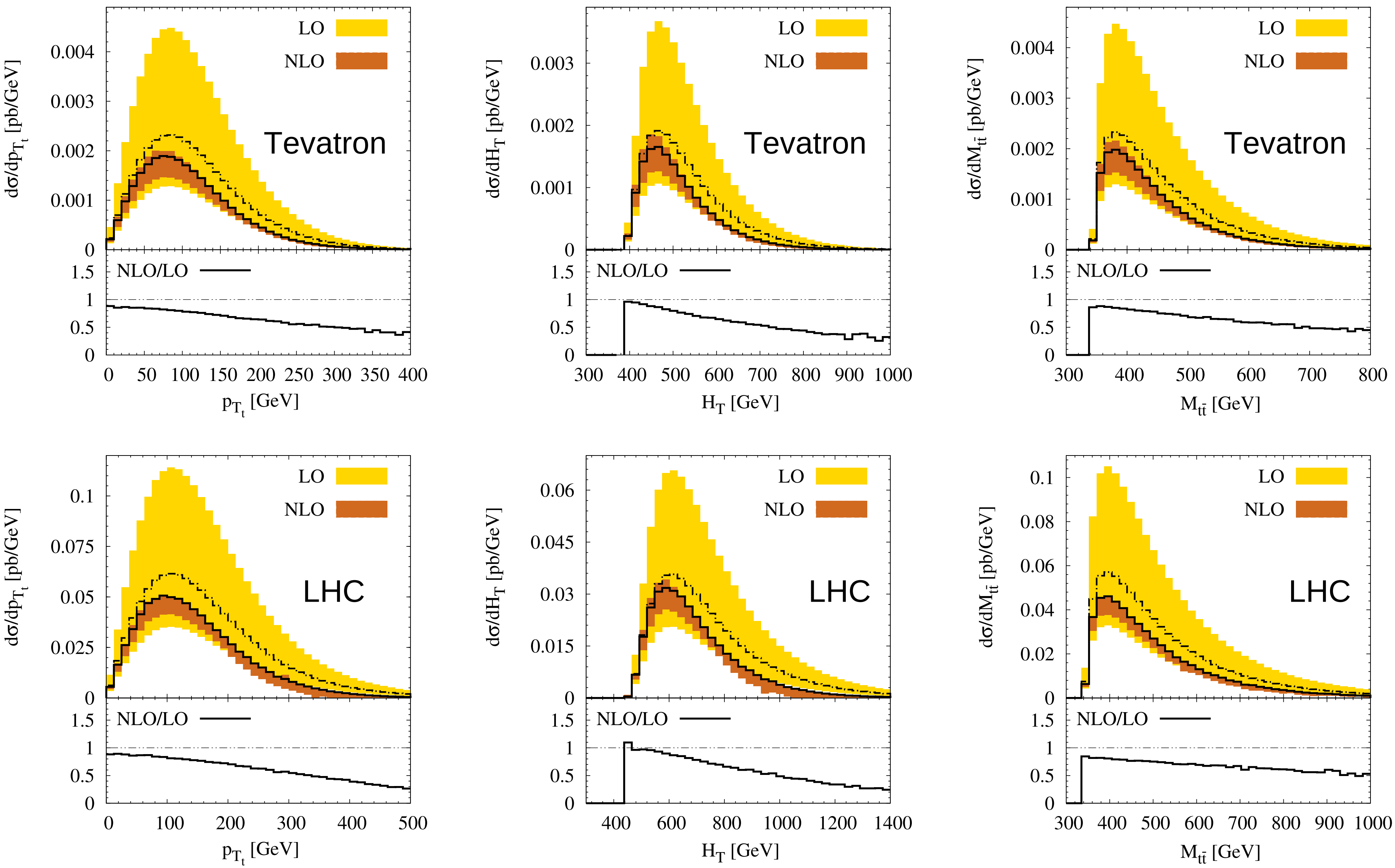
Tevatron $p_{T_j} > 20$ GeV, $|y_i| < 2.0$

ΔR_{jj}	σ_{LO} [pb]	$\sigma_{NLO}^{anti-k_T}$ [pb]	$\sigma_{NLO}^{k_T}$ [pb]	$\sigma_{NLO}^{C/A}$ [pb]
> 0.4	0.3584(1)	0.2709(5)	0.2734(4)	0.2734(4)
> 0.8	0.2876(1)	0.2467(3)	0.2494(3)	0.2491(3)

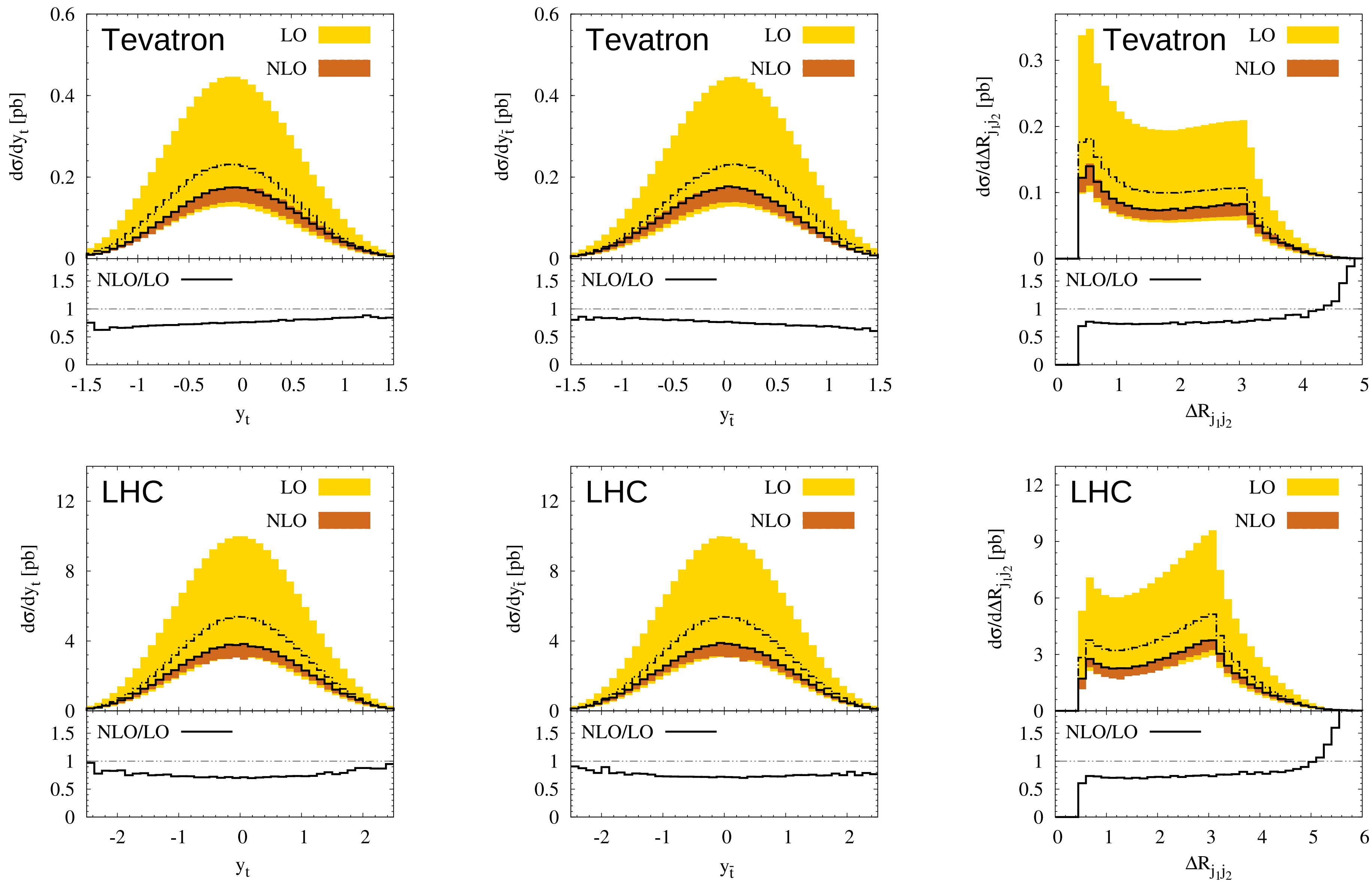
LHC $p_{T_j} > 50$ GeV, $|y_i| < 2.5$

ΔR_{jj}	σ_{LO} [pb]	$\sigma_{NLO}^{anti-k_T}$ [pb]	$\sigma_{NLO}^{k_T}$ [pb]	$\sigma_{NLO}^{C/A}$ [pb]
> 0.5	13.398(4)	9.82(2)	9.86(2)	9.86(2)
> 1.0	11.561(4)	9.95(2)	10.06(2)	10.04(2)

$pp(p\bar{p}) \to t\bar{t}jj$ Bevilacqua,Czakon,Papadopoulos,Worek '11



$pp(p\bar{p}) \to t\bar{t}jj$ Bevilacqua,Czakon,Papadopoulos,Worek ’11



Summary and outlook

- Physics at hadron colliders requires computational control over multi-jet events.
- And this, among other aspects, implies computational control over the hard process to the NLO precision level.
- There has been, and is, considerable progress in this,
  - presented here: $\mathrm{pp}(\mathrm{p\bar{p}}) \to W^+W^-\mathrm{b\bar{b}} \to 4\ell\,\mathrm{b\bar{b}}$
  - presented here: $\mathrm{pp}(\mathrm{p\bar{p}}) \to \mathrm{t\bar{t}jj}$

  Other processes under attack.
- Complete NLO-tool HELAC-NLO will soon be publicly available.
- Interface to parton-shower programs under development.